Ideal MHD Stability Diagram of Simply Connected Magnetic Configurations with Unitary Beta

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Compact Tori are attractive candidates for magnetic fusion propulsion, but…

- Spheromaks are limited by ideal MHD stability to $\beta \sim 0.15$
- FRC's are $\beta \sim 1$, but theoretical understanding of their macroscopic stability remains elusive

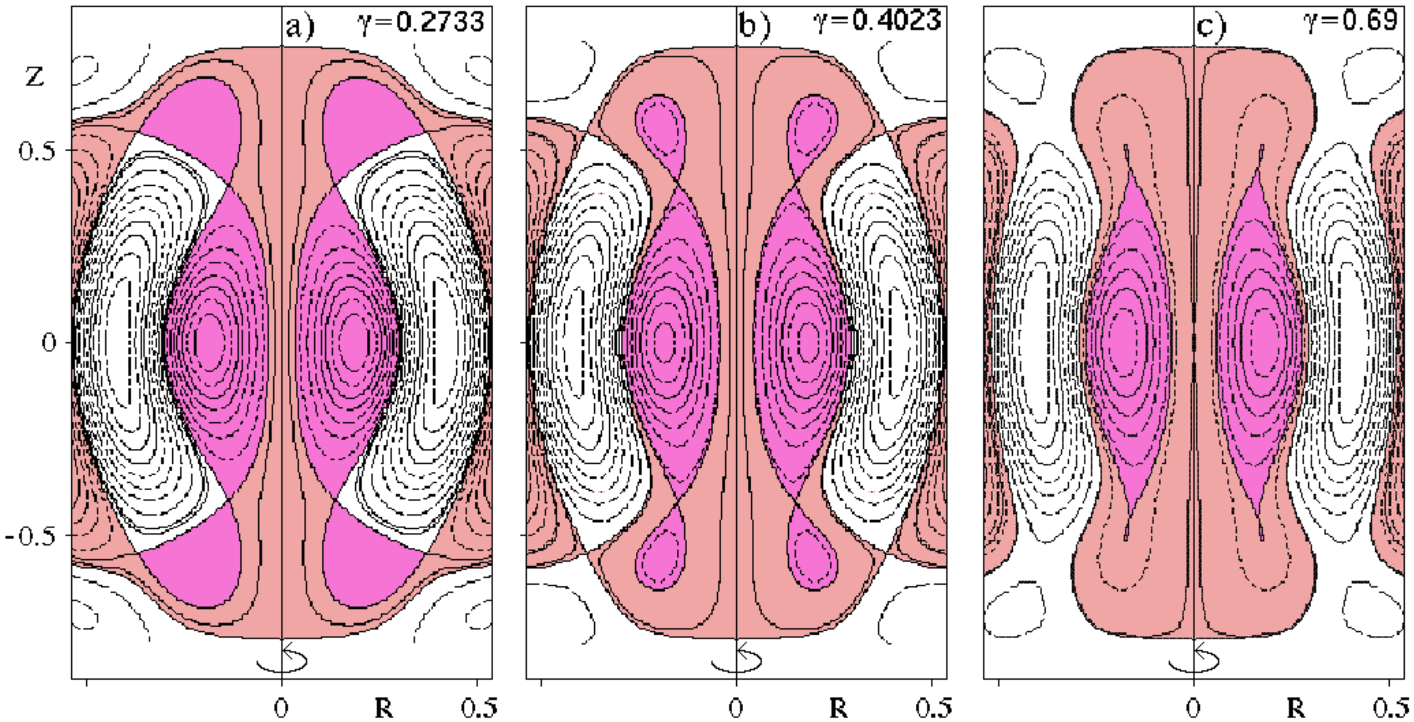
OUTLINE

- **Equilibrium characteristics of a simply connected magnetic confinement scheme: unrelaxed CKF configurations**
- **Ideal MHD stability boundaries of unrelaxed CKF configurations**
- **Preliminary experimental approach to unrelaxed CKF configurations: PROTO-SPHERA**

A simply connected magnetic confinement scheme is obtained superposing two axisymmetric homogeneous force-free fields, both having $\vec{\nabla}\times\vec{\mathbf{B}}=\gamma\vec{\mathbf{B}}$, with the same relaxation parameter γ: the Chandrasekar-Kendall spherical solution and the Furth square-toroids $\psi(r,\theta) = \psi^{CK}_{\gamma,1} + \delta\,\psi^{F}_{\gamma,\gamma}$

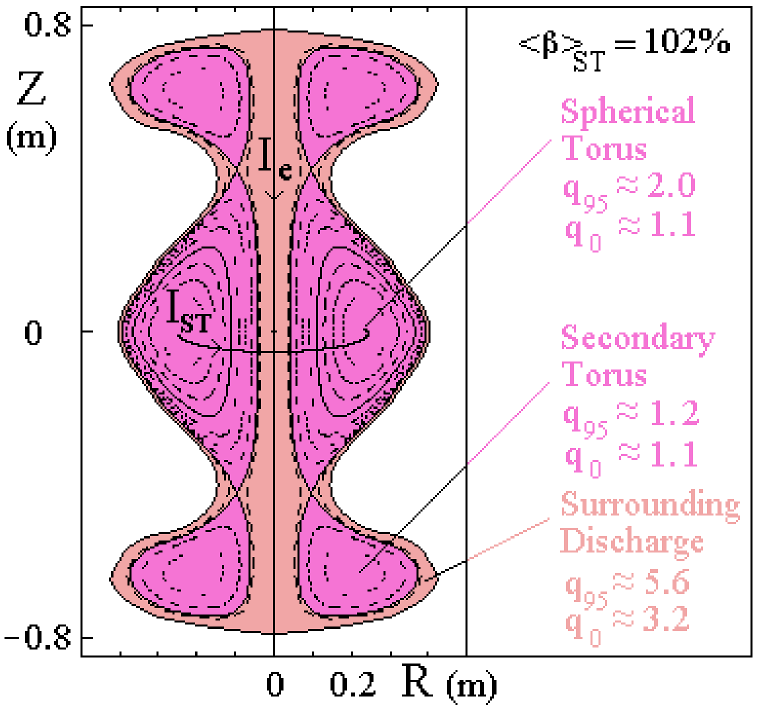
For $\gamma \geq 0.402...$ in a simply connected region the toroidal current density j_{φ} has the same sign:

