NAG Library Routine Document

F07BDF (DGBTRF)

Note: before using this routine, please read the Users' Note for your implementation to check the interpretation of ***bold italicised*** terms and other implementation-dependent details.

1 Purpose

F07BDF (DGBTRF) computes the LU factorization of a real m by n band matrix.

2 Specification

```
SUBROUTINE F07BDF(M, N, KL, KU, AB, LDAB, IPIV, INFO)
INTEGER           M, N, KL, KU, LDAB, IPIV(min(M,N)), INFO
double precision  AB(LDAB,*)
```

The routine may be called by its LAPACK name ***dgbtrf***.

3 Description

F07BDF (DGBTRF) forms the LU factorization of a real m by n band matrix A using partial pivoting, with row interchanges. Usually $m = n$, and then, if A has k_l nonzero subdiagonals and k_u nonzero superdiagonals, the factorization has the form $A = PLU$, where P is a permutation matrix, L is a lower triangular matrix with unit diagonal elements and at most k_l nonzero elements in each column, and U is an upper triangular band matrix with $k_l + k_u$ superdiagonals.

Note that L is not a band matrix, but the nonzero elements of L can be stored in the same space as the subdiagonal elements of A. U is a band matrix but with k_l additional superdiagonals compared with A. These additional superdiagonals are created by the row interchanges.

4 References

Golub G H and Van Loan C F (1996) *Matrix Computations* (3rd Edition) Johns Hopkins University Press, Baltimore

5 Parameters

1: M – INTEGER *Input*

On entry: m, the number of rows of the matrix A.

Constraint: M ≥ 0.

2: N – INTEGER *Input*

On entry: n, the number of columns of the matrix A.

Constraint: N ≥ 0.

3: KL – INTEGER *Input*

On entry: k_l, the number of subdiagonals within the band of the matrix A.

Constraint: KL ≥ 0.

4: KU – INTEGER *Input*

On entry: k_u, the number of superdiagonals within the band of the matrix A.

Constraint: KU ≥ 0.

5: AB(LDAB,$*$) – ***double precision*** array *Input/Output*

Note: the second dimension of the array AB must be at least $\max(1, \mathrm{N})$.

On entry: the m by n coefficient matrix A.

The matrix is stored in rows $k_l + 1$ to $2k_l + k_u + 1$; the first k_l rows need not be set, more precisely, the element A_{ij} must be stored in

$$\mathrm{AB}(k_l + k_u + 1 + i - j, j) = A_{ij} \qquad \text{for } \max(1, j - k_u) \leq i \leq \min(m, j + k_l).$$

On exit: if INFO ≥ 0, AB is overwritten by details of the factorization.

The upper triangular band matrix U, with $k_l + k_u$ superdiagonals, is stored in rows 1 to $k_l + k_u + 1$ of the array, and the multipliers used to form the matrix L are stored in rows $k_l + k_u + 2$ to $2k_l + k_u + 1$.

6: LDAB – INTEGER *Input*

On entry: the first dimension of the array AB as declared in the (sub)program from which F07BDF (DGBTRF) is called.

Constraint: LDAB $\geq 2 \times$ KL + KU + 1.

7: IPIV($\min(\mathrm{M}, \mathrm{N})$) – INTEGER array *Output*

On exit: the pivot indices. Row i of the matrix A was interchanged with row IPIV(i), for $i = 1, 2, \ldots, \min(m, n)$.

8: INFO – INTEGER *Output*

On exit: INFO $= 0$ unless the routine detects an error (see Section 6).

6 Error Indicators and Warnings

Errors or warnings detected by the routine:

INFO < 0

If INFO $= -i$, the ith parameter had an illegal value. An explanatory message is output, and execution of the program is terminated.

INFO > 0

If INFO $= i$, $U(i, i)$ is exactly zero. The factorization has been completed, but the factor U is exactly singular, and division by zero will occur if it is used to solve a system of equations.

7 Accuracy

The computed factors L and U are the exact factors of a perturbed matrix $A + E$, where

$$|E| \leq c(k)\epsilon P|L||U|,$$

$c(k)$ is a modest linear function of $k = k_l + k_u + 1$, and ϵ is the ***machine precision***. This assumes $k \ll \min(m, n)$.

