

Journal Pre-proof

3D tomographic reconstruction of light emission density of plasma in the PROTO-SPHERA experiment

Annamaria Pau, Shayesteh Naghinajad, Franco Alladio,
Paolo Micozzi, Luca Boncagni



PII: S0263-2241(25)01463-0

DOI: <https://doi.org/10.1016/j.measurement.2025.118104>

Reference: MEASUR 118104

To appear in: *Measurement*

Received date: 10 November 2024

Revised date: 4 June 2025

Accepted date: 5 June 2025

Please cite this article as: A. Pau, S. Naghinajad, F. Alladio et al., 3D tomographic reconstruction of light emission density of plasma in the PROTO-SPHERA experiment, *Measurement* (2025), doi: <https://doi.org/10.1016/j.measurement.2025.118104>.

This is a PDF file of an article that has undergone enhancements after acceptance, such as the addition of a cover page and metadata, and formatting for readability, but it is not yet the definitive version of record. This version will undergo additional copyediting, typesetting and review before it is published in its final form, but we are providing this version to give early visibility of the article. Please note that, during the production process, errors may be discovered which could affect the content, and all legal disclaimers that apply to the journal pertain.

© 2025 Published by Elsevier Ltd.

3D tomographic reconstruction of light emission density of plasma in the PROTO-SPHERA experiment

Annamaria Pau (corr.author)^a, Shayesteh Naghinajad^a, Franco Alladio^b,
Paolo Micozzi^c, Luca Boncagni^c

^a*Department of Structural and Geotechnical Engineering, Sapienza University of Rome,
via Eudossiana 18, 00184 Roma, Italy*

^b*INAF-IAPS National Institute of Astrophysics,
Via del Fosso del Cavaliere 100, 00133 Roma, Italy*

^c*Nuclear Research Department, ENEA Italian National Agency for Technology, Energy
and Sustainable Economic Development
Via Enrico Fermi 45, 00044 Frascati (RM), Italy*

Abstract

We address a key challenge in plasma diagnostics: reconstructing the three-dimensional (3D) light emission density from projection data. To this end, we employ a method for reconstructing 3D density functions based on a corollary of the 3D Fourier slice theorem. This corollary establishes a connection between 2D plane projections and plane sections of the 3D Fourier transform, enabling a direct and non-iterative solution to the inverse problem. The method is applied to the PROTO-SPHERA experiment, an innovative magnetic confinement plasma setup for controlled nuclear fusion research. Experimental data were acquired using six high-speed cameras arranged with cylindrical symmetry around the plasma chamber, capturing time-resolved pictures that are assumed to be parallel projections of the visible light emissions from Helium and Hydrogen discharges. After a transformation of coordinates, by computing their inverse 3D Fourier transform, we reconstruct the spatial distribution of light emission density. This technique enabled to reveal dynamic features of plasma self-organization—such as torus formation around a centerpost—and provides internal cross-sectional views of the

Email addresses: annamaria.pau@uniroma1.it (Annamaria Pau (corr.author)),
shayesteh.naghinajad94@gmail.com (Shayesteh Naghinajad),
franco.alladio@protosphaera.it (Franco Alladio), paolo.micozzi@enea.it (Paolo Micozzi), luca.boncagni@enea.it (Luca Boncagni)

plasma, displaying previously obscured spectral components.

Keywords: 3D image reconstruction, magnetically confined plasma, nuclear fusion, monitoring

1. Introduction

Tomographic computed (CT) imaging or scanning is a technique of data treatment which enables visualization of the inner part of objects instead of making real cuts. This has obvious and important applications in the medical field, as well as in the nondestructive monitoring of structures and industrial processes. CT requires the solution of an inverse problem, which can be formulated as follows: given the knowledge of some data called projections, reconstruct the spatial distribution of a density function $f(x, y, x)$ which represents some property (for instance mass density) of an object in the 3D space. Projections are obtained by illuminating the object with stimuli which produce a measurable response, which is measured on the object boundary. Projections are, in practice, a sort of shade that the function $f(x, y, x)$ projects onto lines, or planes. The mathematical demonstration that a density function could be reproduced from an infinite set of its projections was provided by the mathematician J. Radon [37] at the beginning of last century. However, it is quite remarkable that reconstruction of images from projections was not attempted until 1940. In 1972, Hounsfield [21] proposed the first method that allowed to image plane slices of the internal structure of a human body using X-rays projections. The Nobel prize in physiology and medicine in 1979 awarded to Hounsfield and Cormack [8] for their work on the initial computed tomography device signifies the importance that this technique has in the medical field. Since then, the use of computed tomography was extended to a wide number of industrial applications [10], among which food industry [38], materials science [29], geosciences [7] and fusion diagnostics [19, 33, 30, 32].

Medical and industrial computed tomography is based on the same underlying principles but differs in the acquisition system layout, as well as in the stimulus that is used to obtain the projections, which is X-rays in medical CT, but can also be another physical source. In fact, the reflected or transmitted signal of any source which produces a measurable response of the scanned object can be used. The feasibility of using ultrasounds [25, 22, 26], the photo elastic effect [41], or even ground-penetrating radar emitting mi-

crowaves [1] has been proved to be effective to visualize internal patterns of objects. Different sources interact with different properties, which can include, besides mass density, electrical, acoustic, and electromagnetic properties of materials and tissues. For instance, in nuclear medicine, the interest is focused on reconstructing a cross-sectional image of radioactive isotope distributions within the human body [12], while in magnetic resonance the magnetic properties of the object is reconstructed [15]. The photelastic effect instead was used to determine the plane stress distribution in an optical fiber [35]. The most fundamental distinction between medical CT and fusion diagnostics lies in the nature of the signal: while medical tomography is primarily concerned with absorption, fusion diagnostics focuses on emission. The main difference between the mentioned sources is that some of them are non-diffracting, like X-rays, which travel in straight lines [4], while some others are diffracting, like ultrasounds, that is, they are scattered in all directions, resulting in a more complex inverse problem [24].

To convert the measured response data into an image, appropriate processing is necessary. The most popular algorithms that are in use can be classified into two major categories, which are iterative and analytical methods [20]. Iterative methods are numerical reconstruction algorithms that employ a model to predict the measured data from an initial estimate and then refine the image estimate to better fit the measured data. This process is repeated iteratively until the image quality meets a predetermined standard or until further iterations do not significantly change the image [39, 40]. Starting from the Algebraic Reconstruction Technique (ART) [17], research continues to evolve with refinements and developments, such as the customized Simultaneous Algebraic Reconstruction Technique (SART) [13], and the Simultaneous Iterative Reconstruction Technique (SIRT) [42]. Analytical algorithms are based on mathematical models and explicit formulas that directly compute the image from the acquired projection data. They can be further categorized into two groups: the direct or Fourier-Transform-based methods [43, 18] and the Back-Projection-based algorithms, like Simple Back Projection (SBP), Linear Back Projection (LBP) and Filtered Back Projection (FBP) [34, 42]. Fourier-Transform-based algorithms rely on the relationship between the Fourier transform (FT) of the image and the Fourier transform of its projections. More specifically, projections are obtained using Radon transforms [24], that is by computing line integrals of the density function. A more detailed discussion of the literature on the classification of CT algorithms is beyond the scope of this paper. The interested reader can

consult [26, 11] for a more exhaustive presentation of the matter.

2. Application of tomography to fusion diagnostics

In tokamaks, reconstruction of X-ray emission relies on data collected by line-integrated detectors — such as pinhole X-ray cameras, bolometers, and neutron cameras— arranged across a cross-section of the plasma. Unlike medical imaging systems, which benefit from unrestricted access to the subject from all angles, fusion diagnostics is constrained by the geometry of the tokamak. The number and placement of detectors are limited by the physical design of the device, leading to a much lower spatial resolution than that achievable in medical tomography. As a consequence of this sparse and uneven data, plasma tomography requires additional assumptions, such as smoothness or regularity of the emission field, regardless of the inversion technique employed [23].

Under these circumstances, analytical and iterative methods faced various challenges, prompting the development of dedicated numerical techniques specifically tailored for 2D cross-sectional plasma tomography. These numerical approaches rely on an algebraic formulation of the image reconstruction problem, where each equation represents the equality between a known measured integral and the sum of emissions along a corresponding line of sight. However, the resulting system is typically underdetermined and the problem ill-posed, with high sensitivity to data error. To address this issue, various regularization techniques are employed to constrain the solution space, often by promoting smoothness or limiting complexity in the results. Among the robust regularization methods used in plasma tomography, Tikhonov regularization, Maximum Likelihood estimation, and Fisher information-based approaches are particularly noteworthy [32, 30]. In addition, there have been efforts to apply neural networks and deep learning techniques to tomographic reconstruction, offering promising alternatives for handling the inherent challenges of the problem [33].

In this work, we aim to apply tomographic techniques to reconstruct the light emission density recorded during the PROTO-SPHERA experiment deployment [2], conducted at the ENEA laboratories (Italian National Agency for New Technologies, Energy and Sustainable Development) in Frascati, Italy. During the experiment, the plasma emits photons, whose light is captured by six cameras arranged with cylindrical symmetry. PROTO-SPHERA is an innovative magnetic confinement plasma experiment designed for re-

107 search in controlled thermonuclear fusion. Its goal is to form a plasma torus
 108 not around a solid metal centerpost, as in traditional tokamaks, but around a
 109 plasma centerpost. This setup introduces significant differences compared to
 110 the reconstruction of X-ray emission density in tokamaks. Firstly, PROTO-
 111 SPHERA provides unobstructed visual access from any viewing angle. Sec-
 112 ondly, the plasma produced by PROTO-SPHERA has emission concentrated
 113 in the visible and ultraviolet spectrum. Last but not least, the optical sys-
 114 tem uses lenses, producing a sharp image with limited perspective effect, due
 115 to the distance between plasma and cameras. The image that can then be
 116 treated as a parallel projection of light emission.