Chandrasekhar-Kendall-Furth force-free field (CKF)



However CKF force-free fields ($\vec{\nabla} p=0$) are unable to confine plasmas of fusion interest

Unrelaxed ($\vec{\nabla}\mu\neq 0$, $\vec{\nabla} p\neq 0$) MHD free boundary equilibria, similar to CKF force-free fields



$\mu = \mu_0 \dfrac{\vec{j}\cdot\vec{B}}{B^2} = \mu_{edge}$ constant at the plasma edge

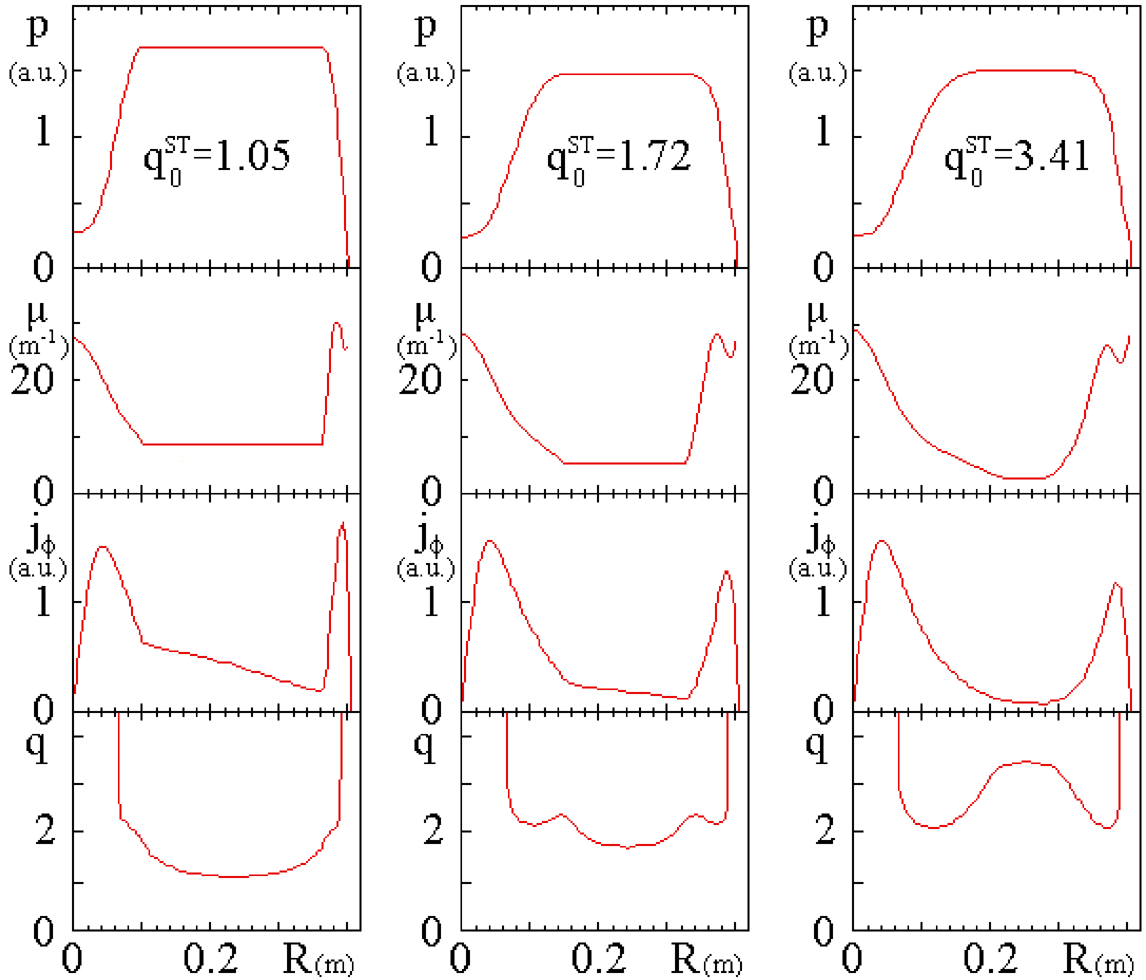
Main parameters of equilibrium:

$I_{ST} / I_e = \dfrac{\text{toroidal current in ST}}{\text{poloidal current in P}}$ determines q_{95} at ST edge

$\Delta\tilde{\mu} = 14.066 \cdot (\mu_{edge} - \mu_{axis}) / \mu_{edge}$ normalized jump of μ between edge of P and axis of ST given the value of I_{ST} / I_e determines q_0 at ST axis

Relaxed & unrelaxed CKF configurations contain:

- a magnetic separatrix with ordinary X-points (B≠0)
- a main spherical torus (ST), 2 secondary tori (SC) and a surrounding discharge (P)
- two degenerate X-points (B=0) are present (top/bottom) on the symmetry axis



Equatorial profiles of pressure p; relaxation parameter □*; toroidal current density j*□*; ST safety factor q*

