

NAG Library Routine Document

F07GJF (DPPTRI)

Note: before using this routine, please read the Users' Note for your implementation to check the interpretation of ***bold italicised*** terms and other implementation-dependent details.

1 Purpose

F07GJF (DPPTRI) computes the inverse of a real symmetric positive-definite matrix A , where A has been factorized by F07GDF (DPPTRF), using packed storage.

2 Specification

```
SUBROUTINE F07GJF(UPLO, N, AP, INFO)
```

```
INTEGER          N, INFO
```

```
double precision AP(*)
```

```
CHARACTER*1      UPLO
```

The routine may be called by its LAPACK name ***dpptri***.

3 Description

F07GJF (DPPTRI) is used to compute the inverse of a real symmetric positive-definite matrix A , the routine must be preceded by a call to F07GDF (DPPTRF), which computes the Cholesky factorization of A , using packed storage.

If UPLO = 'U', $A = U^T U$ and A^{-1} is computed by first inverting U and then forming $(U^{-1})U^{-T}$.

If UPLO = 'L', $A = L L^T$ and A^{-1} is computed by first inverting L and then forming $L^{-T}(L^{-1})$.

4 References

Du Croz J J and Higham N J (1992) Stability of methods for matrix inversion *IMA J. Numer. Anal.* **12** 1–19

5 Parameters

1: UPLO – CHARACTER*1 *Input*

On entry: indicates how A has been factorized.

UPLO = 'U'

$A = U^T U$, where U is upper triangular.

UPLO = 'L'

$A = L L^T$, where L is lower triangular.

Constraint: UPLO = 'U' or 'L'.

2: N – INTEGER *Input*

On entry: n , the order of the matrix A .

Constraint: $N \geq 0$.

3: AP(*) – ***double precision*** array *Input/Output*

Note: the dimension of the array AP must be at least $\max(1, N \times (N + 1)/2)$.

On entry: the Cholesky factor of A stored in packed form, as returned by F07GDF (DPPTRF).

On exit: the factorization is overwritten by the n by n matrix A^{-1} .

More precisely,

if UPLO = 'U', the upper triangle of A^{-1} must be stored with element A_{ij} in $AP(i + j(j-1)/2)$ for $i \leq j$;

if UPLO = 'L', the lower triangle of A^{-1} must be stored with element A_{ij} in $AP(i + (2n-j)(j-1)/2)$ for $i \geq j$.

4: INFO – INTEGER

Output

On exit: INFO = 0 unless the routine detects an error (see Section 6).

6 Error Indicators and Warnings

Errors or warnings detected by the routine:

INFO < 0

If INFO = $-i$, the i th parameter had an illegal value. An explanatory message is output, and execution of the program is terminated.

INFO > 0

If INFO = i , the i th diagonal element of the Cholesky factor is zero; the Cholesky factor is singular and the inverse of A cannot be computed.

7 Accuracy

The computed inverse X satisfies

$$\|XA - I\|_2 \leq c(n)\epsilon\kappa_2(A) \quad \text{and} \quad \|AX - I\|_2 \leq c(n)\epsilon\kappa_2(A),$$

where $c(n)$ is a modest function of n , ϵ is the *machine precision* and $\kappa_2(A)$ is the condition number of A defined by

$$\kappa_2(A) = \|A\|_2 \|A^{-1}\|_2.$$

8 Further Comments

The total number of floating-point operations is approximately $\frac{2}{3}n^3$.

The complex analogue of this routine is F07GWF (ZPPTRI).

9 Example

This example computes the inverse of the matrix A , where

$$A = \begin{pmatrix} 4.16 & -3.12 & 0.56 & -0.10 \\ -3.12 & 5.03 & -0.83 & 1.18 \\ 0.56 & -0.83 & 0.76 & 0.34 \\ -0.10 & 1.18 & 0.34 & 1.18 \end{pmatrix}.$$

Here A is symmetric positive-definite, stored in packed form, and must first be factorized by F07GDF (DPPTRF).

9.1 Program Text

```

*      F07GJF Example Program Text
*      Mark 15 Release. NAG Copyright 1991.
*      .. Parameters ..
      INTEGER          NIN, NOUT
      PARAMETER        (NIN=5,NOUT=6)
      INTEGER          NMAX
      PARAMETER        (NMAX=8)
*      .. Local Scalars ..
      INTEGER          I, IFAIL, INFO, J, N
      CHARACTER        UPLO
*      .. Local Arrays ..
      DOUBLE PRECISION AP(NMAX*(NMAX+1)/2)
*      .. External Subroutines ..
      EXTERNAL         DPPTRF, DPPTRI, X04CCF
*      .. Executable Statements ..
      WRITE (NOUT,*) 'F07GJF Example Program Results'
*      Skip heading in data file
      READ (NIN,*)
      READ (NIN,*) N
      IF (N.LE.NMAX) THEN
*
*          Read A from data file
*
          READ (NIN,*) UPLO
          IF (UPLO.EQ.'U') THEN
              READ (NIN,*) ((AP(I+J*(J-1)/2),J=I,N),I=1,N)
          ELSE IF (UPLO.EQ.'L') THEN
              READ (NIN,*) ((AP(I+(2*N-J)*(J-1)/2),J=1,I),I=1,N)
          END IF
*
*          Factorize A
*
          CALL DPPTRF(UPLO,N,AP,INFO)
*
          WRITE (NOUT,*)
          IF (INFO.EQ.0) THEN
*
*              Compute inverse of A
*
              CALL DPPTRI(UPLO,N,AP,INFO)
*
*              Print inverse
*
              IFAIL = 0
*
              CALL X04CCF(UPLO,'Nonunit',N,AP,'Inverse',IFAIL)
*
          ELSE
              WRITE (NOUT,*) 'A is not positive-definite'
          END IF
      END IF
*
      END

```

9.2 Program Data

```

F07GJF Example Program Data
  4                               :Value of N
  'L'                           :Value of UPLO
  4.16
-3.12    5.03
  0.56   -0.83    0.76
-0.10    1.18    0.34    1.18    :End of matrix A

```

9.3 Program Results

F07GJF Example Program Results

Inverse	1	2	3	4
1	0.6995			
2	0.7769	1.4239		
3	0.7508	1.8255	4.0688	
4	-0.9340	-1.8841	-2.9342	3.4978
