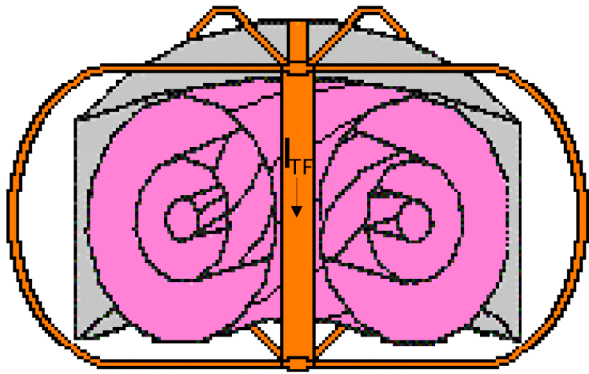


# Ideal MHD Stability Boundaries of the PROTO-SPHERA Configuration

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### Spherical Tokamaks allow to obtain:

- High plasma current  $I_p$  (and high  $\langle n \rangle$ ) with low  $B_T$
- Plasma  $\beta$  much higher than Conventional Tokamaks
- More compact devices

### But, for a reactor/CTF extrapolation:

- No space for central solenoid (Current Drive requirement more severe)
- No neutrons shield for central stack (no superconductor/high dissipation)

Intriguing possibility  $\Rightarrow$  substitute central rod with Screw Pinch plasma ( $I_{TF} \rightarrow I_e$ )

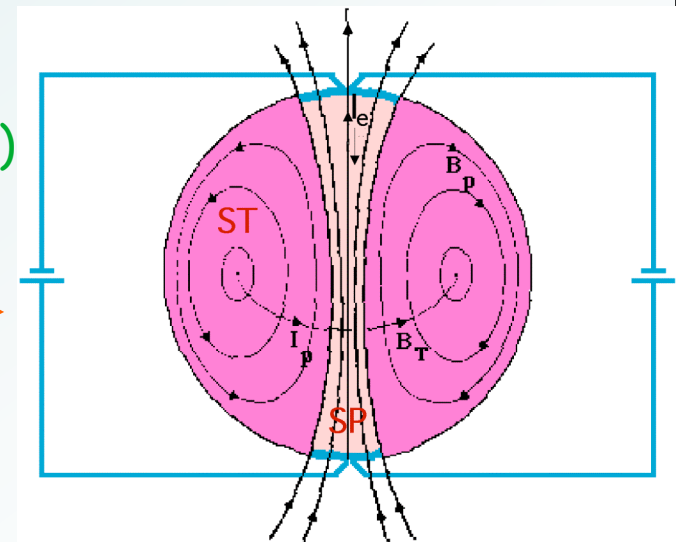
### Potentially two problems solved:

- Simply connected configuration (no conductors inside)
- $I_p$  driven by  $I_e$  (Helicity Injection from SP to ST)

### Flux Core Spheromak (FCS) $\longrightarrow$

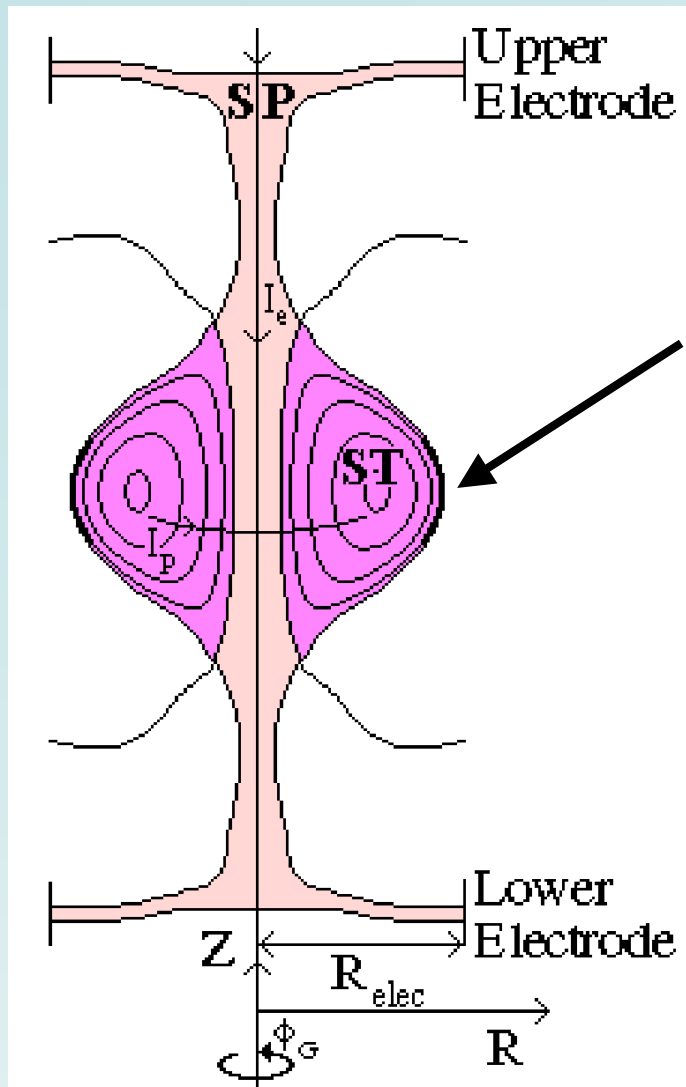
Theory: Taylor & Turner, *Nucl. Fusion* 29, 219 (1989)

Experiment: TS-3; N. Amemiya, et al., *JPSJ* 63, 1552 (1993)



But Flux Core Spheromaks are:

- injected by plasma guns
- formed by  $\sim 10$  kV voltage on electrodes
- high pressure prefilled
- with ST safety factor  $q \leq 1$



*New configuration proposed:*

## PROTO-SPHERA

"Flux Core Spherical Tokamak" (FCST), rather than FCS

Disk-shaped electrode driven Screw Pinch plasma (SP)

Prolated low aspect ratio ST ( $A=R/a \geq 1.2$ ,  $\kappa=b/a \sim 2.3$ )  
to get a Tokamak-like safety factor ( $q_0 \geq 1$ ,  $q_{edge} \sim 3$ )