8 Further Comments

The total number of floating-point operations varies between approximately $2nk_l(k_u + 1)$ and $2nk_l(k_l + k_u + 1)$, depending on the interchanges, assuming $m = n \gg k_l$ and $n \gg k_u$.

A call to F07BDF (DGBTRF) may be followed by calls to the routines:

F07BEF (DGBTRS) to solve $AX = B$ or $A^{\mathrm{T}}X = B$;

F07BGF (DGBCON) to estimate the condition number of A.

The complex analogue of this routine is F07BRF (ZGBTRF).

9 Example

This example computes the LU factorization of the matrix A, where

$$A = \begin{pmatrix} -0.23 & 2.54 & -3.66 & 0.00 \\ -6.98 & 2.46 & -2.73 & -2.13 \\ 0.00 & 2.56 & 2.46 & 4.07 \\ 0.00 & 0.00 & -4.78 & -3.82 \end{pmatrix}.$$

Here A is treated as a band matrix with one subdiagonal and two superdiagonals.

9.1 Program Text

```
*     F07BDF Example Program Text
*     Mark 15 Release. NAG Copyright 1991.
*     .. Parameters ..
      INTEGER          NIN, NOUT
      PARAMETER        (NIN=5,NOUT=6)
      INTEGER          MMAX, NMAX, KLMAX, KUMAX, LDAB
      PARAMETER        (MMAX=8,NMAX=8,KLMAX=8,KUMAX=8,
     +                 LDAB=2*KLMAX+KUMAX+1)
*     .. Local Scalars ..
      INTEGER          I, IFAIL, INFO, J, K, KL, KU, M, N
*     .. Local Arrays ..
      DOUBLE PRECISION AB(LDAB,NMAX)
      INTEGER          IPIV(NMAX)
*     .. External Subroutines ..
      EXTERNAL         DGBTRF, X04CEF
*     .. Intrinsic Functions ..
      INTRINSIC        MAX, MIN
*     .. Executable Statements ..
      WRITE (NOUT,*) 'F07BDF Example Program Results'
*     Skip heading in data file
      READ (NIN,*)
      READ (NIN,*) M, N, KL, KU
      IF (M.LE.MMAX .AND. N.LE.NMAX .AND. KL.LE.KLMAX .AND. KU.LE.KUMAX)
     +    THEN
*
*        Read A from data file
*
         K = KL + KU + 1
         READ (NIN,*) ((AB(K+I-J,J),J=MAX(I-KL,1),MIN(I+KU,N)),I=1,M)
*
*        Factorize A
*
         CALL DGBTRF(M,N,KL,KU,AB,LDAB,IPIV,INFO)
*
*        Print details of factorization
*
         WRITE (NOUT,*)
         IFAIL = 0
*
         CALL X04CEF(M,N,KL,KL+KU,AB,LDAB,'Details of factorization',
     +               IFAIL)
*
*        Print pivot indices
*
         WRITE (NOUT,*)
         WRITE (NOUT,*) 'IPIV'
         WRITE (NOUT,99999) (IPIV(I),I=1,MIN(M,N))
*
         IF (INFO.NE.0) WRITE (NOUT,*) 'The factor U is singular'
```

```
*
      END IF
*
99999 FORMAT ((3X,7I11))
      END
```

9.2 Program Data

```
F07BDF Example Program Data
  4  4  1  2                 :Values of M, N, KL and KU
 -0.23   2.54  -3.66
 -6.98   2.46  -2.73  -2.13
         2.56   2.46   4.07
               -4.78  -3.82   :End of matrix A
```

9.3 Program Results

```
 F07BDF Example Program Results

 Details of factorization
             1          2          3          4
 1     -6.9800     2.4600    -2.7300    -2.1300
 2      0.0330     2.5600     2.4600     4.0700
 3                 0.9605    -5.9329    -3.8391
 4                            0.8057    -0.7269

 IPIV
             2          3          3          4
```