

NAG Library Routine Document

D03NEF

Note: before using this routine, please read the Users' Note for your implementation to check the interpretation of ***bold italicised*** terms and other implementation-dependent details.

1 Purpose

D03NEF computes average values of a continuous function of time over the remaining life of an option. It is used together with D03NDF to value options with time-dependent parameters.

2 Specification

```
SUBROUTINE D03NEF(T0, TMAT, NTD, TD, PHID, PHIIV, WORK, LWORK, IFAIL)
INTEGER          NTD, LWORK, IFAIL
double precision  T0, TMAT, TD(NTD), PHID(NTD), PHIIV(3), WORK(LWORK)
```

3 Description

D03NEF computes the quantities

$$\phi(t_0), \quad \hat{\phi} = \frac{1}{T-t_0} \int_{t_0}^T \phi(\zeta) d\zeta, \quad \bar{\phi} = \left(\frac{1}{T-t_0} \int_{t_0}^T \phi^2(\zeta) d\zeta \right)^{1/2}$$

from a given set of values PHID of a continuous time-dependent function $\phi(t)$ at a set of discrete points TD, where t_0 is the current time and T is the maturity time. Thus $\hat{\phi}$ and $\bar{\phi}$ are first and second order averages of ϕ over the remaining life of an option.

The routine may be used in conjunction with D03NDF in order to value an option in the case where the risk-free interest rate r , the continuous dividend q , or the stock volatility σ is time-dependent and is described by values at a set of discrete times (see Section 8.2). This is illustrated in Section 9.

4 References

None.

5 Parameters

- | | | |
|----|---|--------------|
| 1: | <p>T0 – <i>double precision</i></p> <p><i>On entry:</i> the current time t_0.</p> <p><i>Constraint:</i> $TD(1) \leq T0 \leq TD(NTD)$.</p> | <i>Input</i> |
| 2: | <p>TMAT – <i>double precision</i></p> <p><i>On entry:</i> the maturity time T.</p> <p><i>Constraint:</i> $TD(1) \leq TMAT \leq TD(NTD)$.</p> | <i>Input</i> |
| 3: | <p>NTD – INTEGER</p> <p><i>On entry:</i> the number of discrete times at which ϕ is given.</p> <p><i>Constraint:</i> $NTD \geq 2$.</p> | <i>Input</i> |

- 4: TD(NTD) – *double precision* array *Input*
On entry: the discrete times at which ϕ is specified.
Constraint: TD(1) < TD(2) < ... < TD(NTD).
- 5: PHID(NTD) – *double precision* array *Input*
On entry: PHID(i) must contain the value of ϕ at time TD(i), for $i = 1, 2, \dots, \text{NTD}$.
- 6: PHIIV(3) – *double precision* array *Output*
On exit: PHIIV(1) contains the value of ϕ interpolated to t_0 , PHIIV(2) contains the first-order average $\hat{\phi}$ and PHIIV(3) contains the second-order average $\bar{\phi}$, where:
- $$\hat{\phi} = \frac{1}{T-t_0} \int_{t_0}^T \phi(\zeta) d\zeta, \quad \bar{\phi} = \left(\frac{1}{T-t_0} \int_{t_0}^T \phi^2(\zeta) d\zeta \right)^{1/2}.$$
- 7: WORK(LWORK) – *double precision* array *Workspace*
8: LWORK – INTEGER *Input*
On entry: the dimension of the array WORK as declared in the (sub)program from which D03NEF is called.
Constraint: LWORK $\geq 9 \times \text{NTD} + 24$.
- 9: IFAIL – INTEGER *Input/Output*
On entry: IFAIL must be set to 0, -1 or 1. If you are unfamiliar with this parameter you should refer to Section 3.3 in the Essential Introduction for details.
On exit: IFAIL = 0 unless the routine detects an error (see Section 6).
For environments where it might be inappropriate to halt program execution when an error is detected, the value -1 or 1 is recommended. If the output of error messages is undesirable, then the value 1 is recommended. Otherwise, if you are not familiar with this parameter, the recommended value is 0. **When the value -1 or 1 is used it is essential to test the value of IFAIL on exit.**

6 Error Indicators and Warnings

If on entry IFAIL = 0 or -1, explanatory error messages are output on the current error message unit (as defined by X04AAF).

Errors or warnings detected by the routine:

IFAIL = 1

- On entry, T0 lies outside the range [TD(1), TD(NTD)],
- or T0 lies outside the range [TD(1), TD(NTD)],
- or NTD < 2,
- or TD badly ordered,
- or LWORK < 9 × NTD + 24.