SP electrode current  $I_e = 60$  kA

ST toroidal current  $I_p = 120 \div 240$  kA

ST diameter  $R_{sph} = 0.7$  m

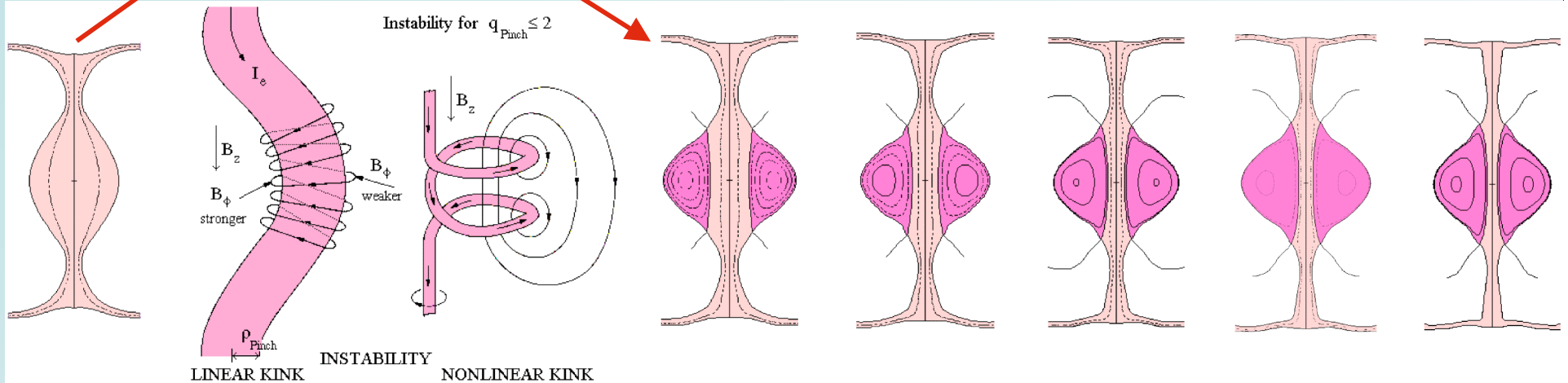


Stability should be improved and helicity drive may be less disruptive than in conventional Flux-Core-Spheromak

# PROTO-SPHERA formation follows TS-3 scheme (SP kink instability)

Tunnelling (ST formation)

ST compression ( $I_p/I_e \uparrow, A \downarrow$ )



T0  
 $I_e = 8.5 \text{ kA}$

$I_e 8.5 \rightarrow 60 \text{ kA}$

T3  
 $I_p = 30 \text{ kA}$   
 $A = 1.8$

T4  
 $I_p = 60 \text{ kA}$   
 $A = 1.5$

T5  
 $I_p = 120 \text{ kA}$   
 $A = 1.3$

T6  
 $I_p = 180 \text{ kA}$   
 $A = 1.25$

TF  
 $I_p = 240 \text{ kA}$   
 $A = 1.2$

- $I_p/I_e$  ratio crucial parameter (strong energy dissipation in SP)
- MHD equilibria computed both with *monotonic (peaked pressure)* as well as *reversal safety factor profiles (flat pressure,  $\mu$  parameterized)*

Some level of low  $n$  resistive instability needed (reconnections to inject helicity from SP to ST)

but

SP+ST must be ideally stable at any time slice



Ideal MHD analysis to assess  $I_p/I_e$  &  $\beta$  limits

# Characteristics of the free-boundary Ideal MHD Stability code

Plasma extends to symmetry axis (R=0) | Open+Closed field lines | Degenerate |B|=0 & Standard X-points

Standard  $\bar{\xi}$  decomposition

$$\bar{\xi} = \xi^\psi \bar{e}_\psi + \eta^\psi \frac{\bar{\mathbf{B}} \wedge \bar{\nabla} \psi_T}{B^2} + \left( \frac{I \eta^\psi}{B^2} - \mu \right) \bar{\mathbf{B}}$$

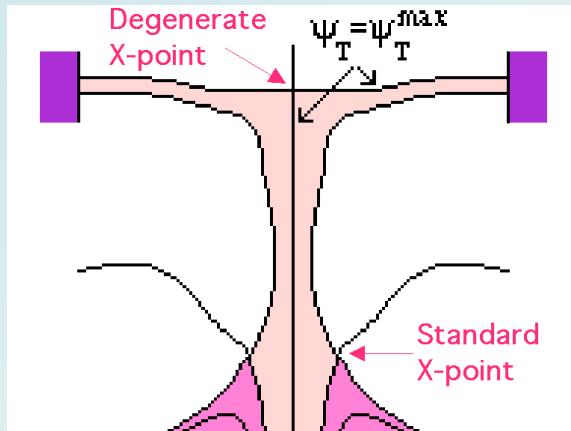
inappropriate

$$\bar{e}_\psi = \frac{\bar{\nabla} \psi_T}{|\bar{\nabla} \psi_T|^2} + \frac{\beta_*}{B^2} \bar{\mathbf{B}} - \frac{\gamma_*}{B^2} (\bar{\mathbf{B}} \wedge \bar{\nabla} \psi_T)$$

$\bar{\nabla} \psi_T \rightarrow 0$  like  $|\bar{\nabla} \psi_T| \propto R$  but, after degenerate X-point ( $|B|=0$ ),  $\psi_T = \psi_T^{\max} \neq R=0$ :  
 $\xi^\psi(\psi_T^{\max})=0$  cannot be imposed

**Solution:**  $\xi^\psi = \xi R^N$  ( $N \geq 1$ );  $\eta^\psi = \eta B$

$$\bar{\xi} = \xi R^N \bar{e}_\psi + \eta \frac{\bar{\mathbf{B}} \wedge \bar{\nabla} \psi_T}{B} + \left( \frac{I \eta}{B} - \mu \right) \bar{\mathbf{B}}$$



**Fourier analysis of:**

$$\xi = \sum_\ell \xi_\ell(\psi_T) \sin(m_\ell \theta - n\phi)$$

$$\eta = \sum_\ell \eta_\ell(\psi_T) \cos(m_\ell \theta - n\phi)$$

$$\mu = \sum_\ell \mu_\ell(\psi_T) \cos(m_\ell \theta - n\phi)$$

**Normal Mode equation**  $\delta \bar{\mathbf{W}} \cdot |\bar{\mathbf{x}}\rangle = \omega^2 \delta \bar{\mathbf{K}} \cdot |\bar{\mathbf{x}}\rangle$

