

NAG Library Routine Document

F07FNF (ZPOSV)

Note: before using this routine, please read the Users' Note for your implementation to check the interpretation of ***bold italicised*** terms and other implementation-dependent details.

1 Purpose

F07FNF (ZPOSV) computes the solution to a complex system of linear equations

$$AX = B,$$

where A is an n by n Hermitian positive-definite matrix and X and B are n by r matrices.

2 Specification

```
SUBROUTINE F07FNF(UPLO, N, NRHS, A, LDA, B, LDB, INFO)
```

```
INTEGER N, NRHS, LDA, LDB, INFO
```

```
complex*16 A(LDA,*), B(LDB,*)
```

```
CHARACTER*1 UPLO
```

The routine may be called by its LAPACK name ***zposv***.

3 Description

F07FNF (ZPOSV) uses the Cholesky decomposition to factor A as $A = U^H U$ if $UPLO = 'U'$ or $A = LL^H$ if $UPLO = 'L'$, where U is an upper triangular matrix and L is a lower triangular matrix. The factored form of A is then used to solve the system of equations $AX = B$.

4 References

Anderson E, Bai Z, Bischof C, Blackford S, Demmel J, Dongarra J J, Du Croz J J, Greenbaum A, Hammarling S, McKenney A and Sorensen D (1999) *LAPACK Users' Guide* (3rd Edition) SIAM, Philadelphia <http://www.netlib.org/lapack/lug>

Golub G H and Van Loan C F (1996) *Matrix Computations* (3rd Edition) Johns Hopkins University Press, Baltimore

5 Parameters

- 1: UPLO – CHARACTER*1 *Input*
On entry: if $UPLO = 'U'$, the upper triangle of A is stored.
 If $UPLO = 'L'$, the lower triangle of A is stored.
Constraint: $UPLO = 'U'$ or $'L'$.
- 2: N – INTEGER *Input*
On entry: n , the number of linear equations, i.e., the order of the matrix A .
Constraint: $N \geq 0$.
- 3: NRHS – INTEGER *Input*
On entry: r , the number of right-hand sides, i.e., the number of columns of the matrix B .
Constraint: $NRHS \geq 0$.

- 4: A(LDA,*) – **complex*16** array *Input/Output*
Note: the second dimension of the array A must be at least $\max(1, N)$.
On entry: the n by n Hermitian matrix A .
 If UPLO = 'U', the upper triangular part of A must be stored and the elements of the array below the diagonal are not referenced.
 If UPLO = 'L', the lower triangular part of A must be stored and the elements of the array above the diagonal are not referenced.
On exit: if INFO = 0, the factor U or L from the Cholesky factorization $A = U^H U$ or $A = L L^H$.
- 5: LDA – INTEGER *Input*
On entry: the first dimension of the array A as declared in the (sub)program from which F07FNF (ZPOSV) is called.
Constraint: $LDA \geq \max(1, N)$.
- 6: B(LDB,*) – **complex*16** array *Input/Output*
Note: the second dimension of the array B must be at least $\max(1, NRHS)$.
 To solve the equations $Ax = b$, where b is a single right-hand side, B may be supplied as a one-dimensional array with length $LDB = \max(1, N)$.
On entry: the n by r right-hand side matrix B .
On exit: if INFO = 0, the n by r solution matrix X .
- 7: LDB – INTEGER *Input*
On entry: the first dimension of the array B as declared in the (sub)program from which F07FNF (ZPOSV) is called.
Constraint: $LDB \geq \max(1, N)$.
- 8: INFO – INTEGER *Output*
On exit: INFO = 0 unless the routine detects an error (see Section 6).

6 Error Indicators and Warnings

Errors or warnings detected by the routine:

INFO < 0

If INFO = $-i$, the i th argument had an illegal value. An explanatory message is output, and execution of the program is terminated.

INFO > 0

If INFO = i , the leading minor of order i of A is not positive-definite, so the factorization could not be completed, and the solution has not been computed.

7 Accuracy

The computed solution for a single right-hand side, $\hat{x}$, satisfies an equation of the form

$$(A + E)\hat{x} = b,$$

where

$$\|E\|_1 = O(\epsilon)\|A\|_1$$

and ϵ is the **machine precision**. An approximate error bound for the computed solution is given by

$$\frac{\|\hat{x} - x\|_1}{\|x\|_1} \leq \kappa(A) \frac{\|E\|_1}{\|A\|_1},$$

where $\kappa(A) = \|A^{-1}\|_1 \|A\|_1$, the condition number of A with respect to the solution of the linear equations. See Section 4.4 of Anderson *et al.* (1999) for further details.