IFAIL = 2

Unexpected failure in internal call to E01BAF or E02BBF.

7 Accuracy

If $\phi \in C^4[t_0, T]$ then the error in the approximation of $\phi(t_0)$ and $\hat{\phi}$ is $O(H^4)$, where $H = \max_i (T(i+1) - T(i))$, for $i = 1, 2, \dots, \text{NTD} - 1$. The approximation is exact for polynomials of degree up to 3.

The third quantity $\bar{\phi}$ is $O(H^2)$, and exact for linear functions.

8 Further Comments

8.1 Timing

The time taken is proportional to NTD.

8.2 Use with D03NDF

Suppose you wish to evaluate the analytic solution of the Black–Scholes equation in the case when the risk-free interest rate r is a known function of time, and is represented as a set of values at discrete times. A call to D03NEF providing these values in PHID produces an output array PHIIV suitable for use as the argument R in a subsequent call to D03NDF.

Time-dependent values of the continuous dividend Q and the volatility σ may be handled in the same way.

8.3 Algorithmic Details

The NTD data points are fitted with a cubic B-spline using the routine E01BAF. Evaluation is then performed using E02BBF, and the definite integrals are computed using direct integration of the cubic splines in each interval. The special case of $T = t_o$ is handled by interpolating ϕ at that point.

9 Example

This example demonstrates the use of the routine in conjunction with D03NDF to solve the Black–Scholes equation for valuation of a 5-month American call option on a non-dividend-paying stock with an exercise price of \$50. The risk-free interest rate varies linearly with time and the stock volatility has a quadratic variation. Since these functions are integrated exactly by D03NEF the solution of the Black–Scholes equation by D03NDF is also exact.

The option is valued at a range of times and stock prices.

9.1 Program Text

```
*      D03NEF Example Program Text
*      Mark 20 Release. NAG Copyright 2001.
*      .. Parameters ..
      INTEGER          NIN, NOUT
      PARAMETER        (NIN=5,NOUT=6)
      INTEGER          NSMAX, NTMAX, NTDMAX, LWMAX
      PARAMETER        (NSMAX=100,NTMAX=100,NTDMAX=10,LWMAX=9*NTDMAX+24)
*      .. Local Scalars ..
      DOUBLE PRECISION DS, DT, TMAT, X
      INTEGER          I, IFAIL, IGREEK, J, KOPT, LWORK, NS, NT, NTD
*      .. Local Arrays ..
      DOUBLE PRECISION DELTA(NSMAX,NTMAX), F(NSMAX,NTMAX),
+                      GAMMA(NSMAX,NTMAX), LAMBDA(NSMAX,NTMAX), Q(3),
+                      RA(3), RD(NTDMAX), RHO(NSMAX,NTMAX), S(NSMAX),
+                      SIGA(3), SIGD(NTDMAX), T(NTMAX), TD(NTDMAX),
+                      THETA(NSMAX,NTMAX), WORK(LWMAX)
      LOGICAL          GPRNT(5), TDPAR(3)
      CHARACTER*6      GNAME(5)
*      .. External Subroutines ..
      EXTERNAL          D03NDF, D03NEF
*      .. Intrinsic Functions ..
      INTRINSIC         DBLE
*      .. Data statements ..
      DATA             GNAME/'Theta ', 'Delta ', 'Gamma ', 'Lambda',
+                      'Rho ' /
      DATA             GPRNT/5*.TRUE./
*      .. Executable Statements ..
      WRITE (NOUT,*) 'D03NEF Example Program Results'
      WRITE (NOUT,*)
*
```

```

*      Skip heading in data file
*
      READ (NIN,*)
*
*      Read problem parameters
*
      READ (NIN,*) KOPT
      READ (NIN,*) X
      READ (NIN,*) TMAT
      READ (NIN,*) NS, NT
      READ (NIN,*) S(1), S(NS)
      READ (NIN,*) T(1), T(NT)
      READ (NIN,*) NTD
      READ (NIN,*) (TD(I), I=1,NTD)
      READ (NIN,*) (RD(I), I=1,NTD)
      READ (NIN,*) (SIGD(I), I=1,NTD)
*
      TDPAR(1) = .TRUE.
      TDPAR(2) = .FALSE.
      TDPAR(3) = .TRUE.
      Q(1) = 0.DO
      LWORK = 9*NTD + 24
*
      IF (NS.LT.2 .OR. NS.GT.NSMAX) THEN
        WRITE (NOUT,*) 'NS invalid.'
      ELSE IF (NT.LT.2 .OR. NT.GT.NTMAX) THEN
        WRITE (NOUT,*) 'NT invalid.'
      ELSE
*
        DS = (S(NS)-S(1))/DBLE(NS-1)
        DT = (T(NT)-T(1))/DBLE(NT-1)
*
*      Loop over times
*
        DO 40 J = 1, NT
*
          T(J) = T(1) + (J-1)*DT
*
*      Find average values of r and sigma
*
          IFAIL = 1
          CALL D03NEF(T(J), TMAT, NTD, TD, RD, RA, WORK, LWORK, IFAIL)
          IF (IFAIL.EQ.0) THEN
            IFAIL = 1
            CALL D03NEF(T(J), TMAT, NTD, TD, SIGD, SIGA, WORK, LWORK, IFAIL)
          END IF
          IF (IFAIL.NE.0) THEN
            WRITE (NOUT,99997) IFAIL
            GO TO 120
          END IF
*
*      Loop over stock prices
*
          DO 20 I = 1, NS
*
            S(I) = S(1) + (I-1)*DS
*
*      Evaluate analytic solution of Black-Scholes equation
*
            IFAIL = 1
            CALL D03NDF(KOPT, X, S(I), T(J), TMAT, TDPAR, RA, Q, SIGA, F(I, J),
+              THETA(I, J), DELTA(I, J), GAMMA(I, J), LAMBDA(I, J),
+              RHO(I, J), IFAIL)
            IF (IFAIL.NE.0) THEN
              WRITE (NOUT,99996) IFAIL
              GO TO 120
            END IF
*
20      CONTINUE
40      CONTINUE
*