□

□ = 1

$I_{ST}/I_e = 3$

Effect of □ jump

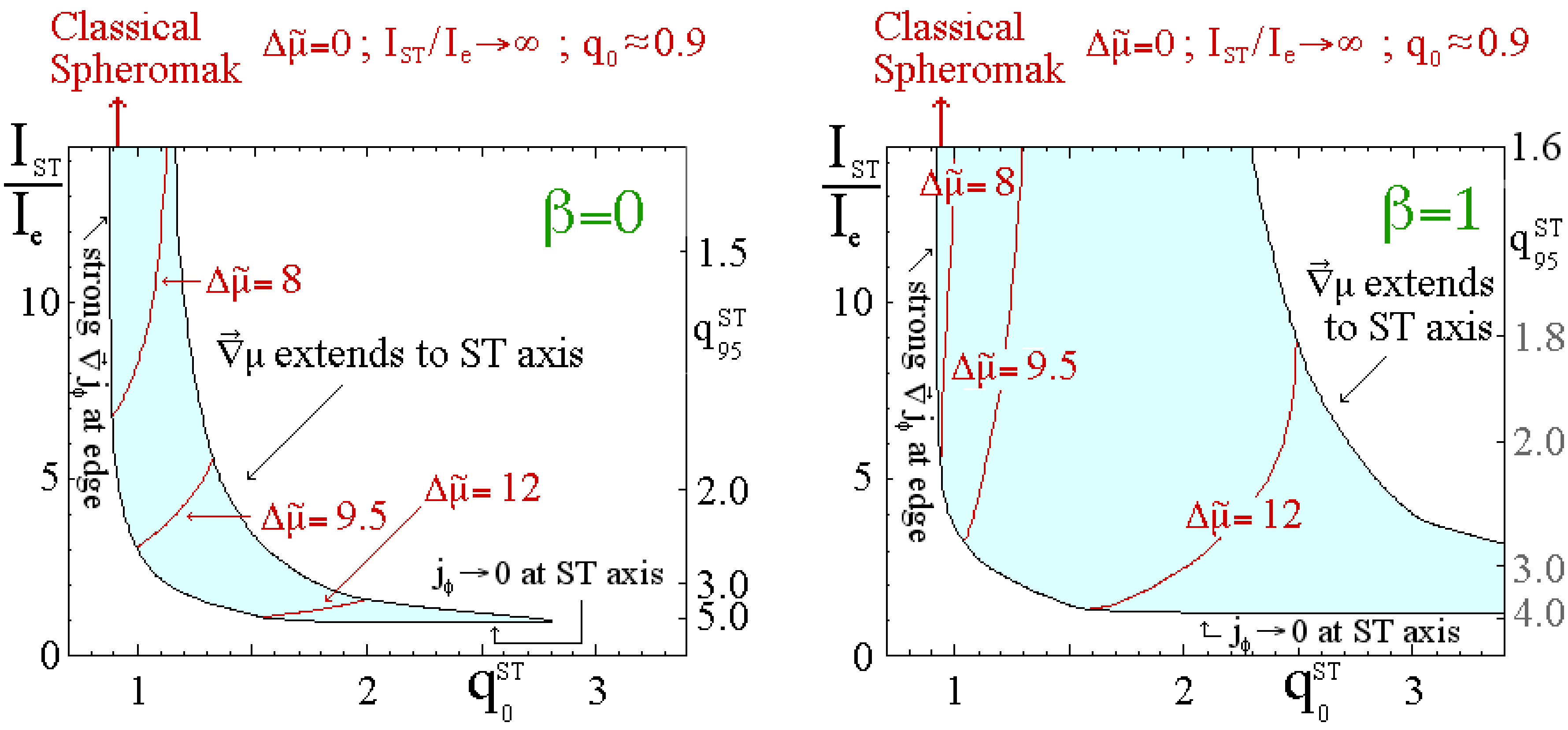
Assumption of the equilibrium scansion:

- same edge shape
- same total toroidal current I_{CKF}

If current flow is sustained in surrounding discharge, magnetic helicity is injected into the ST (X-points), flowing down $\vec{\nabla}$<□>: $\vec{\nabla}$p concentrated in same region

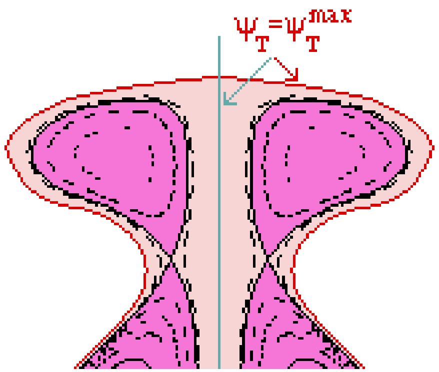
Scan at fixed shape of the plasma edge (& fixed I_{CKF})

Unrelaxed CKF equilibria in terms of $(I_{ST}/I_e,\ q_0^{ST})$



Characteristics of the Ideal MHD Stability Code

- Plasma on the symmetry axis



R=0: $\lim_{R□0} \vec{\nabla}□_T = 0$ □ $□(\psi_T^{max},□,□)=0$

(otherwise potential energy divergences)

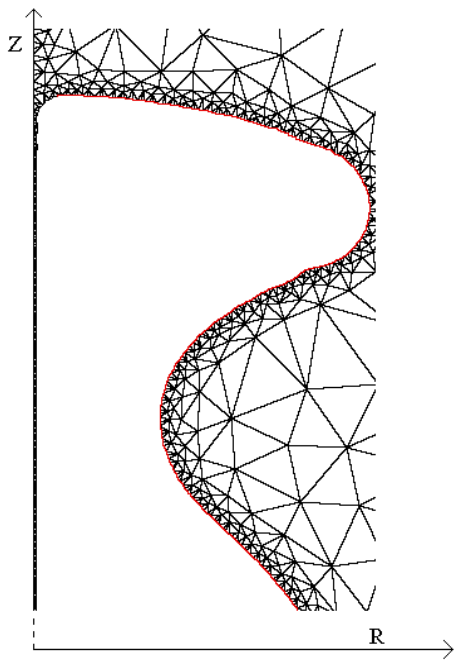
R≠0: $\vec{\nabla}□_T \neq 0$ □ *but if* $□(\psi_T^{max},□,□)=0$

(unphysical fixed-boundary)

Solution □ New variables:

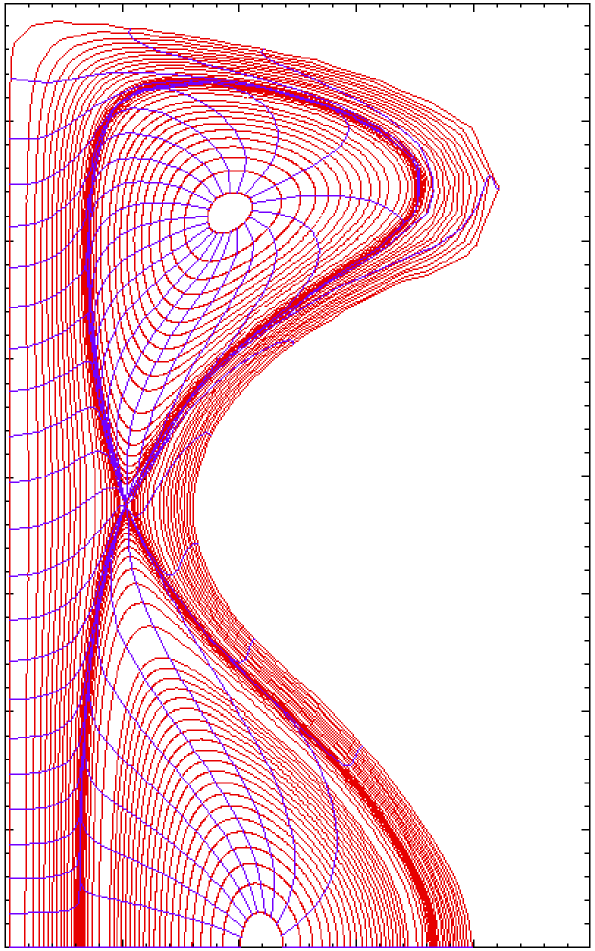
$□=□\cdot\vec{□}\,□_T/R^2$, $□= □\cdot(\vec{□}□-\vec{□}□/q)/B$

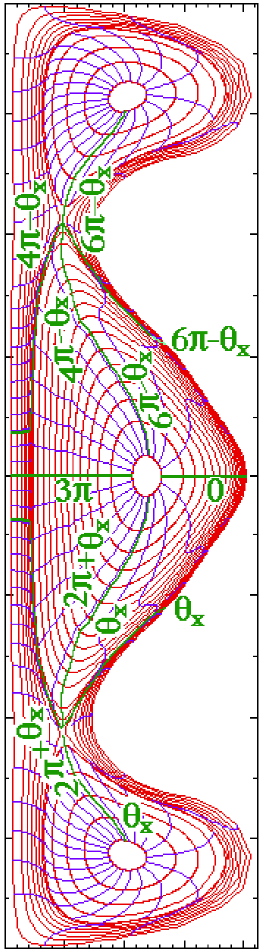
- 2D finite element method for the vacuum energy



Can account for conducting shells of any shape

- Boozer coordinates joined at interfaces





- Global modes:
  ideal MHD conditions at separatrix

Normal displacement: □=□·□ □T/R^2

continuous

Binormal & Parallel: □=□·(□ □-□ □/q)/B,

□=□√g□·□ □

jump

Inside Tori:

□=□$_\ell$ □$_\ell$(□T) sin(m$_\ell$□-n□),

□=□$_\ell$ □$_\ell$(□T) cos(m$_\ell$□-n□),

□=□$_\ell$ □$_\ell$(□T) cos(m$_\ell$□-n□)

Surrounding coupled mode:

□=□$_\ell$ □$_\ell$(□T) sin(m$_\ell$ 3□-n□),

□=□$_\ell$ □$_\ell$(□T) cos(m$_\ell$ 3□-n□),

□=□$_\ell$ □$_\ell$(□T) cos(m$_\ell$ 3□-n□)

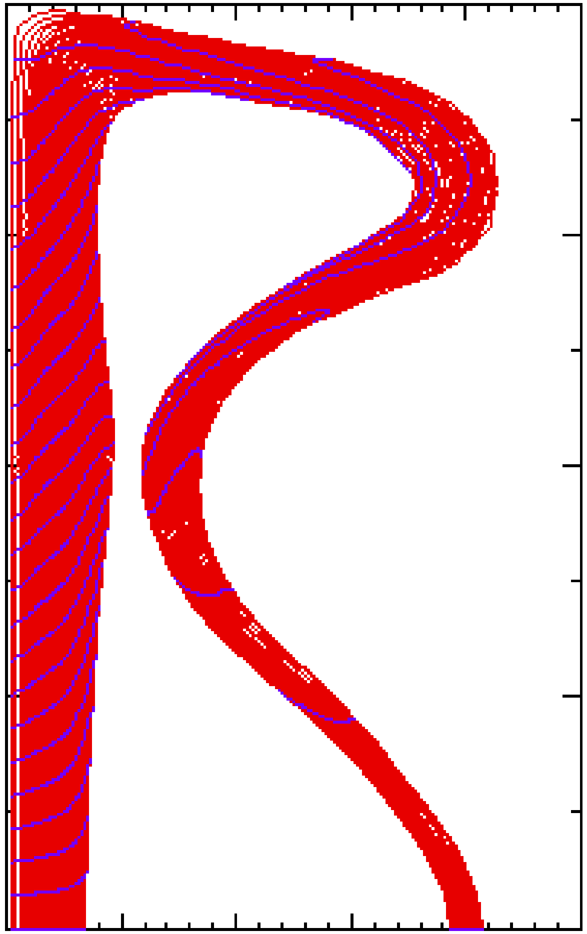
- MHD modes limited
  to the surrounding plasma

Modes that have zero radial displacement at the magnetic separatrix: $□(□_T^X,□,□)=0$
This condition decouples them from tori

