

NAG Library Routine Document

F07ARF (ZGETRF)

Note: before using this routine, please read the Users' Note for your implementation to check the interpretation of ***bold italicised*** terms and other implementation-dependent details.

1 Purpose

F07ARF (ZGETRF) computes the *LU* factorization of a complex *m* by *n* matrix.

2 Specification

```
SUBROUTINE F07ARF(M, N, A, LDA, IPIV, INFO)
INTEGER          M, N, LDA, IPIV(min(M,N)), INFO
complex*16      A(LDA,*)
```

The routine may be called by its LAPACK name *zgetrf*.

3 Description

F07ARF (ZGETRF) forms the *LU* factorization of a complex *m* by *n* matrix *A* as $A = PLU$, where *P* is a permutation matrix, *L* is lower triangular with unit diagonal elements (lower trapezoidal if $m > n$) and *U* is upper triangular (upper trapezoidal if $m < n$). Usually *A* is square ($m = n$), and both *L* and *U* are triangular. The routine uses partial pivoting, with row interchanges.

4 References

Golub G H and Van Loan C F (1996) *Matrix Computations* (3rd Edition) Johns Hopkins University Press, Baltimore

5 Parameters

- 1: M – INTEGER *Input*
On entry: *m*, the number of rows of the matrix *A*.
Constraint: $M \geq 0$.
- 2: N – INTEGER *Input*
On entry: *n*, the number of columns of the matrix *A*.
Constraint: $N \geq 0$.
- 3: A(LDA,*) – **complex*16** array *Input/Output*
Note: the second dimension of the array *A* must be at least $\max(1, N)$.
On entry: the *m* by *n* coefficient matrix *A*.
On exit: the factors *L* and *U* from the factorization $A = PLU$; the unit diagonal elements of *L* are not stored.
- 4: LDA – INTEGER *Input*
On entry: the first dimension of the array *A* as declared in the (sub)program from which F07ARF (ZGETRF) is called.
Constraint: $LDA \geq \max(1, M)$.

- 5: IPIV(min(M,N)) – INTEGER array Output
On exit: the pivot indices. Row i of the matrix A was interchanged with row IPIV(i), for $i = 1, 2, \dots, \min(m, n)$.
- 6: INFO – INTEGER Output
On exit: INFO = 0 unless the routine detects an error (see Section 6).

6 Error Indicators and Warnings

Errors or warnings detected by the routine:

INFO < 0

If INFO = $-i$, the i th parameter had an illegal value. An explanatory message is output, and execution of the program is terminated.

INFO > 0

If INFO = i , $U(i, i)$ is exactly zero. The factorization has been completed, but the factor U is exactly singular, and division by zero will occur if it is used to solve a system of equations.

7 Accuracy

The computed factors L and U are the exact factors of a perturbed matrix $A + E$, where

$$|E| \leq c(\min(m, n))\epsilon P|L||U|,$$

$c(n)$ is a modest linear function of n , and ϵ is the *machine precision*.

8 Further Comments

The total number of real floating-point operations is approximately $\frac{8}{3}n^3$ if $m = n$ (the usual case), $\frac{4}{3}n^2(3m - n)$ if $m > n$ and $\frac{4}{3}m^2(3n - m)$ if $m < n$.

A call to this routine with $m = n$ may be followed by calls to the routines:

F07ASF (ZGETRS) to solve $AX = B$, $A^T X = B$ or $A^H X = B$;

F07AUF (ZGECON) to estimate the condition number of A ;

F07AWF (ZGETRI) to compute the inverse of A .

The real analogue of this routine is F07ADF (DGETRF).

9 Example

This example computes the LU factorization of the matrix A , where

$$A = \begin{pmatrix} -1.34 + 2.55i & 0.28 + 3.17i & -6.39 - 2.20i & 0.72 - 0.92i \\ -0.17 - 1.41i & 3.31 - 0.15i & -0.15 + 1.34i & 1.29 + 1.38i \\ -3.29 - 2.39i & -1.91 + 4.42i & -0.14 - 1.35i & 1.72 + 1.35i \\ 2.41 + 0.39i & -0.56 + 1.47i & -0.83 - 0.69i & -1.96 + 0.67i \end{pmatrix}.$$

9.1 Program Text

```
*      F07ARF Example Program Text
*      Mark 15 Release. NAG Copyright 1991.
*      .. Parameters ..
      INTEGER          NIN, NOUT
      PARAMETER        (NIN=5,NOUT=6)
      INTEGER          MMAX, NMAX, LDA
      PARAMETER        (MMAX=8,NMAX=8,LDA=MMAX)
*      .. Local Scalars ..
```

```

      INTEGER          I, IFAIL, INFO, J, M, N
*    .. Local Arrays ..
      COMPLEX *16      A(LDA,NMAX)
      INTEGER          IPIV(NMAX)
      CHARACTER        CLABS(1), RLABS(1)
*    .. External Subroutines ..
      EXTERNAL          X04DBF, ZGETRF
*    .. Intrinsic Functions ..
      INTRINSIC         MIN
*    .. Executable Statements ..
      WRITE (NOUT,*) 'F07ARF Example Program Results'
*    Skip heading in data file
      READ (NIN,*)
      READ (NIN,*) M, N
      IF (M.LE.MMAX .AND. N.LE.NMAX) THEN
*
*        Read A from data file
*
*        READ (NIN,*) ((A(I,J),J=1,N),I=1,M)
*
*        Factorize A
*
*        CALL ZGETRF(M,N,A,LDA,IPIV,INFO)
*
*        Print details of factorization
*
*        WRITE (NOUT,*)
*        IFAIL = 0
*        CALL X04DBF('General',' ',M,N,A,LDA,'Bracketed','F7.4',
+          'Details of factorization','Integer',RLABS,
+          'Integer',CLABS,80,0,IFAIL)
*
*        Print pivot indices
*
*        WRITE (NOUT,*)
*        WRITE (NOUT,*) 'IPIV'
*        WRITE (NOUT,99999) (IPIV(I),I=1,MIN(M,N))
*
*        IF (INFO.NE.0) WRITE (NOUT,*) 'The factor U is singular'
*
      END IF
*
99999 FORMAT ((1X,I12,3I18))
      END

```

9.2 Program Data

F07ARF Example Program Data

```

  4  4                                     :Values of M and N
(-1.34, 2.55) ( 0.28, 3.17) (-6.39,-2.20) ( 0.72,-0.92)
(-0.17,-1.41) ( 3.31,-0.15) (-0.15, 1.34) ( 1.29, 1.38)
(-3.29,-2.39) (-1.91, 4.42) (-0.14,-1.35) ( 1.72, 1.35)
( 2.41, 0.39) (-0.56, 1.47) (-0.83,-0.69) (-1.96, 0.67) :End of matrix A

```

9.3 Program Results

F07ARF Example Program Results

Details of factorization

```

      1          2          3          4
1  (-3.2900,-2.3900) (-1.9100, 4.4200) (-0.1400,-1.3500) ( 1.7200, 1.3500)
2  ( 0.2376, 0.2560) ( 4.8952,-0.7114) (-0.4623, 1.6966) ( 1.2269, 0.6190)
3  (-0.1020,-0.7010) (-0.6691, 0.3689) (-5.1414,-1.1300) ( 0.9983, 0.3850)
4  (-0.5359, 0.2707) (-0.2040, 0.8601) ( 0.0082, 0.1211) ( 0.1482,-0.1252)

```

IPIV

```

      3          2          3          4

```