NAG Library Routine Document

F07BSF (ZGBTRS)

Note: before using this routine, please read the Users' Note for your implementation to check the interpretation of ***bold italicised*** terms and other implementation-dependent details.

1 Purpose

F07BSF (ZGBTRS) solves a complex band system of linear equations with multiple right-hand sides,

$$AX = B, \quad A^{\mathrm{T}}X = B \quad \text{or} \quad A^{\mathrm{H}}X = B,$$

where A has been factorized by F07BRF (ZGBTRF).

2 Specification

```
SUBROUTINE F07BSF(TRANS, N, KL, KU, NRHS, AB, LDAB, IPIV, B, LDB, INFO)
INTEGER           N, KL, KU, NRHS, LDAB, IPIV(*), LDB, INFO
complex*16        AB(LDAB,*), B(LDB,*)
CHARACTER*1       TRANS
```

The routine may be called by its LAPACK name ***zgbtrs***.

3 Description

F07BSF (ZGBTRS) is used to solve a complex band system of linear equations $AX = B$, $A^{\mathrm{T}}X = B$ or $A^{\mathrm{H}}X = B$, the routine must be preceded by a call to F07BRF (ZGBTRF) which computes the LU factorization of A as $A = PLU$. The solution is computed by forward and backward substitution.

If TRANS = 'N', the solution is computed by solving $PLY = B$ and then $UX = Y$.

If TRANS = 'T', the solution is computed by solving $U^{\mathrm{T}}Y = B$ and then $L^{\mathrm{T}}P^{\mathrm{T}}X = Y$.

If TRANS = 'C', the solution is computed by solving $U^{\mathrm{H}}Y = B$ and then $L^{\mathrm{H}}P^{\mathrm{T}}X = Y$.

4 References

Golub G H and Van Loan C F (1996) *Matrix Computations* (3rd Edition) Johns Hopkins University Press, Baltimore

5 Parameters

1: TRANS – CHARACTER*1 *Input*

On entry: indicates the form of the equations.

TRANS = 'N'
$AX = B$ is solved for X.

TRANS = 'T'
$A^{\mathrm{T}}X = B$ is solved for X.

TRANS = 'C'
$A^{\mathrm{H}}X = B$ is solved for X.

Constraint: TRANS = 'N', 'T' or 'C'.

2: N – INTEGER *Input*

On entry: n, the order of the matrix A.

Constraint: $\mathrm{N} \geq 0$.

3: KL – INTEGER *Input*

On entry: k_l, the number of subdiagonals within the band of the matrix A.

Constraint: $\mathrm{KL} \geq 0$.

4: KU – INTEGER *Input*

On entry: k_u, the number of superdiagonals within the band of the matrix A.

Constraint: $\mathrm{KU} \geq 0$.

5: NRHS – INTEGER *Input*

On entry: r, the number of right-hand sides.

Constraint: $\mathrm{NRHS} \geq 0$.

6: AB(LDAB,$*$) – ***complex*16*** array *Input*

Note: the second dimension of the array AB must be at least $\max(1, \mathrm{N})$.

On entry: the LU factorization of A, as returned by F07BRF (ZGBTRF).

7: LDAB – INTEGER *Input*

On entry: the first dimension of the array AB as declared in the (sub)program from which F07BSF (ZGBTRS) is called.

Constraint: $\mathrm{LDAB} \geq 2 \times \mathrm{KL} + \mathrm{KU} + 1$.

8: IPIV($*$) – INTEGER array *Input*

Note: the dimension of the array IPIV must be at least $\max(1, \mathrm{N})$.

On entry: the pivot indices, as returned by F07BRF (ZGBTRF).

9: B(LDB,$*$) – ***complex*16*** array *Input/Output*

Note: the second dimension of the array B must be at least $\max(1, \mathrm{NRHS})$.

On entry: the n by r right-hand side matrix B.

On exit: the n by r solution matrix X.

10: LDB – INTEGER *Input*

On entry: the first dimension of the array B as declared in the (sub)program from which F07BSF (ZGBTRS) is called.

Constraint: $\mathrm{LDB} \geq \max(1, \mathrm{N})$.

11: INFO – INTEGER *Output*

On exit: $\mathrm{INFO} = 0$ unless the routine detects an error (see Section 6).

6 Error Indicators and Warnings

Errors or warnings detected by the routine:

INFO < 0

If INFO $= -i$, the ith parameter had an illegal value. An explanatory message is output, and execution of the program is terminated.

7 Accuracy

For each right-hand side vector b, the computed solution x is the exact solution of a perturbed system of equations $(A + E)x = b$, where

$$|E| \leq c(k)\epsilon|L||U|,$$

$c(k)$ is a modest linear function of $k = k_l + k_u + 1$, and ϵ is the ***machine precision***. This assumes $k \ll n$.

If $\hat{x}$ is the true solution, then the computed solution x satisfies a forward error bound of the form

$$\frac{\|x - \hat{x}\|_\infty}{\|x\|_\infty} \leq c(k)\,\mathrm{cond}(A, x)\epsilon$$

where $\mathrm{cond}(A, x) = \left\| |A^{-1}||A||x| \right\|_\infty / \|x\|_\infty \leq \mathrm{cond}(A) = \left\| |A^{-1}||A| \right\|_\infty \leq \kappa_\infty(A)$.

Note that $\mathrm{cond}(A, x)$ can be much smaller than $\mathrm{cond}(A)$, and $\mathrm{cond}\left(A^{\mathrm{H}}\right)$ (which is the same as $\mathrm{cond}\left(A^{\mathrm{T}}\right)$) can be much larger (or smaller) than $\mathrm{cond}(A)$.

Forward and backward error bounds can be computed by calling F07BVF (ZGBRFS), and an estimate for $\kappa_\infty(A)$ can be obtained by calling F07BUF (ZGBCON) with NORM = 'I'.

8 Further Comments

The total number of real floating-point operations is approximately $8n(2k_l + k_u)r$, assuming $n \gg k_l$ and $n \gg k_u$.

This routine may be followed by a call to F07BVF (ZGBRFS) to refine the solution and return an error estimate.

The real analogue of this routine is F07BEF (DGBTRS).

9 Example

This example solves the system of equations $AX = B$, where

$$A = \begin{pmatrix} -1.65 + 2.26i & -2.05 - 0.85i & 0.97 - 2.84i & 0.00 + 0.00i \\ 0.00 + 6.30i & -1.48 - 1.75i & -3.99 + 4.01i & 0.59 - 0.48i \\ 0.00 + 0.00i & -0.77 + 2.83i & -1.06 + 1.94i & 3.33 - 1.04i \\ 0.00 + 0.00i & 0.00 + 0.00i & 4.48 - 1.09i & -0.46 - 1.72i \end{pmatrix}$$

and

$$B = \begin{pmatrix} -1.06 + 21.50i & 12.85 + 2.84i \\ -22.72 - 53.90i & -70.22 + 21.57i \\ 28.24 - 38.60i & -20.7 - 31.23i \\ -34.56 + 16.73i & 26.01 + 31.97i \end{pmatrix}.$$

Here A is nonsymmetric and is treated as a band matrix, which must first be factorized by F07BRF (ZGBTRF).

9.1 Program Text

```
*     F07BSF Example Program Text
*     Mark 15 Release. NAG Copyright 1991.
*     .. Parameters ..
      INTEGER          NIN, NOUT
      PARAMETER        (NIN=5,NOUT=6)
      INTEGER          NMAX, KLMAX, KUMAX, LDAB, NRHMAX, LDB
      PARAMETER        (NMAX=8,KLMAX=8,KUMAX=8,LDAB=2*KLMAX+KUMAX+1,
     +                 NRHMAX=NMAX,LDB=NMAX)
      CHARACTER        TRANS
      PARAMETER        (TRANS='N')
*     .. Local Scalars ..
      INTEGER          I, IFAIL, INFO, J, K, KL, KU, N, NRHS
*     .. Local Arrays ..
      COMPLEX *16      AB(LDAB,NMAX), B(LDB,NRHMAX)
      INTEGER          IPIV(NMAX)
      CHARACTER        CLABS(1), RLABS(1)
*     .. External Subroutines ..
      EXTERNAL         X04DBF, ZGBTRF, ZGBTRS
*     .. Intrinsic Functions ..
      INTRINSIC        MAX, MIN
*     .. Executable Statements ..
      WRITE (NOUT,*) 'F07BSF Example Program Results'
*     Skip heading in data file
      READ (NIN,*)
      READ (NIN,*) N, NRHS, KL, KU
      IF (N.LE.NMAX .AND. NRHS.LE.NRHMAX .AND. KL.LE.KLMAX .AND. KU.LE.
     +    KUMAX) THEN
*
*        Read A and B from data file
*
         K = KL + KU + 1
         READ (NIN,*) ((AB(K+I-J,J),J=MAX(I-KL,1),MIN(I+KU,N)),I=1,N)
         READ (NIN,*) ((B(I,J),J=1,NRHS),I=1,N)
*
*        Factorize A
*
         CALL ZGBTRF(N,N,KL,KU,AB,LDAB,IPIV,INFO)
*
         WRITE (NOUT,*)
         IF (INFO.EQ.0) THEN
*
*           Compute solution
*
            CALL ZGBTRS(TRANS,N,KL,KU,NRHS,AB,LDAB,IPIV,B,LDB,INFO)
*
*           Print solution
*
            IFAIL = 0
            CALL X04DBF('General',' ',N,NRHS,B,LDB,'Bracketed','F7.4',
     +                  'Solution(s)','Integer',RLABS,'Integer',CLABS,
     +                  80,0,IFAIL)
         ELSE
            WRITE (NOUT,*) 'The factor U is singular'
         END IF
      END IF
*
      END
```

9.2 Program Data

```
F07BSF Example Program Data
  4  2  1  2                                        :Values of N, NRHS, KL and KU
 (-1.65, 2.26) (-2.05,-0.85) ( 0.97,-2.84)
 ( 0.00, 6.30) (-1.48,-1.75) (-3.99, 4.01) ( 0.59,-0.48)
               (-0.77, 2.83) (-1.06, 1.94) ( 3.33,-1.04)
```

```
                            ( 4.48,-1.09) (-0.46,-1.72)  :End of matrix A
( -1.06, 21.50) ( 12.85,  2.84)
(-22.72,-53.90) (-70.22, 21.57)
( 28.24,-38.60) (-20.73, -1.23)
(-34.56, 16.73) ( 26.01, 31.97)                          :End of matrix B
```

9.3 Program Results

```
 F07BSF Example Program Results

 Solution(s)
                      1                  2
 1  (-3.0000, 2.0000) ( 1.0000, 6.0000)
 2  ( 1.0000,-7.0000) (-7.0000,-4.0000)
 3  (-5.0000, 4.0000) ( 3.0000, 5.0000)
 4  ( 6.0000,-8.0000) (-8.0000, 2.0000)
```