

3D tomographic reconstruction of light emission density of plasma in the PROTO-SPHERA experiment

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Abstract

We apply a method for the reconstruction of 3D density functions based on the formulation of a corollary of the 3D Fourier slice theorem. This corollary, which we state and prove in its general case, connects projections of a 3D density function onto planes to the 3D Fourier transform of the density function itself. By applying a suitable coordinate transformation and computing the 3D inverse Fourier transform of the 2D Fourier transforms of the projections, we directly solve the inverse problem consisting in the reconstruction of a density function based on the knowledge of its projection. This method is employed to represent the visible light emission density distribution of magnetically confined plasma, using experimental data recorded by ENEA (Italian National Agency for Technology, Energy and Sustainable Economic Development) within the ambit of the PROTO-SPHERA experiment. The light emission from Helium and Hydroge plasma is captured by multiple cameras arranged in cylindrical symmetry as part of an ongoing plasma physics experiment. The developed technique enabled observation of the mechanism of self-organization of a plasma torus around a centerpost, which confirmed

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theoretical predictions. It further enabled visualization of cross-sections of plasma, and to disclose its inner organization and emission spectrum as well.

Keywords: 3D image reconstruction, magnetically confined plasma, nuclear fusion, monitoring

1. Introduction

Tomographic computed (CT) imaging or scanning is a technique of data treatment which enables visualization of the inner part of objects instead of making real cuts. This has obvious and important applications in the medical field, as well as in the nondestructive monitoring of structures and industrial processes. CT requires the solution of an inverse problem, which can be formulated as follows: given the knowledge of some data called projections, reconstruct the spatial distribution of a density function $f(x, y, x)$ which represents some property (for instance mass density) of an object in the 3D space. Projections are obtained by illuminating the object with stimuli which produce a measurable response, which is measured on the object boundary. Projections are, in practice, a sort of shade that the function $f(x, y, x)$ projects onto lines, or planes. The mathematical demonstration that a density function could be reproduced from an infinite set of its projections was provided by the mathematician J. Radon [29] at the beginning of last century. However, it is quite remarkable that reconstruction of images from projections was not attempted until 1940. In 1972, Hounsfield [18] proposed the first method that allowed to image plane slices of the internal structure of a human body using X-rays projections. The Nobel prize in physiology and medicine in 1979 awarded to Hounsfield and Cormack [6] for their work on the initial computed tomography device signifies the importance that this technique has in the medical field. Since then, the use of computed tomography was extended to a wide number of industrial applications [8], among which food industry [30], materials science [25], and geosciences [5].

Medical and industrial computed tomography is based on the same underlying principles but differs in the acquisition system layout, as well as in the stimulus that is used to obtain the projections, which is X-rays in medical CT, but can also be another physical source. In fact, the reflected or transmitted signal of any source which produces a measurable response of the scanned object can be used. The feasibility of using ultrasounds [21, 19, 22],

the photo elastic effect [35], or even ground-penetrating radar emitting microwaves [1] has been proved to be effective to visualize internal patterns of objects. Different sources interact with different properties, which can include, besides mass density, electrical, acoustic, and electromagnetic properties of materials and tissues. For instance, in nuclear medicine, the interest is focused on reconstructing a cross-sectional image of radioactive isotope distributions within the human body [10], while in magnetic resonance the magnetic properties of the object is reconstructed [13]. The photelastic effect instead was used to determine the plane stress distribution in an optical fiber [27]. The main difference between the mentioned sources is that some of them are non-diffracting, like X-rays, which travel in straight lines [3], while some others are diffracting, like ultrasounds, that is, they are scattered in all directions, resulting in a more complex inverse problem [20].