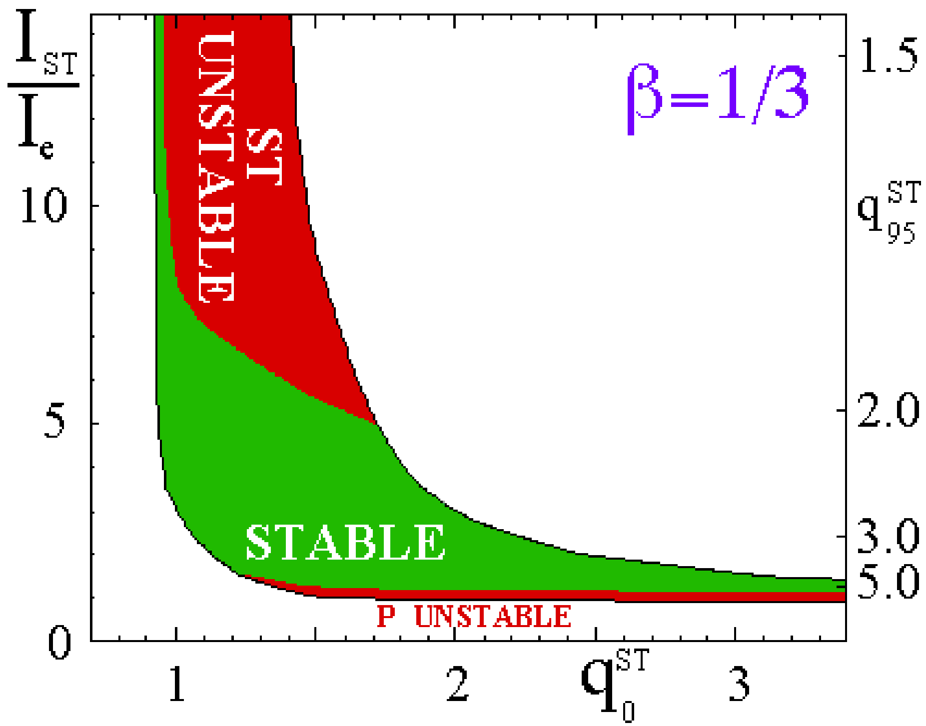
Are anyhow free-boundary modes:
$□(□_T^{max},□,□)\neq 0$

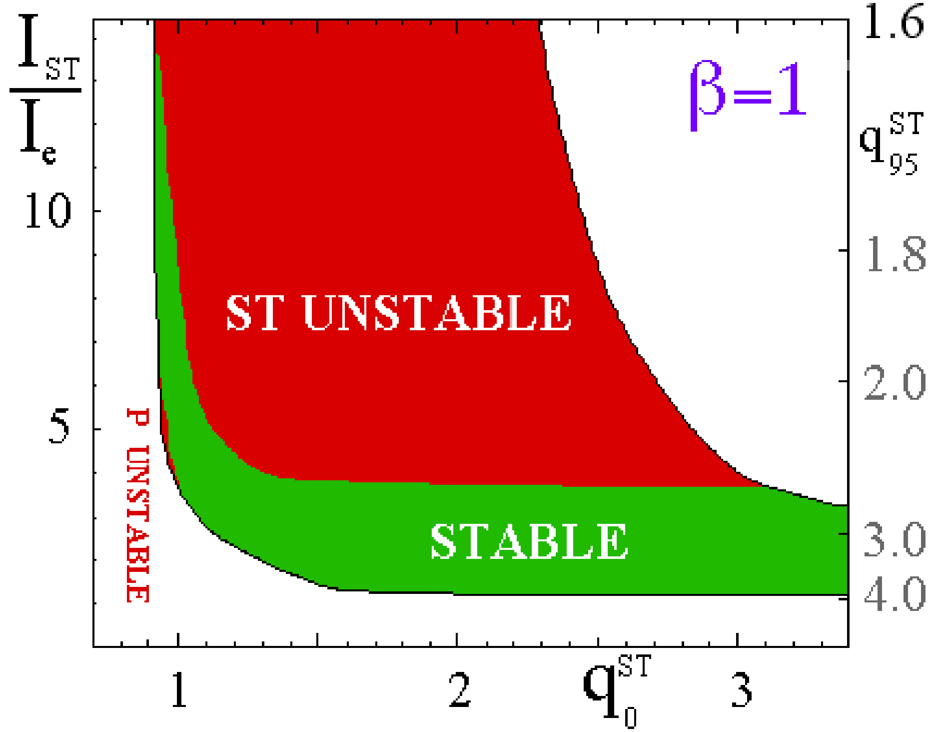
Inside Surronding Discharge:
$□=□_\ell\ □_\ell(□_T)\ \sin(m_\ell□-n□)$,
$□=□_\ell\ □_\ell(□_T)\ \cos(m_\ell□-n□)$,
$□=□_\ell\ □_\ell(□_T)\ \cos(m_\ell□-n□)$

- Boozer coordinates
  in surrounding discharge



Ideal MHD Stability Results (wall at ∞)





