NAG Library Routine Document

F07HEF (DPBTRS)

Note: before using this routine, please read the Users' Note for your implementation to check the interpretation of ***bold italicised*** terms and other implementation-dependent details.

1 Purpose

F07HEF (DPBTRS) solves a real symmetric positive-definite band system of linear equations with multiple right-hand sides,

$$AX = B,$$

where A has been factorized by F07HDF (DPBTRF).

2 Specification

```
SUBROUTINE F07HEF(UPLO, N, KD, NRHS, AB, LDAB, B, LDB, INFO)
INTEGER           N, KD, NRHS, LDAB, LDB, INFO
double precision  AB(LDAB,*), B(LDB,*)
CHARACTER*1       UPLO
```

The routine may be called by its LAPACK name ***dpbtrs***.

3 Description

F07HEF (DPBTRS) is used to solve a real symmetric positive-definite band system of linear equations $AX = B$, the routine must be preceded by a call to F07HDF (DPBTRF) which computes the Cholesky factorization of A. The solution X is computed by forward and backward substitution.

If UPLO = 'U', $A = U^{\mathrm{T}}U$, where U is upper triangular; the solution X is computed by solving $U^{\mathrm{T}}Y = B$ and then $UX = Y$.

If UPLO = 'L', $A = LL^{\mathrm{T}}$, where L is lower triangular; the solution X is computed by solving $LY = B$ and then $L^{\mathrm{T}}X = Y$.

4 References

Golub G H and Van Loan C F (1996) *Matrix Computations* (3rd Edition) Johns Hopkins University Press, Baltimore

5 Parameters

1: UPLO – CHARACTER*1 *Input*

On entry: indicates how A has been factorized.

UPLO = 'U'
$A = U^{\mathrm{T}}U$, where U is upper triangular.

UPLO = 'L'
$A = LL^{\mathrm{T}}$, where L is lower triangular.

Constraint: UPLO = 'U' or 'L'.

2: N – INTEGER *Input*

On entry: n, the order of the matrix A.

Constraint: N ≥ 0.

3: KD – INTEGER *Input*

On entry: k_d, the number of superdiagonals or subdiagonals of the matrix A.

Constraint: KD ≥ 0.

4: NRHS – INTEGER *Input*

On entry: r, the number of right-hand sides.

Constraint: NRHS ≥ 0.

5: AB(LDAB,$*$) – ***double precision*** array *Input*

Note: the second dimension of the array AB must be at least $\max(1, \mathrm{N})$.

On entry: the Cholesky factor of A, as returned by F07HDF (DPBTRF).

6: LDAB – INTEGER *Input*

On entry: the first dimension of the array AB as declared in the (sub)program from which F07HEF (DPBTRS) is called.

Constraint: LDAB $\geq$ KD $+ 1$.

7: B(LDB,$*$) – ***double precision*** array *Input/Output*

Note: the second dimension of the array B must be at least $\max(1, \mathrm{NRHS})$.

On entry: the n by r right-hand side matrix B.

On exit: the n by r solution matrix X.

8: LDB – INTEGER *Input*

On entry: the first dimension of the array B as declared in the (sub)program from which F07HEF (DPBTRS) is called.

Constraint: LDB $\geq \max(1, \mathrm{N})$.

9: INFO – INTEGER *Output*

On exit: INFO $= 0$ unless the routine detects an error (see Section 6).

6 Error Indicators and Warnings

Errors or warnings detected by the routine:

INFO < 0

If INFO $= -i$, the ith parameter had an illegal value. An explanatory message is output, and execution of the program is terminated.

7 Accuracy

For each right-hand side vector b, the computed solution x is the exact solution of a perturbed system of equations $(A + E)x = b$, where

if UPLO = 'U', $|E| \leq c(k+1)\epsilon \left|U^{\mathrm{T}}\right||U|$;

if UPLO = 'L', $|E| \leq c(k+1)\epsilon |L|\left|L^{\mathrm{T}}\right|$,

$c(k+1)$ is a modest linear function of $k+1$, and ϵ is the ***machine precision***.

If $\hat{x}$ is the true solution, then the computed solution x satisfies a forward error bound of the form

$$\frac{\|x-\hat{x}\|_\infty}{\|x\|_\infty} \leq c(k+1)\,\mathrm{cond}(A,x)\epsilon$$

where $\mathrm{cond}(A,x) = \left\||A^{-1}||A||x|\right\|_\infty / \|x\|_\infty \leq \mathrm{cond}(A) = \left\||A^{-1}||A|\right\|_\infty \leq \kappa_\infty(A)$. Note that $\mathrm{cond}(A,x)$ can be much smaller than $\mathrm{cond}(A)$.

