NAG Library Routine Document

F07AUF (ZGECON)

Note: before using this routine, please read the Users' Note for your implementation to check the interpretation of ***bold italicised*** terms and other implementation-dependent details.

1 Purpose

F07AUF (ZGECON) estimates the condition number of a complex matrix A, where A has been factorized by F07ARF (ZGETRF).

2 Specification

```
SUBROUTINE F07AUF(NORM, N, A, LDA, ANORM, RCOND, WORK, RWORK, INFO)
INTEGER            N, LDA, INFO
double precision   ANORM, RCOND, RWORK(2*N)
complex*16         A(LDA,*), WORK(2*N)
CHARACTER*1        NORM
```

The routine may be called by its LAPACK name ***zgecon***.

3 Description

F07AUF (ZGECON) estimates the condition number of a complex matrix A, in either the 1-norm or the ∞-norm:

$$\kappa_1(A) = \|A\|_1 \left\|A^{-1}\right\|_1 \qquad \text{or} \qquad \kappa_\infty(A) = \|A\|_\infty \left\|A^{-1}\right\|_\infty.$$

Note that $\kappa_\infty(A) = \kappa_1\left(A^{\mathrm{H}}\right)$.

Because the condition number is infinite if A is singular, the routine actually returns an estimate of the **reciprocal** of the condition number.

The routine should be preceded by a call to F06UAF to compute $\|A\|_1$ or $\|A\|_\infty$, and a call to F07ARF (ZGETRF) to compute the LU factorization of A. The routine then uses Higham's implementation of Hager's method (see Higham (1988)) to estimate $\left\|A^{-1}\right\|_1$ or $\left\|A^{-1}\right\|_\infty$.

4 References

Higham N J (1988) FORTRAN codes for estimating the one-norm of a real or complex matrix, with applications to condition estimation *ACM Trans. Math. Software* **14** 381–396

5 Parameters

1: NORM – CHARACTER*1 *Input*

On entry: indicates whether $\kappa_1(A)$ or $\kappa_\infty(A)$ is estimated.

NORM = '1' or 'O'
$\kappa_1(A)$ is estimated.

NORM = 'I'
$\kappa_\infty(A)$ is estimated.

Constraint: NORM = '1', 'O' or 'I'.

2: N – INTEGER *Input*

On entry: n, the order of the matrix A.

Constraint: $\mathrm{N} \geq 0$.

3: A(LDA,$*$) – ***complex*16*** array *Input*

Note: the second dimension of the array A must be at least $\max(1, \mathrm{N})$.

On entry: the LU factorization of A, as returned by F07ARF (ZGETRF).

4: LDA – INTEGER *Input*

On entry: the first dimension of the array A as declared in the (sub)program from which F07AUF (ZGECON) is called.

Constraint: $\mathrm{LDA} \geq \max(1, \mathrm{N})$.

5: ANORM – ***double precision*** *Input*

On entry: if NORM = '1' or 'O', the 1-norm of the **original** matrix A.

If NORM = 'I', the ∞-norm of the **original** matrix A.

ANORM may be computed by calling F06UAFwith the same value for the parameter NORM.

ANORM must be computed either **before** calling F07ARF (ZGETRF) or else from a **copy** of the original matrix A (see Section 9).

Constraint: $\mathrm{ANORM} \geq 0.0$.

6: RCOND – ***double precision*** *Output*

On exit: an estimate of the reciprocal of the condition number of A. RCOND is set to zero if exact singularity is detected or the estimate underflows. If RCOND is less than ***machine precision***, A is singular to working precision.

7: WORK($2 \times \mathrm{N}$) – ***complex*16*** array *Workspace*

8: RWORK($2 \times \mathrm{N}$) – ***double precision*** array *Workspace*

9: INFO – INTEGER *Output*

On exit: INFO $= 0$ unless the routine detects an error (see Section 6).

6 Error Indicators and Warnings

Errors or warnings detected by the routine:

INFO < 0

If INFO $= -i$, the ith parameter had an illegal value. An explanatory message is output, and execution of the program is terminated.

7 Accuracy

The computed estimate RCOND is never less than the true value ρ, and in practice is nearly always less than 10ρ, although examples can be constructed where RCOND is much larger.