```

```

*      Output option values.
*
      WRITE (NOUT,*)
      WRITE (NOUT,*) 'Option Values'
      WRITE (NOUT,*) '-----'
      WRITE (NOUT,*) '  Stock Price |   Time to Maturity (months)'
      WRITE (NOUT,99999) '|', (12*(TMAT-T(J)),J=1,NT)
      WRITE (NOUT,'(11A)') '-----',
+      ('-----',J=1,NT)
      DO 60 I = 1, NS
        WRITE (NOUT,99998) S(I), '|', (F(I,J),J=1,NT)
60      CONTINUE
*
      DO 100 IGREEK = 1, 5
*
        IF (GPRNT(IGREEK)) THEN
          WRITE (NOUT,*)
          WRITE (NOUT,*) GNAME(IGREEK)
          WRITE (NOUT,*) '-----'
          WRITE (NOUT,*)
+          '  Stock Price |   Time to Maturity (months)'
          WRITE (NOUT,99999) '|', (12*(TMAT-T(J)),J=1,NT)
          WRITE (NOUT,'(11A)') '-----',
+          ('-----',J=1,NT)
          DO 80 I = 1, NS
            IF (IGREEK.EQ.1) THEN
              WRITE (NOUT,99998) S(I), '|', (THETA(I,J),J=1,NT)
            ELSE IF (IGREEK.EQ.2) THEN
              WRITE (NOUT,99998) S(I), '|', (DELTA(I,J),J=1,NT)
            ELSE IF (IGREEK.EQ.3) THEN
              WRITE (NOUT,99998) S(I), '|', (GAMMA(I,J),J=1,NT)
            ELSE IF (IGREEK.EQ.4) THEN
              WRITE (NOUT,99998) S(I), '|', (LAMBDA(I,J),J=1,NT)
            ELSE IF (IGREEK.EQ.5) THEN
              WRITE (NOUT,99998) S(I), '|', (RHO(I,J),J=1,NT)
            END IF
80          CONTINUE
        END IF
*
100      CONTINUE
*
120      CONTINUE
      END IF
*
99999 FORMAT (16X,A,1X,12(1P,E12.4))
99998 FORMAT (1X,1P,E12.4,3X,A,1X,12(1P,E12.4))
99997 FORMAT (1X,' ** D03NEF returned with IFAIL = ',I5)
99996 FORMAT (1X,' ** D03NDF returned with IFAIL = ',I5)
      END

```

9.2 Program Data

D03NEF Example Program Data		
2		KOPT
50.		X
0.4166667		TMAT
21	4	NS, NT
0.0	100.	S(1), S(NS)
0.0	0.125	T(1), T(NT)
6		NTD
0.00	0.10 0.20	
0.30	0.40 0.50	TD(1) ... TD(NTD)
0.10	0.11 0.12	
0.13	0.14 0.15	RD(1) ... RD(NTD)
0.30	0.46 0.54	
0.54	0.46 0.30	SIGD(1) ... SIGD(NTD)

9.3 Program Results

D03NEF Example Program Results

Option Values

Stock Price	Time to Maturity (months)			
	5.0000E+00	4.5000E+00	4.0000E+00	3.5000E+00
0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
5.0000E+00	3.3671E-13	7.7404E-14	7.3210E-15	2.0179E-16
1.0000E+01	5.2088E-07	2.4281E-07	7.2216E-08	1.1540E-08
1.5000E+01	2.6607E-04	1.6753E-04	8.0943E-05	2.7179E-05
2.0000E+01	8.9697E-03	6.6505E-03	4.1780E-03	2.0942E-03
2.5000E+01	8.3647E-02	6.8467E-02	5.0375E-02	3.2105E-02
3.0000E+01	3.8221E-01	3.3331E-01	2.7117E-01	2.0119E-01
3.5000E+01	1.1298E+00	1.0275E+00	8.9292E-01	7.3146E-01
4.0000E+01	2.5164E+00	2.3541E+00	2.1380E+00	1.8699E+00
4.5000E+01	4.6249E+00	4.4110E+00	4.1267E+00	3.7700E+00
5.0000E+01	7.4287E+00	7.1797E+00	6.8531E+00	6.4449E+00
5.5000E+01	1.0830E+01	1.0564E+01	1.0221E+01	9.7996E+00
6.0000E+01	1.4707E+01	1.4436E+01	1.4097E+01	1.3689E+01
6.5000E+01	1.8937E+01	1.8671E+01	1.8348E+01	1.7968E+01
7.0000E+01	2.3421E+01	2.3164E+01	2.2860E+01	2.2514E+01
7.5000E+01	2.8080E+01	2.7833E+01	2.7550E+01	2.7234E+01
8.0000E+01	3.2857E+01	3.2620E+01	3.2354E+01	3.2064E+01
8.5000E+01	3.7713E+01	3.7484E+01	3.7233E+01	3.6963E+01
9.0000E+01	4.2620E+01	4.2398E+01	4.2158E+01	4.1904E+01
9.5000E+01	4.7561E+01	4.7344E+01	4.7112E+01	4.6868E+01
1.0000E+02	5.2523E+01	5.2310E+01	5.2084E+01	5.1848E+01

Theta

Stock Price	Time to Maturity (months)			
	5.0000E+00	4.5000E+00	4.0000E+00	3.5000E+00
0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
5.0000E+00	-8.9082E-12	-3.4507E-12	-5.0884E-13	-2.1236E-14
1.0000E+01	-7.2097E-06	-5.5915E-06	-2.5721E-06	-6.1830E-07
1.5000E+01	-2.2499E-03	-2.3259E-03	-1.7227E-03	-8.6349E-04
2.0000E+01	-4.9483E-02	-5.9355E-02	-5.6562E-02	-4.1921E-02
2.5000E+01	-3.1200E-01	-4.0620E-01	-4.4765E-01	-4.1683E-01
3.0000E+01	-9.8578E-01	-1.3408E+00	-1.6092E+00	-1.7186E+00
3.5000E+01	-2.0479E+00	-2.8395E+00	-3.5745E+00	-4.1390E+00
4.0000E+01	-3.2501E+00	-4.5165E+00	-5.8147E+00	-7.0323E+00
4.5000E+01	-4.3144E+00	-5.9349E+00	-7.6762E+00	-9.4488E+00
5.0000E+01	-5.0802E+00	-6.8543E+00	-8.7919E+00	-1.0815E+01
5.5000E+01	-5.5225E+00	-7.2603E+00	-9.1500E+00	-1.1104E+01
6.0000E+01	-5.7006E+00	-7.2722E+00	-8.9491E+00	-1.0625E+01
6.5000E+01	-5.7014E+00	-7.0446E+00	-8.4366E+00	-9.7565E+00
7.0000E+01	-5.6037E+00	-6.7093E+00	-7.8142E+00	-8.7951E+00
7.5000E+01	-5.4653E+00	-6.3555E+00	-7.2107E+00	-7.9170E+00
8.0000E+01	-5.3218E+00	-6.0329E+00	-6.6903E+00	-7.1974E+00
8.5000E+01	-5.1920E+00	-5.7627E+00	-6.2736E+00	-6.6481E+00
9.0000E+01	-5.0833E+00	-5.5487E+00	-5.9563E+00	-6.2492E+00
9.5000E+01	-4.9969E+00	-5.3857E+00	-5.7234E+00	-5.9700E+00
1.0000E+02	-4.9306E+00	-5.2651E+00	-5.5570E+00	-5.7797E+00

Delta

Stock Price	Time to Maturity (months)			
	5.0000E+00	4.5000E+00	4.0000E+00	3.5000E+00
0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
5.0000E+00	1.6086E-12	3.8832E-13	3.9572E-14	1.2111E-15
1.0000E+01	8.9933E-07	4.3972E-07	1.4063E-07	2.4884E-08
1.5000E+01	2.3975E-04	1.5810E-04	8.1943E-05	3.0366E-05
2.0000E+01	4.9150E-03	3.8095E-03	2.5596E-03	1.4100E-03
2.5000E+01	3.0345E-02	2.5906E-02	2.0311E-02	1.4153E-02
3.0000E+01	9.6991E-02	8.7980E-02	7.5946E-02	6.1231E-02

3.5000E+01		2.0863E-01	1.9675E-01	1.8053E-01	1.5957E-01
4.0000E+01		3.4875E-01	3.3719E-01	3.2158E-01	3.0109E-01
4.5000E+01		4.9361E-01	4.8480E-01	4.7356E-01	4.5924E-01
5.0000E+01		6.2450E-01	6.1931E-01	6.1363E-01	6.0735E-01
5.5000E+01		7.3200E-01	7.3000E-01	7.2907E-01	7.2954E-01
6.0000E+01		8.1439E-01	8.1462E-01	8.1681E-01	8.2145E-01
6.5000E+01		8.7440E-01	8.7589E-01	8.7961E-01	8.8602E-01
7.0000E+01		9.1650E-01	9.1850E-01	9.2260E-01	9.2911E-01
7.5000E+01		9.4522E-01	9.4726E-01	9.5107E-01	9.5679E-01
8.0000E+01		9.6441E-01	9.6624E-01	9.6946E-01	9.7406E-01
8.5000E+01		9.7704E-01	9.7856E-01	9.8111E-01	9.8461E-01
9.0000E+01		9.8526E-01	9.8646E-01	9.8839E-01	9.9094E-01
9.5000E+01		9.9057E-01	9.9148E-01	9.9290E-01	9.9470E-01
1.0000E+02		9.9397E-01	9.9464E-01	9.9567E-01	9.9691E-01

Gamma

Stock Price		Time to Maturity (months)			
		5.0000E+00	4.5000E+00	4.0000E+00	3.5000E+00
0.0000E+00		0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
5.0000E+00		7.2334E-12	1.8390E-12	2.0276E-13	6.9267E-15
1.0000E+01		1.4139E-06	7.2829E-07	2.5205E-07	4.9786E-08
1.5000E+01		1.8932E-04	1.3153E-04	7.3756E-05	3.0494E-05
2.0000E+01		2.2528E-03	1.8392E-03	1.3360E-03	8.2017E-04
2.5000E+01		8.6933E-03	7.8126E-03	6.6135E-03	5.1251E-03
3.0000E+01		1.8099E-02	1.7264E-02	1.6056E-02	1.4350E-02
3.5000E+01		2.5953E-02	2.5691E-02	2.5315E-02	2.4683E-02
4.0000E+01		2.9260E-02	2.9618E-02	3.0194E-02	3.0968E-02
4.5000E+01		2.8046E-02	2.8736E-02	2.9814E-02	3.1368E-02
5.0000E+01		2.4005E-02	2.4715E-02	2.5793E-02	2.7346E-02
5.5000E+01		1.8950E-02	1.9500E-02	2.0296E-02	2.1401E-02
6.0000E+01		1.4105E-02	1.4449E-02	1.4903E-02	1.5476E-02
6.5000E+01		1.0054E-02	1.0221E-02	1.0396E-02	1.0555E-02
7.0000E+01		6.9401E-03	6.9861E-03	6.9806E-03	6.8890E-03
7.5000E+01		4.6779E-03	4.6538E-03	4.5552E-03	4.3505E-03
8.0000E+01		3.0978E-03	3.0414E-03	2.9096E-03	2.6800E-03
8.5000E+01		2.0250E-03	1.9598E-03	1.8291E-03	1.6205E-03
9.0000E+01		1.3114E-03	1.2499E-03	1.1365E-03	9.6637E-04
9.5000E+01		8.4362E-04	7.9138E-04	7.0024E-04	5.7052E-04
1.0000E+02		5.4033E-04	4.9856E-04	4.2893E-04	3.3442E-04