117 To the best of our knowledge, fusion plasma tomography, was only carried
 118 out on sections of the torus. Here, we employ the imaging technique based on
 119 a corollary to Fourier slice theorem in 3D, also called Fourier Section Theorem
 120 [6, 44] and based on an n th-dimension formulation of the Radon transform,
 121 provided by Radon himself: by calculating 3D inverse Fourier transforms
 122 of 2D Fourier transforms of plane projections, the 3D image of the scanned
 123 object can be reconstructed. This differs from the classical tomographic
 124 approach [28, 14], where a 3D image is reconstructed by assembling stacks of
 125 images deriving from reconstructed images on several parallel planes. This
 126 work represents a first step toward 3D reconstruction of the plasma visible
 127 light emission density, which has never been attempted before. The resulting
 128 images enabled the observation of important aspects of plasma formation
 129 and arrangement.

130 This paper is organized into two sections: Section 3 revises the method of
 131 image reconstruction in 3D based on a corollary of the Fourier slice theorem,
 132 and Section 4 reports the experimental results. For the ease of the reader,
 133 in Subsection 3.1, the 3D continuous Radon transform is recalled, then in
 134 Subsection 3.2 the lemma of the Fourier slice theorem that is at the base of
 135 the image reconstruction method employed is presented. In Subsection 3.3
 136 proof of the effectiveness of the reconstruction method is provided by using
 137 numerical data. Subsection 4.1 describes the PROTO-SPHERA experiments,
 138 and Subsection 4.2 illustrates how the tomographic reconstruction enabled
 139 to prove the sustainment of the plasma torus around the centerpost and
 140 provided further useful information on the dynamics of confined plasma.

141 3. Analytical direct 3D image reconstruction

142 3.1. 3D continuous Radon Transform

143 For simplicity's sake, the 2D Radon transform is first introduced. Let us
 144 consider a plane object - or a slice of a 3D object - with density function
 145 $f(x, y)$ (Figure 1). Its projections onto a line inclined by an angle θ with
 146 respect to the x axis are obtained by integrating $f(x, y)$ on the set of parallel
 147 lines (rays) that are inclined by an angle $\theta + \pi/2$ with respect to the x axis
 148 and at a distance t from the origin of the coordinate system (bold line in
 149 Figure 1). The equation of a ray is:

$$t = x \cos\theta + y \sin\theta. \quad (1)$$

150 The projection of the function $f(x, y)$ is then defined as:

$$p(\theta, t) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x, y) \delta(x \cos\theta + y \sin\theta - t) dx dy, \quad (2)$$

151 where δ is the Dirac's delta function. Equation (2) represents the 2D con-
 152 tinuous Radon transform $[\mathcal{R}_2 f](\theta, t)$ of the function f , which is depicted
 153 schematically in Figure 1 by the blue curve subtending the sky blue area. It
 154 is often preferred to integrate in the rotated coordinate system (t, s) , where
 155 the projection can be written as:

$$p(\theta, t) = \int_{-\infty}^{\infty} f(t, s) ds. \quad (3)$$

156 Geometrically, the continuous 2D Radon transform maps a function in $\mathbb{R}^2$
 157 into the set of its line integrals in $\mathbb{R}^2$. The reconstructed image function
 158 $f(x, y)$ is formally obtained by inverting the Radon transform:

$$f(x, y) = [\mathcal{R}_2^{-1} p](\theta, t). \quad (4)$$

159 Let us now consider a 3D object with density function $f(x, y, z)$ (Figure
 160 2). Its projections are obtained by integrating $f(x, y, z)$ on the plane Σ that
 161 is orthogonal to the versor α and at a distance d from the origin of the coordi-
 162 nate system. The versor has direction cosines $\alpha = (\cos\theta \cos\phi, \sin\theta \cos\phi, \sin\phi)^T$,
 163 where ϕ is the angle that α forms with the z axis, and θ is the angle that the
 164 projection of α onto the xy plane forms with the x axis. The equation of a
 165 plane orthogonal to α and at a distance d from the origin is:

$$d = x \cos\theta \cos\phi + y \sin\theta \cos\phi + z \sin\phi. \quad (5)$$

166 In an analogy with Equation (2), the projection of the function $f(x, y, z)$
 167 associated with the couple (α, d) is then defined as:

$$p(\alpha, d) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x, y, z) \delta(x \cos\theta \cos\phi + y \sin\theta \cos\phi + z \sin\theta - d) dx dy dz. \quad (6)$$

168 The area over which the previous integral is computed is included in the
 169 dashed line in Figure 2. Equation (6) represents the 3D continuous Radon
 170 transform $[\mathcal{R}_3 f](\alpha, t)$ of the function f . Geometrically, the continuous 3D
 171 Radon transform maps a function in $\mathbb{R}^3$ into the set of its plane integrals
 172 in $\mathbb{R}^3$. The reconstructed image function $f(x, y, z)$ is formally obtained by
 173 inverting the Radon transform:

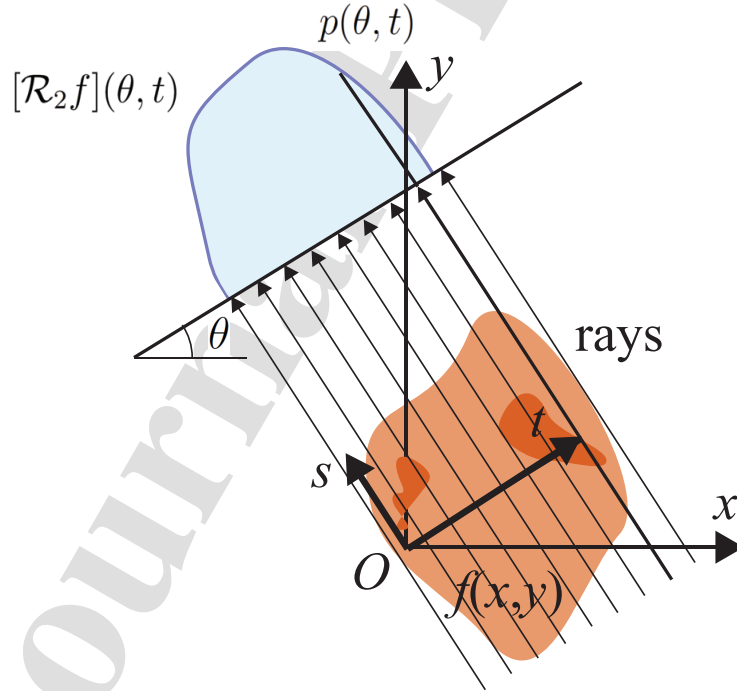


Figure 1: Projection of a plane density function onto a line.

$$f(x, y, z) = [\mathcal{R}_3^{-1}p](\boldsymbol{\alpha}, d). \quad (7)$$

174 It is important to note that in real applications we won't be dealing with plane
 175 integrals. In fact, $\mathcal{R}_3$ can be obtained by computing the $\mathcal{R}_2$ of the intersection
 176 between f and the plane Σ , and then integrating over lines orthogonal to the
 177 rays, which are displayed in Figure 3 as yellow lines. The line integrals are
 178 much easier to be obtained in practical measurements. They correspond to
 179 projections onto detector planes taken from different angles, or, in the present
 180 case, to pictures taken from different cameras.

181 With the definition introduced above, the detector plane still has a rota-
 182 tional degree of freedom around the detector normal. Different from what is
 183 done in [6], here we integrate in the rotated coordinate system (t, s, r) , whose
 184 axes t and s coincide with the tangent lines to the parallels and meridians,
 185 respectively, of an imaginary sphere centered in the origin of the system of
 186 coordinates, at the point of intersection between the sphere and the versor

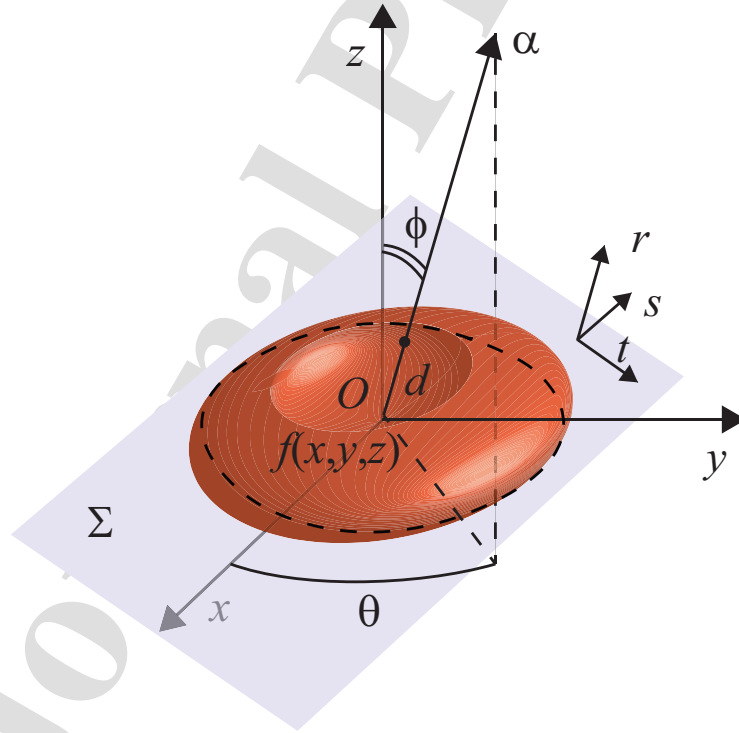


Figure 2: Projection of a 3D density function onto a plane.

