

NAG Library Routine Document

F07PEF (DSPTRS)

Note: before using this routine, please read the Users' Note for your implementation to check the interpretation of ***bold italicised*** terms and other implementation-dependent details.

1 Purpose

F07PEF (DSPTRS) solves a real symmetric indefinite system of linear equations with multiple right-hand sides,

$$AX = B,$$

where A has been factorized by F07PDF (DSPTRF), using packed storage.

2 Specification

```
SUBROUTINE F07PEF(UPLO, N, NRHS, AP, IPIV, B, LDB, INFO)
INTEGER          N, NRHS, IPIV(*), LDB, INFO
double precision AP(*), B(LDB,*)
CHARACTER*1      UPLO
```

The routine may be called by its LAPACK name ***dsptrs***.

3 Description

F07PEF (DSPTRS) is used to solve a real symmetric indefinite system of linear equations $AX = B$, the routine must be preceded by a call to F07PDF (DSPTRF) which computes the Bunch–Kaufman factorization of A , using packed storage.

If $UPLO = 'U'$, $A = PUDU^T P^T$, where P is a permutation matrix, U is an upper triangular matrix and D is a symmetric block diagonal matrix with 1 by 1 and 2 by 2 blocks; the solution X is computed by solving $PUDY = B$ and then $U^T P^T X = Y$.

If $UPLO = 'L'$, $A = PLDL^T P^T$, where L is a lower triangular matrix; the solution X is computed by solving $PLDY = B$ and then $L^T P^T X = Y$.

4 References

Golub G H and Van Loan C F (1996) *Matrix Computations* (3rd Edition) Johns Hopkins University Press, Baltimore

5 Parameters

- 1: UPLO – CHARACTER*1 *Input*
On entry: indicates how A has been factorized.
 UPLO = 'U'
 $A = PUDU^T P^T$, where U is upper triangular.
 UPLO = 'L'
 $A = PLDL^T P^T$, where L is lower triangular.
Constraint: UPLO = 'U' or 'L'.

- 2: N – INTEGER *Input*
On entry: n , the order of the matrix A .
Constraint: $N \geq 0$.
- 3: NRHS – INTEGER *Input*
On entry: r , the number of right-hand sides.
Constraint: $\text{NRHS} \geq 0$.
- 4: AP(*) – **double precision** array *Input*
Note: the dimension of the array AP must be at least $\max(1, N \times (N + 1)/2)$.
On entry: the factorization of A stored in packed form, as returned by F07PDF (DSPTRF).
- 5: IPIV(*) – INTEGER array *Input*
Note: the dimension of the array IPIV must be at least $\max(1, N)$.
On entry: details of the interchanges and the block structure of D , as returned by F07PDF (DSPTRF).
- 6: B(LDB,*) – **double precision** array *Input/Output*
Note: the second dimension of the array B must be at least $\max(1, \text{NRHS})$.
On entry: the n by r right-hand side matrix B .
On exit: the n by r solution matrix X .
- 7: LDB – INTEGER *Input*
On entry: the first dimension of the array B as declared in the (sub)program from which F07PEF (DSPTRS) is called.
Constraint: $\text{LDB} \geq \max(1, N)$.
- 8: INFO – INTEGER *Output*
On exit: $\text{INFO} = 0$ unless the routine detects an error (see Section 6).

6 Error Indicators and Warnings

Errors or warnings detected by the routine:

$\text{INFO} < 0$

If $\text{INFO} = -i$, the i th parameter had an illegal value. An explanatory message is output, and execution of the program is terminated.

7 Accuracy

For each right-hand side vector b , the computed solution x is the exact solution of a perturbed system of equations $(A + E)x = b$, where

$$\text{if UPLO} = 'U', |E| \leq c(n)\epsilon P|U||D||U^T|P^T;$$

$$\text{if UPLO} = 'L', |E| \leq c(n)\epsilon P|L||D||L^T|P^T,$$

$c(n)$ is a modest linear function of n , and ϵ is the **machine precision**.

If $\hat{x}$ is the true solution, then the computed solution x satisfies a forward error bound of the form

$$\frac{\|x - \hat{x}\|_{\infty}}{\|x\|_{\infty}} \leq c(n) \operatorname{cond}(A, x) \epsilon$$

where $\operatorname{cond}(A, x) = \| |A^{-1}| |A| |x| \|_{\infty} / \|x\|_{\infty} \leq \operatorname{cond}(A) = \| |A^{-1}| |A| \|_{\infty} \leq \kappa_{\infty}(A)$.

