

NAG Library Routine Document

F07JAF (DPTSV)

Note: before using this routine, please read the Users' Note for your implementation to check the interpretation of ***bold italicised*** terms and other implementation-dependent details.

1 Purpose

F07JAF (DPTSV) computes the solution to a real system of linear equations

$$AX = B,$$

where A is an n by n symmetric positive-definite tridiagonal matrix, and X and B are n by r matrices.

2 Specification

```
SUBROUTINE F07JAF(N, NRHS, D, E, B, LDB, INFO)
INTEGER          N, NRHS, LDB, INFO
double precision D(*), E(*), B(LDB,*)
```

The routine may be called by its LAPACK name ***dptsv***.

3 Description

F07JAF (DPTSV) factors A as $A = LDL^T$. The factored form of A is then used to solve the system of equations.

4 References

Anderson E, Bai Z, Bischof C, Blackford S, Demmel J, Dongarra J J, Du Croz J J, Greenbaum A, Hammarling S, McKenney A and Sorensen D (1999) *LAPACK Users' Guide* (3rd Edition) SIAM, Philadelphia <http://www.netlib.org/lapack/lug>

Golub G H and Van Loan C F (1996) *Matrix Computations* (3rd Edition) Johns Hopkins University Press, Baltimore

5 Parameters

- 1: N – INTEGER *Input*
On entry: n , the order of the matrix A .
Constraint: $N \geq 0$.
- 2: NRHS – INTEGER *Input*
On entry: r , the number of right-hand sides, i.e., the number of columns of the matrix B .
Constraint: $NRHS \geq 0$.
- 3: D(*) – ***double precision*** array *Input/Output*
Note: the dimension of the array D must be at least $\max(1, N)$.
On entry: the n diagonal elements of the tridiagonal matrix A .
On exit: the n diagonal elements of the diagonal matrix D from the factorization $A = LDL^T$.

- 4: $E(*)$ – **double precision** array *Input/Output*
Note: the dimension of the array E must be at least $\max(1, N - 1)$.
On entry: the $(n - 1)$ subdiagonal elements of the tridiagonal matrix A .
On exit: the $(n - 1)$ subdiagonal elements of the unit bidiagonal factor L from the LDL^T factorization of A . (E can also be regarded as the superdiagonal of the unit bidiagonal factor U from the $U^T DU$ factorization of A .)
- 5: $B(LDB,*)$ – **double precision** array *Input/Output*
Note: the second dimension of the array B must be at least $\max(1, NRHS)$.
On entry: the n by r right-hand side matrix B .
On exit: if $INFO = 0$, the n by r solution matrix X .
- 6: LDB – INTEGER *Input*
On entry: the first dimension of the array B as declared in the (sub)program from which F07JAF (DPTSV) is called.
Constraint: $LDB \geq \max(1, N)$.
- 7: $INFO$ – INTEGER *Output*
On exit: $INFO = 0$ unless the routine detects an error (see Section 6).

6 Error Indicators and Warnings

Errors or warnings detected by the routine:

$INFO < 0$

If $INFO = -i$, the i th argument had an illegal value. An explanatory message is output, and execution of the program is terminated.

$INFO > 0$

If $INFO = i$, the leading minor of order i is not positive-definite, and the solution has not been computed. The factorization has not been completed unless $i = N$.

7 Accuracy

The computed solution for a single right-hand side, $\hat{x}$, satisfies an equation of the form

$$(A + E)\hat{x} = b,$$

where

$$\|E\|_1 = O(\epsilon)\|A\|_1$$

and ϵ is the **machine precision**. An approximate error bound for the computed solution is given by

$$\frac{\|\hat{x} - x\|_1}{\|x\|_1} \leq \kappa(A) \frac{\|E\|_1}{\|A\|_1},$$

where $\kappa(A) = \|A^{-1}\|_1 \|A\|_1$, the condition number of A with respect to the solution of the linear equations. See Section 4.4 of Anderson *et al.* (1999) for further details.

