

NAG Library Routine Document

F07JDF (DPTTRF)

Note: before using this routine, please read the Users' Note for your implementation to check the interpretation of ***bold italicised*** terms and other implementation-dependent details.

1 Purpose

F07JDF (DPTTRF) computes the modified Cholesky factorization of a real n by n symmetric positive-definite tridiagonal matrix A .

2 Specification

```
SUBROUTINE F07JDF(N, D, E, INFO)
  INTEGER          N, INFO
  double precision D(*), E(*)
```

The routine may be called by its LAPACK name *dpstrf*.

3 Description

F07JDF (DPTTRF) factorizes the matrix A as

$$A = LDL^T,$$

where L is a unit lower bidiagonal matrix and D is a diagonal matrix with positive diagonal elements. The factorization may also be regarded as having the form $U^T D U$, where U is a unit upper bidiagonal matrix.

4 References

None.

5 Parameters

- 1: N – INTEGER *Input*
On entry: n , the order of the matrix A .
Constraint: $N \geq 0$.
- 2: D(*) – **double precision** array *Input/Output*
Note: the dimension of the array D must be at least $\max(1, N)$.
On entry: must contain the n diagonal elements of the matrix A .
On exit: is overwritten by the n diagonal elements of the diagonal matrix D from the LDL^T factorization of A .
- 3: E(*) – **double precision** array *Input/Output*
Note: the dimension of the array E must be at least $\max(1, N - 1)$.
On entry: must contain the $(n - 1)$ subdiagonal elements of the matrix A .
On exit: is overwritten by the $(n - 1)$ subdiagonal elements of the lower bidiagonal matrix L . E can also be regarded as containing the $(n - 1)$ superdiagonal elements of the upper bidiagonal matrix U .

4: INFO – INTEGER

Output

On exit: INFO = 0 unless the routine detects an error (see Section 6).

6 Error Indicators and Warnings

Errors or warnings detected by the routine:

INFO < 0

If INFO = $-i$, the i th argument had an illegal value. An explanatory message is output, and execution of the program is terminated.

INFO > 0

If INFO = i , the leading minor of order i is not positive-definite. If $i < N$, the factorization could not be completed, while if $i = N$, the factorization was completed, but $D(N) \leq 0$.

7 Accuracy

The computed factorization satisfies an equation of the form

$$A + E = LDL^T,$$

where

$$\|E\|_{\infty} = O(\epsilon)\|A\|_{\infty}$$

and ϵ is the *machine precision*.

Following the use of this routine, F07JEF (DPTTRS) can be used to solve systems of equations $AX = B$, and F07JGF (DPTCON) can be used to estimate the condition number of A .

8 Further Comments

The total number of floating-point operations required to factorize the matrix A is proportional to n .

The complex analogue of this routine is F07JRF (ZPTTRF).

9 Example

This example factorizes the symmetric positive-definite tridiagonal matrix A given by

$$A = \begin{pmatrix} 4.0 & -2.0 & 0 & 0 & 0 \\ -2.0 & 10.0 & -6.0 & 0 & 0 \\ 0 & -6.0 & 29.0 & 15.0 & 0 \\ 0 & 0 & 15.0 & 25.0 & 8.0 \\ 0 & 0 & 0 & 8.0 & 5.0 \end{pmatrix}.$$

9.1 Program Text

```
*      F07JDF Example Program Text
*      Mark 21 Release. NAG Copyright 2004.
*      .. Parameters ..
      INTEGER          NIN, NOUT
      PARAMETER        (NIN=5,NOUT=6)
      INTEGER          NMAX
      PARAMETER        (NMAX=50)
*      .. Local Scalars ..
      INTEGER          I, INFO, N
*      .. Local Arrays ..
      DOUBLE PRECISION D(NMAX), E(NMAX-1)
*      .. External Subroutines ..
      EXTERNAL        DPTTRF
*      .. Executable Statements ..
      WRITE (NOUT,*) 'F07JDF Example Program Results'
```

```

      WRITE (NOUT,*)
*      Skip heading in data file
      READ (NIN,*)
      READ (NIN,*) N
      IF (N.LE.NMAX) THEN
*
*          Read the lower bidiagonal part of the tridiagonal matrix A from
*          data file
*
*          READ (NIN,*) (D(I),I=1,N)
*          READ (NIN,*) (E(I),I=1,N-1)
*
*          Factorize the tridiagonal matrix A
*
*          CALL DPTTRF(N,D,E,INFO)
*
*          IF (INFO.GT.0) THEN
+            WRITE (NOUT,99999) 'The leading minor of order ', INFO,
+              ' is not positive definite'
            END IF
*
*          Print details of the factorization
*
*          WRITE (NOUT,*) 'Details of factorization'
*          WRITE (NOUT,*)
*          WRITE (NOUT,*) ' The diagonal elements of D'
*          WRITE (NOUT,99998) (D(I),I=1,N)
*          WRITE (NOUT,*)
*          WRITE (NOUT,*)
+          ' Sub-diagonal elements of the Cholesky factor L'
*          WRITE (NOUT,99998) (E(I),I=1,N-1)
*
*          ELSE
*          WRITE (NOUT,*) 'NMAX too small'
*          END IF
*
99999 FORMAT (1X,A,I3,A)
99998 FORMAT (1X,8F9.4)
      END

```

9.2 Program Data

F07JDF Example Program Data

5					:Value of N
4.0	10.0	29.0	25.0	5.0	:End of diagonal D
-2.0	-6.0	15.0	8.0		:End of sub-diagonal E

9.3 Program Results

F07JDF Example Program Results

Details of factorization

The diagonal elements of D

4.0000	9.0000	25.0000	16.0000	1.0000
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Sub-diagonal elements of the Cholesky factor L

-0.5000	-0.6667	0.6000	0.5000
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