

NAG Library Routine Document

F07ANF (ZGESV)

Note: before using this routine, please read the Users' Note for your implementation to check the interpretation of ***bold italicised*** terms and other implementation-dependent details.

1 Purpose

F07ANF (ZGESV) computes the solution to a complex system of linear equations

$$AX = B,$$

where A is an n by n matrix and X and B are n by r matrices.

2 Specification

```
SUBROUTINE F07ANF(N, NRHS, A, LDA, IPIV, B, LDB, INFO)
INTEGER          N, NRHS, LDA, IPIV(N), LDB, INFO
complex*16      A(LDA,*), B(LDB,*)
```

The routine may be called by its LAPACK name ***zgesv***.

3 Description

F07ANF (ZGESV) uses the LU decomposition with partial pivoting and row interchanges to factor A as

$$A = PLU,$$

where P is a permutation matrix, L is unit lower triangular, and U is upper triangular. The factored form of A is then used to solve the system of equations $AX = B$.

4 References

Anderson E, Bai Z, Bischof C, Blackford S, Demmel J, Dongarra J J, Du Croz J J, Greenbaum A, Hammarling S, McKenney A and Sorensen D (1999) *LAPACK Users' Guide* (3rd Edition) SIAM, Philadelphia <http://www.netlib.org/lapack/lug>

Golub G H and Van Loan C F (1996) *Matrix Computations* (3rd Edition) Johns Hopkins University Press, Baltimore

5 Parameters

- 1: N – INTEGER *Input*
On entry: n , the number of linear equations, i.e., the order of the matrix A .
Constraint: $N \geq 0$.
- 2: NRHS – INTEGER *Input*
On entry: r , the number of right-hand sides, i.e., the number of columns of the matrix B .
Constraint: $NRHS \geq 0$.
- 3: A(LDA,*) – ***complex*16*** array *Input/Output*
Note: the second dimension of the array A must be at least $\max(1, N)$.
On entry: the n by n coefficient matrix A .

On exit: the factors L and U from the factorization $A = PLU$; the unit diagonal elements of L are not stored.

4: LDA – INTEGER *Input*

On entry: the first dimension of the array A as declared in the (sub)program from which F07ANF (ZGESV) is called.

Constraint: $LDA \geq \max(1, N)$.

5: IPIV(N) – INTEGER array *Output*

On exit: if no constraints are violated, the pivot indices that define the permutation matrix P ; at the i th step row i of the matrix was interchanged with row $IPIV(i)$. $IPIV(i) = i$ indicates a row interchange was not required.

6: B(LDB,*) – **complex*16** array *Input/Output*

Note: the second dimension of the array B must be at least $\max(1, NRHS)$.

On entry: the n by r right-hand side matrix B .

On exit: if $INFO = 0$, the n by r solution matrix X .

7: LDB – INTEGER *Input*

On entry: the first dimension of the array B as declared in the (sub)program from which F07ANF (ZGESV) is called.

Constraint: $LDB \geq \max(1, N)$.

8: INFO – INTEGER *Output*

On exit: $INFO = 0$ unless the routine detects an error (see Section 6).

6 Error Indicators and Warnings

Errors or warnings detected by the routine:

$INFO < 0$

If $INFO = -i$, the i th argument had an illegal value. An explanatory message is output, and execution of the program is terminated.

$INFO > 0$

If $INFO = i$, u_{ii} is exactly zero. The factorization has been completed, but the factor U is exactly singular, so the solution could not be computed.

7 Accuracy

The computed solution for a single right-hand side, $\hat{x}$, satisfies the equation of the form

$$(A + E)\hat{x} = b,$$

where

$$\|E\|_1 = O(\epsilon)\|A\|_1$$

and ϵ is the **machine precision**. An approximate error bound for the computed solution is given by

$$\frac{\|\hat{x} - x\|_1}{\|x\|_1} \leq \kappa(A) \frac{\|E\|_1}{\|A\|_1}$$

where $\kappa(A) = \|A^{-1}\|_1 \|A\|_1$, the condition number of A with respect to the solution of the linear equations. See Section 4.4 of Anderson *et al.* (1999) for further details.

Following the use of F07ANF (ZGESV), F07AVF (ZGERFS) can be used to estimate the condition number of A and F07AVF (ZGERFS) can be used to obtain approximate error bounds. Alternatives to F07ANF (ZGESV), which return condition and error estimates directly are F04CAF and F07APF (ZGESVX).

8 Further Comments

The total number of floating-point operations is approximately $\frac{8}{3}n^3 + 8n^2r$, where r is the number of right-hand sides.

The real analogue of this routine is F07AAF (DGESV).

