NAG Library Routine Document

F07NSF (ZSYTRS)

Note: before using this routine, please read the Users' Note for your implementation to check the interpretation of ***bold italicised*** terms and other implementation-dependent details.

1 Purpose

F07NSF (ZSYTRS) solves a complex symmetric system of linear equations with multiple right-hand sides,

$$AX = B,$$

where A has been factorized by F07NRF (ZSYTRF).

2 Specification

```
SUBROUTINE F07NSF(UPLO, N, NRHS, A, LDA, IPIV, B, LDB, INFO)
INTEGER            N, NRHS, LDA, IPIV(*), LDB, INFO
complex*16         A(LDA,*), B(LDB,*)
CHARACTER*1        UPLO
```

The routine may be called by its LAPACK name ***zsytrs***.

3 Description

F07NSF (ZSYTRS) is used to solve a complex symmetric system of linear equations $AX = B$, this routine must be preceded by a call to F07NRF (ZSYTRF) which computes the Bunch–Kaufman factorization of A.

If UPLO = 'U', $A = PUDU^{\mathrm{T}}P^{\mathrm{T}}$, where P is a permutation matrix, U is an upper triangular matrix and D is a symmetric block diagonal matrix with 1 by 1 and 2 by 2 blocks; the solution X is computed by solving $PUDY = B$ and then $U^{\mathrm{T}}P^{\mathrm{T}}X = Y$.

If UPLO = 'L', $A = PLDL^{\mathrm{T}}P^{\mathrm{T}}$, where L is a lower triangular matrix; the solution X is computed by solving $PLDY = B$ and then $L^{\mathrm{T}}P^{\mathrm{T}}X = Y$.

4 References

Golub G H and Van Loan C F (1996) *Matrix Computations* (3rd Edition) Johns Hopkins University Press, Baltimore

5 Parameters

1: UPLO – CHARACTER*1 *Input*

On entry: indicates how A has been factorized.

UPLO = 'U'
$A = PUDU^{\mathrm{T}}P^{\mathrm{T}}$, where U is upper triangular.

UPLO = 'L'
$A = PLDL^{\mathrm{T}}P^{\mathrm{T}}$, where L is lower triangular.

Constraint: UPLO = 'U' or 'L'.

2: N – INTEGER *Input*

On entry: n, the order of the matrix A.

Constraint: N ≥ 0.

3: NRHS – INTEGER *Input*

On entry: r, the number of right-hand sides.

Constraint: NRHS ≥ 0.

4: A(LDA,$*$) – ***complex*16*** array *Input*

Note: the second dimension of the array A must be at least $\max(1, \mathrm{N})$.

On entry: details of the factorization of A, as returned by F07NRF (ZSYTRF).

5: LDA – INTEGER *Input*

On entry: the first dimension of the array A as declared in the (sub)program from which F07NSF (ZSYTRS) is called.

Constraint: LDA $\geq \max(1, \mathrm{N})$.

6: IPIV($*$) – INTEGER array *Input*

Note: the dimension of the array IPIV must be at least $\max(1, \mathrm{N})$.

On entry: details of the interchanges and the block structure of D, as returned by F07NRF (ZSYTRF).

7: B(LDB,$*$) – ***complex*16*** array *Input/Output*

Note: the second dimension of the array B must be at least $\max(1, \mathrm{NRHS})$.

On entry: the n by r right-hand side matrix B.

On exit: the n by r solution matrix X.

8: LDB – INTEGER *Input*

On entry: the first dimension of the array B as declared in the (sub)program from which F07NSF (ZSYTRS) is called.

Constraint: LDB $\geq \max(1, \mathrm{N})$.

9: INFO – INTEGER *Output*

On exit: INFO $= 0$ unless the routine detects an error (see Section 6).

6 Error Indicators and Warnings

Errors or warnings detected by the routine:

INFO < 0

If INFO $= -i$, the ith parameter had an illegal value. An explanatory message is output, and execution of the program is terminated.

7 Accuracy

For each right-hand side vector b, the computed solution x is the exact solution of a perturbed system of equations $(A+E)x = b$, where

if UPLO = 'U', $|E| \leq c(n)\epsilon P|U||D||U^{\mathrm{T}}|P^{\mathrm{T}}$;

if UPLO = 'L', $|E| \leq c(n)\epsilon P|L||D||L^{\mathrm{T}}|P^{\mathrm{T}}$,

$c(n)$ is a modest linear function of n, and ϵ is the ***machine precision***.

If $\hat{x}$ is the true solution, then the computed solution x satisfies a forward error bound of the form

$$\frac{\|x - \hat{x}\|_\infty}{\|x\|_\infty} \leq c(n)\,\mathrm{cond}(A, x)\epsilon$$

where $\mathrm{cond}(A, x) = \left\| |A^{-1}||A||x| \right\|_\infty / \|x\|_\infty \leq \mathrm{cond}(A) = \left\| |A^{-1}||A| \right\|_\infty \leq \kappa_\infty(A)$.

Note that $\mathrm{cond}(A, x)$ can be much smaller than $\mathrm{cond}(A)$.