F07FPF (ZPOSVX) is a comprehensive LAPACK driver that returns forward and backward error bounds and an estimate of the condition number. Alternatively, F04CDF solves $Ax = b$ and returns a forward error bound and condition estimate. F04CDF calls F07FNF (ZPOSV) to solve the equations.

8 Further Comments

The total number of floating-point operations is approximately $\frac{4}{3}n^3 + 8n^2r$, where r is the number of right-hand sides.

The real analogue of this routine is F07FAF (DPOSV).

9 Example

This example solves the equations

$$Ax = b,$$

where A is the symmetric positive-definite matrix

$$A = \begin{pmatrix} 3.23 & 1.51 - 1.92i & 1.90 + 0.84i & 0.42 + 2.50i \\ 1.51 + 1.92i & 3.58 & -0.23 + 1.11i & -1.18 + 1.37i \\ 1.90 - 0.84i & -0.23 - 1.11i & 4.09 & 2.33 - 0.14i \\ 0.42 - 2.50i & -1.18 - 1.37i & 2.33 + 0.14i & 4.29 \end{pmatrix}$$

and

$$b = \begin{pmatrix} 3.93 - 6.14i \\ 6.17 + 9.42i \\ -7.17 - 21.83i \\ 1.99 - 14.38i \end{pmatrix}.$$

Details of the Cholesky factorization of A are also output.

9.1 Program Text

```
*      F07FNF Example Program Text
*      Mark 21 Release. NAG Copyright 2004.
*      .. Parameters ..
      INTEGER          NIN, NOUT
      PARAMETER        (NIN=5,NOUT=6)
      INTEGER          NMAX
      PARAMETER        (NMAX=8)
      INTEGER          LDA
      PARAMETER        (LDA=NMAX)
*      .. Local Scalars ..
      INTEGER          I, IFAIL, INFO, J, N
*      .. Local Arrays ..
      COMPLEX *16      A(LDA,NMAX), B(NMAX)
      CHARACTER        CLABS(1), RLABS(1)
*      .. External Subroutines ..
      EXTERNAL         X04DBF, ZPOSV
*      .. Executable Statements ..
      WRITE (NOUT,*) 'F07FNF Example Program Results'
      WRITE (NOUT,*)
*      Skip heading in data file
      READ (NIN,*)
      READ (NIN,*) N
      IF (N.LE.NMAX) THEN
```

```

*
*      Read the upper triangular part of A from data file
*
*      READ (NIN,*) ((A(I,J),J=I,N),I=1,N)
*
*      Read b from data file
*
*      READ (NIN,*) (B(I),I=1,N)
*
*      Solve the equations Ax = b for x
*
*      CALL ZPOSV('Upper',N,1,A,LDA,B,N,INFO)
*
*      IF (INFO.EQ.0) THEN
*
*          Print solution
*
*          WRITE (NOUT,*) 'Solution'
*          WRITE (NOUT,99999) (B(I),I=1,N)
*
*          Print details of factorization
*
*          WRITE (NOUT,*)
*          IFAIL = 0
*          CALL X04DBF('Upper','Non-unit diagonal',N,N,A,LDA,
+                   'Bracketed','F7.4','Cholesky factor U',
+                   'Integer',RLABS,'Integer',CLABS,80,0,IFAIL)
*
*      ELSE
*          WRITE (NOUT,99998) 'The leading minor of order ', INFO,
+          ' is not positive definite'
*      END IF
*      ELSE
*          WRITE (NOUT,*) 'NMAX too small'
*      END IF
*
99999 FORMAT ((3X,4(' (',F7.4,',',F7.4,')',:)))
99998 FORMAT (1X,A,I3,A)
END

```

9.2 Program Data

F07FNF Example Program Data

```

4                                     :Value of N
( 3.23,  0.00) ( 1.51, -1.92) ( 1.90,  0.84) ( 0.42,  2.50)
          ( 3.58,  0.00) (-0.23,  1.11) (-1.18,  1.37)
                  ( 4.09,  0.00) ( 2.33, -0.14)
                          ( 4.29,  0.00) :End of matrix A
( 3.93, -6.14) ( 6.17,  9.42) (-7.17,-21.83) ( 1.99,-14.38) :End of vector b

```

9.3 Program Results

F07FNF Example Program Results

Solution

```
( 1.0000,-1.0000) ( 0.0000, 3.0000) (-4.0000,-5.0000) ( 2.0000, 1.0000)
```

Cholesky factor U

	1	2	3	4
1	(1.7972, 0.0000)	(0.8402,-1.0683)	(1.0572, 0.4674)	(0.2337, 1.3910)
2		(1.3164, 0.0000)	(-0.4702,-0.3131)	(0.0834,-0.0368)
3			(1.5604, 0.0000)	(0.9360,-0.9900)
4				(0.6603, 0.0000)