187 α , with the orientation of the r axis coinciding with that of α (Figure 3,
 188 upper left). The plane (t, s) is the detector plane. The transformation of
 189 coordinates from (x, y, z) to (t, s, r) can be written as:

$$\begin{bmatrix} t \\ s \\ r \end{bmatrix} = \begin{bmatrix} \cos\phi & 0 & \sin\phi \\ 0 & 1 & 0 \\ \sin\phi & 0 & \cos\phi \end{bmatrix} \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}. \quad (8)$$

190 Moreover, the distance d can be expressed in the (t, s, r) system of coordi-
 191 nates as $d = \sqrt{t^2 + r^2}$. The ray integral is then expressed as follows:

$$p(\theta, \phi, t, r) = \int_{-\infty}^{\infty} f(t, s, r) ds. \quad (9)$$

192 The integral $p(\theta, \phi, t, r)$ is in practice a line projection of the 3D function.
 193 The set of all the line projections on the plane (t, r) for given θ and ϕ will
 194 be denoted as $P_{\theta, \phi}$.

195 3.2. 3D Fourier slice theorem

196 The extension of the Fourier slice theorem to higher dimensions, enables
 197 direct reconstruction of 3D objects. The 3D Fourier Slice Theorem can be

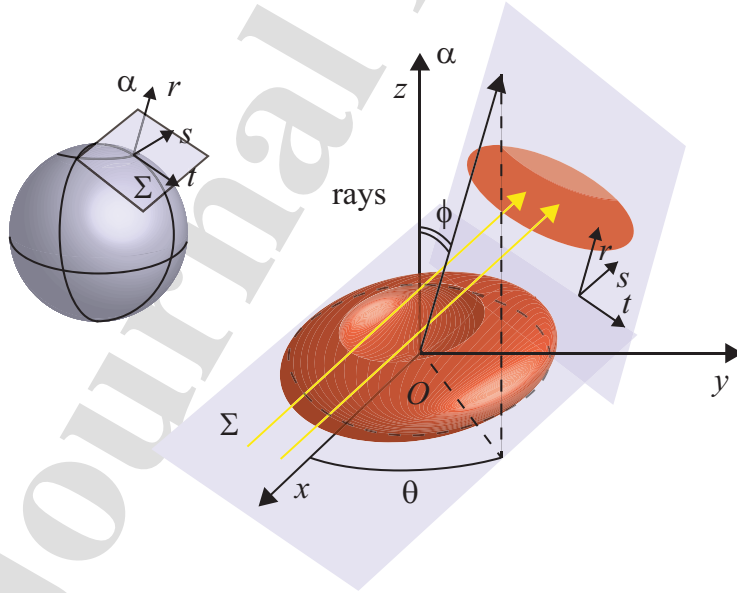


Figure 3: Ray integrals over a 3D function.

198 stated as follows.

199 **Theorem 1.** *A line slice of the 3D Fourier transform of the function $f(x, y, z)$*
 200 *equals the 1D Fourier Transform of the 3D Radon transform $[\mathcal{R}_3 f](\alpha, t)$ at*
 201 *the same angle.*

202 For proof, the interested reader can consult [44, 5]. However, the theorem
 203 in this form is of little use for practical purposes. A useful corollary can be
 204 stated. See for instance [6].

205 **Corollary 1.1.** *A plane slice of the 3D Fourier transform of the density func-*
 206 *tion $f(x, y, z)$ equals the 2D Fourier Transform of the set of line projections*
 207 *at the same angle.*

208 *Proof.* Let us consider the line projection of a 3D density function $P_{\theta, \phi}$, and
 209 take its 2D Fourier Transform ($\mathcal{F}_2$):

$$[\mathcal{F}_2 P_{\theta, \phi}](\mu, \nu) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} P_{\theta, \phi}(t, r) e^{-2\pi i(\mu t + \nu r)} dt dr, \quad (10)$$

210 where μ and ν are the wavenumbers in the t, r directions, respectively and i
 211 is the imaginary unit.

212 By substituting equation (9) into equation (10), we get:

$$[\mathcal{F}_2 P_{\theta, \phi}](\mu, \nu) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \left[\int_{-\infty}^{\infty} f(t, s, r) ds \right] e^{-2\pi i(\mu t + \nu r)} dt dr. \quad (11)$$

213 Based on the transformation (8), the rotated coordinate system (t, s, r) can
 214 be converted back into the initial (x, y, z) system:

$$[\mathcal{F}_2 P_{\theta, \phi}](\mu, \nu) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x, y, z) e^{-2\pi i k} dx dy dz \quad (12)$$

with $k = \mu \cos \phi \cos \theta x + \mu \cos \phi \sin \theta y + \mu \sin \phi z$
 $+ \nu \sin \phi \cos \theta x + \nu \sin \phi \sin \theta y + \nu \cos \phi z.$

215 It is obvious that, with a transformation of coordinates, the right hand side
 216 of (12) represents the 3D Fourier Transform $\mathcal{F}_3$ of f at wavenumbers u
 217 $= \mu \cos \phi \cos \theta + \nu \sin \phi \cos \theta$, $v = \mu \cos \phi \sin \theta + \nu \sin \phi \sin \theta$ and $w = \mu \sin \phi + \nu \cos \phi$:

$$[\mathcal{F}_3 f](u, v, w) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x, y, z) e^{-2\pi i(ux + vy + wz)} dx dy dz, \quad (13)$$

218 which proves the corollary. The quantities u, v and w are wavenumbers in
 219 directions x, y and z , respectively. $\square$

The interesting consequence of this corollary is that, given the plane projections, calculating their 2D Fourier Transform, and then computing the inverse 3D Fourier Transform of the resulting set of 2D Fourier transforms after a transformation of coordinates, the original image is directly reconstructed. Figure 4 reports a flowchart of the described image reconstruction procedure. The following subsection provides numerical proof.

3.3. Numerical example of the 3D direct reconstruction

Let us consider an input density function modeled as a cube with a uniform density of 0.5, side length of 0.6, and centered at the origin of the coordinate system (Figure 5a). Inside this cube, two smaller spheres are embedded: one with a density of 0.3, a radius of 0.1, and centered at coordinates (0.05, 0.05, -0.2); the other with a density of 0.1, a radius of 0.05, and centered at (-0.1, 0, 0.1). These variations in density are used to verify the accuracy of the reconstruction, ensuring it can effectively distinguish between different density regions. Although the spheres are not visible from the exterior of the solid cube, Figure 5b illustrates their positions by rendering the cube with partial transparency. It is important to note that for

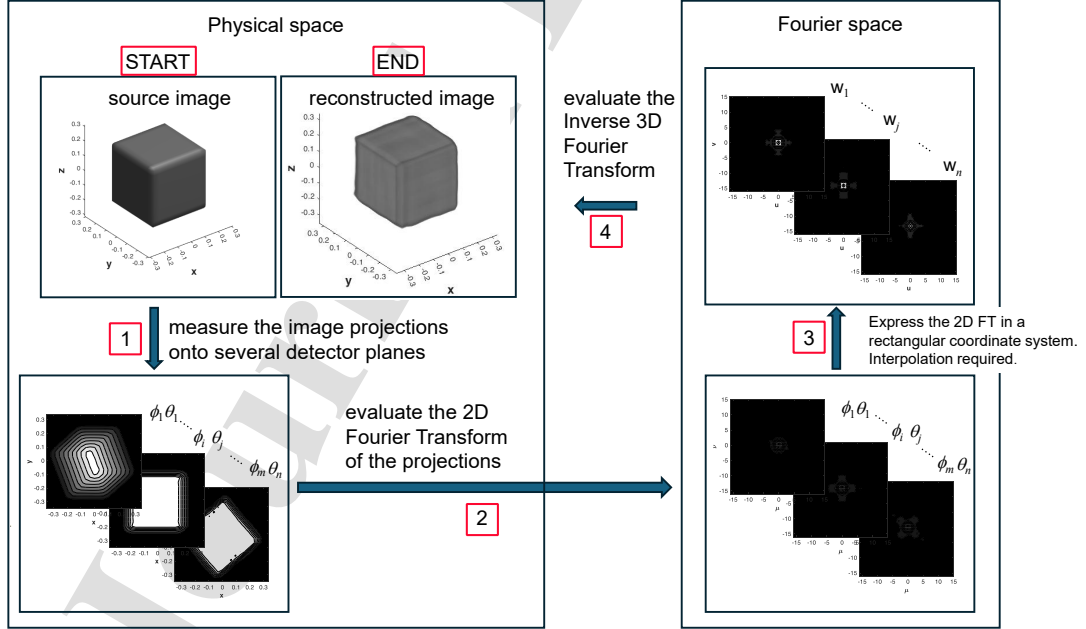


Figure 4: Flowchart of the image reconstruction procedure.

237 this numerical example, the theoretical framework developed for continuous
 238 systems in the previous sections has been adapted for use with discrete data.
 239 In this case, we employed a total of $N = 64 \times 64 \times 64$ sampling points, with
 240 64 points along each side of the cube. Additionally, $N_{deg} = 12 \times 12 = 144$
 241 spherical projections were taken, with 12 projections for each angle θ and ϕ ,
 242 respectively. The angular step size was kept constant over π , set to $\pi/12$.