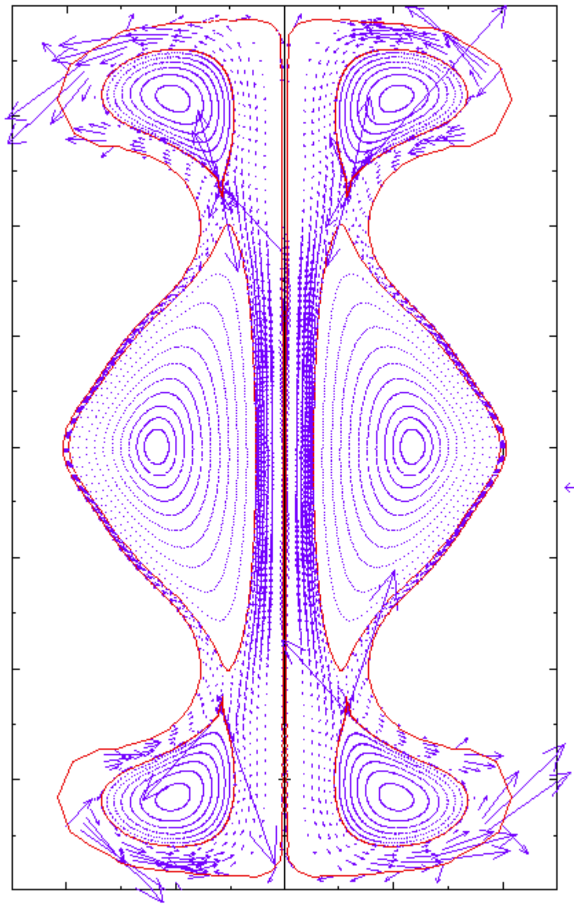
Stability even at β=1

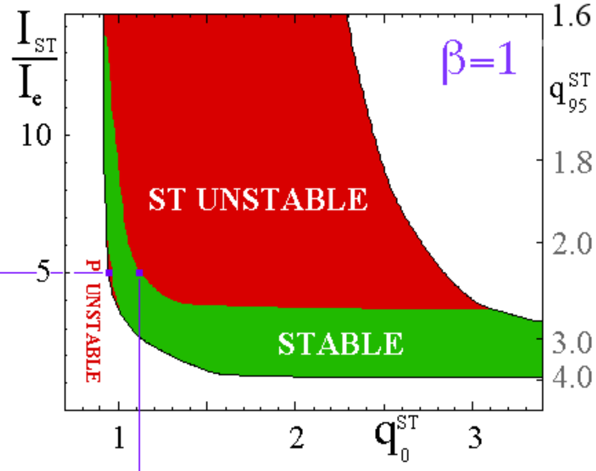
in absence of any conducting shell around the plasma

for toroidal mode number n=1, 2, 3

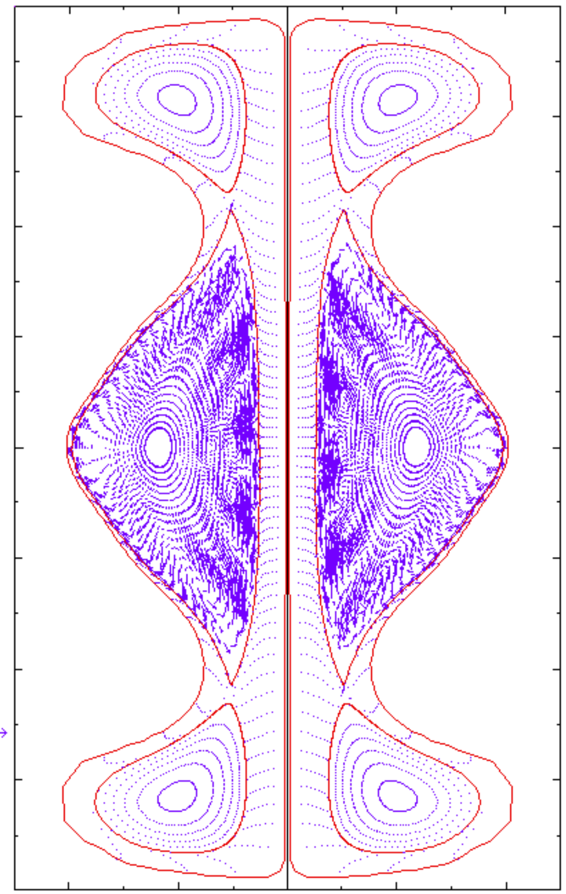
Arrow plots of global unstable modes at $\beta=1$, $I_{ST}/I_e=5$