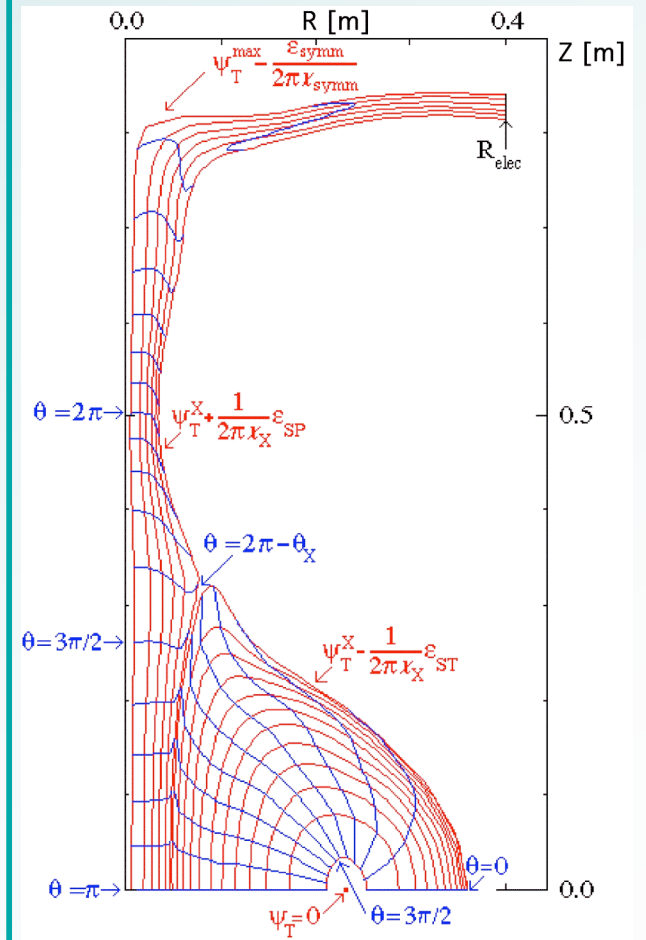
$$|\bar{\mathbf{x}}\rangle \equiv (\xi_\ell^j, \eta_\ell^j, \mu_\ell^j)$$

solved by 1D finite element method

$$\delta \bar{\mathbf{K}} \quad \delta \bar{\mathbf{W}} = \delta \bar{\mathbf{W}}_p^i + \delta \bar{\mathbf{W}}_p^c + \delta \bar{\mathbf{W}}_v$$

Kinetic Energy      Potential Energies

**Boozer magnetic coordinates ( $\psi_T, \theta, \phi$ ) joined at SP-ST interface to guarantee  $\xi^\psi$  continuity**



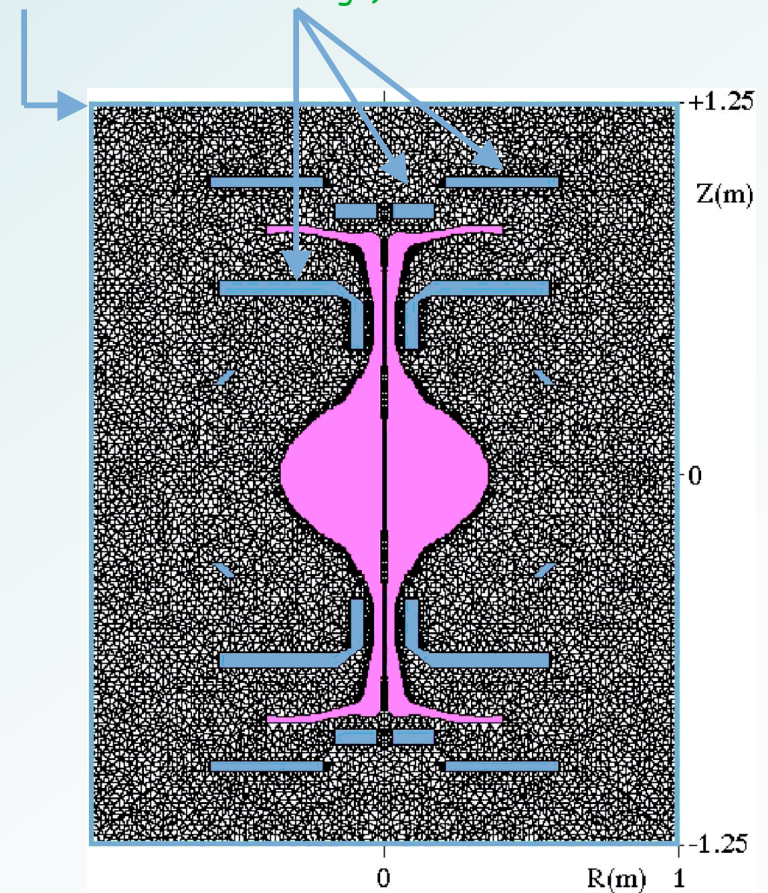
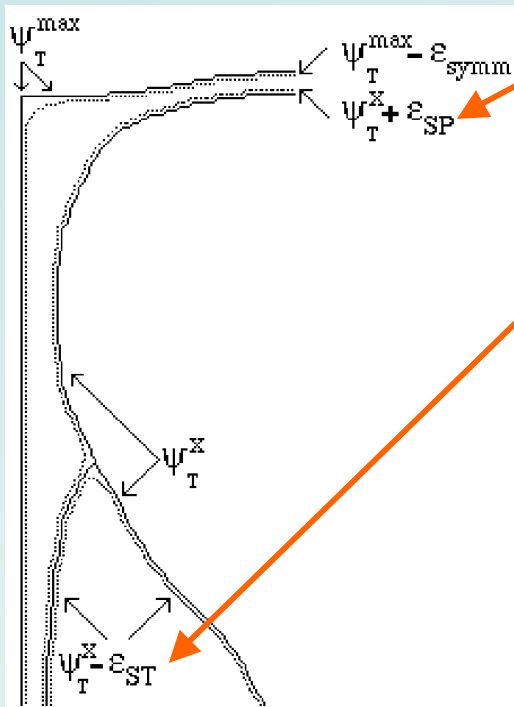


## Vacuum term computation (multiple plasma boundaries)

Using the perturbed scalar magnetic potential  $\Phi$ , the vacuum contribution is expressed as an integral over the plasma surface: 
$$\delta W_v = \frac{\mu_0}{2} \iint_{S_\psi} \Phi \vec{\nabla} \Phi \cdot d\vec{S}_\psi$$

Computation method for  $\delta W_v$  based on 2D finite element: it take into account any stabilizing conductors (vacuum vessel & PF coil casings)

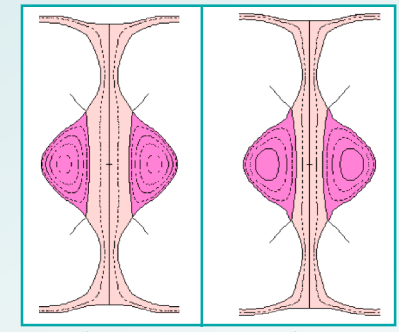
Vacuum contribution to potential energy not only affect  $\psi_T = \psi_T^{\max}$ : contribution even to the radial mesh points  $\psi_T = \psi_T^X - \epsilon_{ST}$  and  $\psi_T^X + \epsilon_{SP}$



