

NAG Library Routine Document

F07PWF (ZHPTRI)

Note: before using this routine, please read the Users' Note for your implementation to check the interpretation of ***bold italicised*** terms and other implementation-dependent details.

1 Purpose

F07PWF (ZHPTRI) computes the inverse of a complex Hermitian indefinite matrix A , where A has been factorized by F07PRF (ZHPTRF), using packed storage.

2 Specification

```
SUBROUTINE F07PWF(UPLO, N, AP, IPIV, WORK, INFO)
```

```
INTEGER          N, IPIV(*), INFO
```

```
complex*16      AP(*), WORK(N)
```

```
CHARACTER*1      UPLO
```

The routine may be called by its LAPACK name *zhptri*.

3 Description

F07PWF (ZHPTRI) is used to compute the inverse of a complex Hermitian indefinite matrix A , the routine must be preceded by a call to F07PRF (ZHPTRF), which computes the Bunch–Kaufman factorization of A , using packed storage.

If UPLO = 'U', $A = PUDU^H P^T$ and A^{-1} is computed by solving $U^H P^T X P U = D^{-1}$ for X .

If UPLO = 'L', $A = PLDL^H P^T$ and A^{-1} is computed by solving $L^H P^T X P L = D^{-1}$ for X .

4 References

Du Croz J J and Higham N J (1992) Stability of methods for matrix inversion *IMA J. Numer. Anal.* **12** 1–19

5 Parameters

- 1: UPLO – CHARACTER*1 *Input*
On entry: indicates how A has been factorized.
UPLO = 'U'
 $A = PUDU^H P^T$, where U is upper triangular.
UPLO = 'L'
 $A = PLDL^H P^T$, where L is lower triangular.
Constraint: UPLO = 'U' or 'L'.
- 2: N – INTEGER *Input*
On entry: n , the order of the matrix A .
Constraint: $N \geq 0$.
- 3: AP(*) – ***complex*16*** array *Input/Output*
Note: the dimension of the array AP must be at least $\max(1, N \times (N + 1)/2)$.
On entry: the factorization of A stored in packed form, as returned by F07PRF (ZHPTRF).

On exit: the factorization is overwritten by the n by n matrix A^{-1} .

More precisely,

if UPLO = 'U', the upper triangle of A^{-1} must be stored with element A_{ij} in $AP(i + j(j - 1)/2)$ for $i \leq j$;

if UPLO = 'L', the lower triangle of A^{-1} must be stored with element A_{ij} in $AP(i + (2n - j)(j - 1)/2)$ for $i \geq j$.

4: IPIV(*) – INTEGER array

Input

Note: the dimension of the array IPIV must be at least $\max(1, N)$.

On entry: details of the interchanges and the block structure of D , as returned by F07PRF (ZHPTRF).

5: WORK(N) – **complex*16** array

Workspace

6: INFO – INTEGER

Output

On exit: INFO = 0 unless the routine detects an error (see Section 6).

6 Error Indicators and Warnings

Errors or warnings detected by the routine:

INFO < 0

If INFO = $-i$, the i th parameter had an illegal value. An explanatory message is output, and execution of the program is terminated.

INFO > 0

If INFO = i , $d(i, i)$ is exactly zero; D is singular and the inverse of A cannot be computed.

7 Accuracy

The computed inverse X satisfies a bound of the form

if UPLO = 'U', $|DU^T P^T X P U - I| \leq c(n)\epsilon(|D||U^T|P^T|X|P|U| + |D||D^{-1}|)$;

if UPLO = 'L', $|DL^T P^T X P L - I| \leq c(n)\epsilon(|D||L^T|P^T|X|P|L| + |D||D^{-1}|)$,

$c(n)$ is a modest linear function of n , and ϵ is the **machine precision**.

8 Further Comments

The total number of real floating-point operations is approximately $\frac{8}{3}n^3$.