UNSTABLE to n=1





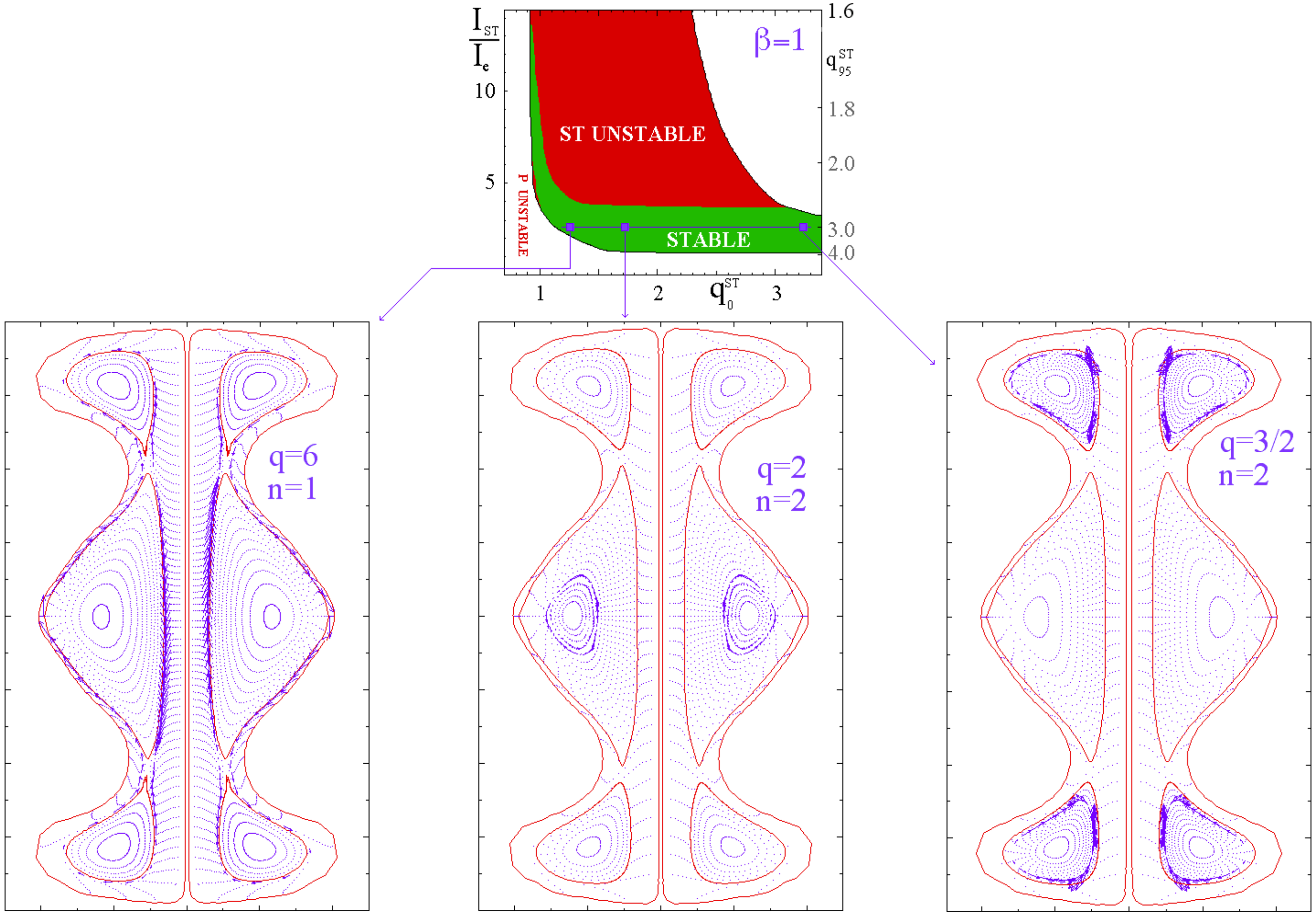
UNSTABLE to n=3



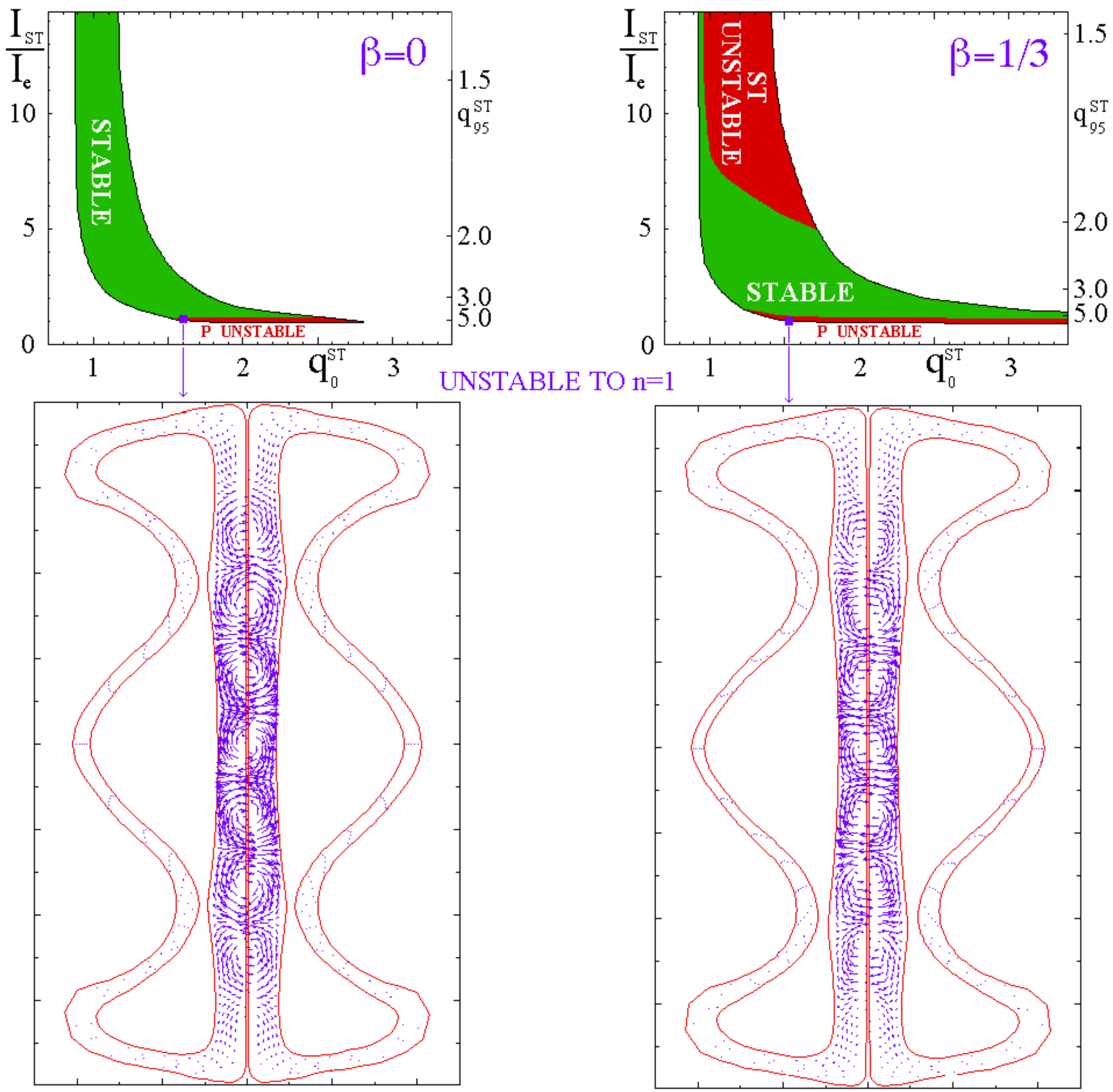
At low q_0^{ST} tilt of the surrounding plasma

At high q_0^{ST} fixed boundary instability of ST

Arrow plots of stable oscillations on resonant q at $\beta=1$, $I_{ST}/I_e=3$



Oscillation on: surr. discharge (P) spherical torus (ST) secondary tori (SC)

