NAG Library Routine Document

F07NRF (ZSYTRF)

Note: before using this routine, please read the Users' Note for your implementation to check the interpretation of ***bold italicised*** terms and other implementation-dependent details.

1 Purpose

F07NRF (ZSYTRF) computes the Bunch–Kaufman factorization of a complex symmetric matrix.

2 Specification

```
SUBROUTINE F07NRF(UPLO, N, A, LDA, IPIV, WORK, LWORK, INFO)
INTEGER          N, LDA, IPIV(*), LWORK, INFO
complex*16       A(LDA,*), WORK(max(1,LWORK))
CHARACTER*1      UPLO
```

The routine may be called by its LAPACK name ***zsytrf***.

3 Description

F07NRF (ZSYTRF) factorizes a complex symmetric matrix A, using the Bunch–Kaufman diagonal pivoting method. A is factorized as either $A = PUDU^{\mathrm{T}}P^{\mathrm{T}}$ if UPLO = 'U' or $A = PLDL^{\mathrm{T}}P^{\mathrm{T}}$ if UPLO = 'L', where P is a permutation matrix, U (or L) is a unit upper (or lower) triangular matrix and D is a symmetric block diagonal matrix with 1 by 1 and 2 by 2 diagonal blocks; U (or L) has 2 by 2 unit diagonal blocks corresponding to the 2 by 2 blocks of D. Row and column interchanges are performed to ensure numerical stability while preserving symmetry.

4 References

Golub G H and Van Loan C F (1996) *Matrix Computations* (3rd Edition) Johns Hopkins University Press, Baltimore

5 Parameters

1: UPLO – CHARACTER*1 *Input*

On entry: indicates whether the upper or lower triangular part of A is stored and how A is to be factorized.

UPLO = 'U'
: The upper triangular part of A is stored and A is factorized as $PUDU^{\mathrm{T}}P^{\mathrm{T}}$, where U is upper triangular.

UPLO = 'L'
: The lower triangular part of A is stored and A is factorized as $PLDL^{\mathrm{T}}P^{\mathrm{T}}$, where L is lower triangular.

Constraint: UPLO = 'U' or 'L'.

2: N – INTEGER *Input*

On entry: n, the order of the matrix A.

Constraint: $\mathrm{N} \geq 0$.

3: A(LDA,$*$) – ***complex*16*** array *Input/Output*

Note: the second dimension of the array A must be at least $\max(1, \mathrm{N})$.

On entry: the n by n symmetric indefinite matrix A.

If UPLO = 'U', the upper triangular part of A must be stored and the elements of the array below the diagonal are not referenced.

If UPLO = 'L', the lower triangular part of A must be stored and the elements of the array above the diagonal are not referenced.

On exit: the upper or lower triangle of A is overwritten by details of the block diagonal matrix D and the multipliers used to obtain the factor U or L as specified by UPLO.

4: LDA – INTEGER *Input*

On entry: the first dimension of the array A as declared in the (sub)program from which F07NRF (ZSYTRF) is called.

Constraint: LDA $\geq \max(1, \mathrm{N})$.

5: IPIV($*$) – INTEGER array *Output*

Note: the dimension of the array IPIV must be at least $\max(1, \mathrm{N})$.

On exit: details of the interchanges and the block structure of D. More precisely,

if IPIV$(i) = k > 0$, d_{ii} is a 1 by 1 pivot block and the ith row and column of A were interchanged with the kth row and column;

if UPLO = 'U' and IPIV$(i-1) =$ IPIV$(i) = -l < 0$, $\begin{pmatrix} d_{i-1,i-1} & \bar{d}_{i,i-1} \\ \bar{d}_{i,i-1} & d_{ii} \end{pmatrix}$ is a 2 by 2 pivot block and the $(i-1)$th row and column of A were interchanged with the lth row and column;

if UPLO = 'L' and IPIV$(i) =$ IPIV$(i+1) = -m < 0$, $\begin{pmatrix} d_{ii} & d_{i+1,i} \\ d_{i+1,i} & d_{i+1,i+1} \end{pmatrix}$ is a 2 by 2 pivot block and the $(i+1)$th row and column of A were interchanged with the mth row and column.

6: WORK(max(1, LWORK)) – ***complex*16*** array *Workspace*

On exit: if INFO $= 0$, WORK(1) contains the minimum value of LWORK required for optimum performance.

7: LWORK – INTEGER *Input*

On entry: the dimension of the array WORK as declared in the (sub)program from which F07NRF (ZSYTRF) is called, unless LWORK $= -1$, in which case a workspace query is assumed and the routine only calculates the optimal dimension of WORK (using the formula given below).

Suggested value: for optimum performance LWORK should be at least N $\times$ nb, where nb is the ***blocksize***.

Constraint: LWORK ≥ 1 or LWORK $= -1$.

8: INFO – INTEGER *Output*

On exit: INFO $= 0$ unless the routine detects an error (see Section 6).

6 Error Indicators and Warnings

Errors or warnings detected by the routine:

INFO < 0

If INFO $= -i$, the ith parameter had an illegal value. An explanatory message is output, and execution of the program is terminated.

INFO > 0

If INFO $= i$, $d(i,i)$ is exactly zero. The factorization has been completed, but the block diagonal matrix D is exactly singular, and division by zero will occur if it is used to solve a system of equations.