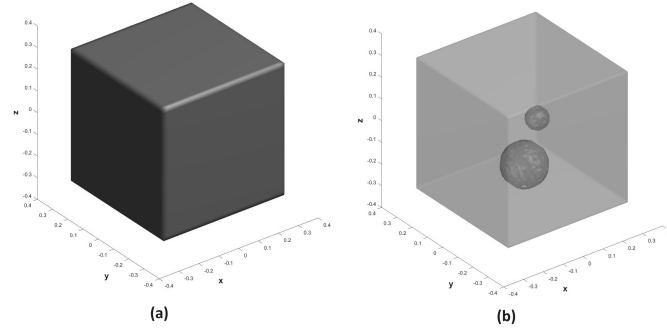


Figure 5: a) 3D density function b) density function made partially transparent to show the embedded spheres

243 Figures 6a and b provide samples of the spherical projections used for
 244 reconstruction, taken at angles $(\theta, \phi) = (\pi/2, 0)$ and $(2\pi/3, 0)$, respectively.
 245 Note that the color scale extends beyond 0.5 because it represents line integrals
 246 of the density function over the entire domain, rather than local values.
 247 Regions of lower density show in fact lower integral values. At this point,
 248 it is necessary that 144 2D Fourier Transforms of the spherical projections
 249 are computed. These Transforms are defined on the plane (μ, ν) (Equa-
 250 tion (11)). However, computing the inverse 3D Fourier Transform requires
 251 knowledge of the data on a Cartesian grid of wavenumbers (u, v, w) (Equa-
 252 tion (12)). Therefore, after the transformation of coordinates, discrete data
 253 available has to be interpolated to obtain the values onto a regularly spaced
 254 Cartesian grid, then 3D inverted. In this way, the reconstructed density
 255 function shown in Figure 7a is obtained, matching the original function from
 256 Figure 5a. Figures 7b, c, and d present cross-sections of the reconstructed
 257 density function along two vertical planes (b,c) and a horizontal plane (d)
 258 passing through the origin, with the corresponding color scale. This Figure
 259 shows that the embedded spheres are exactly reconstructed not only in terms
 260 of position, but also in terms of density. See in fact that the large sphere has

high density, while the small one low.

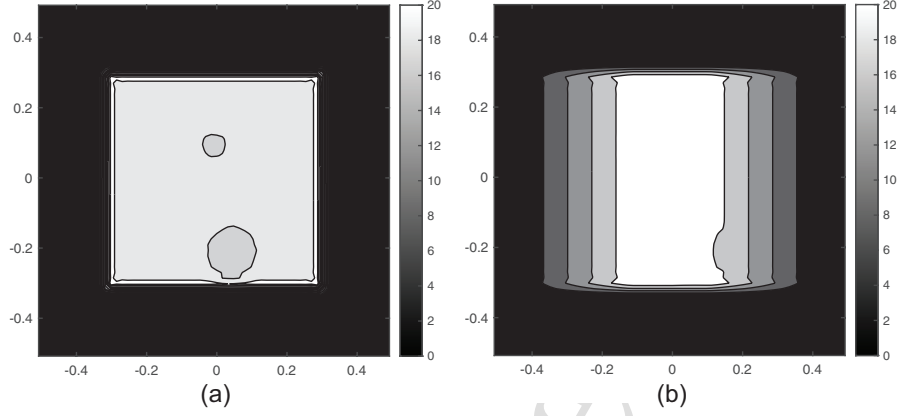


Figure 6: A couple of spherical projections pertaining to angles a) $\theta = \pi/2$, $\phi = 0$ and b) $\theta = 2\pi/3$, $\phi = 0$

The previous example was implemented using a large number of projections. While reducing the number of projections results in a slightly blurred reconstructed image, even a very limited number of projections still provides a sufficiently clear reconstruction. This effect is illustrated in Figure 8, which shows the reconstructed density function with varying numbers of projections. The source density function is depicted in Figure 8 (a). It is shaped as a cube with a unit density, a side length of 0.2, and is centered at the origin of the coordinate system. A sphere with unit density, a diameter larger than the cube's side, and centered at the origin, was subtracted from the density function, causing its volume to intersect with the cube's faces. Although the image quality decreases as the number of projections is reduced, the shape remains distinctly recognizable even with a very limited number of projections, as few as 9 (Figure 8d).

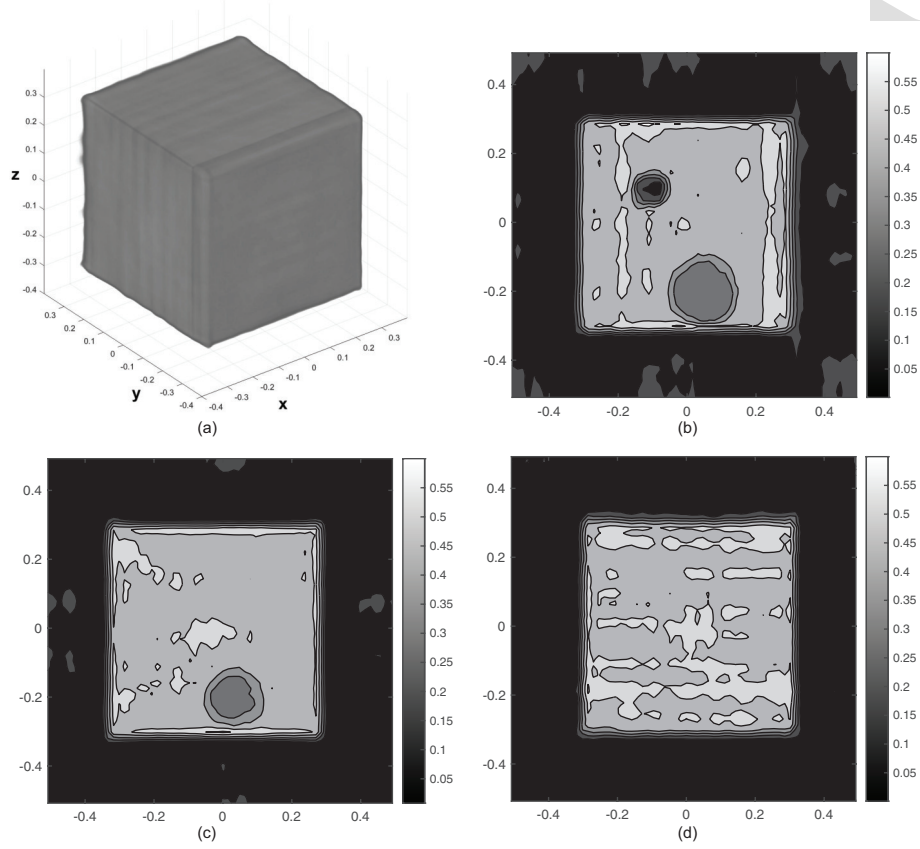


Figure 7: a) Reconstructed image, b) vertical cross section on the xz plane, c) vertical cross section on the yz plane, d) horizontal cross section on the xy plane.

275 4. 3D Reconstruction of light emission density in PROTO-SPHERA 276 experiment

277 4.1. The PROTO-SPHERA experiment

278 PROTO-SPHERA (Spherical Plasma for HElicity Relaxation Assessment)
279 is an innovative magnetic confinement plasma experiment aimed at advancing
280 controlled thermonuclear fusion research. Its goal is to form plasma within
281 a confined environment. Unlike tokamaks, where the plasma forms around a
282 metal centerpost, in PROTO-SPHERA, the plasma forms around a plasma
283 centerpost [3, 27, 31]. Tokamaks, the most extensively studied magnetic fu-
284 sion configurations, have a non-simply connected geometry, where a central

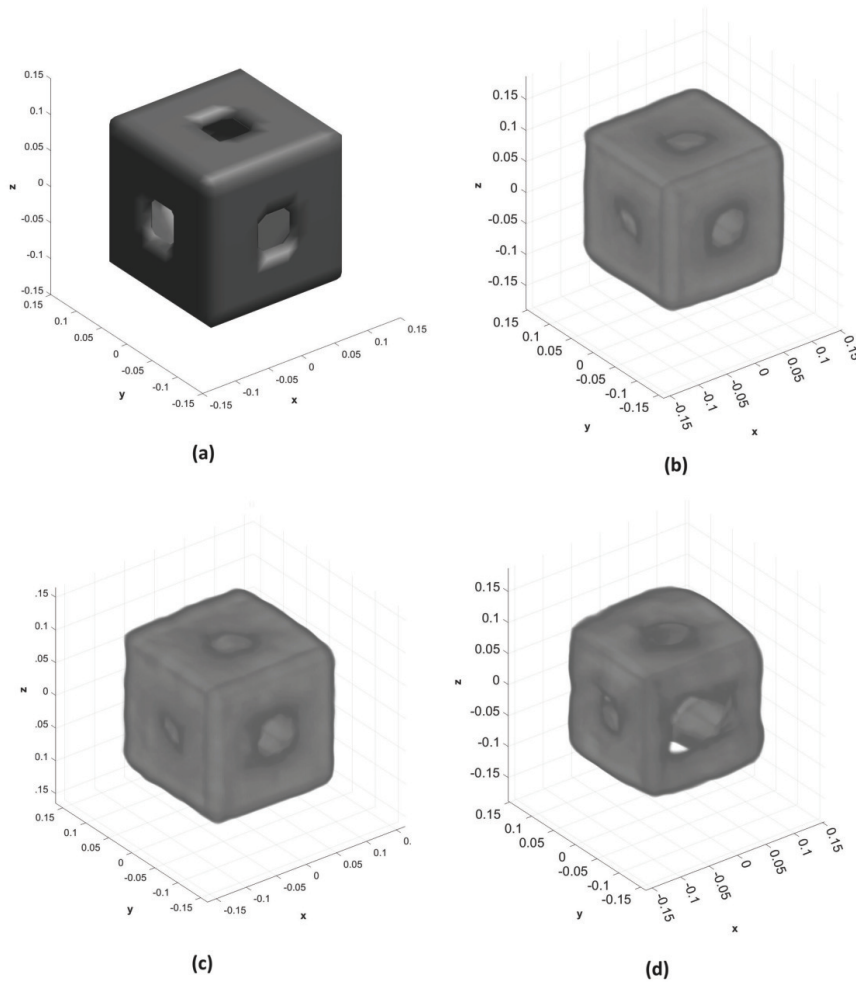


Figure 8: a) Source image and Reconstructed image with b) $12 \times$, c) 6×6 and d) 3×3 projections.