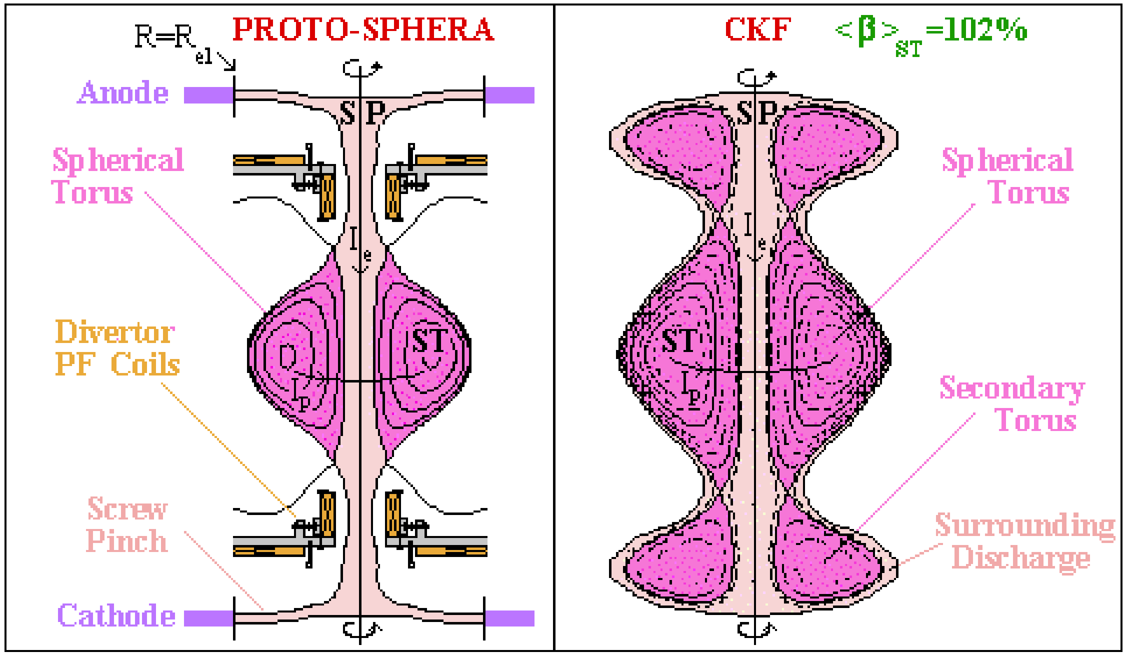


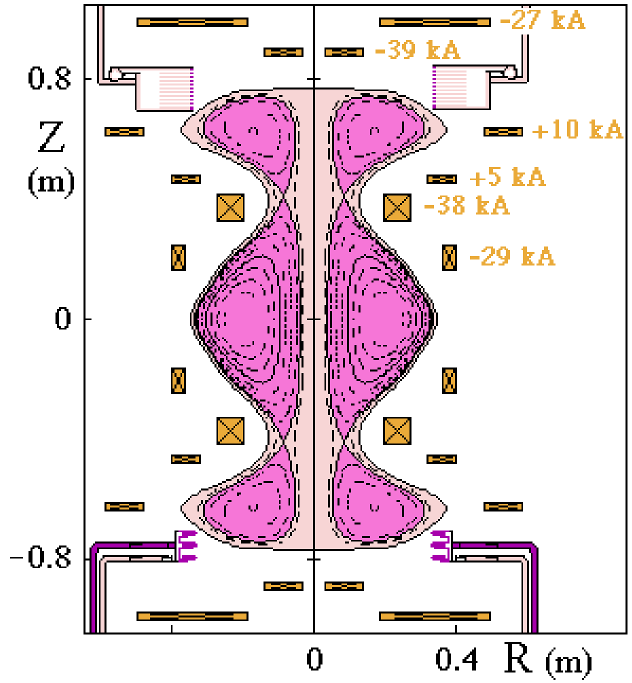
Instabilities limited to the surrounding discharge at □=0 & 1/3, $I_{ST}/I_e=1$

Kink of central column: too large ratio I_e/I_{ST}

PROTO-SPHERA can be viewed as an unrelaxed **CKF** configuration where:

- force-free screw pinch, fed by electrodes, replaces in part the surrounding discharge
- "divertor" poloidal field coils replace the secondary tori





With limited modifications of the load assembly and increasing the number of the PF coil power supplies, PROTO-SPHERA could try the inductive formation of an unrelaxed CKF configuration, after the destabilization of a screw pinch produced by electrodes

Ideal MHD comparison between Unrelaxed CKF & PROTO-SPHERA

Unrelaxed CKF

- With $\beta_{ST}=1$ only flat pressure profiles ($q_0^{ST}\sim 1$) are allowed if $I_{ST}/I_e>4$;
  if $1.5< I_{ST}/I_e<4$ (i.e. $2.7<q_{95}^{ST}<4$) even peaked pressure profiles (high q_0^{ST}) show stability

- At lower β_{ST} the region showing stability with peaked pressure profiles is extended to $1.2<I_{ST}/I_e<5.5$ (i.e. $2<q_{95}^{ST}<5$) and the stability region with flat pressure profiles is enlarged

PROTO-SPHERA [machine parameters: $I_{ST}^{max}/I_e=4$ ($I_{ST}^{max}=240$ kA, $I_e=60$ kA), $q_{95}^{ST}\sim 2.8$]

- If $I_{ST}/I_e\leq 1$ stability for ideal modes with n=1,2,3 has been found up to $\beta_{ST}=25\%$;
  in the range $2<I_{ST}/I_e<4$ the stability limit decreases to $\beta_{ST}=14\div 18\%$

- Instabilities on the ST dominate: the Screw Pinch becames unstable only if $I_{ST}/I_e\geq 4$

$\Downarrow$

Surrounding Discharge plays a crucial role in order to increase the ideal stability of the ST in the unrelaxed CKF configurations