NAG Library Routine Document

F07MAF (DSYSV)

Note: before using this routine, please read the Users' Note for your implementation to check the interpretation of ***bold italicised*** terms and other implementation-dependent details.

1 Purpose

F07MAF (DSYSV) computes the solution to a real system of linear equations

$$AX = B,$$

where A is an n by n symmetric matrix and X and B are n by r matrices.

2 Specification

```
SUBROUTINE F07MAF(UPLO, N, NRHS, A, LDA, IPIV, B, LDB, WORK, LWORK,
1                 INFO)
INTEGER           N, NRHS, LDA, IPIV(*), LDB, LWORK, INFO
double precision  A(LDA,*), B(LDB,*), WORK(max(1,LWORK))
CHARACTER*1       UPLO
```

The routine may be called by its LAPACK name ***dsysv***.

3 Description

F07MAF (DSYSV) uses the diagonal pivoting method to factor A as $A = UDU^{\mathrm{T}}$ if UPLO = 'U' or $A = LDL^{\mathrm{T}}$ if UPLO = 'L', where U (or L) is a product of permutation and unit upper (lower) triangular matrices, and D is symmetric and block diagonal with 1 by 1 and 2 by 2 diagonal blocks. The factored form of A is then used to solve the system of equations $AX = B$.

Note that, in general, different permutations (pivot sequences) and diagonal block structures are obtained for UPLO = 'U' or 'L'

4 References

Anderson E, Bai Z, Bischof C, Blackford S, Demmel J, Dongarra J J, Du Croz J J, Greenbaum A, Hammarling S, McKenney A and Sorensen D (1999) *LAPACK Users' Guide* (3rd Edition) SIAM, Philadelphiahttp://www.netlib.org/lapack/lug

Golub G H and Van Loan C F (1996) *Matrix Computations* (3rd Edition) Johns Hopkins University Press, Baltimore

5 Parameters

1: UPLO – CHARACTER*1 *Input*

On entry: if UPLO = 'U', the upper triangle of A is stored.

If UPLO = 'L', the lower triangle of A is stored.

Constraint: UPLO = 'U' or 'L'.

2: N – INTEGER *Input*

On entry: n, the number of linear equations, i.e., the order of the matrix A.

Constraint: N $\geq$ 0.

3: NRHS – INTEGER *Input*

On entry: r, the number of right-hand sides, i.e., the number of columns of the matrix B.

Constraint: NRHS ≥ 0.

4: A(LDA,$*$) – ***double precision*** array *Input/Output*

Note: the second dimension of the array A must be at least $\max(1, \mathrm{N})$.

On entry: the n by n symmetric matrix A.

If UPLO = 'U', the upper triangular part of A must be stored and the elements of the array below the diagonal are not referenced.

If UPLO = 'L', the lower triangular part of A must be stored and the elements of the array above the diagonal are not referenced.

On exit: if INFO $= 0$, the block diagonal matrix D and the multipliers used to obtain the factor U or L from the factorization $A = UDU^{\mathrm{T}}$ or $A = LDL^{\mathrm{T}}$ as computed by F07MDF (DSYTRF).

5: LDA – INTEGER *Input*

On entry: the first dimension of the array A as declared in the (sub)program from which F07MAF (DSYSV) is called.

Constraint: LDA $\geq \max(1, \mathrm{N})$.

6: IPIV($*$) – INTEGER array *Output*

Note: the dimension of the array IPIV must be at least $\max(1, \mathrm{N})$.

On exit: details of the interchanges and the block structure of D, as determined by F07MDF (DSYTRF).

IPIV$(k) > 0$
Rows and columns k and IPIV(k) were interchanged, and $D(k, k)$ is a 1 by 1 diagonal block.

UPLO = 'U' and IPIV$(k) =$ IPIV$(k-1) < 0$
Rows and columns $k-1$ and $-$IPIV(k) were interchanged and $D(k-1:k, k-1:k)$ is a 2 by 2 diagonal block.

UPLO = 'L' and IPIV$(k) =$ IPIV$(k+1) < 0$
Rows and columns $k+1$ and $-$IPIV(k) were interchanged and $D(k:k+1, k:k+1)$ is a 2 by 2 diagonal block.

7: B(LDB,$*$) – ***double precision*** array *Input/Output*

Note: the second dimension of the array B must be at least $\max(1, \mathrm{NRHS})$.

On entry: the n by r right-hand side matrix B.

On exit: if INFO $= 0$, the n by r solution matrix X.

8: LDB – INTEGER *Input*

On entry: the first dimension of the array B as declared in the (sub)program from which F07MAF (DSYSV) is called.

Constraint: LDB $\geq \max(1, \mathrm{N})$.

9: WORK($\max(1, \mathrm{LWORK})$) – ***double precision*** array *Workspace*

On exit: if INFO $= 0$, WORK(1) returns the optimal LWORK.

10: LWORK – INTEGER *Input*

On entry: the dimension of the array WORK as declared in the (sub)program from which F07MAF (DSYSV) is called.

LWORK ≥ 1, and for best performance LWORK $\geq \max(1, \mathrm{N} \times \mathrm{nb})$, where nb is the optimal blocksize for F07MDF (DSYTRF).

If LWORK $= -1$, a workspace query is assumed; the routine only calculates the optimal size of the WORK array, returns this value as the first entry of the WORK array, and no error message related to LWORK is issued.

11: INFO – INTEGER *Output*

On exit: INFO $= 0$ unless the routine detects an error (see Section 6).

