NAG Library Routine Document

F07HSF (ZPBTRS)

Note: before using this routine, please read the Users' Note for your implementation to check the interpretation of ***bold italicised*** terms and other implementation-dependent details.

1 Purpose

F07HSF (ZPBTRS) solves a complex Hermitian positive-definite band system of linear equations with multiple right-hand sides,

$$AX = B,$$

where A has been factorized by F07HRF (ZPBTRF).

2 Specification

```
SUBROUTINE F07HSF(UPLO, N, KD, NRHS, AB, LDAB, B, LDB, INFO)
INTEGER           N, KD, NRHS, LDAB, LDB, INFO
complex*16        AB(LDAB,*), B(LDB,*)
CHARACTER*1       UPLO
```

The routine may be called by its LAPACK name ***zpbtrs***.

3 Description

F07HSF (ZPBTRS) is used to solve a complex Hermitian positive-definite band system of linear equations $AX = B$, the routine must be preceded by a call to F07HRF (ZPBTRF) which computes the Cholesky factorization of A. The solution X is computed by forward and backward substitution.

If UPLO = 'U', $A = U^{\mathrm{H}}U$, where U is upper triangular; the solution X is computed by solving $U^{\mathrm{H}}Y = B$ and then $UX = Y$.

If UPLO = 'L', $A = LL^{\mathrm{H}}$, where L is lower triangular; the solution X is computed by solving $LY = B$ and then $L^{\mathrm{H}}X = Y$.

4 References

Golub G H and Van Loan C F (1996) *Matrix Computations* (3rd Edition) Johns Hopkins University Press, Baltimore

5 Parameters

1: UPLO – CHARACTER*1 *Input*

On entry: indicates how A has been factorized.

UPLO = 'U'
$A = U^{\mathrm{H}}U$, where U is upper triangular.

UPLO = 'L'
$A = LL^{\mathrm{H}}$, where L is lower triangular.

Constraint: UPLO = 'U' or 'L'.

2: N – INTEGER *Input*

On entry: n, the order of the matrix A.

Constraint: N ≥ 0.

3: KD – INTEGER *Input*

On entry: k_d, the number of superdiagonals or subdiagonals of the matrix A.

Constraint: KD ≥ 0.

4: NRHS – INTEGER *Input*

On entry: r, the number of right-hand sides.

Constraint: NRHS ≥ 0.

5: AB(LDAB,$*$) – ***complex*16*** array *Input*

Note: the second dimension of the array AB must be at least $\max(1, \mathrm{N})$.

On entry: the Cholesky factor of A, as returned by F07HRF (ZPBTRF).

6: LDAB – INTEGER *Input*

On entry: the first dimension of the array AB as declared in the (sub)program from which F07HSF (ZPBTRS) is called.

Constraint: LDAB $\geq$ KD $+ 1$.

7: B(LDB,$*$) – ***complex*16*** array *Input/Output*

Note: the second dimension of the array B must be at least $\max(1, \mathrm{NRHS})$.

On entry: the n by r right-hand side matrix B.

On exit: the n by r solution matrix X.

8: LDB – INTEGER *Input*

On entry: the first dimension of the array B as declared in the (sub)program from which F07HSF (ZPBTRS) is called.

Constraint: LDB $\geq \max(1, \mathrm{N})$.

9: INFO – INTEGER *Output*

On exit: INFO $= 0$ unless the routine detects an error (see Section 6).

6 Error Indicators and Warnings

Errors or warnings detected by the routine:

INFO < 0

If INFO $= -i$, the ith parameter had an illegal value. An explanatory message is output, and execution of the program is terminated.

7 Accuracy

For each right-hand side vector b, the computed solution x is the exact solution of a perturbed system of equations $(A + E)x = b$, where

if UPLO = 'U', $|E| \leq c(k+1)\epsilon \left|U^{\mathrm{H}}\right||U|$;

if UPLO = 'L', $|E| \leq c(k+1)\epsilon |L|\left|L^{\mathrm{H}}\right|$,

$c(k+1)$ is a modest linear function of $k+1$, and ϵ is the ***machine precision***.

If $\hat{x}$ is the true solution, then the computed solution x satisfies a forward error bound of the form

$$\frac{\|x-\hat{x}\|_\infty}{\|x\|_\infty} \leq c(k+1)\,\mathrm{cond}(A,x)\epsilon$$

where $\mathrm{cond}(A,x) = \left\||A^{-1}||A||x|\right\|_\infty / \|x\|_\infty \leq \mathrm{cond}(A) = \left\||A^{-1}||A|\right\|_\infty \leq \kappa_\infty(A)$. Note that $\mathrm{cond}(A,x)$ can be much smaller than $\mathrm{cond}(A)$.

Forward and backward error bounds can be computed by calling F07HVF (ZPBRFS), and an estimate for $\kappa_\infty(A)$ ($=\kappa_1(A)$) can be obtained by calling F07HUF (ZPBCON).

