

NAG Library Routine Document

F07GEF (DPPTRS)

Note: before using this routine, please read the Users' Note for your implementation to check the interpretation of ***bold italicised*** terms and other implementation-dependent details.

1 Purpose

F07GEF (DPPTRS) solves a real symmetric positive-definite system of linear equations with multiple right-hand sides,

$$AX = B,$$

where A has been factorized by F07GDF (DPPTRF), using packed storage.

2 Specification

```
SUBROUTINE F07GEF(UPLO, N, NRHS, AP, B, LDB, INFO)
INTEGER          N, NRHS, LDB, INFO
double precision AP(*), B(LDB,*)
CHARACTER*1      UPLO
```

The routine may be called by its LAPACK name ***dpptrs***.

3 Description

F07GEF (DPPTRS) is used to solve a real symmetric positive-definite system of linear equations $AX = B$, the routine must be preceded by a call to F07GDF (DPPTRF) which computes the Cholesky factorization of A , using packed storage. The solution X is computed by forward and backward substitution.

If UPLO = 'U', $A = U^T U$, where U is upper triangular; the solution X is computed by solving $U^T Y = B$ and then $UX = Y$.

If UPLO = 'L', $A = LL^T$, where L is lower triangular; the solution X is computed by solving $LY = B$ and then $L^T X = Y$.

4 References

Golub G H and Van Loan C F (1996) *Matrix Computations* (3rd Edition) Johns Hopkins University Press, Baltimore

5 Parameters

- 1: UPLO – CHARACTER*1 *Input*
On entry: indicates how A has been factorized.
UPLO = 'U'
 $A = U^T U$, where U is upper triangular.
UPLO = 'L'
 $A = LL^T$, where L is lower triangular.
Constraint: UPLO = 'U' or 'L'.
- 2: N – INTEGER *Input*
On entry: n , the order of the matrix A .
Constraint: $N \geq 0$.

- 3: NRHS – INTEGER *Input*
On entry: r , the number of right-hand sides.
Constraint: $\text{NRHS} \geq 0$.
- 4: AP(*) – **double precision** array *Input*
Note: the dimension of the array AP must be at least $\max(1, N \times (N + 1)/2)$.
On entry: the Cholesky factor of A stored in packed form, as returned by F07GDF (DPPTRF).
- 5: B(LDB,*) – **double precision** array *Input/Output*
Note: the second dimension of the array B must be at least $\max(1, \text{NRHS})$.
On entry: the n by r right-hand side matrix B .
On exit: the n by r solution matrix X .
- 6: LDB – INTEGER *Input*
On entry: the first dimension of the array B as declared in the (sub)program from which F07GEF (DPPTRS) is called.
Constraint: $\text{LDB} \geq \max(1, N)$.
- 7: INFO – INTEGER *Output*
On exit: $\text{INFO} = 0$ unless the routine detects an error (see Section 6).

6 Error Indicators and Warnings

Errors or warnings detected by the routine:

$\text{INFO} < 0$

If $\text{INFO} = -i$, the i th parameter had an illegal value. An explanatory message is output, and execution of the program is terminated.

7 Accuracy

For each right-hand side vector b , the computed solution x is the exact solution of a perturbed system of equations $(A + E)x = b$, where

$$\text{if UPLO} = 'U', |E| \leq c(n)\epsilon |U^T| |U|;$$

$$\text{if UPLO} = 'L', |E| \leq c(n)\epsilon |L| |L^T|,$$

$c(n)$ is a modest linear function of n , and ϵ is the **machine precision**.

If $\hat{x}$ is the true solution, then the computed solution x satisfies a forward error bound of the form

$$\frac{\|x - \hat{x}\|_\infty}{\|x\|_\infty} \leq c(n) \text{cond}(A, x) \epsilon$$

where $\text{cond}(A, x) = \| |A^{-1}| |A| |x| \|_\infty / \|x\|_\infty \leq \text{cond}(A) = \| |A^{-1}| |A| \|_\infty \leq \kappa_\infty(A)$.