6 Error Indicators and Warnings

Errors or warnings detected by the routine:

INFO < 0

If INFO $= -i$, the ith argument had an illegal value. An explanatory message is output, and execution of the program is terminated.

INFO > 0

If INFO $= i$, d_{ii} is exactly zero. The factorization has been completed, but the block diagonal matrix D is exactly singular, so the solution could not be computed.

7 Accuracy

The computed solution for a single right-hand side, $\hat{x}$, satisfies an equation of the form

$$(A + E)\hat{x} = b,$$

where

$$\|E\|_1 = O(\epsilon)\|A\|_1$$

and ϵ is the ***machine precision***. An approximate error bound for the computed solution is given by

$$\frac{\|\hat{x} - x\|_1}{\|x\|_1} \leq \kappa(A)\frac{\|E\|_1}{\|A\|_1},$$

where $\kappa(A) = \left\|A^{-1}\right\|_1 \|A\|_1$, the condition number of A with respect to the solution of the linear equations. See Section 4.4 of Anderson *et al.* (1999) for further details.

F07MBF (DSYSVX) is a comprehensive LAPACK driver that returns forward and backward error bounds and an estimate of the condition number. Alternatively, F04BHF solves $Ax = b$ and returns a forward error bound and condition estimate. F04BHF calls F07MAF (DSYSV) to solve the equations.

8 Further Comments

The total number of floating-point operations is approximately $\frac{1}{3}n^3 + 2n^2r$, where r is the number of right-hand sides.

The complex analogues of F07MAF (DSYSV) are F07MNF (ZHESV) for Hermitian matrices, and F07NNF (ZSYSV) for symmetric matrices.

9 Example

This example solves the equations

$$Ax = b,$$

where A is the symmetric matrix

$$A = \begin{pmatrix} -1.81 & 2.06 & 0.63 & -1.15 \\ 2.06 & 1.15 & 1.87 & 4.20 \\ 0.63 & 1.87 & -0.21 & 3.87 \\ -1.15 & 4.20 & 3.87 & 2.07 \end{pmatrix} \quad \text{and} \quad b = \begin{pmatrix} 0.96 \\ 6.07 \\ 8.38 \\ 9.50 \end{pmatrix}.$$

Details of the factorization of A are also output.

9.1 Program Text

```
*     F07MAF Example Program Text
*     Mark 21 Release. NAG Copyright 2004.
*     .. Parameters ..
      INTEGER          NIN, NOUT
      PARAMETER        (NIN=5,NOUT=6)
      INTEGER          NB, NMAX
      PARAMETER        (NB=64,NMAX=8)
      INTEGER          LDA, LWORK
      PARAMETER        (LDA=NMAX,LWORK=NB*NMAX)
*     .. Local Scalars ..
      INTEGER          I, IFAIL, INFO, J, N
*     .. Local Arrays ..
      DOUBLE PRECISION A(LDA,NMAX), B(NMAX), WORK(LWORK)
      INTEGER          IPIV(NMAX)
*     .. External Subroutines ..
      EXTERNAL         DSYSV, X04CAF
*     .. Executable Statements ..
      WRITE (NOUT,*) 'F07MAF Example Program Results'
      WRITE (NOUT,*)
*     Skip heading in data file
      READ (NIN,*)
      READ (NIN,*) N
      IF (N.LE.NMAX) THEN
*
*        Read the upper triangular part of the matrix A from data file
*
         READ (NIN,*) ((A(I,J),J=I,N),I=1,N)
*
*        Read b from data file
*
         READ (NIN,*) (B(I),I=1,N)
*
*        Solve the equations Ax = b for x
*
         CALL DSYSV('Upper',N,1,A,LDA,IPIV,B,N,WORK,LWORK,INFO)
*
         IF (INFO.EQ.0) THEN
*
*           Print solution
*
            WRITE (NOUT,*) 'Solution'
            WRITE (NOUT,99999) (B(I),I=1,N)
*
*           Print details of factorization
*
            WRITE (NOUT,*)
            IFAIL = 0
            CALL X04CAF('Upper','Non-unit diagonal',N,N,A,LDA,
     +                  'Details of the factorization',IFAIL)
*
*           Print pivot indices
*
            WRITE (NOUT,*)
            WRITE (NOUT,*) 'Pivot indices'
            WRITE (NOUT,99998) (IPIV(I),I=1,N)
*
         ELSE
            WRITE (NOUT,99997) 'The diagonal block ', INFO,
     +        ' of D is zero'
```

```
         END IF
      ELSE
         WRITE (NOUT,*) 'NMAX too small'
      END IF
*
99999 FORMAT ((3X,7F11.4))
99998 FORMAT (1X,7I11)
99997 FORMAT (1X,A,I3,A)
      END
```

9.2 Program Data

```
F07MAF Example Program Data
  4                          :Value of N
 -1.81   2.06   0.63  -1.15
         1.15   1.87   4.20
               -0.21   3.87
                       2.07 :End of matrix A
  0.96   6.07   8.38   9.50 :End of vector b
```

9.3 Program Results

```
 F07MAF Example Program Results

 Solution
       -5.0000    -2.0000     1.0000     4.0000

 Details of the factorization
             1          2          3          4
1       0.4074     0.3031    -0.5960     0.6537
2                 -2.5907     0.8115     0.2230
3                             1.1500     4.2000
4                                        2.0700

 Pivot indices
           1          2         -2         -2
```