To convert the measured response data into an image, appropriate processing is necessary. The most popular algorithms that are in use can be classified into two major categories, which are iterative and analytical methods [17]. Iterative reconstruction algorithms employ a model to predict the measured data from an initial estimate and then refine the image estimate to better fit the measured data. This process is repeated iteratively until the image quality meets a predetermined standard or until further iterations do not significantly change the image [31, 32]. Starting from the Algebraic Reconstruction Technique (ART) [15], research continues to evolve with refinements and developments, such as the customized Simultaneous Algebraic Reconstruction Technique (SART) [11], and the Simultaneous Iterative Reconstruction Technique (SIRT) [36]. Analytical algorithms are based on mathematical models and explicit formulas that directly compute the image from the acquired projection data. They can be further categorized into two groups: the direct or Fourier-Transform-based methods [37, 16] and the Back-Projection-based algorithms, like Simple Back Projection (SBP), Linear Back Projection (LBP) and Filtered Back Projection (FBP) [26, 36]. Fourier-Transform-based algorithms rely on the relationship between the Fourier transform of the image and the Fourier transform of its projections. More specifically, projections are obtained using Radon transforms [20], that is by computing line integrals of the density function. A more detailed discussion of the literature on the classification of CT algorithms is beyond the scope of this paper. The interested reader can consult [22, 9] for a more exhaustive presentation of the matter.

Traditionally, and up to the present day [24, 12], CT 3D scans have been

70 obtained by assembling a stack of 2D images. In the classical tomographic
 71 approach, the 3D image is reconstructed by assembling slices of images de-
 72 riving from a 2D setting of the problem of image reconstruction on several
 73 parallel planes. To do so, the Fourier slice theorem in 2D is employed, which
 74 connects the Fourier transform of the Radon transform to the 2D Fourier
 75 transform of the image itself. Despite a n th-dimension formulation of the
 76 Radon transform was provided by Radon himself, to the best of our knowl-
 77 edge, applications of the 3D Radon transform are limited in the literature.
 78 In most of the cases, for the analytical reconstruction, the 3D Fourier slice
 79 theorem [4] is applied in the form connecting 1D Fourier transforms of line
 80 subsets of the 3D Radon transform [34, 38, 33], with applications also to
 81 image-recognition problems [7]. The sole reference that can be found in the
 82 literature employing the connection between 2D Fourier transforms of plane
 83 projections with central slices of the 3D Fourier volume, is the work by Zosso
 84 et al. [38], where the necessary corollary is formulated and the resulting
 85 method is employed to track bone movements in stereoscopic X-ray image
 86 series of a knee joint.

87 Here, we employ the imaging technique based on the corollary to Fourier
 88 slice theorem in 3D formulated by [38], but demonstrate it for the more gen-
 89 eral case of arbitrary angles. By calculating 3D inverse Fourier transforms
 90 of 2D Fourier transforms of plane projections, the 3D image of the scanned
 91 object can be reconstructed. This technique is fairly general and applies to
 92 the interpretation of the different data, ranging from structural and material
 93 inspection to biological and physical phenomena. The 3D approach is partic-
 94 ularly useful in the case under investigation, where the projections are a given
 95 by a set of digital camera pictures, which will be considered as the discrete 3D
 96 Radon transforms of the visible light emission. We develop an algorithm of
 97 image reconstruction and apply it to the 3D reconstruction of the light emis-
 98 sion density of magnetically confined Helium and Hydrogen plasma. Data
 99 was recorded during the deployment of the PROTO-SHPERA experiment
 100 [2], which took place at ENEA (Italian National Agency for New Technolo-
 101 gies, Energy and Sustainable Development) laboratories in Frascati, Italy.
 102 During the experiment, the plasma emits photons, which light is captured
 103 by six cameras arranged with cylindrical symmetry. The PROTO-SHPERA
 104 experiment is an innovative magnetic confinement plasma experiment for
 105 controlled thermonuclear fusion research, whose aim is to form a plasma
 106 torus not around a metal centerpost, as in tokamaks, but around a plasma
 107 centerpost.

108 This paper is organized into two sections: Section 2 describes the pro-
 109 posed method for image reconstruction, and Section 3 reports the experimen-
 110 tal results. In Subection 2.1, the 3D continuous Radon transform is recalled,
 111 then in Subsection 2.2 the lemma of the Fourier slice theorem that is at the
 112 base of the image reconstruction method employed is proved. In Subsec-
 113 tion 2.3 proof of the effectiveness of the reconstruction method is provided
 114 by using numerical data. Subection 3.1 describes the PROTO-SPHERA ex-
 115 periments, and Subsection 3.2 illustrates how the tomographic reconstruction
 116 enabled to prove the sustainment of the plasma torus around the centerpost
 117 and provided further useful information on the dynamics of confined plasma.