Note that $\text{cond}(A, x)$ can be much smaller than $\text{cond}(A)$.

Forward and backward error bounds can be computed by calling F07GHF (DPPRFS), and an estimate for $\kappa_\infty(A)$ ($= \kappa_1(A)$) can be obtained by calling F07GGF (DPPCON).

8 Further Comments

The total number of floating-point operations is approximately $2n^2r$.

This routine may be followed by a call to F07GHF (DPPRFS) to refine the solution and return an error estimate.

The complex analogue of this routine is F07GSF (ZPPTRS).

9 Example

This example solves the system of equations $AX = B$, where

$$A = \begin{pmatrix} 4.16 & -3.12 & 0.56 & -0.10 \\ -3.12 & 5.03 & -0.83 & 1.18 \\ 0.56 & -0.83 & 0.76 & 0.34 \\ -0.10 & 1.18 & 0.34 & 1.18 \end{pmatrix} \quad \text{and} \quad B = \begin{pmatrix} 8.70 & 8.30 \\ -13.35 & 2.13 \\ 1.89 & 1.61 \\ -4.14 & 5.00 \end{pmatrix}.$$

Here A is symmetric positive-definite, stored in packed form, and must first be factorized by F07GDF (DPPTRF).

9.1 Program Text

```
*      F07GEF Example Program Text
*      Mark 15 Release. NAG Copyright 1991.
*      .. Parameters ..
      INTEGER          NIN, NOUT
      PARAMETER        (NIN=5,NOUT=6)
      INTEGER          NMAX, NRHMAX, LDB
      PARAMETER        (NMAX=8,NRHMAX=NMAX,LDB=NMAX)
*      .. Local Scalars ..
      INTEGER          I, IFAIL, INFO, J, N, NRHS
      CHARACTER        UPLO
*      .. Local Arrays ..
      DOUBLE PRECISION AP(NMAX*(NMAX+1)/2), B(LDB,NRHMAX)
*      .. External Subroutines ..
      EXTERNAL         DPPTRF, DPPTRS, X04CAF
*      .. Executable Statements ..
      WRITE (NOUT,*) 'F07GEF Example Program Results'
*      Skip heading in data file
      READ (NIN,*)
      READ (NIN,*) N, NRHS
      IF (N.LE.NMAX .AND. NRHS.LE.NRHMAX) THEN
*
*      Read A and B from data file
*
      READ (NIN,*) UPLO
      IF (UPLO.EQ.'U') THEN
        READ (NIN,*) ((AP(I+J*(J-1)/2),J=I,N),I=1,N)
      ELSE IF (UPLO.EQ.'L') THEN
        READ (NIN,*) ((AP(I+(2*N-J)*(J-1)/2),J=1,I),I=1,N)
      END IF
      READ (NIN,*) ((B(I,J),J=1,NRHS),I=1,N)
*
*      Factorize A
*
      CALL DPPTRF(UPLO,N,AP,INFO)
*
      WRITE (NOUT,*)
      IF (INFO.EQ.0) THEN
*
*      Compute solution
*
      CALL DPPTRS(UPLO,N,NRHS,AP,B,LDB,INFO)
*
*      Print solution
*
      IFAIL = 0
*
      CALL X04CAF('General',' ',N,NRHS,B,LDB,'Solution(s)',IFAIL)
*
      ELSE

```

```

        WRITE (NOUT,*) 'A is not positive-definite'
      END IF
    END IF
  *
  END

```

9.2 Program Data

F07GEF Example Program Data

```

  4  2      :Values of N and NRHS
  'L'      :Value of UPLO
  4.16
-3.12  5.03
  0.56 -0.83  0.76
-0.10  1.18  0.34  1.18 :End of matrix A
  8.70  8.30
-13.35  2.13
  1.89  1.61
-4.14  5.00      :End of matrix B

```

9.3 Program Results

F07GEF Example Program Results

Solution(s)

	1	2
1	1.0000	4.0000
2	-1.0000	3.0000
3	2.0000	2.0000
4	-3.0000	1.0000
