NAG Library Routine Document

F07AAF (DGESV)

Note: before using this routine, please read the Users' Note for your implementation to check the interpretation of ***bold italicised*** terms and other implementation-dependent details.

1 Purpose

F07AAF (DGESV) computes the solution to a real system of linear equations

$$AX = B,$$

where A is an n by n matrix and X and B are n by r matrices.

2 Specification

```
SUBROUTINE F07AAF(N, NRHS, A, LDA, IPIV, B, LDB, INFO)
INTEGER           N, NRHS, LDA, IPIV(N), LDB, INFO
double precision  A(LDA,*), B(LDB,*)
```

The routine may be called by its LAPACK name ***dgesv***.

3 Description

F07AAF (DGESV) uses the LU decomposition with partial pivoting and row interchanges to factor A as

$$A = PLU,$$

where P is a permutation matrix, L is unit lower triangular, and U is upper triangular. The factored form of A is then used to solve the system of equations $AX = B$.

4 References

Anderson E, Bai Z, Bischof C, Blackford S, Demmel J, Dongarra J J, Du Croz J J, Greenbaum A, Hammarling S, McKenney A and Sorensen D (1999) *LAPACK Users' Guide* (3rd Edition) SIAM, Philadelphiahttp://www.netlib.org/lapack/lug

Golub G H and Van Loan C F (1996) *Matrix Computations* (3rd Edition) Johns Hopkins University Press, Baltimore

5 Parameters

1: N – INTEGER *Input*

On entry: n, the number of linear equations, i.e., the order of the matrix A.

Constraint: $\mathrm{N} \geq 0$.

2: NRHS – INTEGER *Input*

On entry: r, the number of right-hand sides, i.e., the number of columns of the matrix B.

Constraint: $\mathrm{NRHS} \geq 0$.

3: A(LDA,$*$) – ***double precision*** array *Input/Output*

Note: the second dimension of the array A must be at least $\max(1, \mathrm{N})$.

On entry: the n by n coefficient matrix A.

On exit: the factors L and U from the factorization $A = PLU$; the unit diagonal elements of L are not stored.

4: LDA – INTEGER *Input*

On entry: the first dimension of the array A as declared in the (sub)program from which F07AAF (DGESV) is called.

Constraint: LDA $\geq \max(1, \mathrm{N})$.

5: IPIV(N) – INTEGER array *Output*

On exit: if no constraints are violated, the pivot indices that define the permutation matrix P; at the ith step row i of the matrix was interchanged with row IPIV(i). IPIV(i) $= i$ indicates a row interchange was not required.

6: B(LDB,$*$) – ***double precision*** array *Input/Output*

Note: the second dimension of the array B must be at least $\max(1, \mathrm{NRHS})$.

On entry: the n by r right-hand side matrix B.

On exit: if INFO $= 0$, the n by r solution matrix X.

7: LDB – INTEGER *Input*

On entry: the first dimension of the array B as declared in the (sub)program from which F07AAF (DGESV) is called.

Constraint: LDB $\geq \max(1, \mathrm{N})$.

8: INFO – INTEGER *Output*

On exit: INFO $= 0$ unless the routine detects an error (see Section 6).

6 Error Indicators and Warnings

Errors or warnings detected by the routine:

INFO < 0

If INFO $= -i$, the ith argument had an illegal value. An explanatory message is output, and execution of the program is terminated.

INFO > 0

If INFO $= i$, u_{ii} is exactly zero. The factorization has been completed, but the factor U is exactly singular, so the solution could not be computed.

7 Accuracy

The computed solution for a single right-hand side, $\hat{x}$, satisfies the equation of the form

$$(A + E)\hat{x} = b,$$

where

$$\|E\|_1 = O(\epsilon)\|A\|_1$$

and ϵ is the ***machine precision***. An approximate error bound for the computed solution is given by

$$\frac{\|\hat{x} - x\|_1}{\|x\|_1} \leq \kappa(A)\frac{\|E\|_1}{\|A\|_1}$$

where $\kappa(A) = \left\|A^{-1}\right\|_1 \|A\|_1$, the condition number of A with respect to the solution of the linear equations. See Section 4.4 of Anderson *et al.* (1999) for further details.

