

NAG Library Routine Document

F07GRF (ZPPTRF)

Note: before using this routine, please read the Users' Note for your implementation to check the interpretation of ***bold italicised*** terms and other implementation-dependent details.

1 Purpose

F07GRF (ZPPTRF) computes the Cholesky factorization of a complex Hermitian positive-definite matrix, using packed storage.

2 Specification

```
SUBROUTINE F07GRF(UPLO, N, AP, INFO)
```

```
INTEGER          N, INFO
```

```
complex*16      AP(*)
```

```
CHARACTER*1      UPLO
```

The routine may be called by its LAPACK name ***zpptrf***.

3 Description

F07GRF (ZPPTRF) forms the Cholesky factorization of a complex Hermitian positive-definite matrix A either as $A = U^H U$ if UPLO = 'U' or $A = L L^H$ if UPLO = 'L', where U is an upper triangular matrix and L is lower triangular, using packed storage.

4 References

Demmel J W (1989) On floating-point errors in Cholesky *LAPACK Working Note No. 14* University of Tennessee, Knoxville

Golub G H and Van Loan C F (1996) *Matrix Computations* (3rd Edition) Johns Hopkins University Press, Baltimore

5 Parameters

1: UPLO – CHARACTER*1 *Input*

On entry: indicates whether the upper or lower triangular part of A is stored and how A is to be factorized.

UPLO = 'U'

The upper triangular part of A is stored and A is factorized as $U^H U$, where U is upper triangular.

UPLO = 'L'

The lower triangular part of A is stored and A is factorized as $L L^H$, where L is lower triangular.

Constraint: UPLO = 'U' or 'L'.

2: N – INTEGER *Input*

On entry: n , the order of the matrix A .

Constraint: $N \geq 0$.

3: AP(*) – **complex*16** array

Input/Output

Note: the dimension of the array AP must be at least $\max(1, N \times (N + 1)/2)$.

On entry: the n by n Hermitian matrix A , packed by columns.

More precisely,

if UPLO = 'U', the upper triangle of A must be stored with element A_{ij} in $AP(i + j(j - 1)/2)$ for $i \leq j$;

if UPLO = 'L', the lower triangle of A must be stored with element A_{ij} in $AP(i + (2n - j)(j - 1)/2)$ for $i \geq j$.

On exit: if INFO = 0, the factor U or L from the Cholesky factorization $A = U^H U$ or $A = LL^T$, in the same storage format as A .

4: INFO – INTEGER

Output

On exit: INFO = 0 unless the routine detects an error (see Section 6).

6 Error Indicators and Warnings

Errors or warnings detected by the routine:

INFO < 0

If INFO = $-i$, the i th parameter had an illegal value. An explanatory message is output, and execution of the program is terminated.

INFO > 0

If INFO = i , the leading minor of order i is not positive-definite and the factorization could not be completed. Hence A itself is not positive-definite. This may indicate an error in forming the matrix A . To factorize a matrix which is not positive-definite, call F07PRF (ZHPTRF) instead.

7 Accuracy

If UPLO = 'U', the computed factor U is the exact factor of a perturbed matrix $A + E$, where

$$|E| \leq c(n)\epsilon|U^H||U|,$$

$c(n)$ is a modest linear function of n , and ϵ is the **machine precision**.

If UPLO = 'L', a similar statement holds for the computed factor L . It follows that $|e_{ij}| \leq c(n)\epsilon\sqrt{a_{ii}a_{jj}}$.

8 Further Comments

The total number of real floating-point operations is approximately $\frac{4}{3}n^3$.

A call to F07GRF (ZPPTRF) may be followed by calls to the routines:

F07GSF (ZPPTRS) to solve $AX = B$;

F07GUF (ZPPCON) to estimate the condition number of A ;

F07GWF (ZPPTRI) to compute the inverse of A .

The real analogue of this routine is F07GDF (DPPTRF).

9 Example

This example computes the Cholesky factorization of the matrix A , where

$$A = \begin{pmatrix} 3.23 + 0.00i & 1.51 - 1.92i & 1.90 + 0.84i & 0.42 + 2.50i \\ 1.51 + 1.92i & 3.58 + 0.00i & -0.23 + 1.11i & -1.18 + 1.37i \\ 1.90 - 0.84i & -0.23 - 1.11i & 4.09 + 0.00i & 2.33 - 0.14i \\ 0.42 - 2.50i & -1.18 - 1.37i & 2.33 + 0.14i & 4.29 + 0.00i \end{pmatrix}.$$

using packed storage.

9.1 Program Text

```
*      F07GRF Example Program Text
*      Mark 15 Release. NAG Copyright 1991.
*      .. Parameters ..
      INTEGER          NIN, NOUT
      PARAMETER        (NIN=5,NOUT=6)
      INTEGER          NMAX
      PARAMETER        (NMAX=8)
*      .. Local Scalars ..
      INTEGER          I, IFAIL, INFO, J, N
      CHARACTER        UPLO
*      .. Local Arrays ..
      COMPLEX *16      AP(NMAX*(NMAX+1)/2)
      CHARACTER        CLABS(1), RLABS(1)
*      .. External Subroutines ..
      EXTERNAL         X04DDF, ZPPTRF
*      .. Executable Statements ..
      WRITE (NOUT,*) 'F07GRF Example Program Results'
      Skip heading in data file
      READ (NIN,*)
      READ (NIN,*) N
      IF (N.LE.NMAX) THEN
*
*      Read A from data file
*
      READ (NIN,*) UPLO
      IF (UPLO.EQ.'U') THEN
        READ (NIN,*) ((AP(I+J*(J-1)/2),J=I,N),I=1,N)
      ELSE IF (UPLO.EQ.'L') THEN
        READ (NIN,*) ((AP(I+(2*N-J)*(J-1)/2),J=1,I),I=1,N)
      END IF
*
*      Factorize A
*
      CALL ZPPTRF(UPLO,N,AP,INFO)
*
      WRITE (NOUT,*)
      IF (INFO.EQ.0) THEN
*
*      Print factor
*
        IFAIL = 0
        CALL X04DDF(UPLO,'Nonunit',N,AP,'Bracketed','F7.4','Factor',
+              'Integer',RLABS,'Integer',CLABS,80,0,IFAIL)
        ELSE
          WRITE (NOUT,*) 'A is not positive-definite'
        END IF
      END IF
*
      END
```

9.2 Program Data

F07GRF Example Program Data

```

4                                     :Value of N
'L'                                 :Value of UPLO
(3.23, 0.00)
(1.51, 1.92) ( 3.58, 0.00)
(1.90,-0.84) (-0.23,-1.11) ( 4.09, 0.00)
(0.42,-2.50) (-1.18,-1.37) ( 2.33, 0.14) ( 4.29, 0.00) :End of matrix A

```

9.3 Program Results

F07GRF Example Program Results

Factor	1	2	3	4
1	(1.7972, 0.0000)			
2	(0.8402, 1.0683)	(1.3164, 0.0000)		
3	(1.0572,-0.4674)	(-0.4702, 0.3131)	(1.5604, 0.0000)	
4	(0.2337,-1.3910)	(0.0834, 0.0368)	(0.9360, 0.9900)	(0.6603, 0.0000)