7 Accuracy

If UPLO = 'U', the computed factors U and D are the exact factors of a perturbed matrix $A + E$, where

$$|E| \leq c(n)\epsilon P|U||D||U^{\mathrm{T}}|P^{\mathrm{T}},$$

$c(n)$ is a modest linear function of n, and ϵ is the ***machine precision***.

If UPLO = 'L', a similar statement holds for the computed factors L and D.

8 Further Comments

The elements of D overwrite the corresponding elements of A; if D has 2 by 2 blocks, only the upper or lower triangle is stored, as specified by UPLO.

The unit diagonal elements of U or L and the 2 by 2 unit diagonal blocks are not stored. The remaining elements of U or L are stored in the corresponding columns of the array A, but additional row interchanges must be applied to recover U or L explicitly (this is seldom necessary). If IPIV$(i) = i$, for $i = 1, 2, \ldots, n$, then U or L is stored explicitly (except for its unit diagonal elements which are equal to 1).

The total number of real floating-point operations is approximately $\frac{4}{3}n^3$.

A call to F07NRF (ZSYTRF) may be followed by calls to the routines:

F07NSF (ZSYTRS) to solve $AX = B$;

F07NUF (ZSYCON) to estimate the condition number of A;

F07NWF (ZSYTRI) to compute the inverse of A.

The real analogue of this routine is F07MDF (DSYTRF).

9 Example

This example computes the Bunch–Kaufman factorization of the matrix A, where

$$A = \begin{pmatrix} -0.39 - 0.71i & 5.14 - 0.64i & -7.86 - 2.96i & 3.80 + 0.92i \\ 5.14 - 0.64i & 8.86 + 1.81i & -3.52 + 0.58i & 5.32 - 1.59i \\ -7.86 - 2.96i & -3.52 + 0.58i & -2.83 - 0.03i & -1.54 - 2.86i \\ 3.80 + 0.92i & 5.32 - 1.59i & -1.54 - 2.86i & -0.56 + 0.12i \end{pmatrix}.$$

9.1 Program Text

```
*      F07NRF Example Program Text
*      Mark 15 Release. NAG Copyright 1991.
*      .. Parameters ..
       INTEGER          NIN, NOUT
       PARAMETER        (NIN=5,NOUT=6)
       INTEGER          NMAX, LDA, LWORK
       PARAMETER        (NMAX=8,LDA=NMAX,LWORK=64*NMAX)
*      .. Local Scalars ..
```

```
      INTEGER          I, IFAIL, INFO, J, N
      CHARACTER        UPLO
*     .. Local Arrays ..
      COMPLEX *16      A(LDA,NMAX), WORK(LWORK)
      INTEGER          IPIV(NMAX)
      CHARACTER        CLABS(1), RLABS(1)
*     .. External Subroutines ..
      EXTERNAL         X04DBF, ZSYTRF
*     .. Executable Statements ..
      WRITE (NOUT,*) 'F07NRF Example Program Results'
*     Skip heading in data file
      READ (NIN,*)
      READ (NIN,*) N
      IF (N.LE.NMAX) THEN
*
*        Read A from data file
*
         READ (NIN,*) UPLO
         IF (UPLO.EQ.'U') THEN
            READ (NIN,*) ((A(I,J),J=I,N),I=1,N)
         ELSE IF (UPLO.EQ.'L') THEN
            READ (NIN,*) ((A(I,J),J=1,I),I=1,N)
         END IF
*
*        Factorize A
*
         CALL ZSYTRF(UPLO,N,A,LDA,IPIV,WORK,LWORK,INFO)
*
         WRITE (NOUT,*)
*
*        Print details of factorization
*
         IFAIL = 0
         CALL X04DBF(UPLO,'Nonunit',N,N,A,LDA,'Bracketed','F7.4',
     +               'Details of factorization','Integer',RLABS,
     +               'Integer',CLABS,80,0,IFAIL)
*
*        Print pivot indices
*
         WRITE (NOUT,*)
         WRITE (NOUT,*) 'IPIV'
         WRITE (NOUT,99999) (IPIV(I),I=1,N)
*
         IF (INFO.NE.0) WRITE (NOUT,*) 'The factor D is singular'
*
      END IF
*
99999 FORMAT ((1X,I12,3I18))
      END
```

9.2 Program Data

```
F07NRF Example Program Data
  4                                                       :Value of N
  'L'                                                     :Value of UPLO
 (-0.39,-0.71)
 ( 5.14,-0.64) ( 8.86, 1.81)
 (-7.86,-2.96) (-3.52, 0.58) (-2.83,-0.03)
 ( 3.80, 0.92) ( 5.32,-1.59) (-1.54,-2.86) (-0.56, 0.12)  :End of matrix A
```

9.3 Program Results

```
 F07NRF Example Program Results

 Details of factorization
                   1                  2                  3                  4
 1  (-0.3900,-0.7100)
 2  (-7.8600,-2.9600) (-2.8300,-0.0300)
```

```
 3  ( 0.5279,-0.3715) (-0.6078, 0.2811) ( 4.4079, 5.3991)
 4  ( 0.4426, 0.1936) (-0.4823, 0.0150) (-0.1071,-0.3157) (-2.0954,-2.2011)

 IPIV
           -3                  -3                  3                   4
```