

NAG Library Routine Document

F07BGF (DGBCON)

Note: before using this routine, please read the Users' Note for your implementation to check the interpretation of ***bold italicised*** terms and other implementation-dependent details.

1 Purpose

F07BGF (DGBCON) estimates the condition number of a real band matrix A , where A has been factorized by F07BDF (DGBTRF).

2 Specification

```

SUBROUTINE F07BGF(NORM, N, KL, KU, AB, LDAB, IPIV, ANORM, RCOND, WORK,
1 IWORK, INFO)
    INTEGER          N, KL, KU, LDAB, IPIV(*), IWORK(N), INFO
    double precision AB(LDAB,*), ANORM, RCOND, WORK(3*N)
    CHARACTER*1      NORM

```

The routine may be called by its LAPACK name ***dgbcon***.

3 Description

F07BGF (DGBCON) estimates the condition number of a real band matrix A , in either the 1-norm or the ∞ -norm:

$$\kappa_1(A) = \|A\|_1 \|A^{-1}\|_1 \quad \text{or} \quad \kappa_\infty(A) = \|A\|_\infty \|A^{-1}\|_\infty.$$

Note that $\kappa_\infty(A) = \kappa_1(A^T)$.

Because the condition number is infinite if A is singular, the routine actually returns an estimate of the **reciprocal** of the condition number.

The routine should be preceded by a call to F06RBF to compute $\|A\|_1$ or $\|A\|_\infty$, and a call to F07BDF (DGBTRF) to compute the LU factorization of A . The routine then uses Higham's implementation of Hager's method (see Higham (1988)) to estimate $\|A^{-1}\|_1$ or $\|A^{-1}\|_\infty$.

4 References

Higham N J (1988) FORTRAN codes for estimating the one-norm of a real or complex matrix, with applications to condition estimation *ACM Trans. Math. Software* **14** 381–396

5 Parameters

1: NORM – CHARACTER*1 *Input*

On entry: indicates whether $\kappa_1(A)$ or $\kappa_\infty(A)$ is estimated.

NORM = '1' or 'O'
 $\kappa_1(A)$ is estimated.

NORM = 'I'
 $\kappa_\infty(A)$ is estimated.

Constraint: NORM = '1', 'O' or 'I'.

- 2: N – INTEGER *Input*
On entry: n , the order of the matrix A .
Constraint: $N \geq 0$.
- 3: KL – INTEGER *Input*
On entry: k_l , the number of subdiagonals within the band of the matrix A .
Constraint: $KL \geq 0$.
- 4: KU – INTEGER *Input*
On entry: k_u , the number of superdiagonals within the band of the matrix A .
Constraint: $KU \geq 0$.
- 5: AB(LDAB,*) – **double precision** array *Input*
Note: the second dimension of the array AB must be at least $\max(1, N)$.
On entry: the LU factorization of A , as returned by F07BDF (DGBTRF).
- 6: LDAB – INTEGER *Input*
On entry: the first dimension of the array AB as declared in the (sub)program from which F07BGF (DGBCON) is called.
Constraint: $LDAB \geq 2 \times KL + KU + 1$.
- 7: IPIV(*) – INTEGER array *Input*
Note: the dimension of the array IPIV must be at least $\max(1, N)$.
On entry: the pivot indices, as returned by F07BDF (DGBTRF).
- 8: ANORM – **double precision** *Input*
On entry: if $NORM = '1'$ or $'O'$, the 1-norm of the **original** matrix A .
If $NORM = 'I'$, the ∞ -norm of the **original** matrix A .
ANORM may be computed by calling F06RBF with the same value for the parameter NORM.
ANORM must be computed either **before** calling F07BDF (DGBTRF) or else from a **copy** of the original matrix A (see Section 9).
Constraint: $ANORM \geq 0.0$.
- 9: RCOND – **double precision** *Output*
On exit: an estimate of the reciprocal of the condition number of A . RCOND is set to zero if exact singularity is detected or the estimate underflows. If RCOND is less than **machine precision**, A is singular to working precision.
- 10: WORK($3 \times N$) – **double precision** array *Workspace*
- 11: IWORK(N) – INTEGER array *Workspace*
- 12: INFO – INTEGER *Output*
On exit: $INFO = 0$ unless the routine detects an error (see Section 6).

6 Error Indicators and Warnings

Errors or warnings detected by the routine:

INFO < 0

If INFO = $-i$, the i th parameter had an illegal value. An explanatory message is output, and execution of the program is terminated.