Note that $\operatorname{cond}(A, x)$ can be much smaller than $\operatorname{cond}(A)$.

Forward and backward error bounds can be computed by calling F07PHF (DSPRFS), and an estimate for $\kappa_{\infty}(A)$ ($= \kappa_1(A)$) can be obtained by calling F07PGF (DSPCON).

8 Further Comments

The total number of floating-point operations is approximately $2n^2r$.

This routine may be followed by a call to F07PHF (DSPRFS) to refine the solution and return an error estimate.

The complex analogues of this routine are F07PSF (ZHPTRS) for Hermitian matrices and F07QSF (ZSPTRS) for symmetric matrices.

9 Example

This example solves the system of equations $AX = B$, where

$$A = \begin{pmatrix} 2.07 & 3.87 & 4.20 & -1.15 \\ 3.87 & -0.21 & 1.87 & 0.63 \\ 4.20 & 1.87 & 1.15 & 2.06 \\ -1.15 & 0.63 & 2.06 & -1.81 \end{pmatrix} \quad \text{and} \quad B = \begin{pmatrix} -9.50 & 27.85 \\ -8.38 & 9.90 \\ -6.07 & 19.25 \\ -0.96 & 3.93 \end{pmatrix}.$$

Here A is symmetric indefinite, stored in packed form, and must first be factorized by F07PDF (DSPTRF).

9.1 Program Text

```
*      F07PEF Example Program Text
*      Mark 15 Release. NAG Copyright 1991.
*      .. Parameters ..
      INTEGER          NIN, NOUT
      PARAMETER        (NIN=5,NOUT=6)
      INTEGER          NMAX, NRHMAX, LDB
      PARAMETER        (NMAX=8,NRHMAX=NMAX,LDB=NMAX)
*      .. Local Scalars ..
      INTEGER          I, IFAIL, INFO, J, N, NRHS
      CHARACTER        UPLO
*      .. Local Arrays ..
      DOUBLE PRECISION AP(NMAX*(NMAX+1)/2), B(LDB,NRHMAX)
      INTEGER          IPIV(NMAX)
*      .. External Subroutines ..
      EXTERNAL         DSPTRF, DSPTRS, X04CAF
*      .. Executable Statements ..
      WRITE (NOUT,*) 'F07PEF Example Program Results'
*      Skip heading in data file
      READ (NIN,*)
      READ (NIN,*) N, NRHS
      IF (N.LE.NMAX .AND. NRHS.LE.NRHMAX) THEN
*
*      Read A and B from data file
*
      READ (NIN,*) UPLO
      IF (UPLO.EQ.'U') THEN
        READ (NIN,*) ((AP(I+J*(J-1)/2),J=I,N),I=1,N)
      ELSE IF (UPLO.EQ.'L') THEN
        READ (NIN,*) ((AP(I+(2*N-J)*(J-1)/2),J=1,I),I=1,N)
      END IF
      READ (NIN,*) ((B(I,J),J=1,NRHS),I=1,N)
*

```

```

*      Factorize A
*
*      CALL DSPTRF(UPLO,N,AP,IPIV,INFO)
*
*      WRITE (NOUT,*)
*      IF (INFO.EQ.0) THEN
*
*          Compute solution
*
*          CALL DSPTRS(UPLO,N,NRHS,AP,IPIV,B,LDB,INFO)
*
*          Print solution
*
*          IFAIL = 0
*
*          CALL X04CAF('General',' ',N,NRHS,B,LDB,'Solution(s)',IFAIL)
*
*      ELSE
*          WRITE (NOUT,*) 'The factor D is singular'
*      END IF
*  END IF
*
*  END

```

9.2 Program Data

F07PEF Example Program Data

4	2			:Values of N and NRHS
'L'				:Value of UPLO
2.07				
3.87	-0.21			
4.20	1.87	1.15		
-1.15	0.63	2.06	-1.81	:End of matrix A
-9.50	27.85			
-8.38	9.90			
-6.07	19.25			
-0.96	3.93			:End of matrix B

9.3 Program Results

F07PEF Example Program Results

Solution(s)		
	1	2
1	-4.0000	1.0000
2	-1.0000	4.0000
3	2.0000	3.0000
4	5.0000	2.0000
