

NAG Library Routine Document

F07CAF (DGTSV)

Note: before using this routine, please read the Users' Note for your implementation to check the interpretation of ***bold italicised*** terms and other implementation-dependent details.

1 Purpose

F07CAF (DGTSV) computes the solution to a real system of linear equations

$$AX = B,$$

where A is an n by n tridiagonal matrix and X and B are n by r matrices.

2 Specification

```
SUBROUTINE F07CAF(N, NRHS, DL, D, DU, B, LDB, INFO)
INTEGER          N, NRHS, LDB, INFO
double precision DL(*), D(*), DU(*), B(LDB,*)
```

The routine may be called by its LAPACK name ***dgtsv***.

3 Description

F07CAF (DGTSV) uses the Gaussian elimination with partial pivoting and row interchanges to solve the equations $AX = B$. The matrix A is factorized as $A = PLU$, where P is a permutation matrix, L is unit lower triangular with at most one nonzero subdiagonal element per column, and U is an upper triangular band matrix, with two superdiagonals.

Note that the equations $A^T X = B$ may be solved by interchanging the order of the arguments DU and DL .

4 References

Anderson E, Bai Z, Bischof C, Blackford S, Demmel J, Dongarra J J, Du Croz J J, Greenbaum A, Hammarling S, McKenney A and Sorensen D (1999) *LAPACK Users' Guide* (3rd Edition) SIAM, Philadelphia <http://www.netlib.org/lapack/lug>

5 Parameters

- 1: N – INTEGER *Input*
On entry: n , the number of linear equations, i.e., the order of the matrix A .
Constraint: $N \geq 0$.
- 2: NRHS – INTEGER *Input*
On entry: r , the number of right-hand sides, i.e., the number of columns of the matrix B .
Constraint: $NRHS \geq 0$.
- 3: DL(*) – ***double precision*** array *Input/Output*
Note: the dimension of the array DL must be at least $\max(1, N - 1)$.
On entry: must contain the $(n - 1)$ subdiagonal elements of the matrix A .
On exit: if no constraints are violated, DL is overwritten by the $(n - 2)$ elements of the second superdiagonal of the upper triangular matrix U from the LU factorization of A , in $DL(1), DL(2), \dots, DL(n - 2)$.

- 4: $D(*)$ – **double precision** array *Input/Output*
Note: the dimension of the array D must be at least $\max(1, N)$.
On entry: must contain the n diagonal elements of the matrix A .
On exit: if no constraints are violated, D is overwritten by the n diagonal elements of the upper triangular matrix U from the LU factorization of A .
- 5: $DU(*)$ – **double precision** array *Input/Output*
Note: the dimension of the array DU must be at least $\max(1, N - 1)$.
On entry: must contain the $(n - 1)$ superdiagonal elements of the matrix A .
On exit: if no constraints are violated, DU is overwritten by the $(n - 1)$ elements of the first superdiagonal of U .
- 6: $B(LDB,*)$ – **double precision** array *Input/Output*
Note: the second dimension of the array B must be at least $\max(1, NRHS)$.
To solve the equations $Ax = b$, where b is a single right-hand side, B may be supplied as a one-dimensional array with length $LDB = \max(1, N)$.
On entry: the n by r right-hand side matrix B .
On exit: if $INFO = 0$, the n by r solution matrix X .
- 7: LDB – INTEGER *Input*
On entry: the first dimension of the array B as declared in the (sub)program from which F07CAF (DGTSV) is called.
Constraint: $LDB \geq \max(1, N)$.
- 8: $INFO$ – INTEGER *Output*
On exit: $INFO = 0$ unless the routine detects an error (see Section 6).

6 Error Indicators and Warnings

Errors or warnings detected by the routine:

$INFO < 0$

If $INFO = -i$, the i th argument had an illegal value. An explanatory message is output, and execution of the program is terminated.

$INFO > 0$

If $INFO = i$, u_{ii} is exactly zero, and the solution has not been computed. The factorization has not been completed unless $i = N$.

7 Accuracy

The computed solution for a single right-hand side, $\hat{x}$, satisfies an equation of the form

$$(A + E)\hat{x} = b,$$

where

$$\|E\|_1 = O(\epsilon)\|A\|_1$$

and ϵ is the **machine precision**. An approximate error bound for the computed solution is given by

$$\frac{\|\hat{x} - x\|_1}{\|x\|_1} \leq \kappa(A) \frac{\|E\|_1}{\|A\|_1},$$

where $\kappa(A) = \|A^{-1}\|_1 \|A\|_1$, the condition number of A with respect to the solution of the linear equations. See Section 4.4 of Anderson *et al.* (1999) for further details.

Alternatives to F07CAF (DGTSV), which return condition and error estimates are F04BCF and F07CBF (DGTSVX).

8 Further Comments

The total number of floating-point operations required to solve the equations $AX = B$ is proportional to nr .

The complex analogue of this routine is F07CNF (ZGTSV).

9 Example

This example solves the equations

$$Ax = b,$$

where A is the tridiagonal matrix

$$A = \begin{pmatrix} 3.0 & 2.1 & 0 & 0 & 0 \\ 3.4 & 2.3 & -1.0 & 0 & 0 \\ 0 & 3.6 & -5.0 & 1.9 & 0 \\ 0 & 0 & 7.0 & -0.9 & 8.0 \\ 0 & 0 & 0 & -6.0 & 7.1 \end{pmatrix} \quad \text{and} \quad b = \begin{pmatrix} 2.7 \\ -0.5 \\ 2.6 \\ 0.6 \\ 2.7 \end{pmatrix}.$$

9.1 Program Text

```
*      F07CAF Example Program Text
*      Mark 21 Release. NAG Copyright 2004.
*      .. Parameters ..
      INTEGER          NIN, NOUT
      PARAMETER        (NIN=5,NOUT=6)
      INTEGER          NMAX
      PARAMETER        (NMAX=8)
*      .. Local Scalars ..
      INTEGER          I, INFO, N
*      .. Local Arrays ..
      DOUBLE PRECISION B(NMAX), D(NMAX), DL(NMAX-1), DU(NMAX-1)
*      .. External Subroutines ..
      EXTERNAL         DGTSV
*      .. Executable Statements ..
      WRITE (NOUT,*) 'F07CAF Example Program Results'
      WRITE (NOUT,*)
*      Skip heading in data file
      READ (NIN,*)
      READ (NIN,*) N
      IF (N.LE.NMAX) THEN
*
*          Read the tridiagonal matrix A and the right hand side B from
*          data file
*
*          READ (NIN,*) (DU(I),I=1,N-1)
*          READ (NIN,*) (D(I),I=1,N)
*          READ (NIN,*) (DL(I),I=1,N-1)
*          READ (NIN,*) (B(I),I=1,N)
*
*          Solve the equations Ax = b for x
*
*          CALL DGTSV(N,1,DL,D,DU,B,N,INFO)
*
*          IF (INFO.EQ.0) THEN
*
*              Print solution
*
*              WRITE (NOUT,*) 'Solution'
```

```

      WRITE (NOUT,99999) (B(I),I=1,N)
*
      ELSE
        WRITE (NOUT,99998) 'The (', INFO, ', ', INFO, ')',
+      ' element of the factor U is zero'
      END IF
      ELSE
        WRITE (NOUT,*) 'NMAX too small'
      END IF
*
99999 FORMAT ((1X,7F11.4))
99998 FORMAT (1X,A,I3,A,I3,A,A)
      END

```

9.2 Program Data

F07CAF Example Program Data

5					:Value of N
	2.1	-1.0	1.9	8.0	
3.0	2.3	-5.0	-0.9	7.1	
3.4	3.6	7.0	-6.0		:End of matrix A
2.7	-0.5	2.6	0.6	2.7	:End of vector B

9.3 Program Results

F07CAF Example Program Results

Solution				
-4.0000	7.0000	3.0000	-4.0000	-3.0000