8 Further Comments

The total number of real floating-point operations is approximately $16nkr$, assuming $n \gg k$.

This routine may be followed by a call to F07HVF (ZPBRFS) to refine the solution and return an error estimate.

The real analogue of this routine is F07HEF (DPBTRS).

9 Example

This example solves the system of equations $AX = B$, where

$$A = \begin{pmatrix} 9.39+0.00i & 1.08-1.73i & 0.00+0.00i & 0.00+0.00i \\ 1.08+1.73i & 1.69+0.00i & -0.04+0.29i & 0.00+0.00i \\ 0.00+0.00i & -0.04-0.29i & 2.65+0.00i & -0.33+2.24i \\ 0.00+0.00i & 0.00+0.00i & -0.33-2.24i & 2.17+0.00i \end{pmatrix}$$

and

$$B = \begin{pmatrix} -12.42+68.42i & 54.30-56.56i \\ -9.93+0.88i & 18.32+4.76i \\ -27.30-0.01i & -4.40+9.97i \\ 5.31+23.63i & 9.43+1.41i \end{pmatrix}.$$

Here A is Hermitian positive-definite, and is treated as a band matrix, which must first be factorized by F07HRF (ZPBTRF).

9.1 Program Text

```
*     F07HSF Example Program Text
*     Mark 15 Release. NAG Copyright 1991.
*     .. Parameters ..
      INTEGER          NIN, NOUT
      PARAMETER        (NIN=5,NOUT=6)
      INTEGER          NMAX, KDMAX, LDAB, NRHMAX, LDB
      PARAMETER        (NMAX=8,KDMAX=8,LDAB=KDMAX+1,NRHMAX=NMAX,
     +                 LDB=NMAX)
*     .. Local Scalars ..
      INTEGER          I, IFAIL, INFO, J, KD, N, NRHS
      CHARACTER        UPLO
*     .. Local Arrays ..
      COMPLEX *16      AB(LDAB,NMAX), B(LDB,NRHMAX)
      CHARACTER        CLABS(1), RLABS(1)
*     .. External Subroutines ..
      EXTERNAL         X04DBF, ZPBTRF, ZPBTRS
*     .. Intrinsic Functions ..
      INTRINSIC        MAX, MIN
*     .. Executable Statements ..
      WRITE (NOUT,*) 'F07HSF Example Program Results'
*     Skip heading in data file
      READ (NIN,*)
      READ (NIN,*) N, KD, NRHS
      IF (N.LE.NMAX .AND. KD.LE.KDMAX .AND. NRHS.LE.NRHMAX) THEN
*
```

```
*        Read A and B from data file
*
         READ (NIN,*) UPLO
         IF (UPLO.EQ.'U') THEN
            DO 20 I = 1, N
               READ (NIN,*) (AB(KD+1+I-J,J),J=I,MIN(N,I+KD))
   20       CONTINUE
         ELSE IF (UPLO.EQ.'L') THEN
            DO 40 I = 1, N
               READ (NIN,*) (AB(1+I-J,J),J=MAX(1,I-KD),I)
   40       CONTINUE
         END IF
         READ (NIN,*) ((B(I,J),J=1,NRHS),I=1,N)
*
*        Factorize A
*
         CALL ZPBTRF(UPLO,N,KD,AB,LDAB,INFO)
*
         WRITE (NOUT,*)
         IF (INFO.EQ.0) THEN
*
*           Compute solution
*
            CALL ZPBTRS(UPLO,N,KD,NRHS,AB,LDAB,B,LDB,INFO)
*
*           Print solution
*
            IFAIL = 0
            CALL X04DBF('General',' ',N,NRHS,B,LDB,'Bracketed','F7.4',
     +                  'Solution(s)','Integer',RLABS,'Integer',CLABS,
     +                  80,0,IFAIL)
         ELSE
            WRITE (NOUT,*) 'A is not positive-definite'
         END IF
      END IF
*
      END
```

9.2 Program Data

```
F07HSF Example Program Data
  4  1  2                                                :Values of N, KD and NRHS
  'L'                                                        :Value of UPLO
 ( 9.39, 0.00)
 ( 1.08, 1.73) ( 1.69, 0.00)
               (-0.04,-0.29) ( 2.65, 0.00)
                             (-0.33,-2.24) ( 2.17, 0.00)  :End of matrix A
 (-12.42,68.42) (54.30,-56.56)
 ( -9.93, 0.88) (18.32,  4.76)
 (-27.30,-0.01) (-4.40,  9.97)
 (  5.31,23.63) ( 9.43,  1.41)                            :End of matrix B
```

9.3 Program Results

```
 F07HSF Example Program Results

 Solution(s)
                      1                   2
 1  (-1.0000, 8.0000) ( 5.0000,-6.0000)
 2  ( 2.0000,-3.0000) ( 2.0000, 3.0000)
 3  (-4.0000,-5.0000) (-8.0000, 4.0000)
 4  ( 7.0000, 6.0000) (-1.0000,-7.0000)
```