post—containing the inner part of the toroidal magnet and the ohmic trans-
former—connects the plasma torus. The development of simply connected
magnetic configurations relevant for fusion would significantly simplify the
design of fusion reactors. Additionally, the PROTO-SPHERA experiment
seeks to emulate astrophysical plasmas, such as jet and torus structures, in
a laboratory setting [2].

Figure 9 shows a schematic of the PROTO-SPHERA experiment, which consists of a transparent cylindrical vacuum chamber measuring 2.5 meters in height and 2.0 meters in diameter, held by a metal structure (Figure 9a). The vacuum chamber, surrounded by four columns, is equipped with electrodes at its ends and internal poloidal field coils (Figure 9b). The operating sequence proceeds as follows: the tungsten filaments of the cathode are heated to 2600-2900°C over 30 seconds, while the current in the poloidal field coils is ramped up to its final value. Gas is then introduced through a hollow section in the anode, reaching a pressure of 10^{-2} to 10^{-1} mbar, and a total voltage of 100-350 V is applied between the electrodes, yielding a centerpost current up to 10 kAmp. Under these conditions, cold plasma with an energy in the range of 20-30 eV forms, with an emission concentrated in the visible and ultraviolet spectrum.

To study the plasma formation process and the magneto-hydrodynamic phenomena occurring in the experiment, six high-speed cameras are positioned on the equatorial plane of the chamber, spaced 60 degrees apart (Figure 10). It should be noted that in an ideal scenario free from errors, two cameras placed exactly 180 degrees apart would provide redundant information. However, real-world measurements are inevitably affected by several sources of error. These include, but are not limited to, slight misalignments in camera positioning, optical distortions, slight differences in vignetting due to material objects present inside the vessel, thermal expansion, structural vibrations, and electronic noise. In this context, having views from opposite sides of the plasma increases the robustness of the reconstruction and helps mitigate the effects of such errors. Each camera has a resolution of 640x480 pixels and can capture 600 frames per second. Sample images recorded by the cameras at a specific time instant are shown in Figures 11a and 12a for shots with Helium and Hydrogen, respectively.

Combining the data from the cameras with measurements from a Langmuir probe has provided valuable insights into the dynamics of the plasma torus formation, including its size and emission spectrum. Numerous discharges were produced and observed throughout the experimental campaign.

The images captured by the six cameras are assumed to be parallel projections onto vertical planes corresponding to the camera views of the visible-light emission density, which plays the role of the density function f (Figures 11a and 12a). For each time instant, exploiting the corollary of the Fourier Slice Theorem of Section 3, these source images are 2D-Fourier transformed, then 3D-Fourier inverted to obtain the density of the visible light emitted

by the plasma. The only change with respect to the procedure described in Section 3.2 is that, given the positioning of the cameras, the inversion has to be based on a cylindrical set of parallel projections instead of a spherical set, with ϕ fixed. Figures 11b and 12b show the reconstructed plasma light emission distribution derived from the images on the left for Helium and Hydrogen, respectively. It is important to note that the original images contain three layers of information, corresponding to the RGB color channels. The reconstructed images in Figures 11b and 12b are false-color representations, created by combining the RGB channels linearly with the following weighting: 100% Red, 86% Green, and 25% Blue [2].

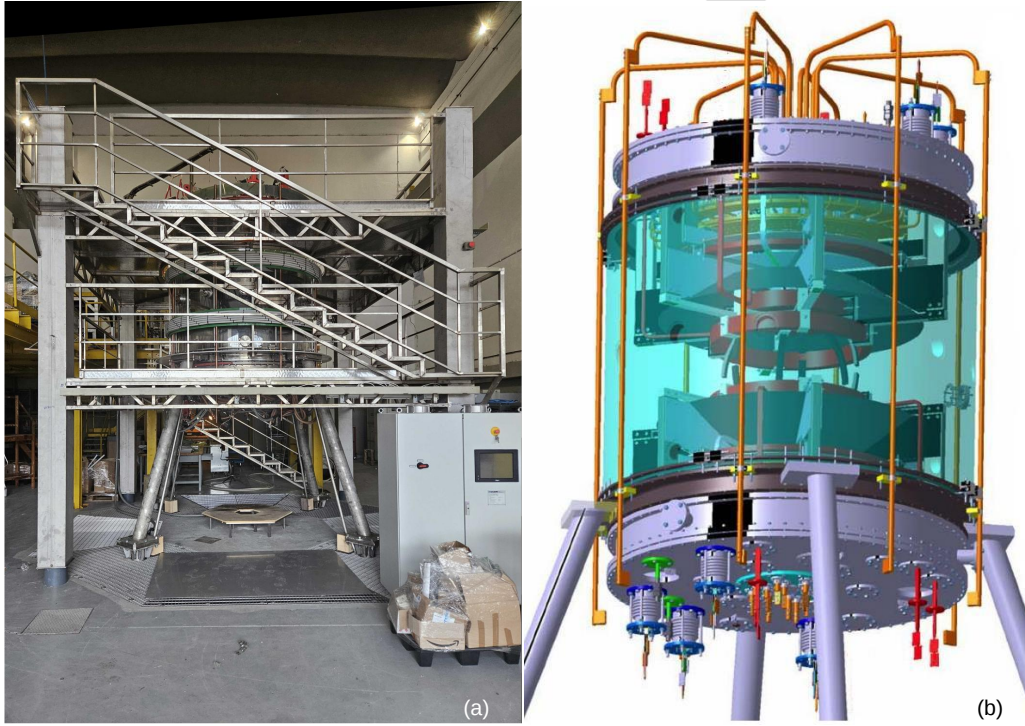


Figure 9: a) Picture of PROTO-SPHERA setup and b) schematic of the machine

4.2. Results and discussion

The proposed 3D tomography technique enabled detailed observation of the plasma torus formation process. The sequence begins with the formation of a plasma column (pinch or centerpost) at a current lower than 10 kAmps.

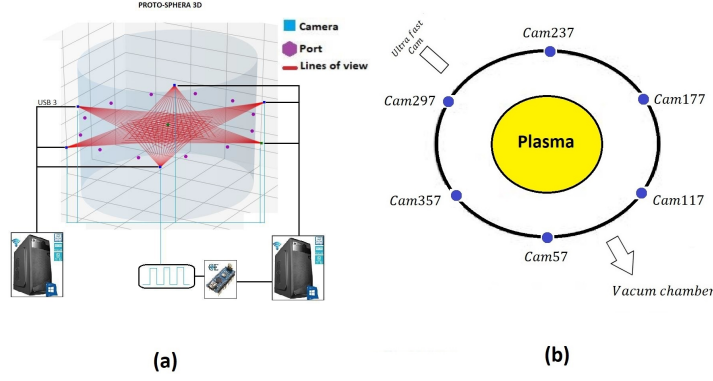


Figure 10: a) Position of the cameras in axonometry b) and plan

As the current reaches this value, the magnetic field lines bend, causing the pinch to become unstable, which leads to the formation of a torus around the centerpost. This progression is visualized in Figure 13 a-h, showing the reconstructed light emission density at eight sequential time steps (each 1/600 s apart) during a Helium plasma discharge. These findings align closely with predictions from magnetohydrodynamic (MHD) models, in particular those presented in the work by García-Martínez et al. [16], whose magnetic field lines at different Lundquist numbers present a striking topological similarity with our emission density reconstruction (see Figure of 8 [16]). The entire torus formation occurs within approximately 8 ms, five time steps intercurring from the arrangements of Figure 13 b to g. The presence of both jets and the torus strongly parallels astrophysical phenomena, as discussed in [2].

Once the spatial distribution of light emission density is known for each time instant, cross-sections can be derived, revealing further insights into plasma dynamics. Figure 14 shows an image of the Helium plasma captured by one of the six cameras (a) and a vertical cross-section (b) of the reconstructed light emission density in true color, taken at the same moment. To create this image, each of the three RGB channels was processed independently to generate a corresponding density function for each color. This cross-sectional view reveals the green color, which was obscured in the camera pictures by the intense orange glow of the 'ionization peel'—the layer between the plasma and the surrounding gas cushion. The correctness of the reconstruction is proved by the appearance of the Langmuir probe and of the copper bars on the right hand side of the picture. These are external

conducting copper bars (3 cm in diameter) that connect the two electrodes.

