

NAG Library Routine Document

F04KLF

Note: before using this routine, please read the Users' Note for your implementation to check the interpretation of ***bold italicised*** terms and other implementation-dependent details.

1 Purpose

F04KLF solves a complex general Gauss–Markov linear (least-squares) model problem.

2 Specification

```
SUBROUTINE F04KLF(M, N, P, A, LDA, B, LDB, D, X, Y, WORK, LWORK, IFAIL)
INTEGER          M, N, P, LDA, LDB, LWORK, IFAIL
complex*16      A(LDA,*), B(LDB,*), D(*), X(*), Y(*),
1               WORK(max(1,LWORK))
```

3 Description

F04KLF solves the complex general Gauss–Markov linear model (GLM) problem

$$\underset{x}{\text{minimize}} \|y\|_2 \quad \text{subject to} \quad d = Ax + By$$

where A is an m by n matrix, B is an m by p matrix and d is an m element vector. It is assumed that $n \leq m \leq n + p$, $\text{rank}(A) = n$ and $\text{rank}(E) = m$, where $E = (A \ B)$. Under these assumptions, the problem has a unique solution x and a minimal 2-norm solution y , which is obtained using a generalized QR factorization of the matrices A and B .

In particular, if the matrix B is square and nonsingular, then the GLM problem is equivalent to the weighted linear least-squares problem

$$\underset{x}{\text{minimize}} \|B^{-1}(d - Ax)\|_2.$$

F04KLF is based on the LAPACK routine CGGGLM/ZGGGLM, see Anderson *et al.* (1999).

4 References

Anderson E, Bai Z, Bischof C, Blackford S, Demmel J, Dongarra J J, Du Croz J J, Greenbaum A, Hammarling S, McKenney A and Sorensen D (1999) *LAPACK Users' Guide* (3rd Edition) SIAM, Philadelphia

Anderson E, Bai Z and Dongarra J (1992) Generalized QR factorization and its applications *Linear Algebra Appl. (Volume 162–164)* 243–271

5 Parameters

- 1: M – INTEGER *Input*
On entry: m , the number of rows of the matrices A and B .
Constraint: $M \geq 0$.
- 2: N – INTEGER *Input*
On entry: n , the number of columns of the matrix A .
Constraint: $0 \leq N \leq M$.

- 3: P – INTEGER *Input*
On entry: p , the number of columns of the matrix B .
Constraint: $P \geq M - N$.
- 4: A(LDA,*) – **complex*16** array *Input/Output*
Note: the second dimension of the array A must be at least $\max(1, N)$.
On entry: the m by n matrix A .
On exit: A is overwritten.
- 5: LDA – INTEGER *Input*
On entry: the first dimension of the array A as declared in the (sub)program from which F04KLF is called.
Constraint: $LDA \geq \max(1, M)$.
- 6: B(LDB,*) – **complex*16** array *Input/Output*
Note: the second dimension of the array B must be at least $\max(1, P)$.
On entry: the m by p matrix B .
On exit: B is overwritten.
- 7: LDB – INTEGER *Input*
On entry: the first dimension of the array B as declared in the (sub)program from which F04KLF is called.
Constraint: $LDB \geq \max(1, M)$.
- 8: D(*) – **complex*16** array *Input/Output*
Note: the dimension of the array D must be at least $\max(1, M)$.
On entry: the left-hand side vector d of the GLM equation.
On exit: D is overwritten.
- 9: X(*) – **complex*16** array *Output*
Note: the dimension of the array X must be at least $\max(1, N)$.
On exit: the solution vector x of the GLM problem.
- 10: Y(*) – **complex*16** array *Output*
Note: the dimension of the array Y must be at least $\max(1, P)$.
On exit: the solution vector y of the GLM problem.
- 11: WORK(max(1, LWORK)) – **complex*16** array *Workspace*
On exit: if IFAIL = 0, WORK(1) contains the minimum value of LWORK required for optimum performance.
- 12: LWORK – INTEGER *Input*
On entry: the dimension of the array WORK as declared in the subprogram from which F04KLF is called unless LWORK = -1, in which case a workspace query is assumed and the routine only calculates the optimal dimension of WORK (using the formula given below).

Suggested value: for optimum performance LWORK should be at least $N + \min(M, P) + \max(M, P) \times nb$, where *nb* is the **blocksize**.

Constraint: $LWORK \geq \max(1, M + N + P)$ or $LWORK = -1$.

13: IFAIL – INTEGER

Input/Output

On entry: IFAIL must be set to 0, -1 or 1. If you are unfamiliar with this parameter you should refer to Section 3.3 in the Essential Introduction for details.

On exit: IFAIL = 0 unless the routine detects an error (see Section 6).

For environments where it might be inappropriate to halt program execution when an error is detected, the value -1 or 1 is recommended. If the output of error messages is undesirable, then the value 1 is recommended. Otherwise, if you are not familiar with this parameter, the recommended value is 0. **When the value -1 or 1 is used it is essential to test the value of IFAIL on exit.**

6 Error Indicators and Warnings

If on entry IFAIL = 0 or -1, explanatory error messages are output on the current error message unit (as defined by X04AAF).

Errors or warnings detected by the routine:

IFAIL = 1

On entry, $M < 0$,
or $N < 0$,
or $N > M$,
or $P < 0$,
or $P < M - N$,
or $LDA < \max(1, M)$,
or $LDB < \max(1, M)$,
or $LWORK < \max(1, M + N + P)$ and $LWORK \neq -1$.

