NAG Library Routine Document

F07PJF (DSPTRI)

Note: before using this routine, please read the Users' Note for your implementation to check the interpretation of ***bold italicised*** terms and other implementation-dependent details.

1 Purpose

F07PJF (DSPTRI) computes the inverse of a real symmetric indefinite matrix A, where A has been factorized by F07PDF (DSPTRF), using packed storage.

2 Specification

```
SUBROUTINE F07PJF(UPLO, N, AP, IPIV, WORK, INFO)
INTEGER           N, IPIV(*), INFO
double precision  AP(*), WORK(N)
CHARACTER*1       UPLO
```

The routine may be called by its LAPACK name ***dsptri***.

3 Description

F07PJF (DSPTRI) is used to compute the inverse of a real symmetric indefinite matrix A, the routine must be preceded by a call to F07PDF (DSPTRF), which computes the Bunch–Kaufman factorization of A, using packed storage.

If UPLO = 'U', $A = PUDU^{\mathrm{T}}P^{\mathrm{T}}$ and A^{-1} is computed by solving $U^{\mathrm{T}}P^{\mathrm{T}}XPU = D^{-1}$.

If UPLO = 'L', $A = PLDL^{\mathrm{T}}P^{\mathrm{T}}$ and A^{-1} is computed by solving $L^{\mathrm{T}}P^{\mathrm{T}}XPL = D^{-1}$.

4 References

Du Croz J J and Higham N J (1992) Stability of methods for matrix inversion *IMA J. Numer. Anal.* **12** 1–19

5 Parameters

1: UPLO – CHARACTER*1 *Input*

On entry: indicates how A has been factorized.

UPLO = 'U'
$A = PUDU^{\mathrm{T}}P^{\mathrm{T}}$, where U is upper triangular.

UPLO = 'L'
$A = PLDL^{\mathrm{T}}P^{\mathrm{T}}$, where L is lower triangular.

Constraint: UPLO = 'U' or 'L'.

2: N – INTEGER *Input*

On entry: n, the order of the matrix A.

Constraint: $\mathrm{N} \geq 0$.

3: AP$(*)$ – ***double precision*** array *Input/Output*

Note: the dimension of the array AP must be at least $\max(1, \mathrm{N} \times (\mathrm{N}+1)/2)$.

On entry: the factorization of A stored in packed form, as returned by F07PDF (DSPTRF).

On exit: the factorization is overwritten by the n by n matrix A^{-1}.

More precisely,

if UPLO = 'U', the upper triangle of A^{-1} must be stored with element A_{ij} in AP$(i+j(j-1)/2)$ for $i \leq j$;

if UPLO = 'L', the lower triangle of A^{-1} must be stored with element A_{ij} in AP$(i+(2n-j)(j-1)/2)$ for $i \geq j$.

4: IPIV$(*)$ – INTEGER array *Input*

Note: the dimension of the array IPIV must be at least max$(1,\mathrm{N})$.

On entry: details of the interchanges and the block structure of D, as returned by F07PDF (DSPTRF).

5: WORK(N) – ***double precision*** array *Workspace*

6: INFO – INTEGER *Output*

On exit: INFO $= 0$ unless the routine detects an error (see Section 6).

6 Error Indicators and Warnings

Errors or warnings detected by the routine:

INFO < 0

If INFO $= -i$, the ith parameter had an illegal value. An explanatory message is output, and execution of the program is terminated.

INFO > 0

If INFO $= i$, $d(i,i)$ is exactly zero; D is singular and the inverse of A cannot be computed.

7 Accuracy

The computed inverse X satisfies a bound of the form

if UPLO = 'U', $\left|DU^{\mathrm{T}}P^{\mathrm{T}}XPU - I\right| \leq c(n)\epsilon\left(|D|\left|U^{\mathrm{T}}\right|P^{\mathrm{T}}|X|P|U| + |D|\left|D^{-1}\right|\right)$;

if UPLO = 'L', $\left|DL^{\mathrm{T}}P^{\mathrm{T}}XPL - I\right| \leq c(n)\epsilon\left(|D|\left|L^{\mathrm{T}}\right|P^{\mathrm{T}}|X|P|L| + |D|\left|D^{-1}\right|\right)$,

$c(n)$ is a modest linear function of n, and ϵ is the ***machine precision***.

8 Further Comments

The total number of floating-point operations is approximately $\frac{2}{3}n^3$.

The complex analogues of this routine are F07PWF (ZHPTRI) for Hermitian matrices and F07QWF (ZSPTRI) for symmetric matrices.

9 Example

This example computes the inverse of the matrix A, where

$$A = \begin{pmatrix} 2.07 & 3.87 & 4.20 & -1.15 \\ 3.87 & -0.21 & 1.87 & 0.63 \\ 4.20 & 1.87 & 1.15 & 2.06 \\ -1.15 & 0.63 & 2.06 & -1.81 \end{pmatrix}.$$

Here A is symmetric indefinite, stored in packed form, and must first be factorized by F07PDF (DSPTRF).

9.1 Program Text

```
*     F07PJF Example Program Text
*     Mark 15 Release. NAG Copyright 1991.
*     .. Parameters ..
      INTEGER          NIN, NOUT
      PARAMETER        (NIN=5,NOUT=6)
      INTEGER          NMAX
      PARAMETER        (NMAX=8)
*     .. Local Scalars ..
      INTEGER          I, IFAIL, INFO, J, N
      CHARACTER        UPLO
*     .. Local Arrays ..
      DOUBLE PRECISION AP(NMAX*(NMAX+1)/2), WORK(NMAX)
      INTEGER          IPIV(NMAX)
*     .. External Subroutines ..
      EXTERNAL         DSPTRF, DSPTRI, X04CCF
*     .. Executable Statements ..
      WRITE (NOUT,*) 'F07PJF Example Program Results'
*     Skip heading in data file
      READ (NIN,*)
      READ (NIN,*) N
      IF (N.LE.NMAX) THEN
*
*        Read A from data file
*
         READ (NIN,*) UPLO
         IF (UPLO.EQ.'U') THEN
            READ (NIN,*) ((AP(I+J*(J-1)/2),J=I,N),I=1,N)
         ELSE IF (UPLO.EQ.'L') THEN
            READ (NIN,*) ((AP(I+(2*N-J)*(J-1)/2),J=1,I),I=1,N)
         END IF
*
*        Factorize A
*
         CALL DSPTRF(UPLO,N,AP,IPIV,INFO)
*
         WRITE (NOUT,*)
         IF (INFO.EQ.0) THEN
*
*           Compute inverse of A
*
            CALL DSPTRI(UPLO,N,AP,IPIV,WORK,INFO)
*
*           Print inverse
*
            IFAIL = 0
*
            CALL X04CCF(UPLO,'Nonunit',N,AP,'Inverse',IFAIL)
*
         ELSE
            WRITE (NOUT,*) 'The factor D is singular'
         END IF
      END IF
*
      END
```

9.2 Program Data

```
F07PJF Example Program Data
  4                          :Value of N
  'L'                        :Value of UPLO
  2.07
  3.87  -0.21
  4.20   1.87   1.15
 -1.15   0.63   2.06  -1.81   :End of matrix A
```

9.3 Program Results

```
 F07PJF Example Program Results

 Inverse
              1          2          3          4
 1       0.7485
 2       0.5221    -0.1605
 3      -1.0058    -0.3131     1.3501
 4      -1.4386    -0.7440     2.0667     2.4547
```