## Stability results for time slices T3 & T4

Both times ideally stable ( $\omega^2 / \omega_A^2 > 0$ ) for  $n=1, 2, 3$   
 ( $q$  profile monotonic & shear reversed)

→ Oscillations on resonant surfaces



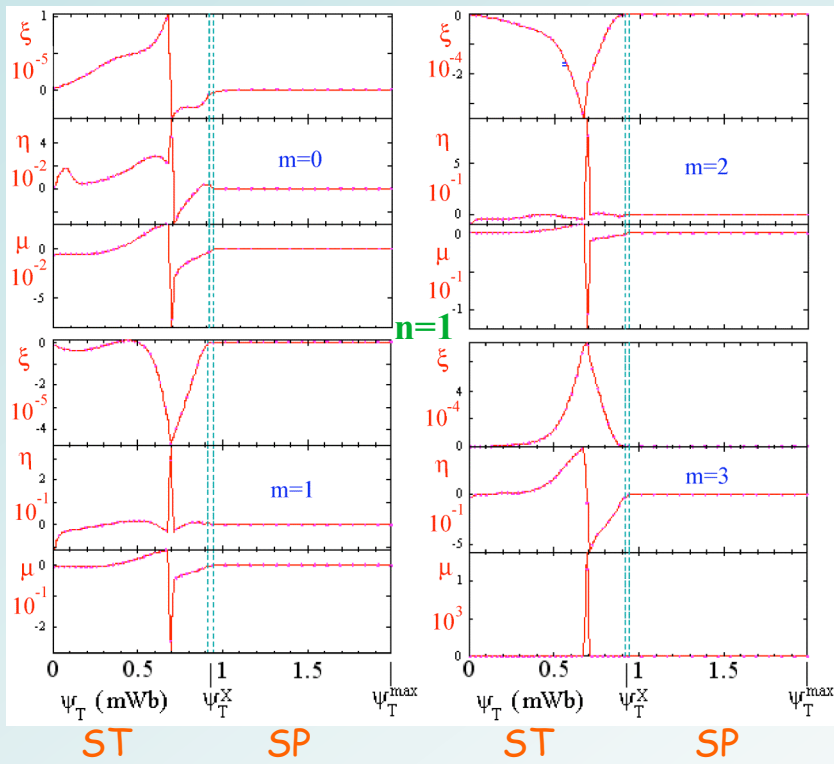
$I_p/I_e=0.5$

$I_p/I_e=1$

### Equilibrium parameters:

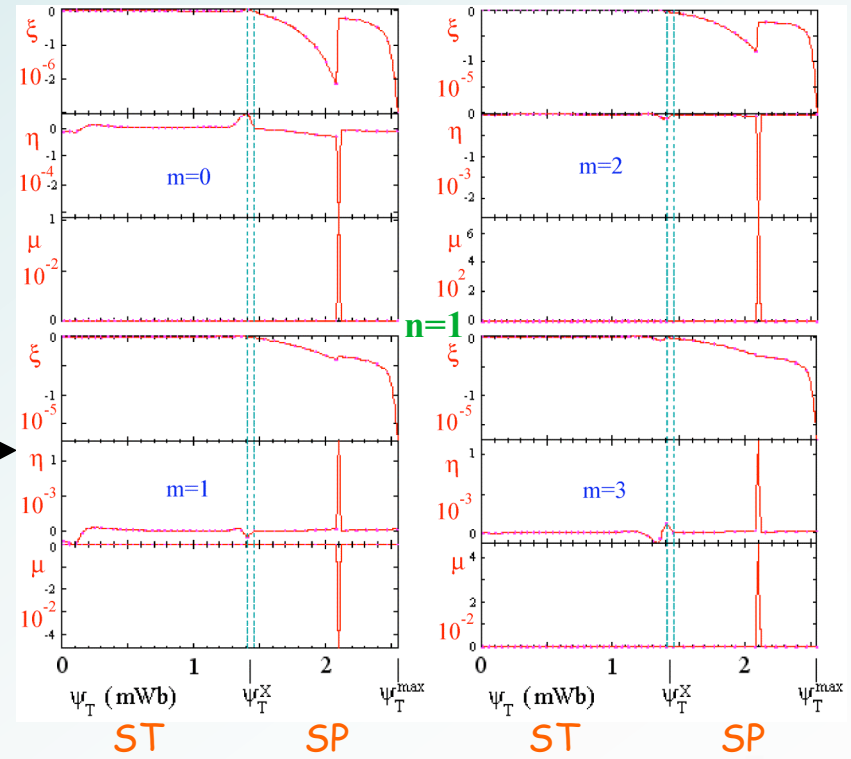
**T3:**  $I_p=30$  kA,  $A=1.8(1.9)$ ,  $\kappa=2.2(2.4)$ ,  $q_{95}=3.4(3.3)$ ,  $q_0=1.2(2.1)$ ,  $\beta_p=1.15$  and  $\beta=22(24)\%$

**T4:**  $I_p=60$  kA,  $A=1.5(1.6)$ ,  $\kappa=2.1(2.4)$ ,  $q_{95}=2.9(3.1)$ ,  $q_0=1.1(3.1)$ ,  $\beta_p=0.5$  and  $\beta=21(26)\%$



**T3**

**T4**



## Stability results for time slices T5

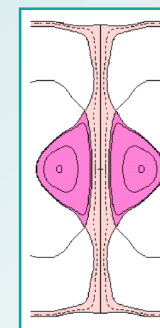
With "reference"  $\beta_p=0.3 \Rightarrow n=1$  stable,  $n=2$  &  $3$  unstable

Equilibrium parameters:

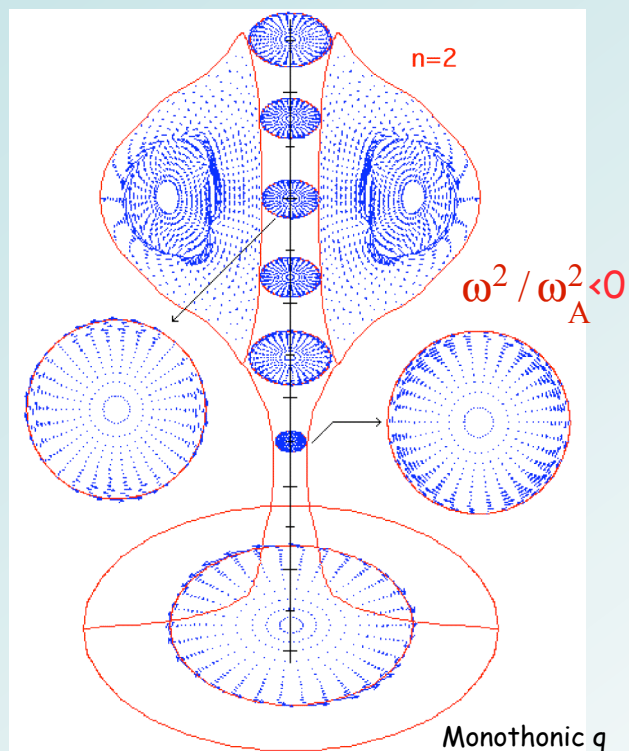
T5 (monothonic q):  $I_p=120$  kA,  $A=1.3$ ,  $\kappa=2.1$ ,  $q_{95}=2.8$ ,  $q_0=1.0$ ,  $\beta=25\%$