The real analogue of this routine is F07PJF (DSPTRI).

9 Example

This example computes the inverse of the matrix A , where

$$A = \begin{pmatrix} -1.36 + 0.00i & 1.58 + 0.90i & 2.21 - 0.21i & 3.91 + 1.50i \\ 1.58 - 0.90i & -8.87 + 0.00i & -1.84 - 0.03i & -1.78 + 1.18i \\ 2.21 + 0.21i & -1.84 + 0.03i & -4.63 + 0.00i & 0.11 + 0.11i \\ 3.91 - 1.50i & -1.78 - 1.18i & 0.11 - 0.11i & -1.84 + 0.00i \end{pmatrix}.$$

Here A is Hermitian indefinite, stored in packed form, and must first be factorized by F07PRF (ZHPTRF).

9.1 Program Text

```

*      F07PWF Example Program Text
*      Mark 15 Release. NAG Copyright 1991.
*      .. Parameters ..
      INTEGER          NIN, NOUT
      PARAMETER        (NIN=5,NOUT=6)
      INTEGER          NMAX
      PARAMETER        (NMAX=8)
*      .. Local Scalars ..
      INTEGER          I, IFAIL, INFO, J, N
      CHARACTER        UPLO
*      .. Local Arrays ..
      COMPLEX *16      AP(NMAX*(NMAX+1)/2), WORK(NMAX)
      INTEGER          IPIV(NMAX)
      CHARACTER        CLABS(1), RLABS(1)
*      .. External Subroutines ..
      EXTERNAL         X04DDF, ZHPTRF, ZHPTRI
*      .. Executable Statements ..
      WRITE (NOUT,*) 'F07PWF Example Program Results'
*      Skip heading in data file
      READ (NIN,*)
      READ (NIN,*) N
      IF (N.LE.NMAX) THEN
*
*          Read A from data file
*
*          READ (NIN,*) UPLO
*          IF (UPLO.EQ.'U') THEN
*              READ (NIN,*) ((AP(I+J*(J-1)/2),J=I,N),I=1,N)
*          ELSE IF (UPLO.EQ.'L') THEN
*              READ (NIN,*) ((AP(I+(2*N-J)*(J-1)/2),J=1,I),I=1,N)
*          END IF
*
*          Factorize A
*
*          CALL ZHPTRF(UPLO,N,AP,IPIV,INFO)
*
*          WRITE (NOUT,*)
*          IF (INFO.EQ.0) THEN
*
*              Compute inverse of A
*
*              CALL ZHPTRI(UPLO,N,AP,IPIV,WORK,INFO)
*
*              Print inverse
*
*              IFAIL = 0
*              CALL X04DDF(UPLO,'Nonunit',N,AP,'Bracketed','F7.4',
+                  'Inverse','Integer',RLABS,'Integer',CLABS,80,0,
+                  IFAIL)
*              ELSE
*                  WRITE (NOUT,*) 'The factor D is singular'
*              END IF
*          END IF
*
*          END

```

9.2 Program Data

F07PWF Example Program Data

```

4                                     :Value of N
'L'                                 :Value of UPLO
(-1.36, 0.00)
( 1.58,-0.90) (-8.87, 0.00)
( 2.21, 0.21) (-1.84, 0.03) (-4.63, 0.00)
( 3.91,-1.50) (-1.78,-1.18) ( 0.11,-0.11) (-1.84, 0.00) :End of matrix A

```

9.3 Program Results

F07PWF Example Program Results

```
Inverse
          1          2          3          4
1  ( 0.0826, 0.0000)
2  (-0.0335, 0.0440) (-0.1408, 0.0000)
3  ( 0.0603,-0.0105) ( 0.0422,-0.0222) (-0.2007, 0.0000)
4  ( 0.2391,-0.0926) ( 0.0304, 0.0203) ( 0.0982,-0.0635) ( 0.0073, 0.0000)
```