Figure 15 presents vertical cross-sections of the plasma light emission density at the same time instants as Figure 13. These images further confirm the sequence of events leading to the formation of the torus, as described in the previous paragraph. Specifically, Figure 15 illustrates the emergence of a stable pinch (a-b), its progression towards instability (c-f), and ultimately the arrangement of the torus around the centerpost (d-h). Similar patterns, with comparable duration, were observed in other plasma discharges [2, 36]. Figure 16 presents a horizontal cross-section at the equatorial level, enabling observation of the same phenomena from a different perspective. In Figures 16d and 16e, the pinch's instability is particularly evident, as shown by the loss of axial symmetry and the presence of green regions on the right side of the images. The phenomena described, featuring a centerpost around which a toroidal structure forms, show similar morphology with some theoretical results presented in [16], concerning MHD models, in particular, with those displaying the ratio between current density and magnetic field of an equatorial section. Further experimental confirmation of our finding comes from the PROTO-SPHERA research group, who obtained similar results using different reconstruction techniques. Specifically, Figure 17 reproduces a Figure of the work [9], and displays the plasma luminosity obtained through tomographic inversion using Zernike polynomials. The data in this figure refers to earlier experiments conducted with Argon gas and are shown in false colors. Data support the presence of the toroidal light emission structure and of the centerpost observed in our study. In the experiments reported in [9] not all the details of the setup had been finalized.

Additionally, the horizontal cross-section analysis provides insights into plasma stability and torus thickness. Figure 18 illustrates selected horizontal cross-sections of Helium plasma at the equatorial level across sequential times, clearly depicting the torus encircling the centerpost. Notably, the light emission intensity varies over time, suggesting potential correlations with plasma density and stability. These images also allowed for the measurement of torus thickness, defined as the difference between the outer and inner circumferences' radii, which was observed to be between 0.25 and 0.35 m. Note that, in Figure 18, the angle brilliance visible at the four corners is likely due to the reflection of light from the four metal columns supporting the vessel. Similar behaviors were noted for Hydrogen plasma discharges, with an equatorial horizontal cross-section provided in Figure 19 for reference.

404 5. Conclusions

405 An imaging technique based on a corollary of the Fourier slice theorem
 406 was applied to reconstruct the light emission distribution of magnetically
 407 confined plasma in three dimensions. The derived algorithm was used on pic-
 408 tures capturing the visible light emission from Helium and Hydrogen plasmas,
 409 recorded by six cameras as part of the PROTO-SPHERA experiment at the
 410 ENEA Frascati laboratories. The results demonstrate that six images taken
 411 by cameras arranged in cylindrical symmetry enable an accurate 3D recon-
 412 struction of the plasma's spatial distribution at different time instants during
 413 the experiment. We observed that the plasma torus forms in approximately
 414 8 ms, evolving from a kinked filament caused by the destabilization of the
 415 plasma centerpost. This 3D reconstruction allows for cross-sectional anal-
 416 ysis, revealing previously hidden green light emission from the core, which
 417 had been obscured by the outer orange ionized layer. Additionally, equa-
 418 torial cross-sections enabled the measurement of the torus thickness, which
 419 was found to range between 0.25 and 0.35 meters.

420 6. Acknowledgement

421 Annamaria Pau gratefully acknowledges the funding contribution from
 422 National Center on HPC, Big Data and Quantum Computing CN00000013.
 423 This manuscript reflects only the authors' views and opinions, neither the
 424 European Union nor the European Commission can be considered responsible
 425 for them.

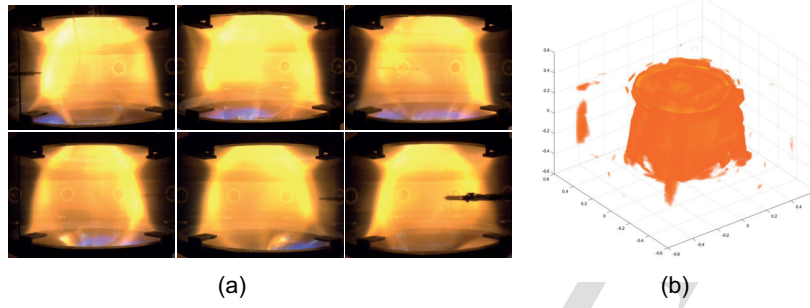


Figure 11: a) Pictures recorded by the six cameras at a given time instant (267 msec) of the PROTO-SPHERA experiment from shot 2505 of Helium b) 3D reconstruction of plasma light emission density

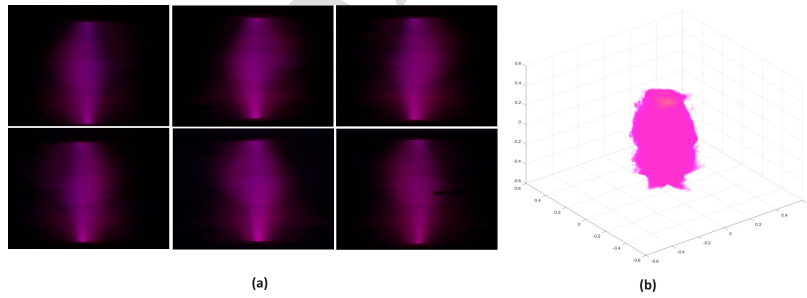


Figure 12: a) Pictures recorded by the six cameras at a given time instant (233msec) of the PROTO-SPHERA experiment from shot 2154 of Hydrogen b) 3D reconstruction of plasma light emission density

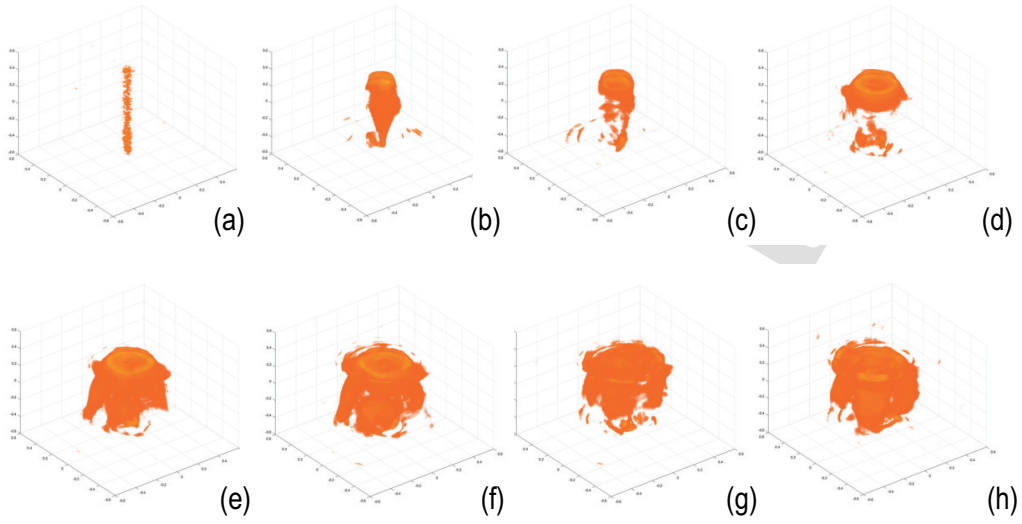


Figure 13: Reconstructed light emission density of Helium discharge 2505 at eight sequential time steps: stable pinch (a-b), pinch instability (c-f), formation of the plasma torus around the pinch (g-h)

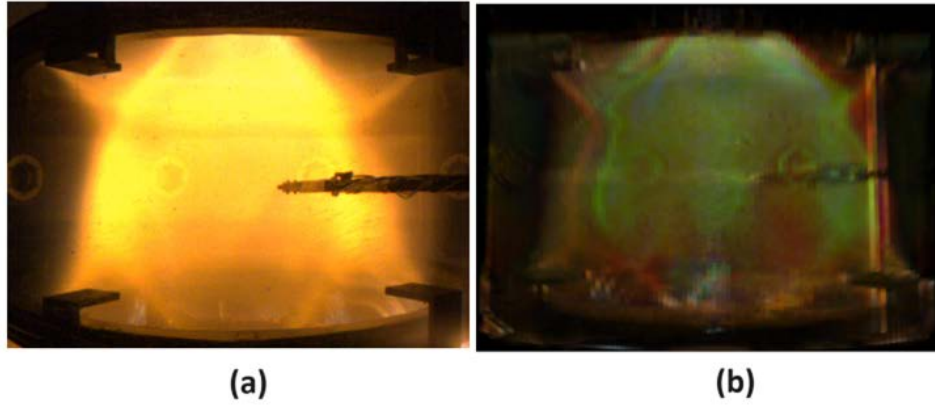


Figure 14: A projection of Helium plasma visible light emission captured by a camera during shot 2141 (a) and cross-section of the reconstructed light emission density in true colors (b).