118 2. Analytical direct 3D image reconstruction

119 2.1. 3D continuous Radon Transform

120 For simplicity's sake, the 2D Radon transform is first introduced. Let us
 121 consider a plane object - or a slice of a 3D object - with density function
 122 $f(x, y)$ (Figure 1). Its projections onto a line inclined by an angle θ with
 123 respect to the x axis are obtained by integrating $f(x, y)$ on the set of parallel
 124 lines (rays) that are inclined by an angle $\theta + \pi/2$ with respect to the x axis
 125 and at a distance t from the origin of the coordinate system (bold line in 1).
 126 The equation of a ray is:

$$t = x \cos\theta + y \sin\theta. \quad (1)$$

127 The projection of the function $f(x, y)$ is then defined as:

$$p(\theta, t) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x, y) \delta(x \cos\theta + y \sin\theta - t) dx dy, \quad (2)$$

128 where δ is the Dirac's delta function. Equation (2) represents the 2D con-
 129 tinuous Radon transform $[\mathcal{R}_2 f](\theta, t)$ of the function f , which is depicted
 130 schematically in Figure 1 by the blue curve subtending the sky blue area. It
 131 is often preferred to integrate in the rotated coordinate system (t, s) , where
 132 the projection can be written as:

$$p(\theta, t) = \int_{-\infty}^{\infty} f(t, s) ds. \quad (3)$$

133 Geometrically, the continuous 2D Radon transform maps a function in $\mathbb{R}^2$
 134 into the set of its line integrals in $\mathbb{R}^2$. The reconstructed image function
 135 $f(x, y)$ is formally obtained by inverting the Radon transform:

$$f(x, y) = [\mathcal{R}_2^{-1}p](\theta, t). \quad (4)$$

Let us now consider a 3D object with density function $f(x, y, z)$ (Figure 2). Its projections are obtained by integrating $f(x, y, z)$ on the plane Σ that is orthogonal to the versor $\boldsymbol{\alpha}$ and at a distance d from the origin of the coordinate system. The versor has direction cosines $\boldsymbol{\alpha} = (\cos\theta \cos\phi, \sin\theta \cos\phi, \sin\phi)^T$, where ϕ is the angle that $\boldsymbol{\alpha}$ forms with the z axis, and θ is the angle that the projection of $\boldsymbol{\alpha}$ onto the xy plane forms with the x axis. The equation of a plane orthogonal to $\boldsymbol{\alpha}$ and at a distance d from the origin is:

$$d = x \cos\theta \cos\phi + y \sin\theta \cos\phi + z \sin\phi. \quad (5)$$

In an analogy with Equation (2), the projection of the function $f(x, y, z)$ associated with the couple $(\boldsymbol{\alpha}, d)$ is then defined as:

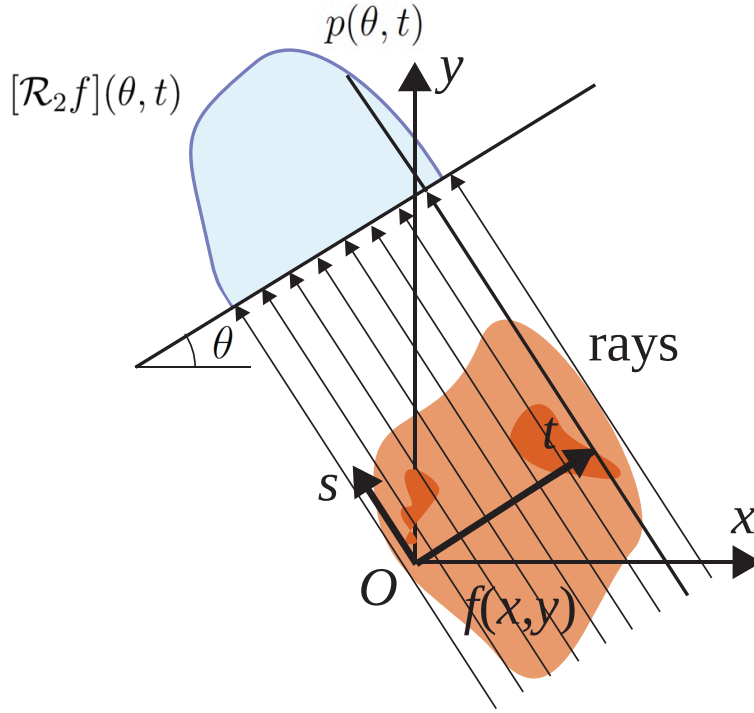


Figure 1: Projection of a plane density function onto a line.