F07JBF (DPTSVX) is a comprehensive LAPACK driver that returns forward and backward error bounds and an estimate of the condition number. Alternatively, F04BGF solves $Ax = b$ and returns a forward error bound and condition estimate. F04BGF calls F07JAF (DPTSV) to solve the equations.

8 Further Comments

The number of floating-point operations required for the factorization of A is proportional to n , and the number of floating-point operations required for the solution of the equations is proportional to nr , where r is the number of right-hand sides.

The complex analogue of this routine is F07JNF (ZPTSV).

9 Example

This example solves the equations

$$Ax = b,$$

where A is the symmetric positive-definite tridiagonal matrix

$$A = \begin{pmatrix} 4.0 & -2.0 & 0 & 0 & 0 \\ -2.0 & 10.0 & -6.0 & 0 & 0 \\ 0 & -6.0 & 29.0 & 15.0 & 0 \\ 0 & 0 & 15.0 & 25.0 & 8.0 \\ 0 & 0 & 0 & 8.0 & 5.0 \end{pmatrix} \quad \text{and} \quad b = \begin{pmatrix} 6.0 \\ 9.0 \\ 2.0 \\ 14.0 \\ 7.0 \end{pmatrix}.$$

Details of the LDL^T factorization of A are also output.

9.1 Program Text

```
*      F07JAF Example Program Text
*      Mark 21 Release. NAG Copyright 2004.
*      .. Parameters ..
      INTEGER          NIN, NOUT
      PARAMETER        (NIN=5,NOUT=6)
      INTEGER          NMAX
      PARAMETER        (NMAX=8)
*      .. Local Scalars ..
      INTEGER          I, INFO, N
*      .. Local Arrays ..
      DOUBLE PRECISION B(NMAX), D(NMAX), E(NMAX-1)
*      .. External Subroutines ..
      EXTERNAL         DPTSV
*      .. Executable Statements ..
      WRITE (NOUT,*) 'F07JAF Example Program Results'
      WRITE (NOUT,*)
*      Skip heading in data file
      READ (NIN,*)
      READ (NIN,*) N
      IF (N.LE.NMAX) THEN
*
*         Read the lower bidiagonal part of the tridiagonal matrix A and
*         the right hand side b from data file
*
*
      READ (NIN,*) (D(I),I=1,N)
      READ (NIN,*) (E(I),I=1,N-1)
      READ (NIN,*) (B(I),I=1,N)
*
*         Solve the equations Ax = b for x
*
      CALL DPTSV(N,1,D,E,B,N,INFO)
*
      IF (INFO.EQ.0) THEN
*
*         Print solution
*
      WRITE (NOUT,*) 'Solution'
      WRITE (NOUT,99999) (B(I),I=1,N)
*
*         Print details of factorization
*
      WRITE (NOUT,*)
```

```

        WRITE (NOUT,*) 'Diagonal elements of the diagonal matrix D'
        WRITE (NOUT,99999) (D(I),I=1,N)
        WRITE (NOUT,*)
        WRITE (NOUT,*)
+       'Sub-diagonal elements of the Cholesky factor L'
        WRITE (NOUT,99999) (E(I),I=1,N-1)
*
        ELSE
        WRITE (NOUT,99998) 'The leading minor of order ', INFO,
+       ' is not positive definite'
        END IF
        ELSE
        WRITE (NOUT,*) 'NMAX too small'
        END IF
*
99999 FORMAT (1X,7F11.4)
99998 FORMAT (1X,A,I3,A)
        END

```

9.2 Program Data

F07JAF Example Program Data

```

5           :Value of N
4.0  10.0  29.0  25.0  5.0 :End of diagonal D
-2.0 -6.0  15.0  8.0   :End of sub-diagonal E
6.0  9.0  2.0  14.0  7.0 :End of vector b

```

9.3 Program Results

F07JAF Example Program Results

```

Solution
2.5000      2.0000      1.0000     -1.0000      3.0000

Diagonal elements of the diagonal matrix D
4.0000      9.0000     25.0000     16.0000      1.0000

Sub-diagonal elements of the Cholesky factor L
-0.5000     -0.6667      0.6000      0.5000

```