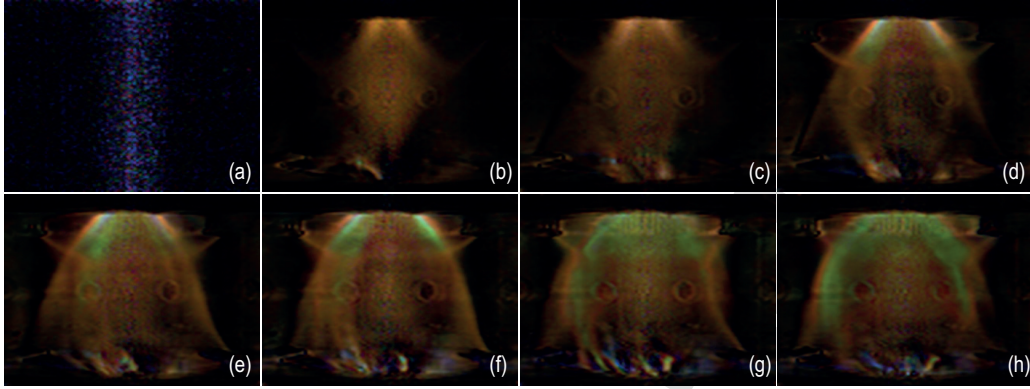


Figure 15: Vertical cross-sections of the reconstructed light emission density of Helium discharge 2505 at the same eight sequential time instants as Figure 12: stable pinch (a-b), pinch instability (c-f), formation of the plasma torus around the pinch (g-h)

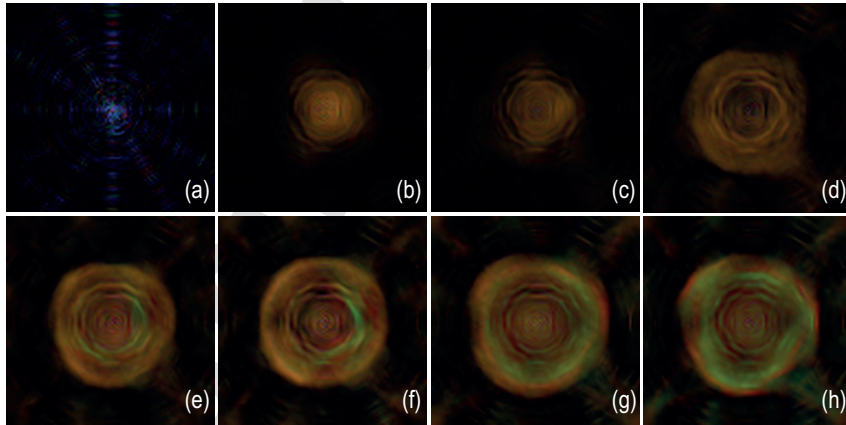


Figure 16: Horizontal cross-sections of the reconstructed light emission density of Helium discharge 2505 at the same eight sequential time instants as Figure 12: stable pinch (a-b), pinch instability (c-f), formation of the plasma torus around the pinch (g-h)

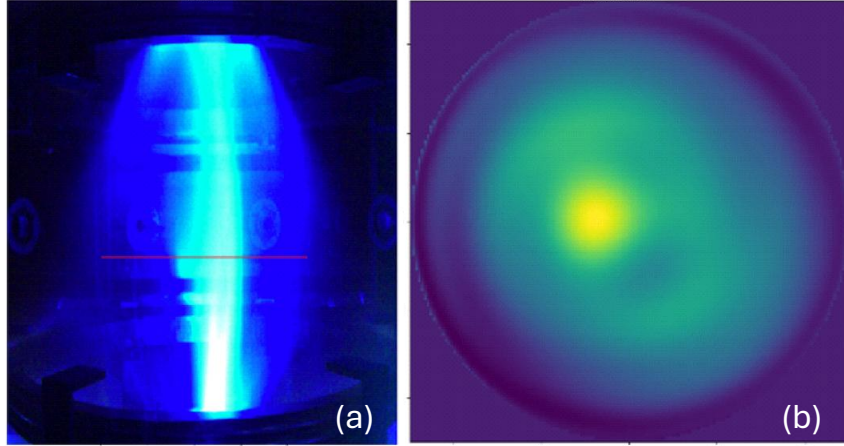


Figure 17: Picture and related equatorial distribution of Argon plasma luminosity obtained by Zernike polynomials inversion [9]

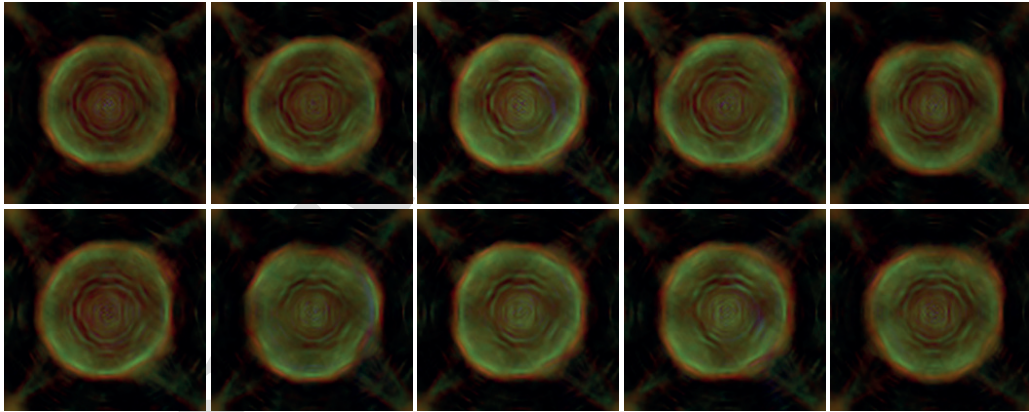


Figure 18: Sequence of reconstructed equatorial cross-sections of light emission density of Helium discharge 2505 at 10 time instants

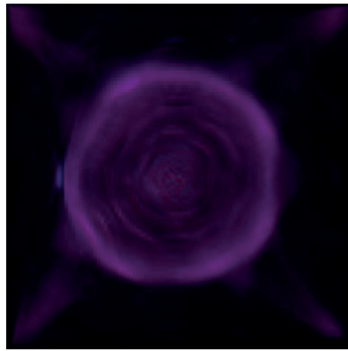


Figure 19: Equatorial cross-section of light emission density of Hydrogen discharge 2154

References

- [1] Amir M. Alani, Francesco Soldovieri, Ilaria Catapano, Iraklis Giannakis, Gianluca Gennarelli, Livia Lantini, Giovanni Ludeno, and Fabio Tosti. The use of ground penetrating radar and microwave tomography for the detection of decay and cavities in tree trunks. *Remote Sensing*, 11(18), 2019.
- [2] F. Alladio, P. Micozzi, L. Boncagni, A. Pau, S. Naghinajad, S. Macera, Y. Damizia, P. Buratti, F. Filippi, G. Galatola Teka, F. Giammanco, E. Giovannozzi, M. Iafrati, A. Lampasi, and P. Marsili. PROTO-SPHERA: a magnetic confinement experiment which emulates the jet + torus astrophysical plasmas. *Plasma Physics and Controlled Fusion*, 66(3), 2024.
- [3] Franco Alladio, Paolo Costa, Alessandro Mancuso, Paolo Micozzi, Stamos Papastergiou, and Francois Rogier. Design of the PROTO-SPHERA experiment and of its first step (MULTI-PINCH). *Nuclear Fusion*, 46:S613–S624, 2006.
- [4] Habib Ammari. Tomographic imaging with non-diffracting sources. an introduction. *Mathematics of Emerging Biomedical Imaging*, 28(10):95–106, 2008.
- [5] Amir Averbuch and Yoel Shkolnisky. 3D Fourier based discrete Radon transform. *Applied and Computational Harmonic Analysis*, 15(1):33 – 69, 2003.
- [6] T. Buzug. *Computed Tomography*. Springer: Berlin, Heidelberg, 2008.
- [7] V. Cnudde and M.N. Boone. High-resolution X-ray computed tomography in geosciences: A review of the current technology and applications. *Earth-Science Reviews*, 123:1 – 17, 2013.
- [8] A. M. Cormack. Early two-dimensional reconstruction and recent topics stemming from it. *Science*, 209:1482–1486, 1980.
- [9] Yacopo Damizia, Matteo Iafrati, Davide Liuzza, Luca Boncagni, Paolo Micozzi, and Franco Alladio. Optical tomography of the plasma on the PROTO-SPHERA experiment. *Fusion Engineering and Design*, 169, 2021.

- [10] L. De Chiffre, S. Carmignato, J.-P. Kruth, R. Schmitt, and A. Weckenmann. Industrial applications of computed tomography. *CIRP Annals - Manufacturing Technology*, 63(2):655 – 677, 2014.
- [11] S.R. Deans. *The Radon Transform and some of its applications*. Wiley: New York, 1983.
- [12] Musa Sani Musa Niyazi Şentürk Fatih Veysel Nurçin Dilber Uzun Ozsahin, Berna Uzun and Ilker Ozsahin. Evaluating nuclear medicine imaging devices using fuzzy PROMETHEE method. *Procedia computer science*, 120:699–705, 2017.
- [13] S. Donato, L. Brombal, L.M. Arana Peña, F. Arfelli, A. Contillo, P. De-logu, F. Di Lillo, V. Di Trapani, V. Fanti, R. Longo, P. Oliva, L. Rigon, L. Stori, G. Tromba, and B. Golosio. Optimization of a customized simultaneous algebraic reconstruction technique algorithm for phase-contrast breast computed tomography. *Physics in Medicine and Biology*, 67(9), 2022.
- [14] Mario D’Acunto, Antonio Benassi, Davide Moroni, and Ovidio Salvetti. 3D image reconstruction using Radon transform. *Signal, Image and Video Processing*, 10(1):1 – 8, 2016.
- [15] Jeffrey Fessler. Optimization methods for magnetic resonance image reconstruction: Key models and optimization algorithms. *IEEE signal processing magazine*, 37(1):33–40, 2020.
- [16] Pablo Luis García-Martínez, Leandro Gabriel Lampugnani, and Ricardo Farengo. Effect of the helicity injection rate and the Lundquist number on spheromak sustainment. *Physics of Plasmas*, 21(12), 2014.
- [17] Richard Gordon, Robert Bender, and Gabor T. Herman. Algebraic reconstruction techniques (ART) for three-dimensional electron microscopy and X-ray photography. *Journal of Theoretical Biology*, 29(3):471–476, IN1–IN2, 477–481, 1970.
- [18] David Gottlieb, Bertil Gustaffson, and Patrick Forssen. On the direct Fourier method for computed tomography. *IEEE Transactions on Medical Imaging*, 19(3):223–232, 2000.

