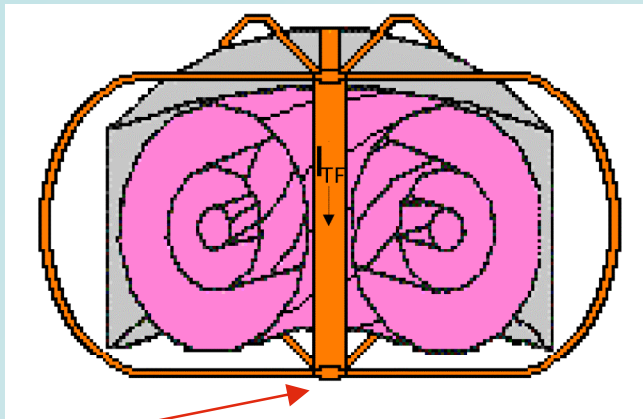


Ideal MHD Stability Boundaries of the PROTO-SPHERA Configuration

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Spherical Tokamaks allow to obtain:

- High plasma current I_p (and high $\langle n \rangle$) with low B_T
- Plasma β much higher than Conventional Tokamaks
- More compact devices

But, for a reactor/CTF extrapolation:

- No space for central solenoid (Current Drive requirement more severe)
- No neutrons shield for central stack (no superconductor/high dissipation)

Intriguing possibility $\Rightarrow$ substitute central rod with Screw Pinch plasma ($I_{TF} \rightarrow I_e$)

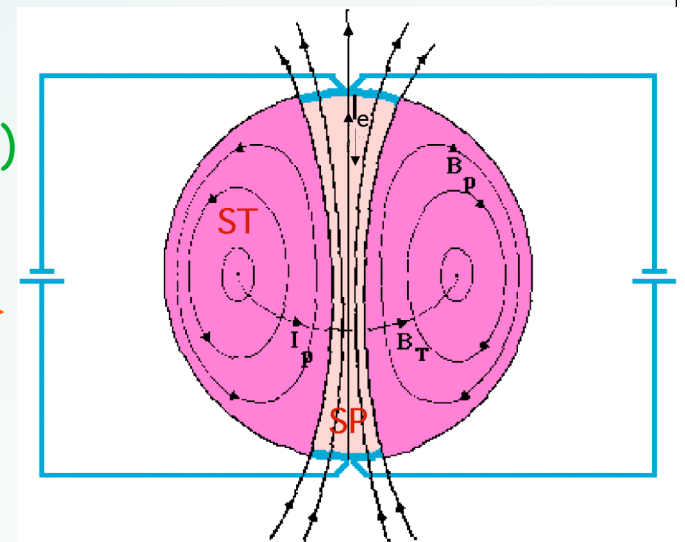
Potentially two problems solved:

- Simply connected configuration (no conductors inside)
- I_p driven by I_e (Helicity Injection from SP to ST)

Flux Core Spheromak (FCS) $\longrightarrow$

Theory: Taylor & Turner, Nucl. Fusion 29, 219 (1989)

Experiment: TS-3; N. Amemiya, et al., JPSJ 63, 1552 (1993)



But Flux Core Spheromaks are:

- injected by plasma guns
- formed by ~ 10 kV voltage on electrodes
- high pressure prefilled
- with ST safety factor $q \leq 1$

New configuration proposed:

PROTO-SPHERA

"Flux Core Spherical Tokamak" (FCST), rather than FCS

Disk-shaped electrode driven Screw Pinch plasma (SP)

Prolated low aspect ratio ST ($A=R/a \geq 1.2$, $\kappa=b/a \sim 2.3$) to get a Tokamak-like safety factor ($q_0 \geq 1$, $q_{edge} \sim 3$)

SP electrode current

$$I_e = 60 \text{ kA}$$

ST toroidal current

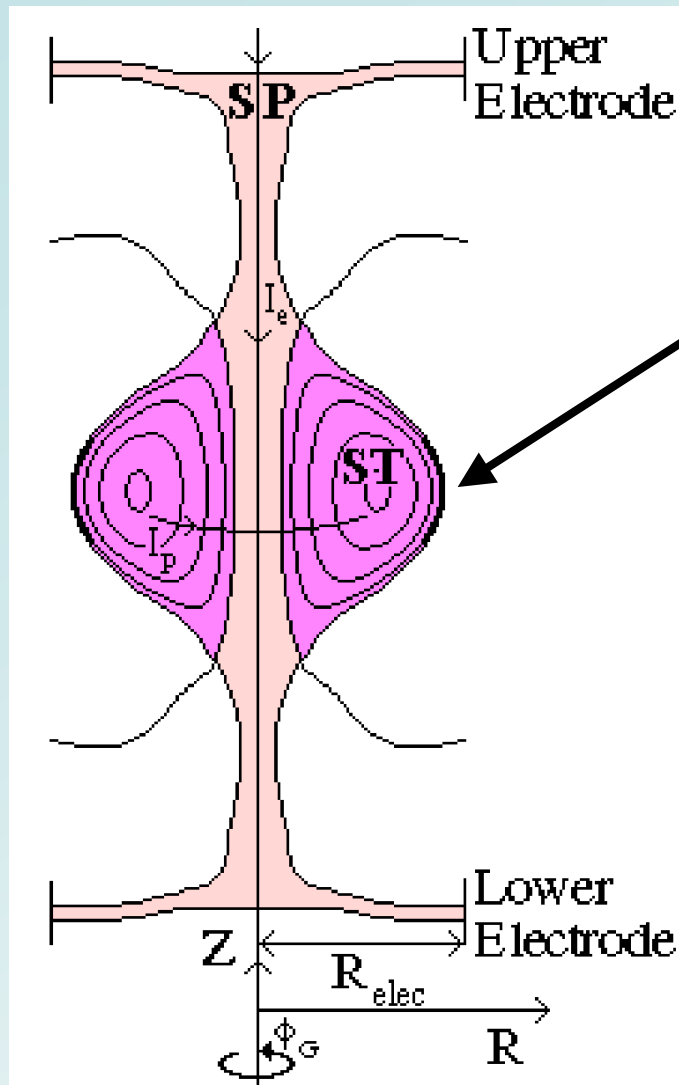
$$I_p = 120 \div 240 \text{ kA}$$

ST diameter

$$R_{sph} = 0.7 \text{ m}$$



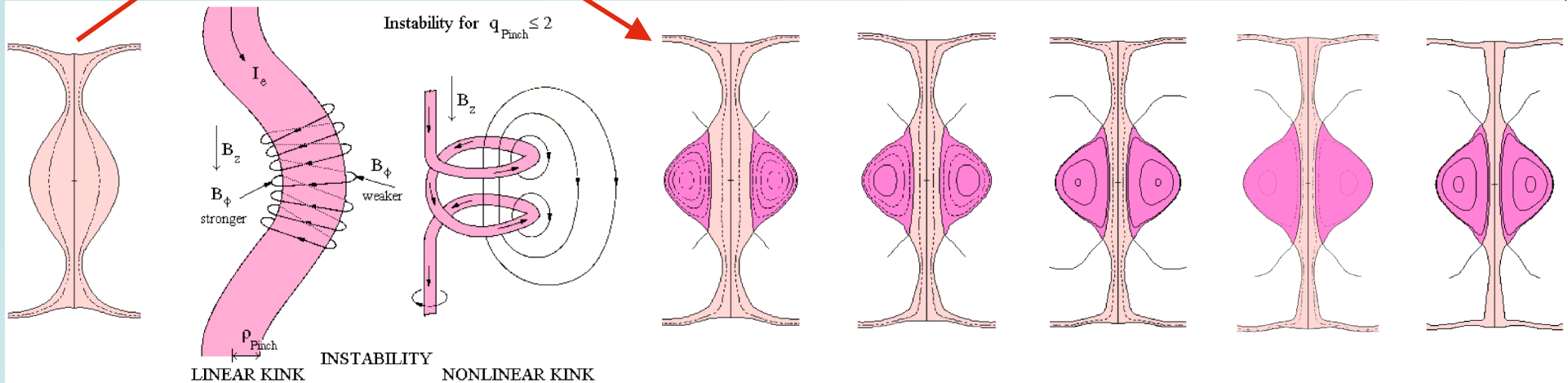
Stability should be improved and helicity drive may be less disruptive than in conventional Flux-Core-Spheromak



PROTO-SPHERA formation follows TS-3 scheme (SP kink instability)

Tunnelling (ST formation)

ST compression ($I_p/I_e \uparrow, A \downarrow$)



T0
 $I_e = 8.5 \text{ kA}$

$I_e 8.5 \rightarrow 60 \text{ kA}$

T3
 $I_p = 30 \text{ kA}$
 $A = 1.8$

T4
 $I_p = 60 \text{ kA}$
 $A = 1.5$

T5
 $I_p = 120 \text{ kA}$
 $A = 1.3$

T6
 $I_p = 180 \text{ kA}$
 $A = 1.25$

TF
 $I_p = 240 \text{ kA}$
 $A = 1.2$

- I_p/I_e ratio crucial parameter (strong energy dissipation in SP)
- MHD equilibria computed both with *monotonic (peaked pressure)* as well as *reversal safety factor profiles* (flat pressure, $\mu = \mathbf{J} \cdot \mathbf{B} / B^2$ parameterized)

Some level of low n resistive instability needed (reconnections to inject helicity from SP to ST)

but

SP+ST must be ideally stable at any time slice



Ideal MHD analysis to assess I_p/I_e & β limits

Characteristics of the free-boundary Ideal MHD Stability code

Plasma extends to symmetry axis (R=0) | Open+Closed field lines | Degenerate |B|=0 & Standard X-points

Standard $\bar{\xi}$ decomposition

$$\bar{\xi} = \xi^\psi \bar{\mathbf{e}}_\psi + \eta^\psi \frac{\bar{\mathbf{B}} \wedge \bar{\nabla} \psi_T}{B^2} + \left(\frac{I \eta^\psi}{B^2} - \mu \right) \bar{\mathbf{B}}$$

inappropriate

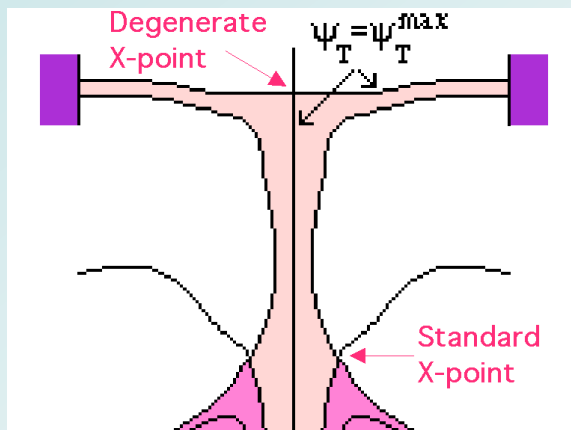
$$\bar{\mathbf{e}}_\psi = \frac{\bar{\nabla} \psi_T}{|\bar{\nabla} \psi_T|^2} + \frac{\beta_*}{B^2} \bar{\mathbf{B}} - \frac{\gamma_*}{B^2} (\bar{\mathbf{B}} \wedge \bar{\nabla} \psi_T)$$

$\bar{\nabla} \psi_T \rightarrow 0$ like $|\bar{\nabla} \psi_T| \propto R$ but, after degenerate X-point ($|B|=0$), $\psi_T = \psi_T^{\max} \neq R=0$:
 $\xi^\psi(\psi_T^{\max})=0$ cannot be imposed

Solution: $\xi^\psi = \xi R^N$ ($N \geq 1$); $\eta^\psi = \eta B$



$$\bar{\xi} = \xi R^N \bar{\mathbf{e}}_\psi + \eta \frac{\bar{\mathbf{B}} \wedge \bar{\nabla} \psi_T}{B} + \left(\frac{I \eta}{B} - \mu \right) \bar{\mathbf{B}}$$



Fourier analysis of:

$$\xi = \sum_\ell \xi_\ell(\psi_T) \sin(m_\ell \theta - n\phi)$$

$$\eta = \sum_\ell \eta_\ell(\psi_T) \cos(m_\ell \theta - n\phi)$$

$$\mu = \sum_\ell \mu_\ell(\psi_T) \cos(m_\ell \theta - n\phi)$$

Normal Mode equation $\delta \bar{\mathbf{W}} \cdot \langle \bar{\mathbf{x}} \rangle = \omega^2 \delta \bar{\mathbf{K}} \cdot \langle \bar{\mathbf{x}} \rangle$

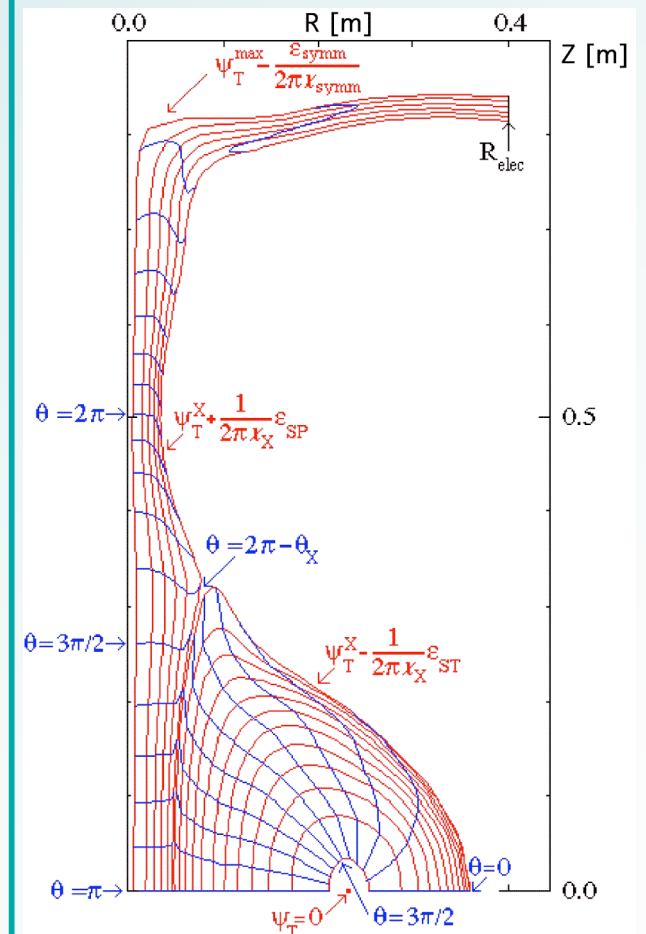
$$\langle \bar{\mathbf{x}} \rangle \equiv (\xi_\ell^j, \eta_\ell^j, \mu_\ell^j)$$

solved by 1D finite element method

$$\delta \bar{\mathbf{K}} \quad \delta \bar{\mathbf{W}} = \delta \bar{\mathbf{W}}_p^i + \delta \bar{\mathbf{W}}_p^c + \delta \bar{\mathbf{W}}_v$$

Kinetic Energy Potential Energies

Boozer magnetic coordinates (ψ_T, θ, ϕ) joined at SP-ST interface to guarantee ξ^ψ continuity



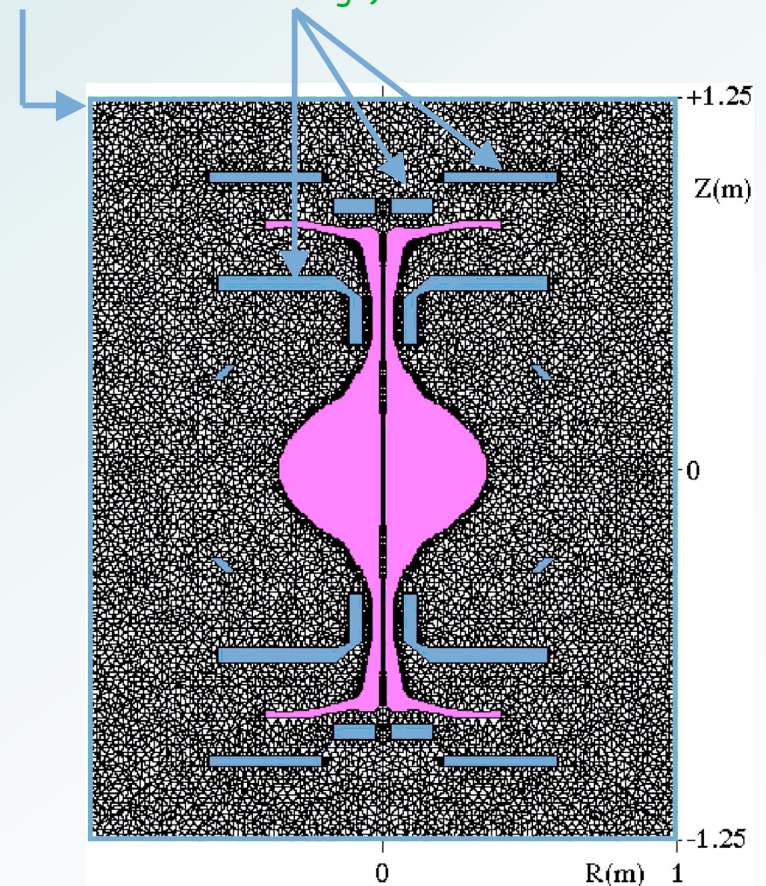
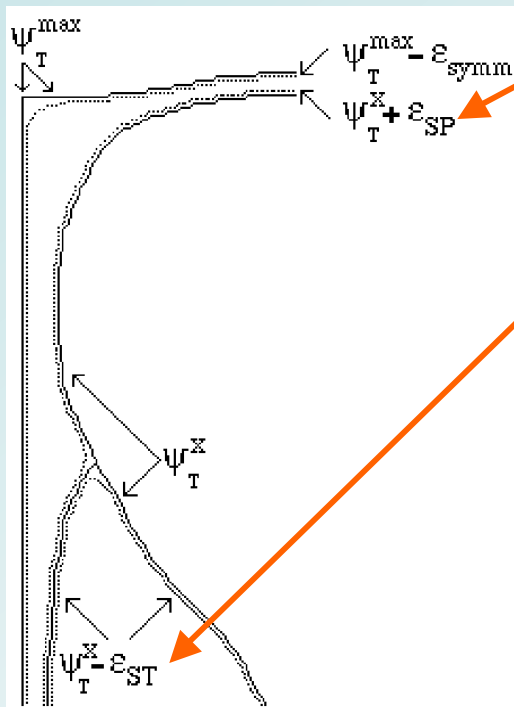
Vacuum term computation (multiple plasma boundaries)

Using the perturbed scalar magnetic potential Φ , the vacuum contribution

is expressed as an integral over the plasma surface:
$$\delta W_v = \frac{\mu_0}{2} \oint_{S_\psi} \Phi \vec{\nabla} \Phi \cdot d\vec{S}_\psi$$

Computation method for δW_v based on 2D finite element:
it take into account any stabilizing conductors
(vacuum vessel & PF coil casings)

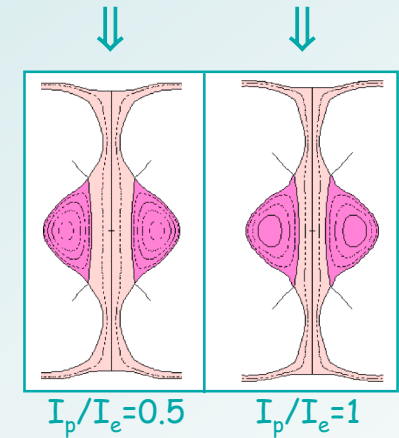
Vacuum contribution to potential energy not only affect $\psi_T = \psi_T^{\max}$
contribution even to the radial mesh points $\psi_T = \psi_T^x - \epsilon_{ST}$ and $\psi_T^x + \epsilon_{SP}$



Stability results for time slices T3 & T4

Both times ideally stable ($\omega^2 / \omega_A^2 > 0$) for $n=1,2,3$
(q profile monotonic & shear reversed)

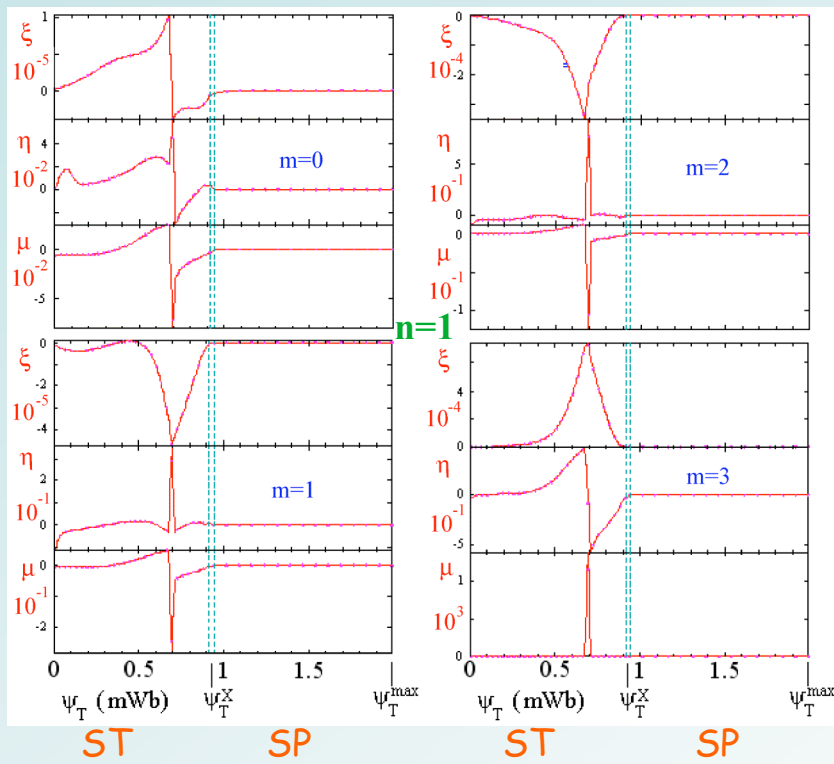
→ Oscillations on resonant surfaces



Equilibrium parameters:

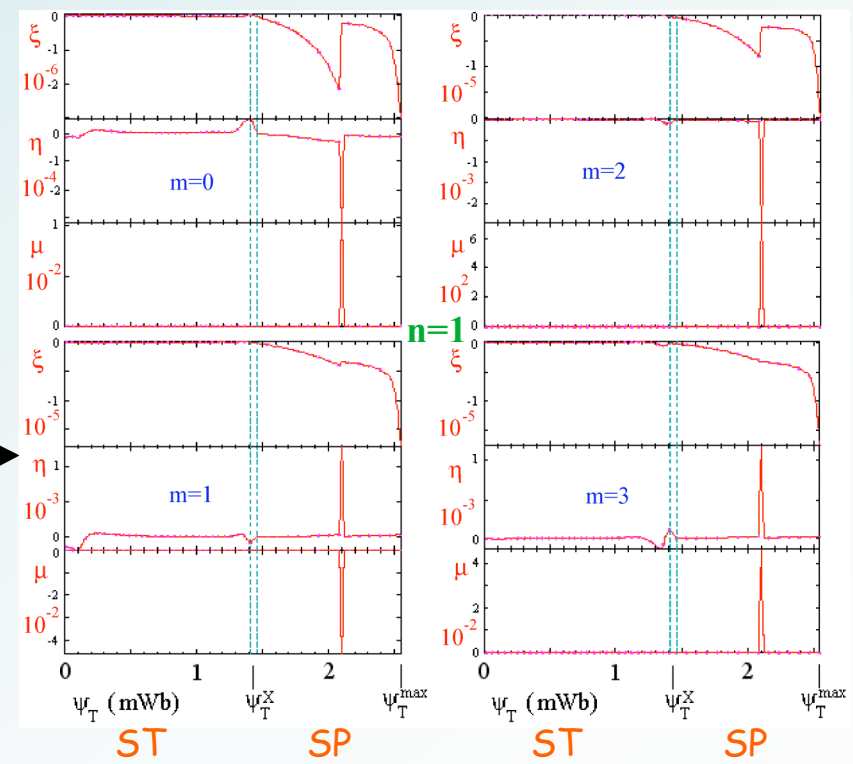
T3: $I_p=30$ kA, $A=1.8(1.9)$, $\kappa=2.2(2.4)$, $q_{95}=3.4(3.3)$, $q_0=1.2(2.1)$, $\beta_p=1.15$ and $\beta=22(24)\%$

T4: $I_p=60$ kA, $A=1.5(1.6)$, $\kappa=2.1(2.4)$, $q_{95}=2.9(3.1)$, $q_0=1.1(3.1)$, $\beta_p=0.5$ and $\beta=21(26)\%$



T3

T4



Stability results for time slices T5

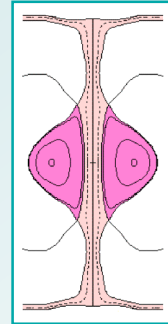
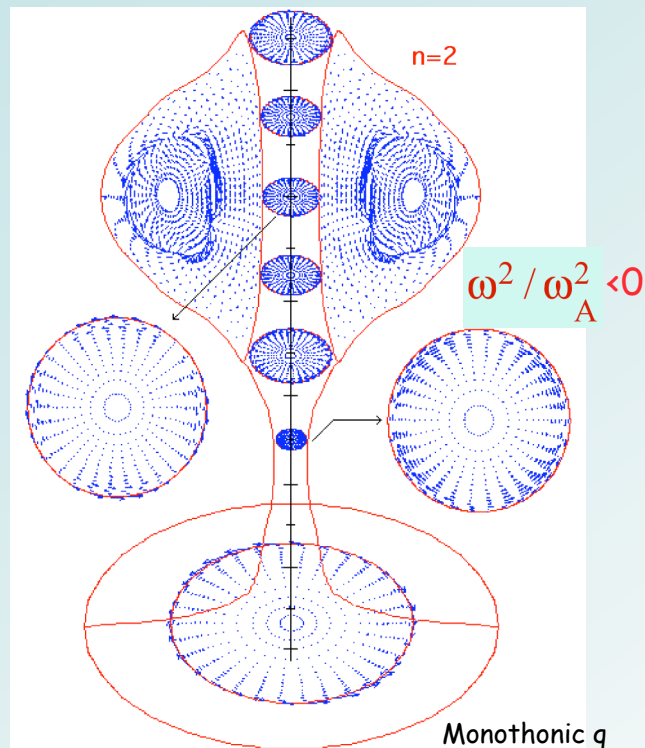
With "reference" $\beta_p=0.3 \Rightarrow n=1$ stable, $n=2$ & 3 unstable

Equilibrium parameters:

T5 (monothonic q): $I_p=120$ kA, $A=1.3$, $\kappa=2.1$, $q_{95}=2.8$, $q_0=1.0$, $\beta=25\%$

T5 (reversed q): $I_p=120$ kA, $A=1.4$, $\kappa=2.5$, $q_{95}=3.5$, $q_0=2.8$, $\beta=33\%$

ST drives instability: only perturbed motion on the ST/SP interface



$I_p/I_e=2$

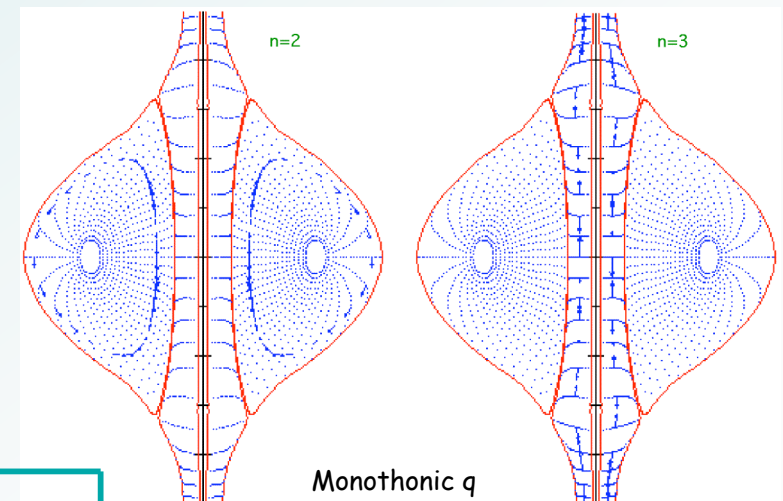
Stability restored with $\beta_p=0.2$

Equilibrium parameters:

T5 (monothonic q): $I_p=120$ kA, $A=1.4$, $\kappa=2.2$, $q_{95}=2.7$, $q_0=1.2$, $\beta=16\%$

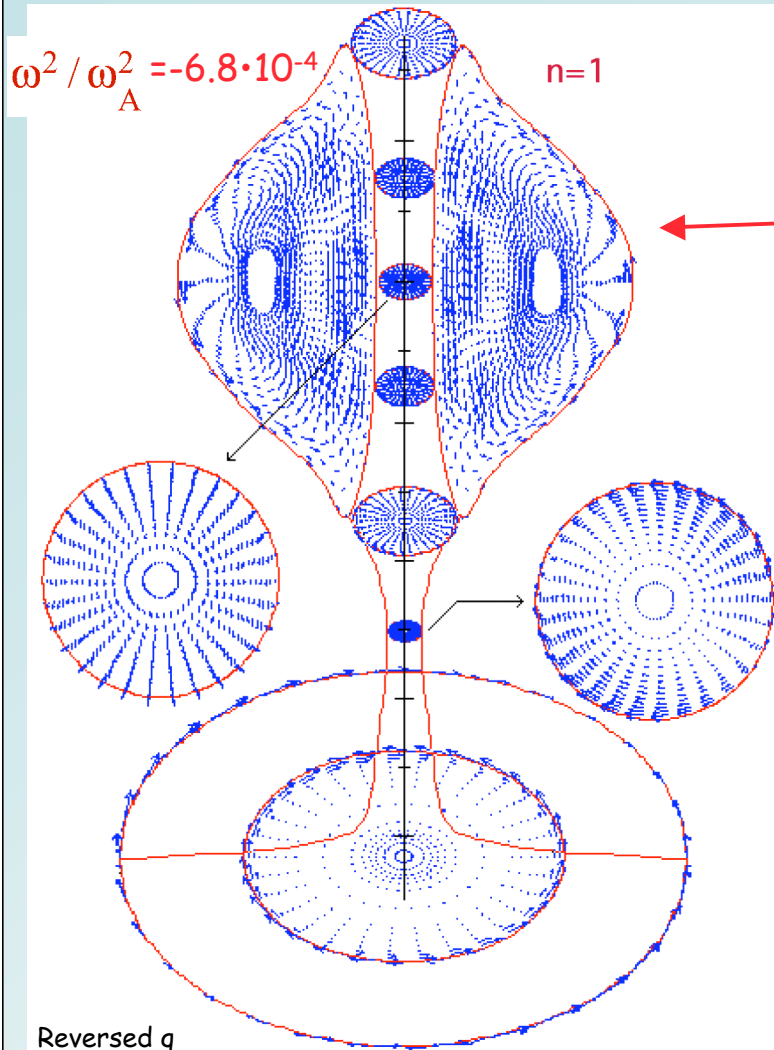
T5 (reversed q): $I_p=120$ kA, $A=1.4$, $\kappa=2.4$, $q_{95}=2.7$, $q_0=1.9$, $\beta=18\%$

Stable oscillation on the resonant q surfaces



Stability results for time slices T6

Screw Pinch drives instability:
ST tilt induced by SP kink



With "reference" $\beta_p=0.225$:

Monothonic $q \rightarrow n=1$ stable, $n=2$ & 3 unstable

Equilibrium parameters:

T6: $I_p=180$ kA, $A=1.25$, $\kappa=2.2$, $q_{95}=2.6$, $q_0=0.96$, $\beta=25\%$

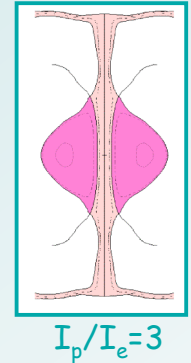
Reversed $q \rightarrow n=1$, $n=2$ & 3 unstable

Equilibrium parameters:

T6: $I_p=180$ kA, $A=1.29$, $\kappa=2.5$, $q_{95}=3.2$, $q_0=2.3$, $\beta=33\%$

Weak effect of vacuum term:

for $n=1$ $\omega / \omega_A^2 -6.8 \cdot 10^{-4} \rightarrow -7 \cdot 10^{-4}$ if PF coil casings suppressed



With "lower" $\beta_p=0.15$:

Monothonic $q \rightarrow n=1,2,3$ stable

Equilibrium parameters:

T6: $I_p=180$ kA, $A=1.29$, $\kappa=2.2$, $q_{95}=2.5$, $q_0=1.12$, $\beta=15\%$

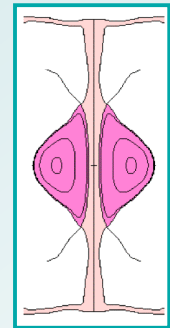
Reversed $q \rightarrow n=1,2,3$ stable

Equilibrium parameters:

T6: $I_p=180$ kA, $A=1.32$, $\kappa=2.5$, $q_{95}=2.5$, $q_0=1.83$, $\beta=19\%$

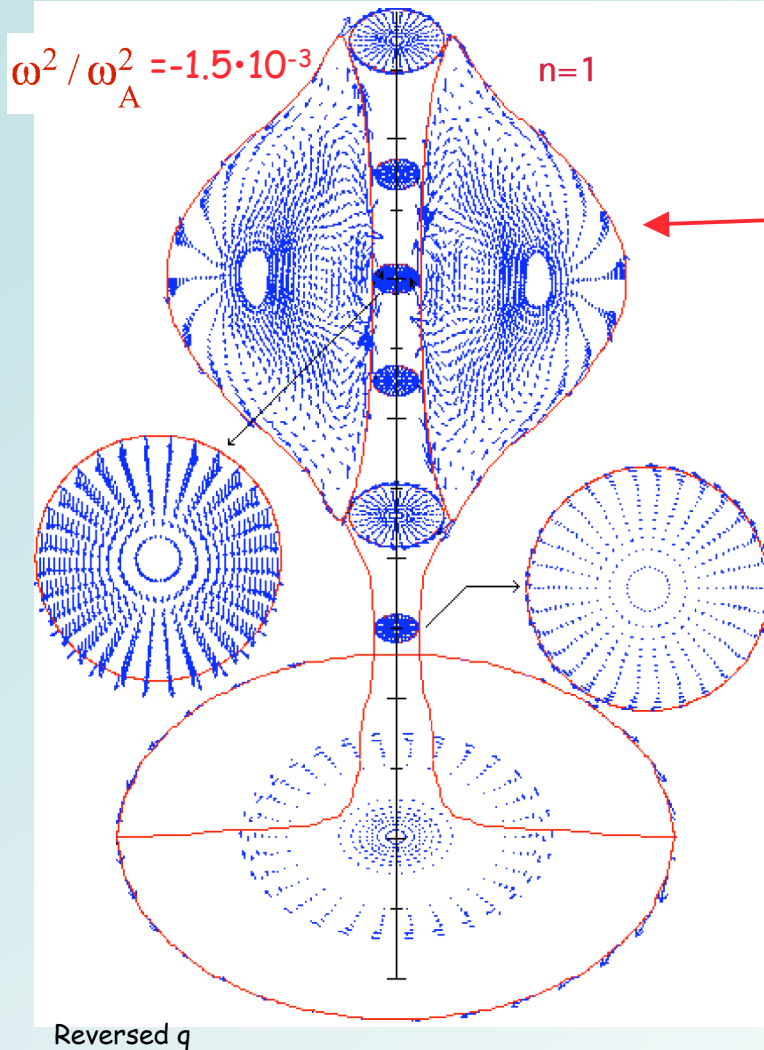
Screw Pinch drives instability:
ST tilt induced by SP kink
(kink more extended with respect to T6)

Stability results for time slices TF



$$I_p/I_e=4$$

10



With "reference" $\beta_p=0.225$:

Monothonic $q \rightarrow n=1$ stable, $n=2$ & 3 unstable

Equilibrium parameters:

TF: $I_p=240$ kA, $A=1.22$, $\kappa=2.2$, $q_{95}=2.65$, $q_0=1.04$, $\beta=19\%$

Reversed $q \rightarrow n=1$ & 2 unstable, $n=3$ stable

Equilibrium parameters:

TF: $I_p=240$ kA, $A=1.24$, $\kappa=2.4$, $q_{95}=2.89$, $q_0=1.82$, $\beta=23\%$

With "lower" $\beta_p=0.12$

Monothonic $q \rightarrow n=1,2,3$ stable

Equilibrium parameters:

TF: $I_p=240$ kA, $A=1.24$, $\kappa=2.3$, $q_{95}=2.55$, $q_0=1.13$, $\beta=16\%$

With further lowered $\beta_p=0.10$

Reversed $q \rightarrow n=1,2,3$ stable

Equilibrium parameters:

TF: $I_p=240$ kA, $A=1.26$, $\kappa=2.4$, $q_{95}=2.55$, $q_0=1.64$, $\beta=14\%$

Reversed shear profiles less effective in stabilizing SP kink

Effect of ST elongation on I_p/I_e limits

PROTO-SPHERA ($b/a \approx 3$)

Stable for $n=1,2,3$

Equilibrium parameters:

$I_p=329$ kA

$I_e=60$ kA

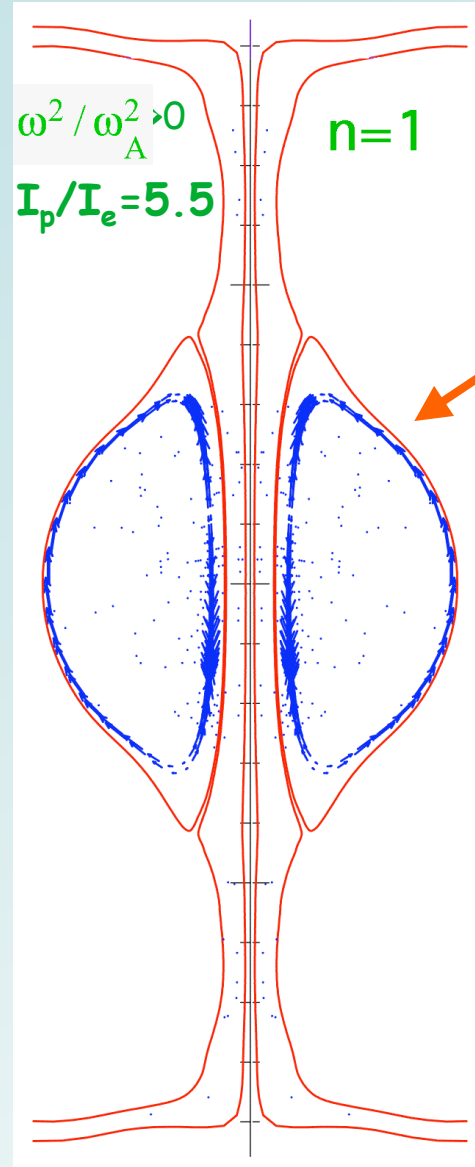
$A=1.23$

$\kappa=3.0$

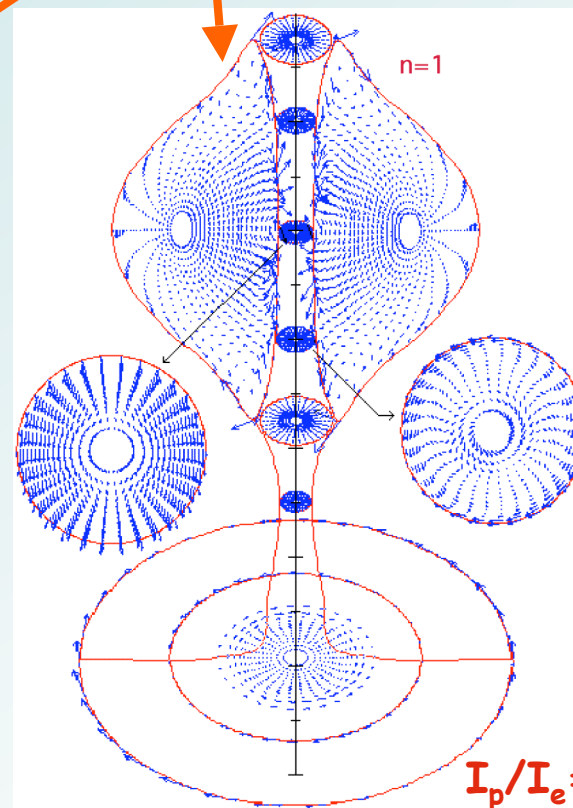
$q_{95}=2.99, q_0=1.42$

$\beta=13\%$

(monothonic q)



Increasing κ allow for higher I_p/I_e ratio



PROTO-SPHERA (standard b/a)

Unstable for $n=1$
Stable for $n=2$ & 3

Equilibrium parameters:

$I_p=300$ kA

$I_e=60$ kA

$A=1.20$

$\kappa=2.3$

$q_{95}=2.7, q_0=1.15$

$\beta=15\%$

(monothonic q)

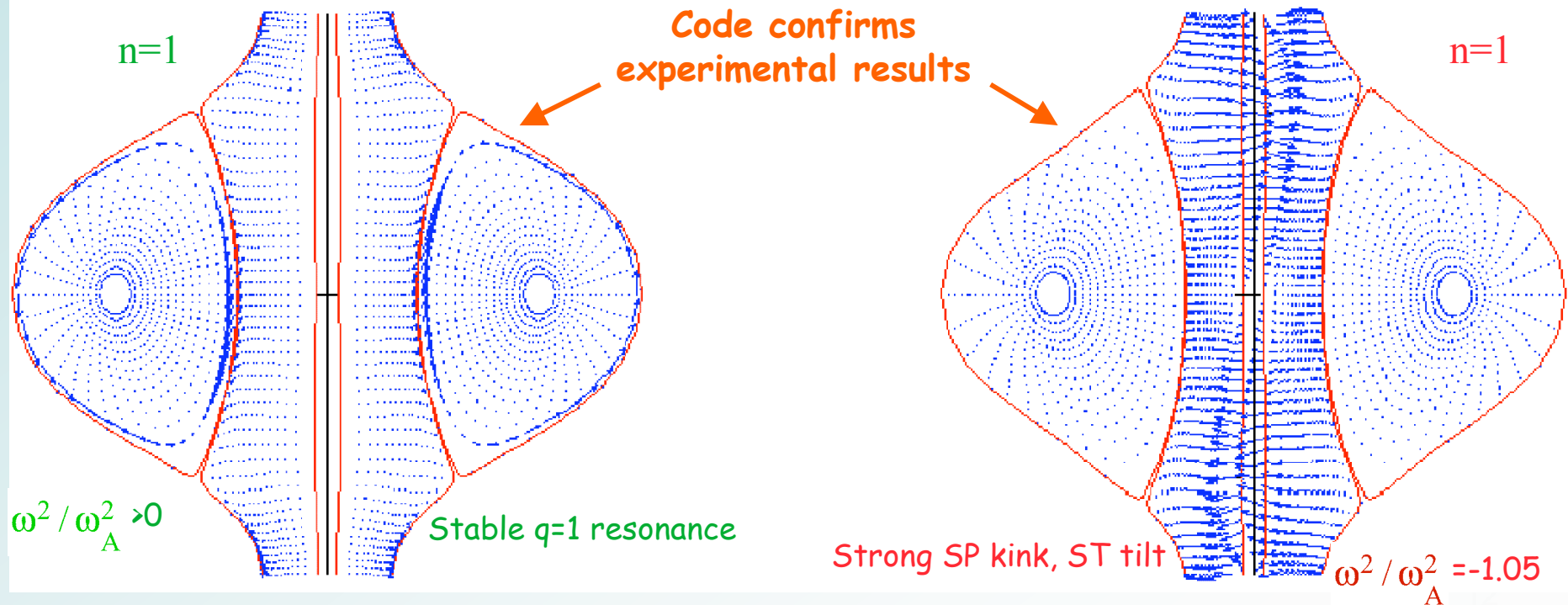
Comparison with TS-3 (1)

Tokio Device had:

- Simple "linear" electrodes
- Oblated Spherical Torus
- $q < 1$ all over the ST (Spheromak)

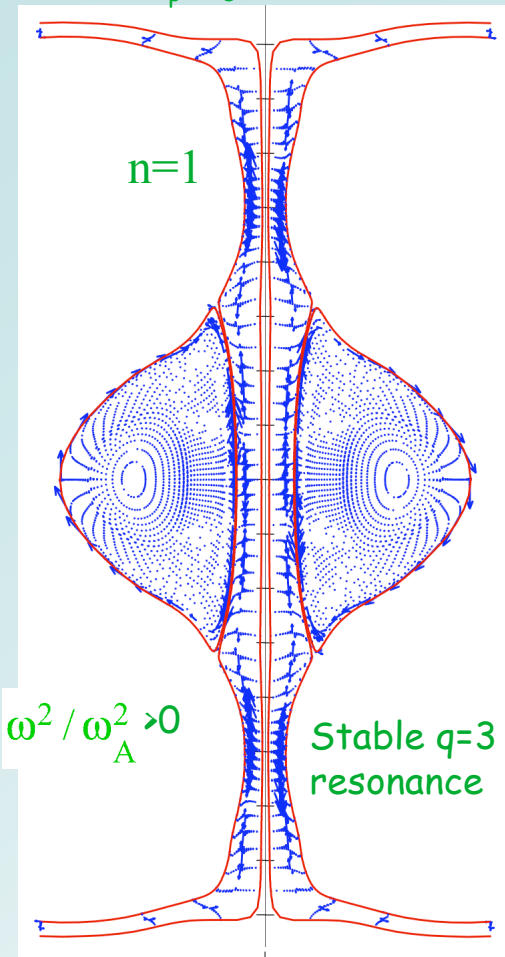
$I_p = 50 \text{ kA}$, $I_e = 40 \text{ kA}$
 $I_p/I_e \sim 1$, $A \sim 1.8$

$I_p = 100 \text{ kA}$, $I_e = 40 \text{ kA}$
 $I_p/I_e \sim 2$, $A \sim 1.5$



Comparison with TS-3 (2) (effect of the SP shape)

T5 ($\beta=16\%$)
 $I_p=120$ kA, $I_e=60$ kA
 $I_p/I_e=2$, $A\sim 1.3$

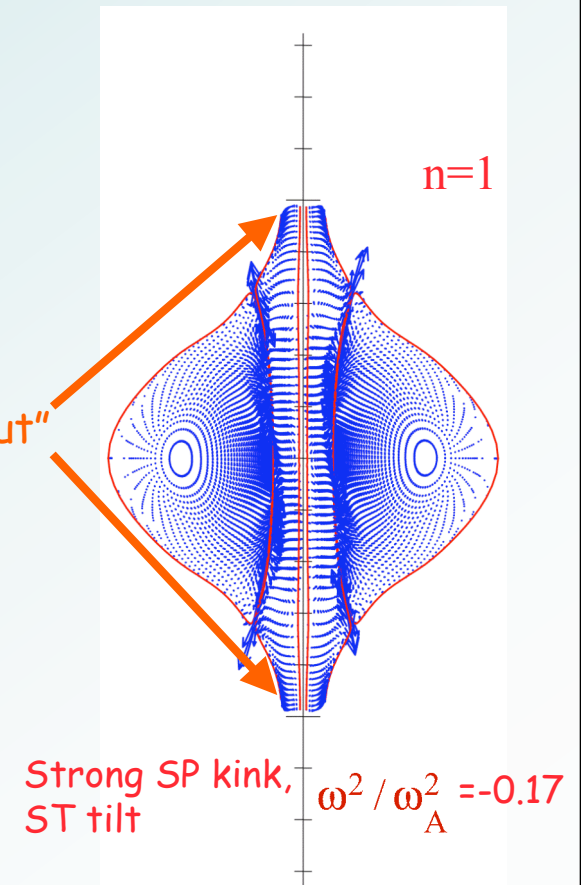


If the fully stable T5 is "artificially cut"
to remove degenerate X-points
as well as disk-shaped SP



Strong $n=1$ instability appears,
despite higher κ & q_{95}

T5-cut ($\beta=16\%$)
 $I_p=120$ kA, $I_e=60$ kA
 $I_p/I_e=2$, $A\sim 1.3$



Conclusions

Ideal MHD stability results for PROTO-SPHERA

- PROTO-SPHERA stable at full β 21÷26% for $I_p/I_e=0.5$ & 1, down to 14÷16% for $I_p/I_e=4$ (depending upon profiles inside the ST)
Comparison with the conventional Spherical Tokamak with central rod:
 $\beta_{TO}=28\div29\%$ for $I_p/I_e=0.5$ to $\beta_{TO}=72\div84\%$ for $I_p/I_e=4$
- Spherical Torus dominates instability up to $I_p/I_e \approx 3$; beyond this level of I_p/I_e , dominant instability is the SP kink (that gives rise to ST tilt motion)
- Spherical Torus elongation κ plays a key role in increasing I_p/I_e
- Comparison with TS-3 experimental results:
disk-shaped Screw Pinch plasma important for the configuration stability

Ideal MHD stability of Flux Core Spherical Torus rather insensitive to internal ST profiles
 $\Rightarrow$ configuration quite robust from an ideal point of view

Resistive instabilities behaviour is the main experimental point of PROTO-SPHERA