Following the use of F07AAF (DGESV), F07AGF (DGECON) can be used to estimate the condition number of A and F07AHF (DGERFS) can be used to obtain approximate error bounds. Alternatives to F07AAF (DGESV), which return condition and error estimates directly are F04BAF and F07ABF (DGESVX).

8 Further Comments

The total number of floating-point operations is approximately $\frac{2}{3}n^3 + 2n^2r$, where r is the number of right-hand sides.

The complex analogue of this routine is F07ANF (ZGESV).

9 Example

This example solves the equations

$$Ax = b,$$

where A is the general matrix

$$A = \begin{pmatrix} 1.80 & 2.88 & 2.05 & -0.89 \\ 5.25 & -2.95 & -0.95 & -3.80 \\ 1.58 & -2.69 & -2.90 & -1.04 \\ -1.11 & -0.66 & -0.59 & 0.80 \end{pmatrix} \quad \text{and} \quad b = \begin{pmatrix} 9.52 \\ 24.35 \\ 0.77 \\ -6.22 \end{pmatrix}.$$

Details of the LU factorization of A are also output.

9.1 Program Text

```
*     F07AAF Example Program Text
*     Mark 21 Release. NAG Copyright 2004.
*     .. Parameters ..
      INTEGER          NIN, NOUT
      PARAMETER        (NIN=5,NOUT=6)
      INTEGER          NMAX
      PARAMETER        (NMAX=8)
      INTEGER          LDA, LDB
      PARAMETER        (LDA=NMAX,LDB=NMAX)
*     .. Local Scalars ..
      INTEGER          I, IFAIL, INFO, J, N
*     .. Local Arrays ..
      DOUBLE PRECISION A(LDA,NMAX), B(LDB)
      INTEGER          IPIV(NMAX)
*     .. External Subroutines ..
      EXTERNAL         DGESV, X04CAF
*     .. Executable Statements ..
      WRITE (NOUT,*) 'F07AAF Example Program Results'
      WRITE (NOUT,*)
*     Skip heading in data file
      READ (NIN,*)
      READ (NIN,*) N
      IF (N.LE.NMAX) THEN
*
*        Read A and B from data file
*
         READ (NIN,*) ((A(I,J),J=1,N),I=1,N)
         READ (NIN,*) (B(I),I=1,N)
*
*        Solve the equations Ax = b for x
*
         CALL DGESV(N,1,A,LDA,IPIV,B,LDB,INFO)
*
         IF (INFO.EQ.0) THEN
*
*           Print solution
*
            WRITE (NOUT,*) 'Solution'
```

```
            WRITE (NOUT,99999) (B(I),I=1,N)
*
*           Print details of factorization
*
            WRITE (NOUT,*)
            IFAIL = 0
            CALL X04CAF('General',' ',N,N,A,LDA,
     +                  'Details of factorization',IFAIL)
*
*           Print pivot indices
*
            WRITE (NOUT,*)
            WRITE (NOUT,*) 'Pivot indices'
            WRITE (NOUT,99998) (IPIV(I),I=1,N)
*
         ELSE
            WRITE (NOUT,99997) 'The (', INFO, ',', INFO, ')',
     +        ' element of the factor U is zero'
         END IF
       ELSE
          WRITE (NOUT,*) 'NMAX too small'
       END IF
*
99999 FORMAT ((3X,7F11.4))
99998 FORMAT ((3X,7I11))
99997 FORMAT (1X,A,I3,A,I3,A,A)
      END
```

9.2 Program Data

```
F07AAF Example Program Data

  4                          :Value of N

  1.80   2.88   2.05  -0.89
  5.25  -2.95  -0.95  -3.80
  1.58  -2.69  -2.90  -1.04
 -1.11  -0.66  -0.59   0.80   :End of matrix A

  9.52  24.35   0.77  -6.22   :End of vector b
```

9.3 Program Results

```
 F07AAF Example Program Results

 Solution
        1.0000    -1.0000     3.0000    -5.0000

 Details of factorization
             1          2          3          4
 1      5.2500    -2.9500    -0.9500    -3.8000
 2      0.3429     3.8914     2.3757     0.4129
 3      0.3010    -0.4631    -1.5139     0.2948
 4     -0.2114    -0.3299     0.0047     0.1314

 Pivot indices
             2          2          3          4
```