- [19] R.S. Granetz and P. Smeulders. X-ray tomography on JET. *Nuclear Fusion*, 28(3):457 – 476, 1988.
- [20] B Gustaffson. Mathematics for computed tomography. *Physica Scripta*, T61:38– 43, 1996.
- [21] G. N. Hounsfield. Computerized transverse axial scanning (tomography): Part I. description of system. *British Journal of Radiology*, 46(3):1016–1022, 1973.
- [22] P. Huthwaite. Guided wave tomography with an improved scattering model. *Proceedings of the Royal Society A: Mathematical, Physical and Eng. Sciences*, 472(2195), 2016.
- [23] A. Jardin, J. Bielecki, D. Mazon, Y. Peysson, K. Król, D. Dworak, and M. Scholz. Implementing an X-ray tomography method for fusion devices. *European Physical Journal Plus*, 136(7), 2021.
- [24] A. C. Kak and M. Slaney. *Principles of Computerized Tomographic Imaging*. IEEE press, 1988.
- [25] Avinash C. Kak. Computerized tomography with X-ray, emission, and ultrasound sources. *Proceedings of the IEEE*, 67(9):1245 – 1272, 1979.
- [26] Mohd Taufiq Mohd Khairi, Sallehuddin Ibrahim, Mohd Amri Md Yunus, M. Faramarzi, Goh Pei Sean, Jaysuman Pusppanathan, and A. Abid. Ultrasound computed tomography for material inspection: Principles, design and applications. *Measurement: Journal of the International Measurement Confederation*, 146:490 – 523, 2019.
- [27] Alessandro Lampasi, Giuseppe Maffia, Franco Alladio, Luca Boncagni, Federica Causa, Edmondo Giovannozzi, Luigi Andrea Grosso, Alessandro Mancuso, Paolo Micozzi, Valerio Piergotti, Giuliano Rocchi, Alessandro Sibio, Benedetto Tilia, and Vincenzo Zanza. Progress of the plasma centerpost for the PROTO-SPHERA spherical tokamak. *Energies*, 9(7), 2016.
- [28] Xin Li, Yudong Lu, Xiaozhou Zhang, Wen Fan, Yangchun Lu, and Wangsheng Pan. Quantification of macropores of malan loess and the hydraulic significance on slope stability by x-ray computed tomography. *Environmental Earth Sciences*, 78(16), 2019.

- [29] E. Maire and P.J. Withers. Quantitative X-ray tomography. *Int. Materials Reviews*, 59(1):1 – 43, 2014.
- [30] Francisco A. Matos, Diogo R. Ferreira, and Pedro J. Carvalho. Deep learning for plasma tomography using the bolometer system at JET. *Fusion Engineering and Design*, 114:18 – 25, 2017.
- [31] Paolo Micozzi, Franco Alladio, Alessandro Mancuso, Vincenzo Zanza, Gerarda Apruzzese, Francesca Bpmbarda, Luca Boncagni, Paolo Burratti, Francesco Filippi, Giuselle Galatola Teka, Francesco Giammarco, Edmondo Giovannozzi, Andrea Grosso, Matteo Iafrati, Alessandro Lampasi, Violeta Lazic, Simone Magagnino, Simone Mannori, Paolo Marsili, Paolo Piergotti, Giuliano Rocchi, Alessandro Sibio, Benedetto Tilia, and Onofrio Tudisco. Final results of the first phase of the PROTO-SPHERA experiment: obtainment of the full current stable screw pinch and first evidences of the jet + torus combined plasma configuration. *Plasma Science and Technology*, 26:025103, 2024.
- [32] Jan Mlynar, Teddy Craciunescu, Diogo R. Ferreira, Pedro Carvalho, Ondrej Ficker, Ondrej Grover, Martin Imrisek, and Jakub Svoboda. Current Research into Applications of Tomography for Fusion Diagnostics. *Journal of Fusion Energy*, 38(3-4):458 – 466, 2019.
- [33] M. Odstreil, J. Mlynar, T. Odstreil, B. Alper, and A. Murari. Modern numerical methods for plasma tomography optimisation. *Nuclear Instruments and Methods in Physics Research, Section A: Accelerators, Spectrometers, Detectors and Associated Equipment*, 686:156 – 161, 2012.
- [34] Tugba Ozge Onur. An application of filtered back projection method for computed tomography images. *International Review of Applied Sciences and Engineering*, 12(2):194 – 200, 2021.
- [35] Yongwoo Park, Un-Chul Paek, and Dug Young Kim. Characterization of a stress-applied polarization-maintaining (PM) fiberthrough photoelastic tomography. *Journal of lightwave technology*, 21(4):997 – 1004, 2003.
- [36] A. Pau, S. Naghinajad, F. Alladio, P. Micozzi, and L. Boncagni. 3D tomography of Hydrogen and Helium plasma produced in th PROTO-SPHERA experiment. In *Proceedings of the ASME 2024 51st Annual*

- 554 *Review Progress in Quantitative Nondestructive Evaluation QNDE2024*,
555 pages 1–7, Denver, Colorado, July 2024.
- 556 [37] Johann Radon. On the determination of functions from their integral
557 values along certain manifolds (translation from the original German
558 text published in *Berichte der Sächsischen Akademie der Wissenschaft*,
559 vol. 69, pp. 262-277, 1917). *IEEE Transactions on Medical Imaging*,
560 MI-5(4):170–176, 1986.
- 561 [38] L. Schoeman, P. Williams, A. du Plessis, and M. Manley. X-ray micro-
562 computed tomography (μ CT) for non-destructive characterisation of
563 food microstructure. *Trends in Food Science and Technology*, 47:10 –
564 24, 2016.
- 565 [39] James Anthony Seibert. Iterative reconstruction: how it works, how to
566 apply it. *Pediatric Radiology*, 44(3):431 – 439, 2014.
- 567 [40] Wolfram Stiller. Basics of iterative reconstruction methods in computed
568 tomography: A vendor-independent overview. *European Journal of Ra-
569 diology*, 109:147 – 154, 2018.
- 570 [41] M.L.L. Wijerathne, Kenji Oguni, and Muneo Hori. Tensor field tomog-
571 raphy based on 3D photoelasticity. *Mechanics of Materials*, 34(9):533 –
572 545, 2002.
- 573 [42] Tao Zhang, Yuxiang Xing, Li Zhang, Xin Jin, Hwei Gao, and Zhiqiang
574 Chen. Stationary computed tomography with source and detector in lin-
575 ear symmetric geometry: Direct filtered backprojection reconstruction.
576 *Medical physics*, 47(5):2222–2236, 2020.
- 577 [43] Yingying Zhang-O’Connor and Jeffrey A. Fessler. Fourier-based for-
578 ward and back-projectors in iterative fan-beam tomographic image re-
579 construction. *IEEE Transactions on Medical Imaging*, 25(5):582 – 589,
580 2006.
- 581 [44] Dominique Zosso, Benoît Le Callennec, Meritxell Bach Cuadra, Kamiar
582 Aminian, Brigitte M. Jolles, and Jean-Philippe Thiran. Bi-planar 2D-to-
583 3D registration in fourier domain for stereoscopic X-ray motion tracking.
584 volume 6914, 2008.

Title Page

Title:

*3D tomographic reconstruction of light emission density of plasma in the
PROTO-SPHERA experiment*

Author Information

Author names:

Annamaria Pau^a, corresponding author, annamaria.pau@uniroma1.it
Shayesteh Naghinajad^a, shayesteh.naghinajad94@gmail.com
Franco Alladio^b, franco.alladio@protosphaera.it
Paolo Micozzi^c, paolo.micozzi@enea.it
Luca Boncagni^c, luca.boncagni@enea.it

Affiliations:

^a Department of Structural and Geotechnical Engineering, Sapienza University of Rome, via
Eudossiana 18, 00184 Roma, Italy,

^bINAF-IAPS National Institute of Astrophysics, Via del Fosso del Cavaliere 100, 00133 Roma, Italy

^cNuclear Research Department, ENEA Italian National Agency for Technology, Energy and
Sustainable Economic Development, Via Enrico Fermi 45, 00044 Frascati (RM), Italy

Corresponding author:

Annamaria Pau, annamaria.pau@uniroma1.it

Annamaria Pau gratefully acknowledges the funding contribution from National Center on HPC, Big Data and Quantum Computing CN00000013. This manuscript reflects only the authors' views and opinions, neither the European Union nor the European Commission can be considered responsible for them.

- 3D reconstruction is performed using a corollary of the Fourier slice theorem
- The method is applied to light emission from plasma in the PROTO-SPHERA experiment
- A six-camera setup enables accurate time-resolved tomography of plasma discharges
- Tomographic inversion reveals hidden core emissions masked by outer plasma layers
- Formation of a plasma torus around the centerpost is resolved within 8 milliseconds

Declaration of interests

☒ The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

☐ The authors declare the following financial interests/personal relationships which may be considered as potential competing interests: