

NAG Library Routine Document

F07GDF (DPPTRF)

Note: before using this routine, please read the Users' Note for your implementation to check the interpretation of ***bold italicised*** terms and other implementation-dependent details.

1 Purpose

F07GDF (DPPTRF) computes the Cholesky factorization of a real symmetric positive-definite matrix, using packed storage.

2 Specification

```
SUBROUTINE F07GDF(UPLO, N, AP, INFO)
```

```
INTEGER          N, INFO
```

```
double precision AP(*)
```

```
CHARACTER*1      UPLO
```

The routine may be called by its LAPACK name ***dpptrf***.

3 Description

F07GDF (DPPTRF) forms the Cholesky factorization of a real symmetric positive-definite matrix A either as $A = U^T U$ if UPLO = 'U' or $A = LL^T$ if UPLO = 'L', where U is an upper triangular matrix and L is lower triangular, using packed storage.

4 References

Demmel J W (1989) On floating-point errors in Cholesky *LAPACK Working Note No. 14* University of Tennessee, Knoxville

Golub G H and Van Loan C F (1996) *Matrix Computations* (3rd Edition) Johns Hopkins University Press, Baltimore

5 Parameters

1: UPLO – CHARACTER*1 *Input*

On entry: indicates whether the upper or lower triangular part of A is stored and how A is to be factorized.

UPLO = 'U'

The upper triangular part of A is stored and A is factorized as $U^T U$, where U is upper triangular.

UPLO = 'L'

The lower triangular part of A is stored and A is factorized as LL^T , where L is lower triangular.

Constraint: UPLO = 'U' or 'L'.

2: N – INTEGER *Input*

On entry: n , the order of the matrix A .

Constraint: $N \geq 0$.

3: AP(*) – **double precision** array Input/Output

Note: the dimension of the array AP must be at least $\max(1, N \times (N + 1)/2)$.

On entry: the n by n symmetric matrix A , packed by columns.

More precisely,

if UPLO = 'U', the upper triangle of A must be stored with element A_{ij} in $AP(i + j(j - 1)/2)$ for $i \leq j$;

if UPLO = 'L', the lower triangle of A must be stored with element A_{ij} in $AP(i + (2n - j)(j - 1)/2)$ for $i \geq j$.

On exit: if INFO = 0, the factor U or L from the Cholesky factorization $A = U^T U$ or $A = LL^H$, in the same storage format as A .

4: INFO – INTEGER Output

On exit: INFO = 0 unless the routine detects an error (see Section 6).

6 Error Indicators and Warnings

Errors or warnings detected by the routine:

INFO < 0

If INFO = $-i$, the i th parameter had an illegal value. An explanatory message is output, and execution of the program is terminated.

INFO > 0

If INFO = i , the leading minor of order i is not positive-definite and the factorization could not be completed. Hence A itself is not positive-definite. This may indicate an error in forming the matrix A . To factorize a matrix which is not positive-definite, call F07PDF (DSPTRF) instead.

7 Accuracy

If UPLO = 'U', the computed factor U is the exact factor of a perturbed matrix $A + E$, where

$$|E| \leq c(n)\epsilon|U^T||U|,$$

$c(n)$ is a modest linear function of n , and ϵ is the **machine precision**.

If UPLO = 'L', a similar statement holds for the computed factor L . It follows that $|e_{ij}| \leq c(n)\epsilon\sqrt{a_{ii}a_{jj}}$.

8 Further Comments

The total number of floating-point operations is approximately $\frac{1}{3}n^3$.

A call to F07GDF (DPPTRF) may be followed by calls to the routines:

F07GEF (DPPTRS) to solve $AX = B$;

F07GGF (DPPCON) to estimate the condition number of A ;

F07GJF (DPPTRI) to compute the inverse of A .

The complex analogue of this routine is F07GRF (ZPPTRF).

9 Example

This example computes the Cholesky factorization of the matrix A , where

$$A = \begin{pmatrix} 4.16 & -3.12 & 0.56 & -0.10 \\ -3.12 & 5.03 & -0.83 & 1.18 \\ 0.56 & -0.83 & 0.76 & 0.34 \\ -0.10 & 1.18 & 0.34 & 1.18 \end{pmatrix},$$

using packed storage.

9.1 Program Text

```
*      F07GDF Example Program Text
*      Mark 15 Release. NAG Copyright 1991.
*      .. Parameters ..
      INTEGER          NIN, NOUT
      PARAMETER        (NIN=5,NOUT=6)
      INTEGER          NMAX
      PARAMETER        (NMAX=8)
*      .. Local Scalars ..
      INTEGER          I, IFAIL, INFO, J, N
      CHARACTER        UPLO
*      .. Local Arrays ..
      DOUBLE PRECISION AP(NMAX*(NMAX+1)/2)
*      .. External Subroutines ..
      EXTERNAL         DPPTRF, X04CCF
*      .. Executable Statements ..
      WRITE (NOUT,*) 'F07GDF Example Program Results'
*      Skip heading in data file
      READ (NIN,*)
      READ (NIN,*) N
      IF (N.LE.NMAX) THEN
*
*          Read A from data file
*
*          READ (NIN,*) UPLO
*          IF (UPLO.EQ.'U') THEN
*              READ (NIN,*) ((AP(I+J*(J-1)/2),J=I,N),I=1,N)
*          ELSE IF (UPLO.EQ.'L') THEN
*              READ (NIN,*) ((AP(I+(2*N-J)*(J-1)/2),J=1,I),I=1,N)
*          END IF
*
*          Factorize A
*
*          CALL DPPTRF(UPLO,N,AP,INFO)
*
*          WRITE (NOUT,*)
*          IF (INFO.EQ.0) THEN
*
*              Print factor
*
*              IFAIL = 0
*
*              CALL X04CCF(UPLO,'Nonunit',N,AP,'Factor',IFAIL)
*
*          ELSE
*              WRITE (NOUT,*) 'A is not positive-definite'
*          END IF
*      END IF
*
      END
```

9.2 Program Data

F07GDF Example Program Data

```
4                               :Value of N
'L'                             :Value of UPLO
4.16
-3.12    5.03
0.56    -0.83    0.76
-0.10    1.18    0.34    1.18    :End of matrix A
```

9.3 Program Results

F07GDF Example Program Results

Factor	1	2	3	4
1	2.0396			
2	-1.5297	1.6401		
3	0.2746	-0.2500	0.7887	
4	-0.0490	0.6737	0.6617	0.5347