7 Accuracy

For an error analysis, see Anderson *et al.* (1992).

8 Further Comments

When $p = m \geq n$, the total number of real floating-point operations is approximately $\frac{8}{3}(2m^3 - n^3) + 16nm^2$; when $p = m = n$, the total number of real floating-point operations is approximately $\frac{56}{3}m^3$.

9 Example

This example solves the weighted least-squares problem

$$\underset{x}{\text{minimize}} \|B^{-1}(d - Ax)\|_2,$$

where

$$B = \begin{pmatrix} 0.5 - 1.0i & 0.0 + 0.0i & 0.0 + 0.0i & 0.0 + 0.0i \\ 0.0 + 0.0i & 1.0 - 2.0i & 0.0 + 0.0i & 0.0 + 0.0i \\ 0.0 + 0.0i & 0.0 + 0.0i & 2.0 - 3.0i & 0.0 + 0.0i \\ 0.0 + 0.0i & 0.0 + 0.0i & 0.0 + 0.0i & 5.0 - 4.0i \end{pmatrix},$$

$$d = \begin{pmatrix} 6.01 - 0.37i \\ -5.27 + 0.90i \\ 2.72 - 2.13i \\ -1.34 - 2.80i \end{pmatrix}$$

and

$$A = \begin{pmatrix} 0.96 - 0.81i & -0.03 + 0.96i & -0.91 + 2.06i \\ -0.98 + 1.98i & -1.20 + 0.19i & -0.66 + 0.42i \\ 0.62 - 0.46i & 1.01 + 0.02i & 0.63 - 0.17i \\ 1.08 - 0.28i & 0.20 - 0.12i & -0.07 + 1.23i \end{pmatrix}.$$

9.1 Program Text

```
*      F04KLF Example Program Text
*      Mark 20 Revised. NAG Copyright 2001.
*      .. Parameters ..
      INTEGER          NIN, NOUT
      PARAMETER        (NIN=5,NOUT=6)
      INTEGER          NMAX, MMAX, PMAX, LDA, LDB, LWORK
      PARAMETER        (NMAX=10,MMAX=10,PMAX=10,LDA=MMAX,LDB=MMAX,
+                      LWORK=NMAX+MMAX+64*(MMAX+PMAX))
*      .. Local Scalars ..
      INTEGER          I, IFAIL, J, M, N, P
*      .. Local Arrays ..
      COMPLEX *16      A(LDA,NMAX), B(LDB,PMAX), D(MMAX), WORK(LWORK),
+                      X(NMAX), Y(PMAX)
*      .. External Subroutines ..
      EXTERNAL         F04KLF
*      .. Intrinsic Functions ..
      INTRINSIC        AIMAG, DBLE
*      .. Executable Statements ..
      WRITE (NOUT,*) 'F04KLF Example Program Results'
*      Skip heading in data file
      READ (NIN,*)
      READ (NIN,*) M, N, P
      IF (M.LE.MMAX .AND. N.LE.NMAX .AND. P.LE.PMAX) THEN
*
*      Read A, B and D from data file
*
      READ (NIN,*) ((A(I,J),J=1,N),I=1,M)
      READ (NIN,*) ((B(I,J),J=1,P),I=1,M)
      READ (NIN,*) (D(I),I=1,M)
*
*      Solve the weighted least-squares problem
*
*      minimize ||inv(B)*(D-A*X)|| (in the 2-norm)
*
      IFAIL = 1
*
      CALL F04KLF(M,N,P,A,LDA,B,LDB,D,X,Y,WORK,LWORK,IFAIL)
*
*      Print least-squares solution
*
      IF (IFAIL.EQ.0) THEN
        WRITE (NOUT,*)
        WRITE (NOUT,*) 'Least-squares solution'
        WRITE (NOUT,99999) (' (' ,DBLE(X(I)),',',',',AIMAG(X(I)),')',I=1,
+                          N)
      ELSE
        WRITE (NOUT,99998) IFAIL
      END IF
      END IF
*
99999 FORMAT ((3X,4(A,F7.4,A,F7.4,A,:)))
99998 FORMAT (1X,/1X,' ** F04KLF returned with IFAIL = ',I5)
      END
```

9.2 Program Data

F04KLF Example Program Data

```

  4 3 4                                     :Values of M, N and P
( 0.96,-0.81) (-0.03, 0.96) (-0.91, 2.06)
(-0.98, 1.98) (-1.20, 0.19) (-0.66, 0.42)
( 0.62,-0.46) ( 1.01, 0.02) ( 0.63,-0.17)
( 1.08,-0.28) ( 0.20,-0.12) (-0.07, 1.23)      :End of matrix A
( 0.50,-1.00) ( 0.00, 0.00) ( 0.00, 0.00) ( 0.00, 0.00)
( 0.00, 0.00) ( 1.00,-2.00) ( 0.00, 0.00) ( 0.00, 0.00)
( 0.00, 0.00) ( 0.00, 0.00) ( 2.00,-3.00) ( 0.00, 0.00)
( 0.00, 0.00) ( 0.00, 0.00) ( 0.00, 0.00) ( 5.00,-4.00) :End of matrix B
( 6.01,-0.37)
(-5.27, 0.90)
( 2.72,-2.13)
(-1.34,-2.80)                                :End of D

```

9.3 Program Results

F04KLF Example Program Results

Least-squares solution

```
(-1.0000, 2.0000) ( 4.0000,-5.0000) (-3.0000, 1.0000)
```