Forward and backward error bounds can be computed by calling F07HHF (DPBRFS), and an estimate for $\kappa_\infty(A)$ ($=\kappa_1(A)$) can be obtained by calling F07HGF (DPBCON).

8 Further Comments

The total number of floating-point operations is approximately $4nkr$, assuming $n \gg k$.

This routine may be followed by a call to F07HHF (DPBRFS) to refine the solution and return an error estimate.

The complex analogue of this routine is F07HSF (ZPBTRS).

9 Example

This example solves the system of equations $AX = B$, where

$$A = \begin{pmatrix} 5.49 & 2.68 & 0.00 & 0.00 \\ 2.68 & 5.63 & -2.39 & 0.00 \\ 0.00 & -2.39 & 2.60 & -2.22 \\ 0.00 & 0.00 & -2.22 & 5.17 \end{pmatrix} \quad \text{and} \quad B = \begin{pmatrix} 22.09 & 5.10 \\ 9.31 & 30.81 \\ -5.24 & -25.82 \\ 11.83 & 22.90 \end{pmatrix}.$$

Here A is symmetric and positive-definite, and is treated as a band matrix, which must first be factorized by F07HDF (DPBTRF).

9.1 Program Text

```
*     F07HEF Example Program Text
*     Mark 15 Release. NAG Copyright 1991.
*     .. Parameters ..
      INTEGER          NIN, NOUT
      PARAMETER        (NIN=5,NOUT=6)
      INTEGER          NMAX, KDMAX, LDAB, NRHMAX, LDB
      PARAMETER        (NMAX=8,KDMAX=8,LDAB=KDMAX+1,NRHMAX=NMAX,
     +                 LDB=NMAX)
*     .. Local Scalars ..
      INTEGER          I, IFAIL, INFO, J, KD, N, NRHS
      CHARACTER        UPLO
*     .. Local Arrays ..
      DOUBLE PRECISION AB(LDAB,NMAX), B(LDB,NRHMAX)
*     .. External Subroutines ..
      EXTERNAL         DPBTRF, DPBTRS, X04CAF
*     .. Intrinsic Functions ..
      INTRINSIC        MAX, MIN
*     .. Executable Statements ..
      WRITE (NOUT,*) 'F07HEF Example Program Results'
*     Skip heading in data file
      READ (NIN,*)
      READ (NIN,*) N, KD, NRHS
      IF (N.LE.NMAX .AND. KD.LE.KDMAX .AND. NRHS.LE.NRHMAX) THEN
*
*        Read A and B from data file
*
         READ (NIN,*) UPLO
         IF (UPLO.EQ.'U') THEN
            DO 20 I = 1, N
               READ (NIN,*) (AB(KD+1+I-J,J),J=I,MIN(N,I+KD))
   20       CONTINUE
         ELSE IF (UPLO.EQ.'L') THEN
```

```
            DO 40 I = 1, N
               READ (NIN,*) (AB(1+I-J,J),J=MAX(1,I-KD),I)
   40       CONTINUE
         END IF
         READ (NIN,*) ((B(I,J),J=1,NRHS),I=1,N)
*
*        Factorize A
*
         CALL DPBTRF(UPLO,N,KD,AB,LDAB,INFO)
*
         WRITE (NOUT,*)
         IF (INFO.EQ.0) THEN
*
*           Compute solution
*
            CALL DPBTRS(UPLO,N,KD,NRHS,AB,LDAB,B,LDB,INFO)
*
*           Print solution
*
            IFAIL = 0
*
            CALL X04CAF('General',' ',N,NRHS,B,LDB,'Solution(s)',IFAIL)
*
         ELSE
            WRITE (NOUT,*) 'A is not positive-definite'
         END IF
      END IF
*
      END
```

9.2 Program Data

```
F07HEF Example Program Data
  4  1  2                     :Values of N, KD and NRHS
  'L'                         :Value of UPLO
  5.49
  2.68   5.63
        -2.39   2.60
                -2.22   5.17   :End of matrix A
 22.09   5.10
  9.31  30.81
 -5.24 -25.82
 11.83  22.90                 :End of matrix B
```

9.3 Program Results

```
 F07HEF Example Program Results

 Solution(s)
              1          2
 1      5.0000    -2.0000
 2     -2.0000     6.0000
 3     -3.0000    -1.0000
 4      1.0000     4.0000
```