9 Example

This example solves the equations

$$Ax = b,$$

where A is the general matrix

$$A = \begin{pmatrix} -1.34 + 2.55i & 0.28 + 3.17i & -6.39 - 2.20i & 0.72 - 0.92i \\ -0.17 - 1.41i & 3.31 - 0.15i & -0.15 + 1.34i & 1.29 + 1.38i \\ -3.29 - 2.39i & -1.91 + 4.42i & -0.14 - 1.35i & 1.72 + 1.35i \\ 2.41 + 0.39i & -0.56 + 1.47i & -0.83 - 0.69i & -1.96 + 0.67i \end{pmatrix} \quad \text{and} \quad b = \begin{pmatrix} 26.26 + 51.78i \\ 6.43 - 8.68i \\ -5.75 + 25.31i \\ 1.16 + 2.57i \end{pmatrix}.$$

Details of the LU factorization of A are also output.

9.1 Program Text

```
*      F07ANF Example Program Text
*      Mark 21 Release. NAG Copyright 2004.
*      .. Parameters ..
      INTEGER          NIN, NOUT
      PARAMETER        (NIN=5,NOUT=6)
      INTEGER          NMAX
      PARAMETER        (NMAX=8)
      INTEGER          LDA, LDB
      PARAMETER        (LDA=NMAX,LDB=NMAX)
*      .. Local Scalars ..
      INTEGER          I, IFAIL, INFO, J, N
*      .. Local Arrays ..
      COMPLEX *16      A(LDA,NMAX), B(LDB)
      INTEGER          IPIV(NMAX)
      CHARACTER        CLABS(1), RLABS(1)
*      .. External Subroutines ..
      EXTERNAL          X04DBF, ZGESV
*      .. Executable Statements ..
      WRITE (NOUT,*) 'F07ANF Example Program Results'
      WRITE (NOUT,*)
*      Skip heading in data file
      READ (NIN,*)
      READ (NIN,*) N
      IF (N.LE.NMAX) THEN
*
*          Read A and B from data file
*
*          READ (NIN,*) ((A(I,J),J=1,N),I=1,N)
*          READ (NIN,*) (B(I),I=1,N)
*
*          Solve the equations Ax = b for x
*
*          CALL ZGESV(N,1,A,LDA,IPIV,B,LDB,INFO)
*
*          IF (INFO.EQ.0) THEN
*
*              Print solution
*
*          
```

```

        WRITE (NOUT,*) 'Solution'
        WRITE (NOUT,99999) (B(I),I=1,N)
*
*      Print details of factorization
*
        WRITE (NOUT,*)
        IFAIL = 0
        CALL X04DBF('General',' ',N,N,A,LDA,'Bracketed','F7.4',
+                  'Details of factorization','Integer',RLABS,
+                  'Integer',CLABS,80,0,IFAIL)
*
*      Print pivot indices
*
        WRITE (NOUT,*)
        WRITE (NOUT,*) 'Pivot indices'
        WRITE (NOUT,99998) (IPIV(I),I=1,N)
    ELSE
        WRITE (NOUT,99997) 'The (', INFO, ', ', INFO, ')',
+        ' element of the factor U is zero'
    END IF
    ELSE
        WRITE (NOUT,*) 'NMAX too small'
    END IF
*
99999 FORMAT ((3X,4(' (',F7.4,',',F7.4,')',:)))
99998 FORMAT (1X,7I11)
99997 FORMAT (1X,A,I3,A,I3,A,A)
    END

```

9.2 Program Data

F07ANF Example Program Data

```

4                                     :Value of N

(-1.34, 2.55) ( 0.28, 3.17) (-6.39,-2.20) ( 0.72,-0.92)
(-0.17,-1.41) ( 3.31,-0.15) (-0.15, 1.34) ( 1.29, 1.38)
(-3.29,-2.39) (-1.91, 4.42) (-0.14,-1.35) ( 1.72, 1.35)
( 2.41, 0.39) (-0.56, 1.47) (-0.83,-0.69) (-1.96, 0.67) :End of matrix A

(26.26,51.78) ( 6.43,-8.68) (-5.75,25.31) ( 1.16, 2.57) :End of vector b

```

9.3 Program Results

F07ANF Example Program Results

Solution
 (1.0000, 1.0000) (2.0000,-3.0000) (-4.0000,-5.0000) (0.0000, 6.0000)

Details of factorization

	1	2	3	4
1	(-3.2900,-2.3900)	(-1.9100, 4.4200)	(-0.1400,-1.3500)	(1.7200, 1.3500)
2	(0.2376, 0.2560)	(4.8952,-0.7114)	(-0.4623, 1.6966)	(1.2269, 0.6190)
3	(-0.1020,-0.7010)	(-0.6691, 0.3689)	(-5.1414,-1.1300)	(0.9983, 0.3850)
4	(-0.5359, 0.2707)	(-0.2040, 0.8601)	(0.0082, 0.1211)	(0.1482,-0.1252)

Pivot indices

	3	2	3	4
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