NAG Library Routine Document

F07MWF (ZHETRI)

Note: before using this routine, please read the Users' Note for your implementation to check the interpretation of ***bold italicised*** terms and other implementation-dependent details.

1 Purpose

F07MWF (ZHETRI) computes the inverse of a complex Hermitian indefinite matrix A, where A has been factorized by F07MRF (ZHETRF).

2 Specification

```
SUBROUTINE F07MWF(UPLO, N, A, LDA, IPIV, WORK, INFO)
INTEGER            N, LDA, IPIV(*), INFO
complex*16         A(LDA,*), WORK(N)
CHARACTER*1        UPLO
```

The routine may be called by its LAPACK name ***zhetri***.

3 Description

F07MWF (ZHETRI) is used to compute the inverse of a complex Hermitian indefinite matrix A, the routine must be preceded by a call to F07MRF (ZHETRF), which computes the Bunch–Kaufman factorization of A.

If UPLO = 'U', $A = PUDU^{\mathrm{H}}P^{\mathrm{T}}$ and A^{-1} is computed by solving $U^{\mathrm{H}}P^{\mathrm{T}}XPU = D^{-1}$ for X.

If UPLO = 'L', $A = PLDL^{\mathrm{H}}P^{\mathrm{T}}$ and A^{-1} is computed by solving $L^{\mathrm{H}}P^{\mathrm{T}}XPL = D^{-1}$ for X.

4 References

Du Croz J J and Higham N J (1992) Stability of methods for matrix inversion *IMA J. Numer. Anal.* **12** 1–19

5 Parameters

1: UPLO – CHARACTER*1 *Input*

On entry: indicates how A has been factorized.

UPLO = 'U'
$A = PUDU^{\mathrm{H}}P^{\mathrm{T}}$, where U is upper triangular.

UPLO = 'L'
$A = PLDL^{\mathrm{H}}P^{\mathrm{T}}$, where L is lower triangular.

Constraint: UPLO = 'U' or 'L'.

2: N – INTEGER *Input*

On entry: n, the order of the matrix A.

Constraint: N ≥ 0.

3: A(LDA,$*$) – ***complex*16*** array *Input/Output*

Note: the second dimension of the array A must be at least max(1, N).

On entry: details of the factorization of A, as returned by F07MRF (ZHETRF).

On exit: the factorization is overwritten by the n by n Hermitian matrix A^{-1}.

If UPLO = 'U', the upper triangle of A^{-1} is stored in the upper triangular part of the array.

If UPLO = 'L', the lower triangle of A^{-1} is stored in the lower triangular part of the array.

4: LDA – INTEGER *Input*

On entry: the first dimension of the array A as declared in the (sub)program from which F07MWF (ZHETRI) is called.

Constraint: LDA $\geq \max(1, \mathrm{N})$.

5: IPIV$(*)$ – INTEGER array *Input*

Note: the dimension of the array IPIV must be at least $\max(1, \mathrm{N})$.

On entry: details of the interchanges and the block structure of D, as returned by F07MRF (ZHETRF).

6: WORK(N) – ***complex*16*** array *Workspace*

7: INFO – INTEGER *Output*

On exit: INFO = 0 unless the routine detects an error (see Section 6).

6 Error Indicators and Warnings

Errors or warnings detected by the routine:

INFO < 0

If INFO $= -i$, the ith parameter had an illegal value. An explanatory message is output, and execution of the program is terminated.

INFO > 0

If INFO $= i$, $d(i,i)$ is exactly zero; D is singular and the inverse of A cannot be computed.

7 Accuracy

The computed inverse X satisfies a bound of the form

if UPLO = 'U', $\left|DU^{\mathrm{H}}P^{\mathrm{T}}XPU - I\right| \leq c(n)\epsilon\left(|D|\left|U^{\mathrm{H}}\right|P^{\mathrm{T}}|X|P|U| + |D|\left|D^{-1}\right|\right)$;

if UPLO = 'L', $\left|DL^{\mathrm{H}}P^{\mathrm{T}}XPL - I\right| \leq c(n)\epsilon\left(|D|\left|L^{\mathrm{H}}\right|P^{\mathrm{T}}|X|P|L| + |D|\left|D^{-1}\right|\right)$,

$c(n)$ is a modest linear function of n, and ϵ is the ***machine precision***.