T5 (reversed q):  $I_p=120$  kA,  $A=1.4$ ,  $\kappa=2.5$ ,  $q_{95}=3.5$ ,  $q_0=2.8$ ,  $\beta=33\%$

ST drives instability: only perturbed motion on the ST/SP interface



$$I_p/I_e=2$$



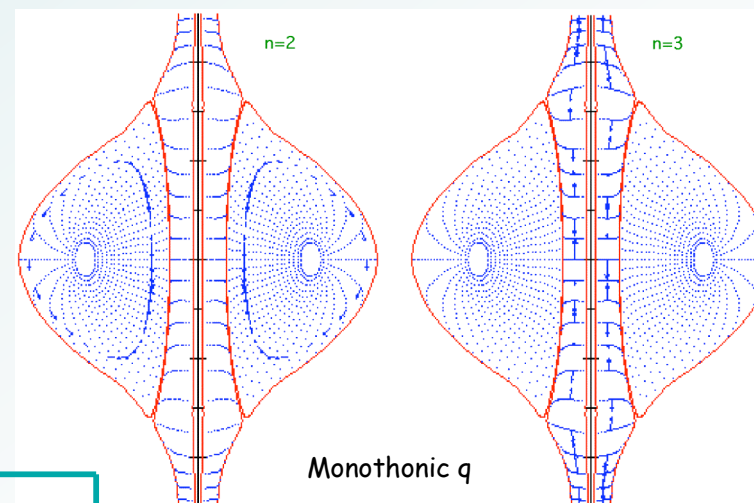
Stability restored with  $\beta_p=0.2$

Equilibrium parameters:

T5 (monothonic q):  $I_p=120$  kA,  $A=1.4$ ,  $\kappa=2.2$ ,  $q_{95}=2.7$ ,  $q_0=1.2$ ,  $\beta=16\%$

T5 (reversed q):  $I_p=120$  kA,  $A=1.4$ ,  $\kappa=2.4$ ,  $q_{95}=2.7$ ,  $q_0=1.9$ ,  $\beta=18\%$

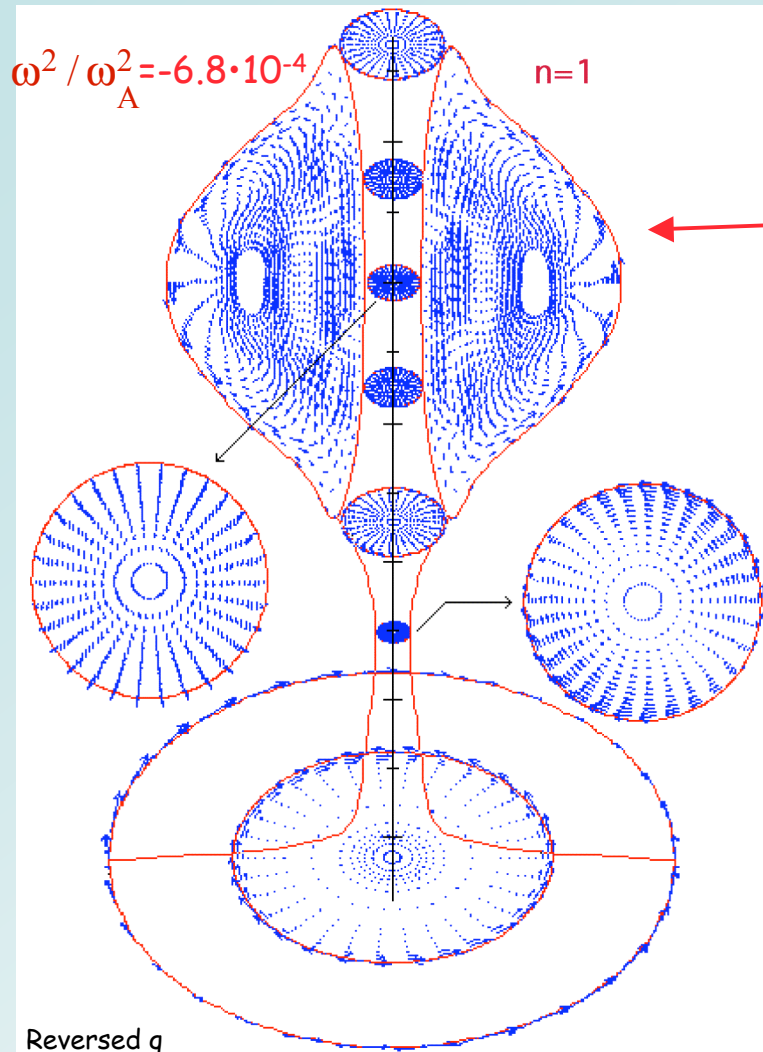
Stable oscillation on the resonant q surfaces





## Stability results for time slices T6

Screw Pinch drives instability:  
ST tilt induced by SP kink



With "reference"  $\beta_p=0.225$ :

Monothonic q  $\rightarrow$   $n=1$  stable,  $n=2$  &  $3$  unstable

Equilibrium parameters:

T6:  $I_p=180$  kA,  $A=1.25$ ,  $\kappa=2.2$ ,  $q_{95}=2.6$ ,  $q_0=0.96$ ,  $\beta=25\%$

Reversed q  $\rightarrow$   $n=1$ ,  $n=2$  &  $3$  unstable

Equilibrium parameters:

T6:  $I_p=180$  kA,  $A=1.29$ ,  $\kappa=2.5$ ,  $q_{95}=3.2$ ,  $q_0=2.3$ ,  $\beta=33\%$

Weak effect of vacuum term:

for  $n=1$   $\omega / \omega_A^2 -6.8 \cdot 10^{-4} \rightarrow -7 \cdot 10^{-4}$  if PF coil casings suppressed