Forward and backward error bounds can be computed by calling F07NVF (ZSYRFS), and an estimate for $\kappa_\infty(A)$ ($= \kappa_1(A)$) can be obtained by calling F07NUF (ZSYCON).

8 Further Comments

The total number of real floating-point operations is approximately $8n^2r$.

This routine may be followed by a call to F07NVF (ZSYRFS) to refine the solution and return an error estimate.

The real analogue of this routine is F07MEF (DSYTRS).

9 Example

This example solves the system of equations $AX = B$, where

$$A = \begin{pmatrix} -0.39 - 0.71i & 5.14 - 0.64i & -7.86 - 2.96i & 3.80 + 0.92i \\ 5.14 - 0.64i & 8.86 + 1.81i & -3.52 + 0.58i & 5.32 - 1.59i \\ -7.86 - 2.96i & -3.52 + 0.58i & -2.83 - 0.03i & -1.54 - 2.86i \\ 3.80 + 0.92i & 5.32 - 1.59i & -1.54 - 2.86i & -0.56 + 0.12i \end{pmatrix}$$

and

$$B = \begin{pmatrix} -55.64 + 41.22i & -19.09 - 35.97i \\ -48.18 + 66.00i & -12.08 - 27.02i \\ -0.49 - 1.47i & 6.95 + 20.49i \\ -6.43 + 19.24i & -4.59 - 35.53i \end{pmatrix}.$$

Here A is symmetric and must first be factorized by F07NRF (ZSYTRF).

9.1 Program Text

```
*     F07NSF Example Program Text
*     Mark 15 Release. NAG Copyright 1991.
*     .. Parameters ..
      INTEGER          NIN, NOUT
      PARAMETER        (NIN=5,NOUT=6)
      INTEGER          NMAX, LDA, LWORK, NRHMAX, LDB
      PARAMETER        (NMAX=8,LDA=NMAX,LWORK=64*NMAX,NRHMAX=NMAX,
     +                 LDB=NMAX)
*     .. Local Scalars ..
      INTEGER          I, IFAIL, INFO, J, N, NRHS
      CHARACTER        UPLO
*     .. Local Arrays ..
      COMPLEX *16      A(LDA,NMAX), B(LDB,NRHMAX), WORK(LWORK)
      INTEGER          IPIV(NMAX)
      CHARACTER        CLABS(1), RLABS(1)
*     .. External Subroutines ..
      EXTERNAL         X04DBF, ZSYTRF, ZSYTRS
*     .. Executable Statements ..
      WRITE (NOUT,*) 'F07NSF Example Program Results'
*     Skip heading in data file
      READ (NIN,*)
      READ (NIN,*) N, NRHS
      IF (N.LE.NMAX .AND. NRHS.LE.NRHMAX) THEN
*
*        Read A and B from data file
```

```
*
         READ (NIN,*) UPLO
         IF (UPLO.EQ.'U') THEN
            READ (NIN,*) ((A(I,J),J=I,N),I=1,N)
         ELSE IF (UPLO.EQ.'L') THEN
            READ (NIN,*) ((A(I,J),J=1,I),I=1,N)
         END IF
         READ (NIN,*) ((B(I,J),J=1,NRHS),I=1,N)
*
*        Factorize A
*
         CALL ZSYTRF(UPLO,N,A,LDA,IPIV,WORK,LWORK,INFO)
*
         WRITE (NOUT,*)
         IF (INFO.EQ.0) THEN
*
*           Compute solution
*
            CALL ZSYTRS(UPLO,N,NRHS,A,LDA,IPIV,B,LDB,INFO)
*
*           Print solution
*
            IFAIL = 0
            CALL X04DBF('General',' ',N,NRHS,B,LDB,'Bracketed','F7.4',
     +                  'Solution(s)','Integer',RLABS,'Integer',CLABS,
     +                  80,0,IFAIL)
         ELSE
            WRITE (NOUT,*) 'The factor D is singular'
         END IF
      END IF
*
      END
```

9.2 Program Data

```
F07NSF Example Program Data
  4  2                                                     :Values of N and NRHS
  'L'                                                      :Value of UPLO
 (-0.39,-0.71)
 ( 5.14,-0.64) ( 8.86, 1.81)
 (-7.86,-2.96) (-3.52, 0.58) (-2.83,-0.03)
 ( 3.80, 0.92) ( 5.32,-1.59) (-1.54,-2.86) (-0.56, 0.12)  :End of matrix A
 (-55.64, 41.22) (-19.09,-35.97)
 (-48.18, 66.00) (-12.08,-27.02)
 ( -0.49, -1.47) (  6.95, 20.49)
 ( -6.43, 19.24) ( -4.59,-35.53)                           :End of matrix B
```

9.3 Program Results

```
 F07NSF Example Program Results

 Solution(s)
                    1                 2
 1  ( 1.0000,-1.0000) (-2.0000,-1.0000)
 2  (-2.0000, 5.0000) ( 1.0000,-3.0000)
 3  ( 3.0000,-2.0000) ( 3.0000, 2.0000)
 4  (-4.0000, 3.0000) (-1.0000, 1.0000)
```