8 Further Comments

The total number of real floating-point operations is approximately $\frac{8}{3}n^3$.

The real analogue of this routine is F07MJF (DSYTRI).

9 Example

This example computes the inverse of the matrix A, where

$$A = \begin{pmatrix} -1.36 + 0.00i & 1.58 + 0.90i & 2.21 - 0.21i & 3.91 + 1.50i \\ 1.58 - 0.90i & -8.87 + 0.00i & -1.84 - 0.03i & -1.78 + 1.18i \\ 2.21 + 0.21i & -1.84 + 0.03i & -4.63 + 0.00i & 0.11 + 0.11i \\ 3.91 - 1.50i & -1.78 - 1.18i & 0.11 - 0.11i & -1.84 + 0.00i \end{pmatrix}.$$

Here A is Hermitian indefinite and must first be factorized by F07MRF (ZHETRF).

9.1 Program Text

```
*     F07MWF Example Program Text
*     Mark 15 Release. NAG Copyright 1991.
*     .. Parameters ..
      INTEGER          NIN, NOUT
      PARAMETER        (NIN=5,NOUT=6)
      INTEGER          NMAX, LDA, LWORK
      PARAMETER        (NMAX=8,LDA=NMAX,LWORK=64*NMAX)
*     .. Local Scalars ..
      INTEGER          I, IFAIL, INFO, J, N
      CHARACTER        UPLO
*     .. Local Arrays ..
      COMPLEX *16      A(LDA,NMAX), WORK(LWORK)
      INTEGER          IPIV(NMAX)
      CHARACTER        CLABS(1), RLABS(1)
*     .. External Subroutines ..
      EXTERNAL         X04DBF, ZHETRF, ZHETRI
*     .. Executable Statements ..
      WRITE (NOUT,*) 'F07MWF Example Program Results'
*     Skip heading in data file
      READ (NIN,*)
      READ (NIN,*) N
      IF (N.LE.NMAX) THEN
*
*        Read A from data file
*
         READ (NIN,*) UPLO
         IF (UPLO.EQ.'U') THEN
            READ (NIN,*) ((A(I,J),J=I,N),I=1,N)
         ELSE IF (UPLO.EQ.'L') THEN
            READ (NIN,*) ((A(I,J),J=1,I),I=1,N)
         END IF
*
*        Factorize A
*
         CALL ZHETRF(UPLO,N,A,LDA,IPIV,WORK,LWORK,INFO)
*
         WRITE (NOUT,*)
         IF (INFO.EQ.0) THEN
*
*           Compute inverse of A
*
            CALL ZHETRI(UPLO,N,A,LDA,IPIV,WORK,INFO)
*
*           Print inverse
*
            IFAIL = 0
            CALL X04DBF(UPLO,'Nonunit',N,N,A,LDA,'Bracketed','F7.4',
     +                  'Inverse','Integer',RLABS,'Integer',CLABS,80,0,
     +                  IFAIL)
         ELSE
            WRITE (NOUT,*) 'The factor D is singular'
         END IF
       END IF
*
       END
```

9.2 Program Data

```
F07MWF Example Program Data
  4                                                          :Value of N
  'L'                                                        :Value of UPLO
 (-1.36, 0.00)
 ( 1.58,-0.90) (-8.87, 0.00)
 ( 2.21, 0.21) (-1.84, 0.03) (-4.63, 0.00)
 ( 3.91,-1.50) (-1.78,-1.18) ( 0.11,-0.11) (-1.84, 0.00)  :End of matrix A
```

9.3 Program Results

```
 F07MWF Example Program Results

 Inverse
                     1                 2                 3                 4
 1  ( 0.0826, 0.0000)
 2  (-0.0335, 0.0440) (-0.1408, 0.0000)
 3  ( 0.0603,-0.0105) ( 0.0422,-0.0222) (-0.2007, 0.0000)
 4  ( 0.2391,-0.0926) ( 0.0304, 0.0203) ( 0.0982,-0.0635) ( 0.0073, 0.0000)
```