8 Further Comments

A call to F07AUF (ZGECON) involves solving a number of systems of linear equations of the form $Ax = b$ or $A^{\mathrm{H}}x = b$; the number is usually 5 and never more than 11. Each solution involves approximately $8n^2$ real floating-point operations but takes considerably longer than a call to F07ASF

(ZGETRS) with one right-hand side, because extra care is taken to avoid overflow when A is approximately singular.

The real analogue of this routine is F07AGF (DGECON).

9 Example

This example estimates the condition number in the 1-norm of the matrix A, where

$$A = \begin{pmatrix} -1.34 + 2.55i & 0.28 + 3.17i & -6.39 - 2.20i & 0.72 - 0.92i \\ -0.17 - 1.41i & 3.31 - 0.15i & -0.15 + 1.34i & 1.29 + 1.38i \\ -3.29 - 2.39i & -1.91 + 4.42i & -0.14 - 1.35i & 1.72 + 1.35i \\ 2.41 + 0.39i & -0.56 + 1.47i & -0.83 - 0.69i & -1.96 + 0.67i \end{pmatrix}.$$

Here A is nonsymmetric and must first be factorized by F07ARF (ZGETRF). The true condition number in the 1-norm is 231.86.

9.1 Program Text

```
*     F07AUF Example Program Text
*     Mark 15 Release. NAG Copyright 1991.
*     .. Parameters ..
      INTEGER          NIN, NOUT
      PARAMETER        (NIN=5,NOUT=6)
      INTEGER          NMAX, LDA
      PARAMETER        (NMAX=8,LDA=NMAX)
      CHARACTER        NORM
      PARAMETER        (NORM='1')
*     .. Local Scalars ..
      DOUBLE PRECISION ANORM, RCOND
      INTEGER          I, INFO, J, N
*     .. Local Arrays ..
      COMPLEX *16      A(LDA,NMAX), WORK(2*NMAX)
      DOUBLE PRECISION RWORK(2*NMAX)
      INTEGER          IPIV(NMAX)
*     .. External Functions ..
      DOUBLE PRECISION F06UAF, X02AJF
      EXTERNAL         F06UAF, X02AJF
*     .. External Subroutines ..
      EXTERNAL         ZGECON, ZGETRF
*     .. Executable Statements ..
      WRITE (NOUT,*) 'F07AUF Example Program Results'
*     Skip heading in data file
      READ (NIN,*)
      READ (NIN,*) N
      IF (N.LE.NMAX) THEN
*
*        Read A from data file
*
         READ (NIN,*) ((A(I,J),J=1,N),I=1,N)
*
*        Compute norm of A
*
         ANORM = F06UAF(NORM,N,N,A,LDA,RWORK)
*
*        Factorize A
*
         CALL ZGETRF(N,N,A,LDA,IPIV,INFO)
*
         WRITE (NOUT,*)
         IF (INFO.EQ.0) THEN
*
*           Estimate condition number
*
            CALL ZGECON(NORM,N,A,LDA,ANORM,RCOND,WORK,RWORK,INFO)
*
            IF (RCOND.GE.X02AJF()) THEN
               WRITE (NOUT,99999) 'Estimate of condition number =',
```

```
     +           1.0D0/RCOND
            ELSE
               WRITE (NOUT,*) 'A is singular to working precision'
            END IF
         ELSE
            WRITE (NOUT,*) 'The factor U is singular'
         END IF
       END IF
*
99999 FORMAT (1X,A,1P,E10.2)
      END
```

9.2 Program Data

```
F07AUF Example Program Data
  4                                                          :Value of N
 (-1.34, 2.55) ( 0.28, 3.17) (-6.39,-2.20) ( 0.72,-0.92)
 (-0.17,-1.41) ( 3.31,-0.15) (-0.15, 1.34) ( 1.29, 1.38)
 (-3.29,-2.39) (-1.91, 4.42) (-0.14,-1.35) ( 1.72, 1.35)
 ( 2.41, 0.39) (-0.56, 1.47) (-0.83,-0.69) (-1.96, 0.67)  :End of matrix A
```

9.3 Program Results

```
 F07AUF Example Program Results

 Estimate of condition number =  1.50E+02
```