

# NAG Library Routine Document

## C05PDF/C05PDA

**Note:** before using this routine, please read the Users' Note for your implementation to check the interpretation of ***bold italicised*** terms and other implementation-dependent details.

### 1 Purpose

C05PDF/C05PDA is a comprehensive reverse communication routine to find a solution of a system of nonlinear equations by a modification of the Powell hybrid method. You must provide the Jacobian.

C05PDA is a version of C05PDF that has additional parameters in order to make it safe for use in multithreaded applications (see Section 5).

### 2 Specification

#### 2.1 Specification for C05PDF

```

SUBROUTINE C05PDF(IREVCM, N, X, FVEC, FJAC, LDFJAC, XTOL, DIAG, MODE,
1                FACTOR, R, LR, QTF, W, IFAIL)
    INTEGER      IREVCM, N, LDFJAC, MODE, LR, IFAIL
    double precision X(N), FVEC(N), FJAC(LDFJAC,N), XTOL, DIAG(N), FACTOR,
1                R(LR), QTF(N), W(N,4)

```

#### 2.2 Specification for C05PDA

```

SUBROUTINE C05PDA(IREVCM, N, X, FVEC, FJAC, LDFJAC, XTOL, DIAG, MODE,
1                FACTOR, R, LR, QTF, W, LWSAV, IWSAV, RWSAV, IFAIL)
    INTEGER      IREVCM, N, LDFJAC, MODE, LR, IWSAV(15), IFAIL
    double precision X(N), FVEC(N), FJAC(LDFJAC,N), XTOL, DIAG(N), FACTOR,
1                R(LR), QTF(N), W(N,4), RWSAV(10)
    LOGICAL      LWSAV(5)

```

### 3 Description

The system of equations is defined as:

$$f_i(x_1, x_2, \dots, x_n) = 0, \quad i = 1, 2, \dots, n.$$

C05PDF/C05PDA is based on the MINPACK routine HYBRJ (see Moré *et al.* (1980)). It chooses the correction at each step as a convex combination of the Newton and scaled gradient directions. Under reasonable conditions this guarantees global convergence for starting points far from the solution and a fast rate of convergence. The Jacobian is updated by the rank-1 method of Broyden. For more details see Powell (1970).

### 4 References

Moré J J, Garbow B S and Hillstom K E (1980) User guide for MINPACK-1 *Technical Report ANL-80-74* Argonne National Laboratory

Powell M J D (1970) A hybrid method for nonlinear algebraic equations *Numerical Methods for Nonlinear Algebraic Equations* (ed P Rabinowitz) Gordon and Breach

## 5 Parameters

**Note:** this routine uses **reverse communication**. Its use involves an initial entry, intermediate exits and re-entries, and a final exit, as indicated by the **parameter IREVCM**. Between intermediate exits and re-entries, **all parameters other than FVEC and FJAC must remain unchanged**.

- 1: IREVCM – INTEGER *Input/Output*  
*On initial entry:* must have the value 0.  
*On intermediate exit:* specifies what action you must take before re-entering C05PDF/C05PDA **with IREVCM unchanged**. The value of IREVCM should be interpreted as follows:  
 IREVCM = 1  
     Indicates the start of a new iteration. No action is required by you, but X and FVEC are available for printing.  
 IREVCM = 2  
     Indicates that before re-entry to C05PDF/C05PDA, FVEC must contain the function value  $f_i(x)$ .  
 IREVCM = 3  
     Indicates that before re-entry to C05PDF/C05PDA, FJAC( $i, j$ ) must contain the value of  $\frac{\partial f_i}{\partial x_j}$  at the point  $x$ , for  $i, j = 1, 2, \dots, n$ .  
*On final exit:* IREVCM = 0, and the algorithm has terminated.  
*Constraint:* IREVCM = 0, 1, 2 or 3.
- 2: N – INTEGER *Input*  
*On initial entry:*  $n$ , the number of equations.  
*Constraint:*  $N > 0$ .
- 3: X(N) – **double precision** array *Input/Output*  
*On initial entry:* an initial guess at the solution vector.  
*On intermediate exit:* contains the current point.  
*On final exit:* the final estimate of the solution vector.
- 4: FVEC(N) – **double precision** array *Input/Output*  
*On initial entry:* need not be set.  
*On intermediate re-entry:* if IREVCM  $\neq 2$ , FVEC must not be changed.  
 If IREVCM = 2, FVEC must be set to the values of the functions computed at the current point X.  
*On final exit:* the function values at the final point, X.
- 5: FJAC(LDFJAC,N) – **double precision** array *Input/Output*  
*On initial entry:* must be set to the values of the Jacobian evaluated at the initial point X.  
*On intermediate re-entry:* if IREVCM  $\neq 3$ , FJAC must not be changed.  
 If IREVCM = 3, FJAC must be set to the value of the Jacobian computed at the current point X.  
*On final exit:* the orthogonal matrix  $Q$  produced by the  $QR$  factorization of the final approximate Jacobian.

- 6: LDFJAC – INTEGER *Input*  
*On initial entry:* the first dimension of the array FJAC as declared in the (sub)program from which C05PDF/C05PDA is called.  
*Constraint:*  $LDFJAC \geq N$ .
- 7: XTOL – **double precision** *Input*  
*On initial entry:* the accuracy in X to which the solution is required.  
*Suggested value:* the square root of the **machine precision**.  
*Constraint:*  $XTOL \geq 0.0$ .
- 8: DIAG(N) – **double precision** array *Input/Output*  
*On initial entry:* if  $MODE = 2$ , DIAG must contain multiplicative scale factors for the variables.  
*Constraint:*  $DIAG(i) > 0.0$ , for  $i = 1, 2, \dots, n$ .  
*On intermediate exit:* the scale factors actually used (computed internally if  $MODE \neq 2$ ).
- 9: MODE – INTEGER *Input*  
*On initial entry:* indicates whether or not you have provided scaling factors in DIAG. If  $MODE = 2$  the scale factors must be supplied in DIAG. Otherwise, the variables will be scaled internally.
- 10: FACTOR – **double precision** *Input*  
*On initial entry:* a quantity to be used in determining the initial step bound. In most cases, FACTOR should lie between 0.1 and 100.0. (The step bound is  $FACTOR \times \|DIAG \times X\|_2$  if this is nonzero; otherwise the bound is FACTOR.)  
*Suggested value:*  $FACTOR = 100.0$ .  
*Constraint:*  $FACTOR > 0.0$ .
- 11: R(LR) – **double precision** array *Input/Output*  
*On initial entry:* need not be set.  
*On intermediate exit:* must not be changed.  
*On final exit:* the upper triangular matrix  $R$  produced by the  $QR$  factorization of the final approximate Jacobian, stored row-wise.
- 12: LR – INTEGER *Input*  
*On initial entry:* the dimension of the array R as declared in the (sub)program from which C05PDF/C05PDA is called.  
*Constraint:*  $LR \geq N \times (N + 1)/2$ .
- 13: QTF(N) – **double precision** array *Input/Output*  
*On initial entry:* need not be set.  
*On intermediate exit:* must not be changed.  
*On final exit:* the vector  $Q^T f$ .
- 14: W(N,4) – **double precision** array *Communication Array*

## 15: IFAIL – INTEGER

Input/Output

**Note:** for C05PDA, IFAIL does not occur in this position in the parameter list. See the additional parameters described below.

On initial entry: IFAIL must be set to 0, -1 or 1. If you are unfamiliar with this parameter you should refer to Section 3.3 in the Essential Introduction for details.

On final exit: IFAIL = 0 unless the routine detects an error (see Section 6).

For environments where it might be inappropriate to halt program execution when an error is detected, the value -1 or 1 is recommended. If the output of error messages is undesirable, then the value 1 is recommended. Otherwise, because for this routine the values of the output parameters may be useful even if IFAIL  $\neq$  0 on exit, the recommended value is -1. **When the value -1 or 1 is used it is essential to test the value of IFAIL on exit.**

**Note:** the following are additional parameters for specific use with C05PDA. Users of C05PDF therefore need not read the remainder of this description.

## 15: LWSAV(5) – LOGICAL array

Communication Array

## 16: IWSAV(15) – INTEGER array

Communication Array

17: RWSAV(10) – *double precision* array

Communication Array

The arrays LWSAV, IWSAV and RWSAV **must not** be altered between calls to C05PDA.

## 18: IFAIL – INTEGER

Input/Output

**Note:** see the parameter description for IFAIL above.

## 6 Error Indicators and Warnings

If on entry IFAIL = 0 or -1, explanatory error messages are output on the current error message unit (as defined by X04AAF).

Errors or warnings detected by the routine:

IFAIL = 1

On entry,  $N \leq 0$ ,  
 or  $XTOL < 0.0$ ,  
 or  $FACTOR \leq 0.0$ ,  
 or  $LDFJAC < N$ ,  
 or  $LR < N \times (N + 1)/2$ ,  
 or  $MODE = 2$  and  $DIAG(i) \leq 0.0$  for some  $i$ ,  $i = 1, 2, \dots, N$ .

IFAIL = 2

On entry,  $IREVCM < 0$  or  $IREVCM > 3$ .

IFAIL = 3

No further improvement in the approximate solution X is possible; XTOL is too small.

IFAIL = 4

The iteration is not making good progress, as measured by the improvement from the last five Jacobian evaluations.

IFAIL = 5

The iteration is not making good progress, as measured by the improvement from the last 10 iterations.

The values  $\text{IFAIL} = 4$  and  $\text{IFAIL} = 5$  may indicate that the system does not have a zero, or that the solution is very close to the origin (see Section 7). Otherwise, rerunning C05PDF/C05PDA from a different starting point may avoid the region of difficulty.

## 7 Accuracy

If  $\hat{x}$  is the true solution and  $D$  denotes the diagonal matrix whose entries are defined by the array DIAG, then C05PDF/C05PDA tries to ensure that

$$\|D(x - \hat{x})\|_2 \leq \text{XTOL} \times \|D\hat{x}\|_2.$$

If this condition is satisfied with  $\text{XTOL} = 10^{-k}$ , then the larger components of  $Dx$  have  $k$  significant decimal digits. There is a danger that the smaller components of  $Dx$  may have large relative errors, but the fast rate of convergence of C05PDF/C05PDA usually avoids this possibility.

If  $\text{XTOL}$  is less than *machine precision* and the above test is satisfied with the *machine precision* in place of  $\text{XTOL}$ , then the routine exits with  $\text{IFAIL} = 3$ .

**Note:** this convergence test is based purely on relative error, and may not indicate convergence if the solution is very close to the origin.

The test assumes that the functions and the Jacobian are coded consistently and that the functions are reasonably well behaved. If these conditions are not satisfied, then C05PDF/C05PDA may incorrectly indicate convergence. The coding of the Jacobian can be checked using C05ZAF. If the Jacobian is coded correctly, then the validity of the answer can be checked by rerunning C05PDF/C05PDA with a tighter tolerance.

## 8 Further Comments

The time required by C05PDF/C05PDA to solve a given problem depends on  $n$ , the behaviour of the functions, the accuracy requested and the starting point. The number of arithmetic operations executed by C05PDF/C05PDA is about  $11.5 \times n^2$  to process each evaluation of the functions and about  $1.3 \times n^3$  to process each evaluation of the Jacobian. The timing of C05PDF/C05PDA is strongly influenced by the time spent in the evaluation of the functions and the Jacobian.

Ideally the problem should be scaled so that, at the solution, the function values are of comparable magnitude.

## 9 Example

This example determines the values  $x_1, \dots, x_9$  which satisfy the tridiagonal equations:

$$\begin{aligned} (3 - 2x_1)x_1 - 2x_2 &= -1, \\ -x_{i-1} + (3 - 2x_i)x_i - 2x_{i+1} &= -1, \quad i = 2, 3, \dots, 8 \\ -x_8 + (3 - 2x_9)x_9 &= -1. \end{aligned}$$

### 9.1 Program Text

```
*      C05PDF Example Program Text
*      Mark 14 Release. NAG Copyright 1989.
*      .. Parameters ..
      INTEGER          N, LDFJAC, LR
      PARAMETER        (N=9,LDFJAC=N,LR=(N*(N+1))/2)
      INTEGER          NOUT
      PARAMETER        (NOUT=6)
      DOUBLE PRECISION ZERO, ONE, TWO, THREE, FOUR
      PARAMETER        (ZERO=0.0D0,ONE=1.0D0,TWO=2.0D0,THREE=3.0D0,
+                     FOUR=4.0D0)
*      .. Local Scalars ..
      DOUBLE PRECISION FACTOR, FNORM, XTOL
      INTEGER          ICOUNT, IFAIL, IREVCN, J, K, MODE
*      .. Local Arrays ..
      DOUBLE PRECISION DIAG(N), FJAC(LDFJAC,N), FVEC(N), QTF(N), R(LR),
+                     W(N,4), X(N)
*      .. External Functions ..
```

```

      DOUBLE PRECISION F06EJF, X02AJF
      EXTERNAL          F06EJF, X02AJF
*      .. External Subroutines ..
      EXTERNAL          C05PDF
*      .. Intrinsic Functions ..
      INTRINSIC          SQRT
*      .. Executable Statements ..
      WRITE (NOUT,*) 'C05PDF Example Program Results'
*      The following starting values provide a rough solution.
      DO 20 J = 1, N
        X(J) = -1.0D0
20    CONTINUE
      XTOL = SQRT(X02AJF())
      DO 40 J = 1, N
        DIAG(J) = 1.0D0
40    CONTINUE
      MODE = 2
      FACTOR = 100.0D0
      ICOUNT = 0
      IFAIL = 1
      IREVCM = 0
*
      60 CALL C05PDF(IREVCM,N,X,FVEC,FJAC,LDFJAC,XTOL,DIAG,MODE,FACTOR,R,
        +           LR,QTF,W,IFAIL)
*
      IF (IREVCM.EQ.1) THEN
        ICOUNT = ICOUNT + 1
*      Insert print statements here to monitor progress if desired
      GO TO 60
      ELSE IF (IREVCM.EQ.2) THEN
*      Evaluate functions at current point
      DO 80 K = 1, N
        FVEC(K) = (THREE-TWO*X(K))*X(K) + ONE
        IF (K.GT.1) FVEC(K) = FVEC(K) - X(K-1)
        IF (K.LT.N) FVEC(K) = FVEC(K) - TWO*X(K+1)
80    CONTINUE
      GO TO 60
      ELSE IF (IREVCM.EQ.3) THEN
*      Evaluate Jacobian at current point
      DO 120 K = 1, N
        DO 100 J = 1, N
          FJAC(K,J) = ZERO
100    CONTINUE
        FJAC(K,K) = THREE - FOUR*X(K)
        IF (K.NE.1) FJAC(K,K-1) = -ONE
        IF (K.NE.N) FJAC(K,K+1) = -TWO
120    CONTINUE
      GO TO 60
      END IF
*
      WRITE (NOUT,*)
      IF (IFAIL.EQ.0) THEN
        FNORM = F06EJF(N,FVEC,1)
        WRITE (NOUT,99999) 'Final 2 norm of the residuals after',
        + ICOUNT, ' iterations is ', FNORM
        WRITE (NOUT,*)
        WRITE (NOUT,*) 'Final approximate solution'
        WRITE (NOUT,99998) (X(J),J=1,N)
      ELSE IF (IFAIL.LT.0) THEN
        WRITE (NOUT,*)
        WRITE (NOUT,99997) ' ** C05PDF returned with IFAIL = ', IFAIL
      ELSE
        WRITE (NOUT,99999) 'IFAIL =', IFAIL
        IF (IFAIL.GT.2) THEN
          WRITE (NOUT,*) 'Approximate solution'
          WRITE (NOUT,99998) (X(J),J=1,N)
        END IF
      END IF

```

```
*
99999 FORMAT (1X,A,I4,A,E12.4)
99998 FORMAT (5X,3F12.4)
99997 FORMAT (1X,A,I5)
      END
```

## 9.2 Program Data

None.

## 9.3 Program Results

C05PDF Example Program Results

Final 2 norm of the residuals after 11 iterations is 0.1193E-07

Final approximate solution

|         |         |         |
|---------|---------|---------|
| -0.5707 | -0.6816 | -0.7017 |
| -0.7042 | -0.7014 | -0.6919 |
| -0.6658 | -0.5960 | -0.4164 |

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