With "lower"  $\beta_p=0.15$ :

Monothonic q  $\rightarrow$   $n=1,2,3$  stable

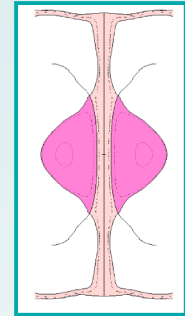
Equilibrium parameters:

T6:  $I_p=180$  kA,  $A=1.29$ ,  $\kappa=2.2$ ,  $q_{95}=2.5$ ,  $q_0=1.12$ ,  $\beta=15\%$

Reversed q  $\rightarrow$   $n=1,2,3$  stable

Equilibrium parameters:

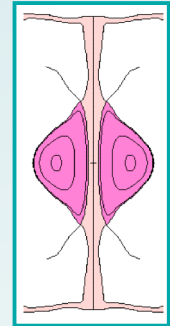
T6:  $I_p=180$  kA,  $A=1.32$ ,  $\kappa=2.5$ ,  $q_{95}=2.5$ ,  $q_0=1.83$ ,  $\beta=19\%$



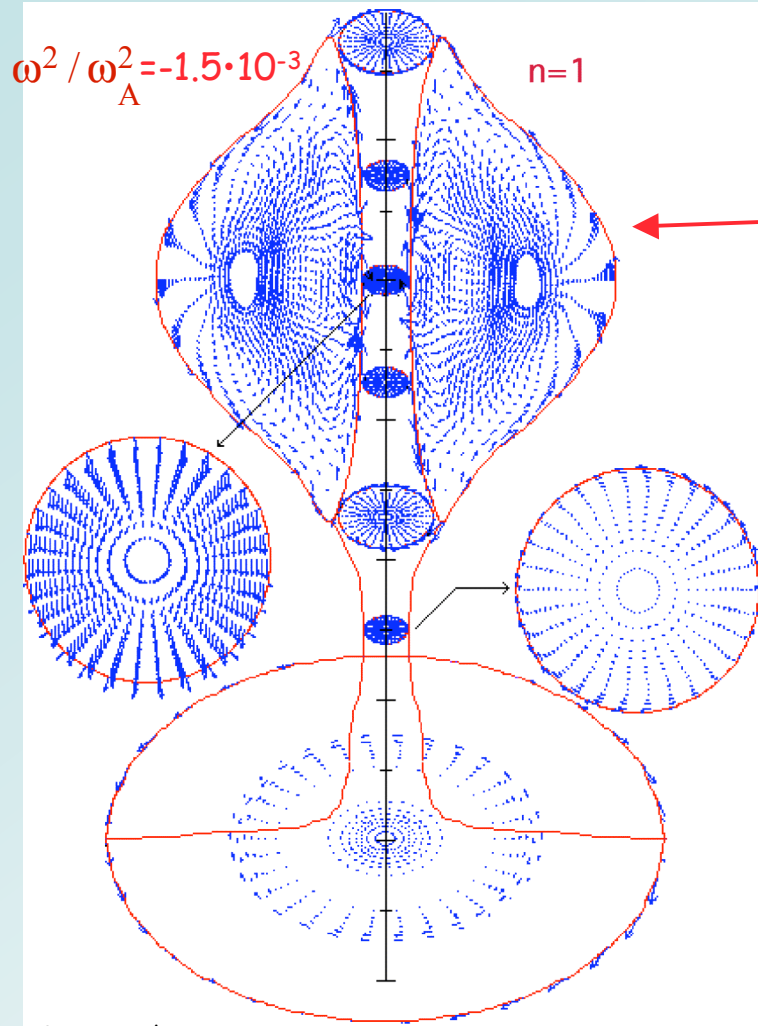
$I_p/I_e=3$

Screw Pinch drives instability:  
 ST tilt induced by SP kink  
 (kink more extended with respect to T6)

## Stability results for time slices TF



$$I_p/I_e=4$$



$$\frac{\omega^2}{\omega_A^2} = -1.5 \cdot 10^{-3}$$

$n=1$

With "reference"  $\beta_p=0.225$ :

Monothonic  $q \rightarrow n=1$  stable,  $n=2$  &  $3$  unstable

Equilibrium parameters:

TF:  $I_p=240$  kA,  $A=1.22$ ,  $\kappa=2.2$ ,  $q_{95}=2.65$ ,  $q_0=1.04$ ,  $\beta=19\%$

Reversed  $q \rightarrow n=1$  &  $2$  unstable,  $n=3$  stable

Equilibrium parameters:

TF:  $I_p=240$  kA,  $A=1.24$ ,  $\kappa=2.4$ ,  $q_{95}=2.89$ ,  $q_0=1.82$ ,  $\beta=23\%$

With "lower"  $\beta_p=0.12$

Monothonic  $q \rightarrow n=1,2,3$  stable

Equilibrium parameters:

TF:  $I_p=240$  kA,  $A=1.24$ ,  $\kappa=2.3$ ,  $q_{95}=2.55$ ,  $q_0=1.13$ ,  $\beta=16\%$

With further lowered  $\beta_p=0.10$

Reversed  $q \rightarrow n=1,2,3$  stable

Equilibrium parameters:

TF:  $I_p=240$  kA,  $A=1.26$ ,  $\kappa=2.4$ ,  $q_{95}=2.55$ ,  $q_0=1.64$ ,  $\beta=14\%$

Reversed shear profiles less effective in stabilizing SP kink

# Effect of ST elongation on $I_p/I_e$ limits

**SPHERA**  
(2xPROTO-SPHERA)

Stable for  $n=1,2,3$

Equilibrium parameters:

$I_p=2$  MA

$I_e=365$  kA

$A=1.23$

$\kappa=3.0$

$q_{95}=2.99, q_0=1.42$

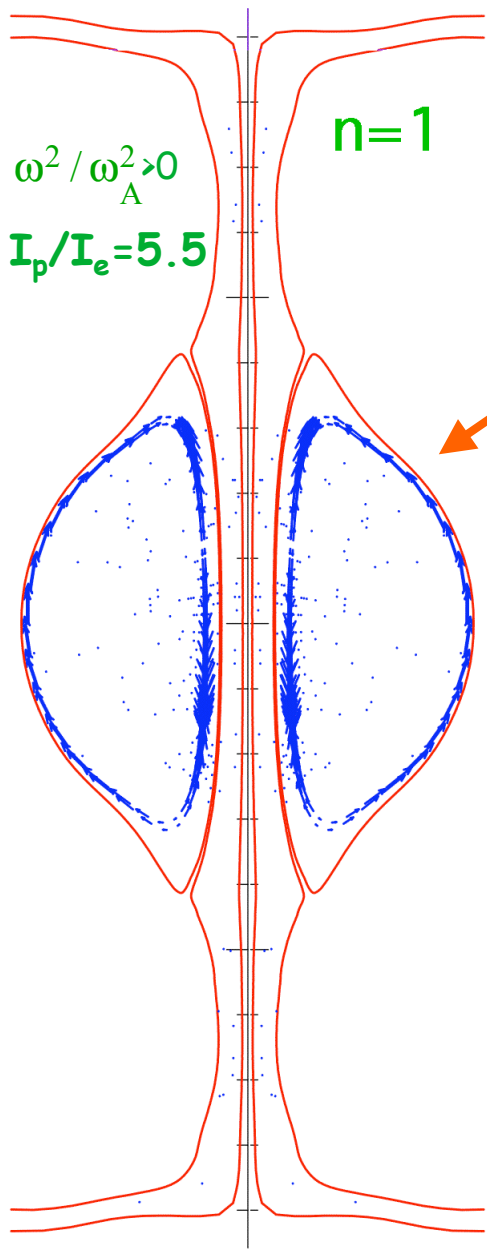
$\beta=13\%$

(*monothonic q*)

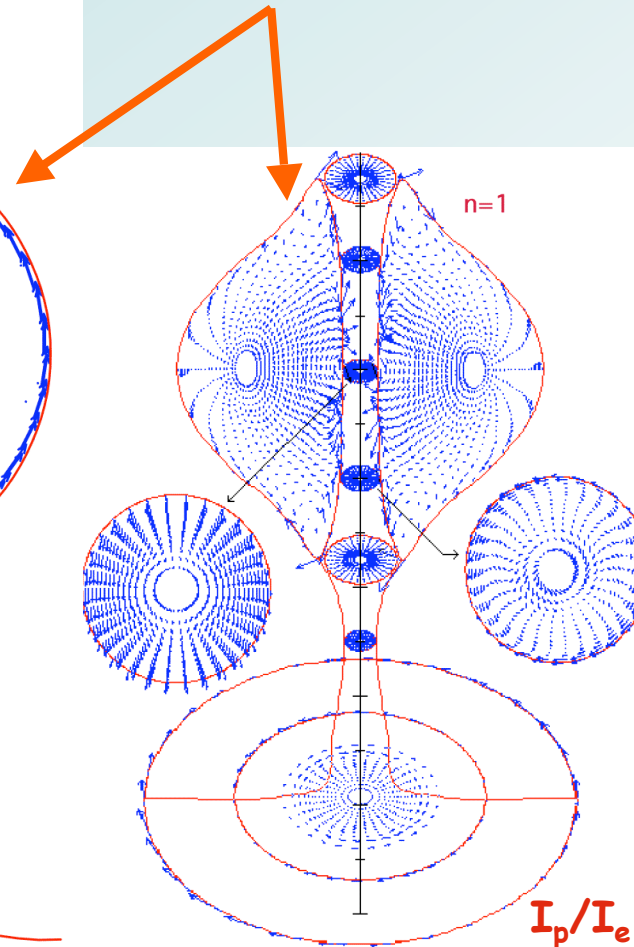
$$\omega^2 / \omega_A^2 > 0$$

$$I_p/I_e = 5.5$$

$n=1$



Increasing  $\kappa$  allow for higher  $I_p/I_e$  ratio



**PROTO-SPHERA**

Unstable for  $n=1$

Stable for  $n=2$  & 3

Equilibrium parameters:

$I_p=300$  kA

$I_e=60$  kA

$A=1.20$

$\kappa=2.3$

$q_{95}=2.7, q_0=1.15$

$\beta=15\%$

(*monothonic q*)

$$I_p/I_e = 5$$

$$\omega^2 / \omega_A^2 = -4.4 \cdot 10^{-2}$$

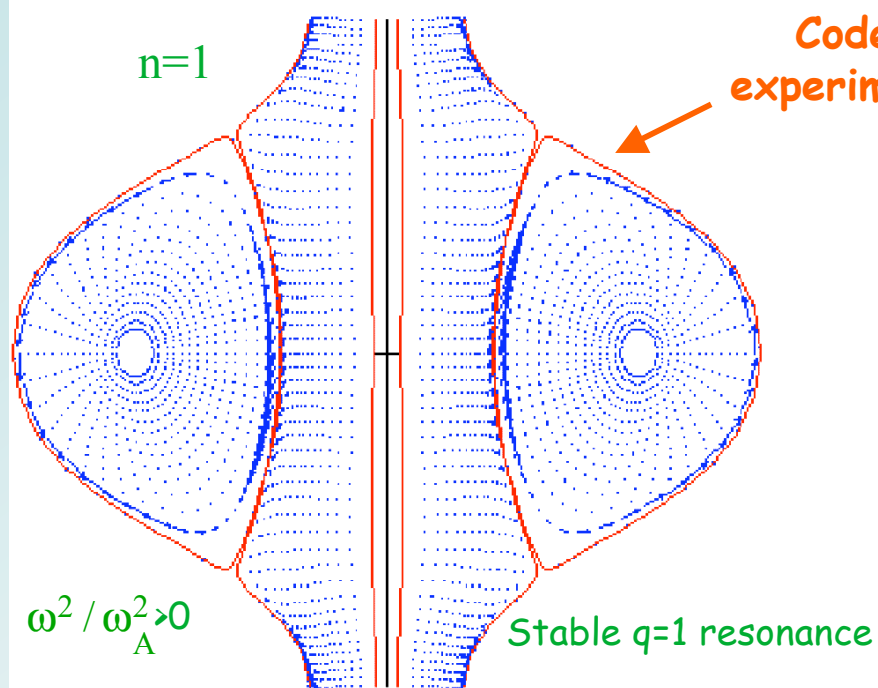
## Comparison with TS-3 (1)

Tokio Device had:

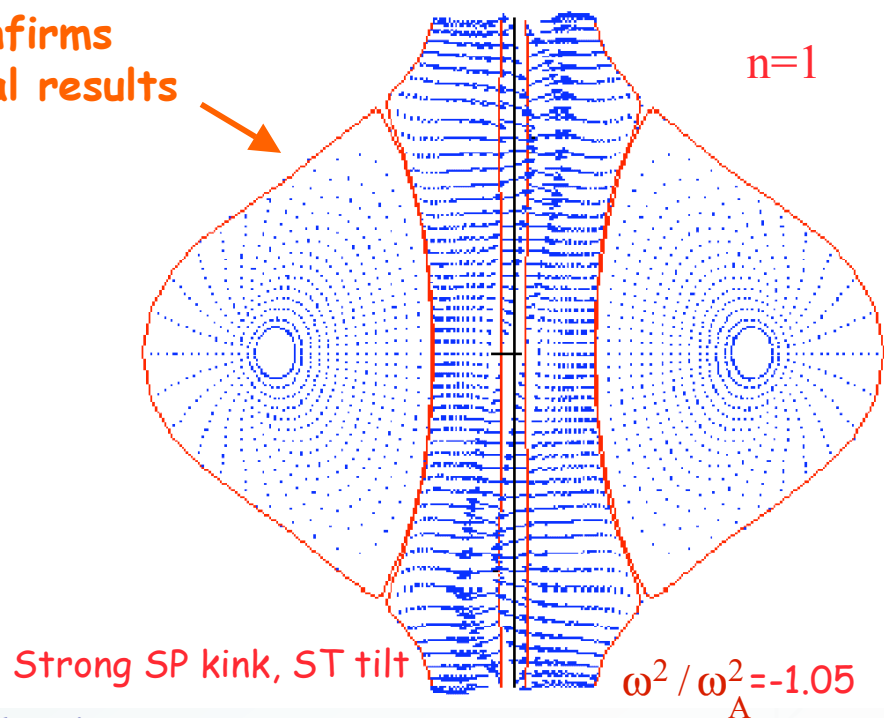
- Simple "linear" electrodes
- Oblated Spherical Torus
- $q < 1$  all over the ST (Spheromak)

$I_p = 50 \text{ kA}$ ,  $I_e = 40 \text{ kA}$   
 $I_p/I_e \sim 1$ ,  $A \sim 1.8$

$I_p = 100 \text{ kA}$ ,  $I_e = 40 \text{ kA}$   
 $I_p/I_e \sim 2$ ,  $A \sim 1.5$



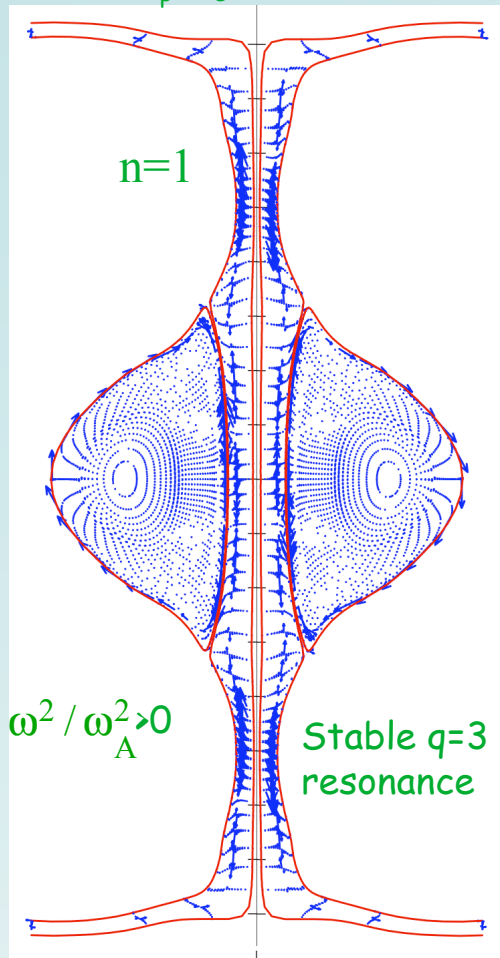
Code confirms  
 experimental results





## Comparison with TS-3 (2) (effect of the SP shape)

T5 ( $\beta=16\%$ )  
 $I_p=120$  kA,  $I_e=60$  kA  
 $I_p/I_e=2$ ,  $A\sim 1.3$

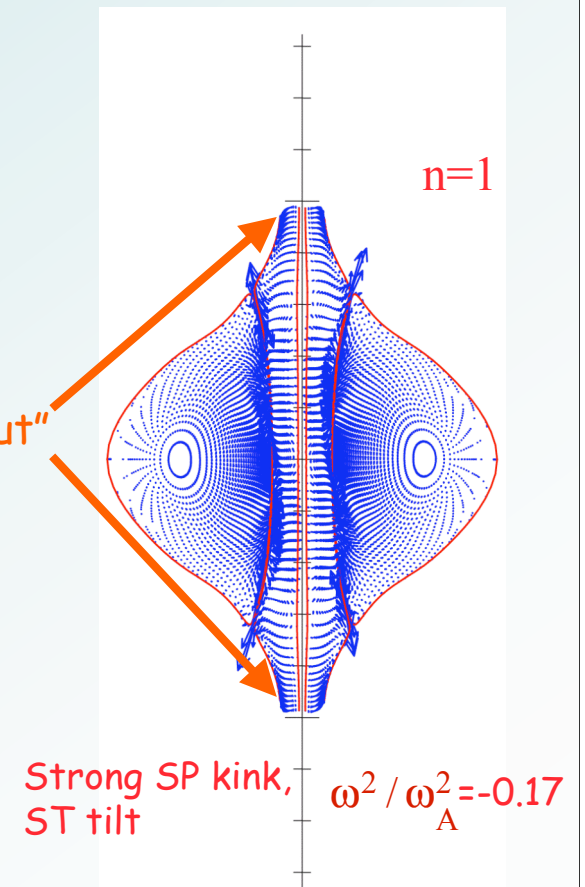


If the fully stable T5 is "artificially cut"  
 to remove degenerate X-points  
 as well as disk-shaped SP



Strong  $n=1$  instability appears,  
 despite higher  $\kappa$  &  $q_{95}$

T5-cut ( $\beta=16\%$ )  
 $I_p=120$  kA,  $I_e=60$  kA  
 $I_p/I_e=2$ ,  $A\sim 1.3$



# Conclusions

## Ideal MHD stability results for PROTO-SPHERA

- PROTO-SPHERA stable at full  $\beta$  21÷26% for  $I_p/I_e=0.5$  & 1, down to 14÷16% for  $I_p/I_e=4$  (depending upon profiles inside the ST)  
*Comparison with the conventional Spherical Tokamak with central rod:*  
 $\beta_{TO}=28\div29\%$  for  $I_p/I_e=0.5$  to  $\beta_{TO}=72\div84\%$  for  $I_p/I_e=4$
- Spherical Torus dominates instability up to  $I_p/I_e \approx 3$ ; beyond this level of  $I_p/I_e$ , dominant instability is the SP kink (that gives rise to ST tilt motion)
- Spherical Torus elongation  $\kappa$  plays a key role in increasing  $I_p/I_e$
- Comparison with TS-3 experimental results:  
 disk-shaped Screw Pinch plasma important for the configuration stability