3D Modeling of NSTX Vertical Displacement Events with M3D-C1

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Outline

• 3D extended MHD equations solved by M3D-C1
• Multi-region resistive wall model of arbitrary thickness
• Comparison of thin-wall model and M3D-C1 multi-region model
• Initial 2D simulation to compare with DINA and TSC
• 3D linear RWM Analytic Benchmark
• Full 3D simulation of a VDE in NSTX
• Summary of 3D results
• Near-term code development plans
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3D Extended MHD Equations in M3D-C1

\[
\frac{\partial n}{\partial t} + \nabla \cdot (n \mathbf{V}) = S_n
\]

\[
\frac{\partial \mathbf{B}}{\partial t} = -\nabla \times \mathbf{E} \quad \mathbf{B} = \nabla \times \mathbf{A} \quad \mathbf{J} = \nabla \times \mathbf{B}
\]

\[
nM_i \left( \frac{\partial \mathbf{V}}{\partial t} + \mathbf{V} \cdot \nabla \mathbf{V} \right) + \nabla p = \mathbf{J} \times \mathbf{B} - \nabla \cdot \Pi_i + S_m
\]

\[
\mathbf{E} + \mathbf{V} \times \mathbf{B} = \frac{1}{ne} \left( \mathbf{R}_e + \mathbf{J} \times \mathbf{B} - \nabla p_e - \nabla \cdot \Pi_e \right) - \frac{m_e}{e} \left( \frac{\partial \mathbf{V}_e}{\partial t} + \mathbf{V}_e \cdot \nabla \mathbf{V}_e \right) + S_{CD}
\]

\[
\frac{3}{2} \left[ \frac{\partial p_e}{\partial t} + \nabla \cdot (p_e \mathbf{V}) \right] = -p_e \nabla \cdot \mathbf{V} + \frac{\mathbf{J}}{ne} \cdot \left[ \frac{3}{2} \nabla p_e - \frac{5}{2} \frac{p_e}{n} \nabla n + \mathbf{R}_e \right] + \nabla \left( \frac{\mathbf{J}}{ne} \right) : \Pi_e - \nabla \cdot \mathbf{q}_e + Q_\Delta + S_{EE}
\]

\[
\frac{3}{2} \left[ \frac{\partial p_i}{\partial t} + \nabla \cdot (p_i \mathbf{V}) \right] = -p_i \nabla \cdot \mathbf{V} - \Pi_i : \nabla \mathbf{V} - \nabla \cdot \mathbf{q}_i - Q_\Delta + S_{iE}
\]

\[
\mathbf{R}_e = \eta ne \mathbf{J}, \quad \Pi_i = -\mu \left[ \nabla \mathbf{V} + \nabla \mathbf{V}^\dagger \right] - 2(\mu_c - \mu)(\nabla \cdot \mathbf{V}) \mathbf{I} + \Pi_i^{GV}
\]

\[
\Pi_e = (\mathbf{B} / B^2) \nabla \cdot \left[ \lambda_h \nabla \left( \mathbf{J} \cdot \mathbf{B} / B^2 \right) \right], \quad Q_\Delta = 3m_e (p_i - p_e) / (M_i \tau_e)
\]

Full set of MHD equations are solved: NOT Reduced MHD
Resistive MHD in black, 2-Fluid terms are in Blue
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M3D-C\textsuperscript{1} has been extended to 3 regions for RW*.

- **Vacuum (J=0)**
- **RW (E = \eta W J)**
- **Plasma (X-MHD)**

**BC:**
- \(\mathbf{v}, p, n\) set at inner wall
- \(\mathbf{B}\) set at outer (ideal) wall
- No boundary conditions on \(\mathbf{B}\) or \(\mathbf{J}\) at the resistive wall
- Current can flow into and through the wall

Wall can be of arbitrary thickness

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*Ferraro, et al., Sherwood 2014
Submitted to JCP 2015*
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Comparison of thin-wall and M3D-C1 multi-region models

**Thin Wall**
- Plasma equations
- B.C. on $B_T$?
- All grid points touching wall are coupled to each other in matrix
- Magnetic boundary conditions applied using thin-wall approximation
- Halo current involves approximations
- Thin wall only
- Constant wall resistivity

**M3D-C1 MR**
- Grid points are coupled to only nearest neighbors in matrix
- No magnetic boundary conditions at PW or WV interface
- Halo current occurs naturally
- Wall can be of arbitrary thickness
- Wall resistance can vary with angles to represent ports, etc.
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Initial 2D simulation of VDE in NSTX with M3D-C$^1$

- Initial results from 2D low-resolution calculation (similar to TSC)
- Both **Positive** and **Negative** (counter-current) currents are found
- We have now extended these results to 3D and realistic $\eta_W$

Toroidal current density at 5 times in VDE simulation
M3D-C$^1$ $J_\varphi$, $p$, and $I=RB_T$ at a late time

Note halo currents
Dependence of NSTX VDE on $\eta_W$ in 2D

Linear growth rate scales inversely with wall resistivity $\eta_W$ as expected.

However, nonlinear time to a given offset scales weaker than linearly.
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3D linear RWM Analytic Benchmark\textsuperscript{1,2}

Circular cylindrical plasma

<table>
<thead>
<tr>
<th>Plasma region</th>
<th>Vacuum region</th>
<th>Wall</th>
</tr>
</thead>
<tbody>
<tr>
<td>$J_z = J_0$</td>
<td>$J_z = 0$</td>
<td>$J_z = 0$</td>
</tr>
<tr>
<td>$B_z = B_0$</td>
<td>$B_z = B_0$</td>
<td>$B_z = B_0$</td>
</tr>
<tr>
<td>$\rho = \rho_0$</td>
<td>$\rho = 0$</td>
<td>$\eta = \eta_w$</td>
</tr>
</tbody>
</table>

Reduced MHD Model

\[ \delta B = \nabla \psi \times \hat{z} \quad \psi = \bar{\psi}(r)e^{i(m\theta - nz/R_0) + \gamma t} \]

\[ \delta V = \nabla \phi \times \hat{z} \quad \phi = \bar{\phi}(r)e^{i(m\theta - nz/R_0) + \gamma t} \]

Note that:

\[ \delta J = \frac{1}{\mu_0} \nabla \times \delta B = -\frac{1}{\mu_0} \left( \hat{z} \nabla_{\perp}^2 \psi + \frac{in}{R_0} \nabla_{\perp} \psi \right) \]

Must allow non-zero current in vacuum in this model

Thick Wall Dispersion Relation

\[ \frac{m/m}{m - nq_0} - \frac{1}{1 - F \left( \frac{r_0}{r_w} \right)^2} = \frac{\left( \gamma \tau_A \right)^2}{2} \frac{q_0^2}{\left( m - nq_0^2 \right)^2} \]

where

\[ F = \frac{I_{|m|-1}(\rho_b)K_{|m|-1}(\rho_a) - I_{|m|-1}(\rho_a)K_{|m|-1}(\rho_b)}{I_{|m|-1}(\rho_b)K_{|m|+1}(\rho_a) - I_{|m|+1}(\rho_a)K_{|m|-1}(\rho_b)} \]

here:

\[ \rho_a = \sqrt{2\gamma \tau_w r_w / d}, \quad \rho_b = (1 + d / r_w) \rho_a \]

\[ \tau_w = \mu_0 r_w d / 2\eta_w, \quad \tau_A = \sqrt{\mu_0 \rho_0 R_0 / B_0}, \]

\[ q_0 = 2B_0 / (R_0 \mu_0 J_0) \]

also, thin wall and ideal wall limits


\textsuperscript{2}Generalized to thick wall in Ferraro, et al. Submitted to JCP (2015)
Results of 3D Analytic Benchmark (1)

Growth rate of the external kink as a function of the radius of an ideal conformal wall.

Growth rate of the resistive wall mode as a function of the resistive diffusion time of the wall in the thin wall limit:

\[ d / r_w = .02 \]
Results of 3D Analytic Benchmark (2)

RWM growth rate as a function of the wall thickness $d$ compared to general solution and thin wall solution.

Note: High resolution in wall and near plasma-vacuum boundary.
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NSTX Shot 132859  \( \eta_w=0.00025 \)

First 3D M3D-C1 simulation of NSTX VDE with resistive vessel embedded in 3D finite element mesh.
NSTX Shot 132859 \( \eta_w = 0.00025 \)

Drift phase:

\( 0 < t < 2700 \)

\( \gamma \tau_A = 0.00135 \)
Drift phase:
$0 < t < 2700$
$\gamma \tau_A = 0.00135$
NSTX Shot 132859 \( \eta_w = 0.00025 \)

Disruption phase \( 2700 < t < 2950 \)

Contours of RBT show halo currents

\( \gamma \tau_A = 0.132 \)

\( \gamma \tau_A = 0.024 \)
NSTX Shot 132859  \( \eta_w = 0.00025 \)

\[ \frac{\partial (R J_\varphi)}{\partial \varphi} \]

Disruption phase \( 2700 < t < 2950 \)

Magnitude of the toroidal derivative of the toroidal current at one poloidal plane at the 5 times shown. Each color scale is adjusted to maximum range.

\( t = 2856 \quad t = 2868 \quad t = 2887 \quad t = 2900 \quad t = 2912 \)
NSTX Shot 132859  \( \eta_w = 0.00025 \)

Disruption phase \( 2700 < t < 2950 \)

Top is magnitude of the toroidal current at one poloidal plane at the 5 times shown. Each color scale is adjusted to maximum range. Bottom is values along horizontal line of maximum current as shown.
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NSTX Shot 132859  $\eta_w=0.00025$

Summary of first 3D M3D-C1 simulation of VDE in NSTX

- Plasma drifts downward with linear growth rate $\gamma \tau_A = 0.00135$ for entire drift phase: $0 < t < 2700 \tau_A$

- $q$-profile remains fixed during this phase and plasma nearly axisymmetric

- Slow $n=1$ mode with $\gamma \tau_A = 0.024$ begins to grow at $t=2800 \tau_A$ (RWM?) and this mode accelerates to $\gamma \tau_A = 0.132$ at $t=2850 \tau_A$ (external kink?)

- Wall current is initially negative (to repel plasma) and then becomes positive as plasma current decays. Halo currents begin to form at about at $t=2825 \tau_A$ when plasma makes contact with vessel.

- $n=1$ mode mostly external with $m \sim nq$. Continues to growth in amplitude until plasma disappears

- Strong shielding currents develop once plasma makes contact with vessel. (These should reduce in size with smaller value of vessel resistivity)
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Near-term Code development plans for M3D-C1

• Fully 3D resistive wall
  • Wall resistivity to vary with $\theta$ and $\varphi$ to model ports
  • Diagnostic output of 3D wall forces and halo currents
  • Study of island penetration and locking $\Rightarrow$ disruptions?

• Impurity transport and radiation
  • Neutral and impurity transport and radiation package now being added
  • Role of impurities in thermal quench, density limit

• Kinetic closures
  • Continuum closure based in Ramos formulation (underway)
  • Demonstrated correct neoclassical resistivity, bootstrap current
  • Efficient PIC closure will enable high-accuracy EP mode studies

• Improved preconditioners
  • Present preconditioners (PC) motivated by resistive MHD physics
  • New PC will improve efficiency of 2F runs with large $\kappa_{||}$, high resolution

• Optimize for Knights Landing Processor (Cori at NERSC)
  • Implementing 3 levels of parallelism: MPI, OpenMP, vector
  • M3D-C1 one of 2 fusion codes selected for early access program NESAP