Lambda

Stock Price		Time to Maturity (months)			
		5.0000E+00	4.5000E+00	4.0000E+00	3.5000E+00
0.0000E+00		0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
5.0000E+00		3.6558E-11	8.6441E-12	8.6672E-13	2.6259E-14
1.0000E+01		2.8583E-05	1.3693E-05	4.3098E-06	7.5495E-07
1.5000E+01		8.6115E-03	5.5645E-03	2.8375E-03	1.0404E-03
2.0000E+01		1.8217E-01	1.3832E-01	9.1376E-02	4.9748E-02
2.5000E+01		1.0984E+00	9.1808E-01	7.0676E-01	4.8574E-01
3.0000E+01		3.2931E+00	2.9214E+00	2.4708E+00	1.9584E+00
3.5000E+01		6.4272E+00	5.9173E+00	5.3025E+00	4.5851E+00
4.0000E+01		9.4643E+00	8.9101E+00	8.2604E+00	7.5135E+00
4.5000E+01		1.1481E+01	1.0941E+01	1.0323E+01	9.6323E+00
5.0000E+01		1.2132E+01	1.1617E+01	1.1026E+01	1.0367E+01
5.5000E+01		1.1588E+01	1.1091E+01	1.0498E+01	9.8169E+00
6.0000E+01		1.0265E+01	9.7801E+00	9.1734E+00	8.4486E+00
6.5000E+01		8.5872E+00	8.1198E+00	7.5104E+00	6.7621E+00
7.0000E+01		6.8747E+00	6.4363E+00	5.8487E+00	5.1188E+00
7.5000E+01		5.3194E+00	4.9219E+00	4.3812E+00	3.7109E+00
8.0000E+01		4.0081E+00	3.6599E+00	3.1840E+00	2.6009E+00
8.5000E+01		2.9578E+00	2.6623E+00	2.2597E+00	1.7754E+00
9.0000E+01		2.1474E+00	1.9036E+00	1.5741E+00	1.1870E+00
9.5000E+01		1.5392E+00	1.3429E+00	1.0806E+00	7.8078E-01
1.0000E+02		1.0923E+00	9.3740E-01	7.3341E-01	5.0711E-01

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Stock Price	Time to Maturity (months)			
	5.0000E+00	4.5000E+00	4.0000E+00	3.5000E+00
0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
5.0000E+00	3.2110E-12	6.9908E-13	6.3513E-14	1.7073E-15
1.0000E+01	3.5302E-06	1.5579E-06	4.4470E-07	6.9214E-08
1.5000E+01	1.3876E-03	8.2648E-04	3.8273E-04	1.2492E-04
2.0000E+01	3.7221E-02	2.6077E-02	1.5671E-02	7.6142E-03
2.5000E+01	2.8124E-01	2.1719E-01	1.5247E-01	9.3836E-02
3.0000E+01	1.0531E+00	8.6478E-01	6.6907E-01	4.7709E-01
3.5000E+01	2.5718E+00	2.1971E+00	1.8086E+00	1.4156E+00
4.0000E+01	4.7641E+00	4.1750E+00	3.5750E+00	2.9673E+00
4.5000E+01	7.3281E+00	6.5270E+00	5.7279E+00	4.9280E+00
5.0000E+01	9.9152E+00	8.9196E+00	7.9427E+00	6.9774E+00
5.5000E+01	1.2262E+01	1.1095E+01	9.9592E+00	8.8448E+00
6.0000E+01	1.4232E+01	1.2915E+01	1.1637E+01	1.0383E+01
6.5000E+01	1.5791E+01	1.4348E+01	1.2942E+01	1.1557E+01
7.0000E+01	1.6973E+01	1.5424E+01	1.3907E+01	1.2403E+01
7.5000E+01	1.7838E+01	1.6204E+01	1.4594E+01	1.2987E+01
8.0000E+01	1.8457E+01	1.6755E+01	1.5067E+01	1.3376E+01
8.5000E+01	1.8890E+01	1.7135E+01	1.5387E+01	1.3629E+01
9.0000E+01	1.9189E+01	1.7393E+01	1.5599E+01	1.3790E+01
9.5000E+01	1.9393E+01	1.7567E+01	1.5738E+01	1.3891E+01
1.0000E+02	1.9531E+01	1.7683E+01	1.5827E+01	1.3954E+01