7 Accuracy

The computed estimate RCOND is never less than the true value ρ , and in practice is nearly always less than 10ρ , although examples can be constructed where RCOND is much larger.

8 Further Comments

A call to F07BGF (DGBCON) involves solving a number of systems of linear equations of the form $Ax = b$ or $A^T x = b$; the number is usually 4 or 5 and never more than 11. Each solution involves approximately $2n(2k_l + k_u)$ floating-point operations (assuming $n \gg k_l$ and $n \gg k_u$) but takes considerably longer than a call to F07BEF (DGBTRS) with one right-hand side, because extra care is taken to avoid overflow when A is approximately singular.

The complex analogue of this routine is F07BUF (ZGBCON).

9 Example

This example estimates the condition number in the 1-norm of the matrix A , where

$$A = \begin{pmatrix} -0.23 & 2.54 & -3.66 & 0.00 \\ -6.98 & 2.46 & -2.73 & -2.13 \\ 0.00 & 2.56 & 2.46 & 4.07 \\ 0.00 & 0.00 & -4.78 & -3.82 \end{pmatrix}.$$

Here A is nonsymmetric and is treated as a band matrix, which must first be factorized by F07BDF (DGBTRF). The true condition number in the 1-norm is 56.40.

9.1 Program Text

```
*      F07BGF Example Program Text
*      Mark 15 Release. NAG Copyright 1991.
*      .. Parameters ..
      INTEGER          NIN, NOUT
      PARAMETER        (NIN=5,NOUT=6)
      INTEGER          NMAX, KLMAX, KUMAX, LDAB
      PARAMETER        (NMAX=8, KLMAX=8, KUMAX=8, LDAB=2*KLMAX+KUMAX+1)
      CHARACTER        NORM
      PARAMETER        (NORM='1')
*      .. Local Scalars ..
      DOUBLE PRECISION ANORM, RCOND
      INTEGER          I, INFO, J, K, KL, KU, N
*      .. Local Arrays ..
      DOUBLE PRECISION AB(LDAB,NMAX), WORK(3*NMAX)
      INTEGER          IPIV(NMAX), IWORK(NMAX)
*      .. External Functions ..
      DOUBLE PRECISION F06RBF, X02AJF
      EXTERNAL         F06RBF, X02AJF
*      .. External Subroutines ..
      EXTERNAL         DGBCON, DGBTRF
*      .. Intrinsic Functions ..
      INTRINSIC        MAX, MIN
*      .. Executable Statements ..
      WRITE (NOUT,*) 'F07BGF Example Program Results'
*      Skip heading in data file
      READ (NIN,*)
```

```

      READ (NIN,*) N, KL, KU
      IF (N.LE.NMAX .AND. KL.LE.KLMAX .AND. KU.LE.KUMAX) THEN
*
*       Read A from data file
*
      K = KL + KU + 1
      READ (NIN,*) ((AB(K+I-J,J),J=MAX(I-KL,1),MIN(I+KU,N)),I=1,N)
*
*       Compute norm of A
*
      ANORM = F06RBF(NORM,N,KL,KU,AB(KL+1,1),LDAB,WORK)
*
*       Factorize A
*
      CALL DGBTRF(N,N,KL,KU,AB,LDAB,IPIV,INFO)
*
      WRITE (NOUT,*)
      IF (INFO.EQ.0) THEN
*
*       Estimate condition number
*
      CALL DGBCON(NORM,N,KL,KU,AB,LDAB,IPIV,ANORM,RCOND,WORK,
+          IWORK,INFO)
*
      IF (RCOND.GE.X02AJF()) THEN
          WRITE (NOUT,99999) 'Estimate of condition number =',
+              1.0D0/RCOND
      ELSE
          WRITE (NOUT,*) 'A is singular to working precision'
      END IF
      ELSE
          WRITE (NOUT,*) 'The factor U is singular'
      END IF
      END IF
*
99999 FORMAT (1X,A,1P,E10.2)
      END

```

9.2 Program Data

F07BGF Example Program Data

```

  4  1  2           :Values of N, KL and KU
-0.23  2.54 -3.66
-6.98  2.46 -2.73 -2.13
      2.56  2.46  4.07
      -4.78 -3.82   :End of matrix A

```

9.3 Program Results

F07BGF Example Program Results

Estimate of condition number = 5.64E+01