$$p(\boldsymbol{\alpha}, d) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x, y, z) \delta(x \cos \theta \cos \phi + y \cos \theta \sin \phi + z \sin \theta - d) dx dy dz. \quad (6)$$

145 The area over which the previous integral is computed is included in the
 146 dashed line in Figure 2. Equation (6) represents the 3D continuous Radon
 147 transform $[\mathcal{R}_3 f](\boldsymbol{\alpha}, t)$ of the function f . Geometrically, the continuous 3D
 148 Radon transform maps a function in $\mathbb{R}^3$ into the set of its plane integrals
 149 in $\mathbb{R}^3$. The reconstructed image function $f(x, y, z)$ is formally obtained by
 150 inverting the Radon transform:

$$f(x, y, z) = [\mathcal{R}_3^{-1} p](\boldsymbol{\alpha}, d). \quad (7)$$

151 It is important to note that in real applications we won't be dealing with plane
 152 integrals. In fact, $\mathcal{R}_3$ can be obtained by computing the $\mathcal{R}_2$ of the intersection

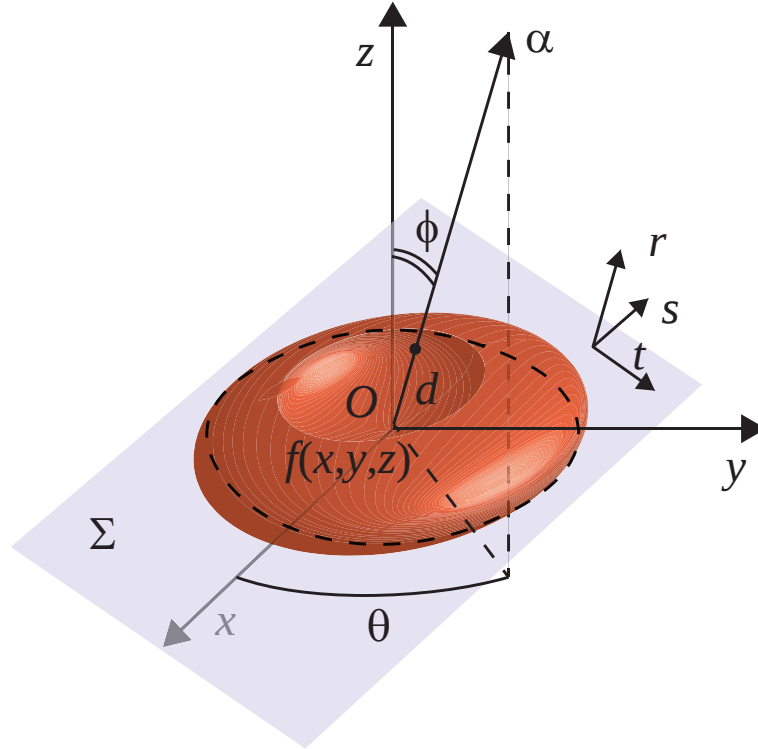


Figure 2: Projection of a 3D density function onto a plane.

153 between f and the plane Σ , and then integrating over lines orthogonal to the
 154 rays, which are displayed in Figure 3 as yellow lines. The line integrals are
 155 much easier to be obtained in practical measurements. They correspond
 156 to X-ray projections taken from different angles, or, in the present case, to
 157 pictures taken from different cameras.

158 Let us assume that the integration is carried out in the rotated coordinate
 159 system (t, s, r) , whose axes t and s coincide with the tangent lines to the
 160 parallels and meridians, respectively, of an imaginary sphere centered in the
 161 origin of the system of coordinates, at the point of intersection between the
 162 sphere and the versor α , with the orientation of the r axis coinciding with
 163 that of α (Figure 3, upper left). The transformation of coordinates from
 164 (x, y, z) to (t, s, r) can be written as:

$$\begin{bmatrix} t \\ s \\ r \end{bmatrix} = \begin{bmatrix} \cos\phi & 0 & \sin\phi \\ 0 & 1 & 0 \\ \sin\phi & 0 & \cos\phi \end{bmatrix} \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} \quad (8)$$

165 Moreover, the distance d can be expressed in the (t, s, r) system of coordi-
 166 nates as $d = \sqrt{t^2 + r^2}$. The ray integral is then expressed as follows:

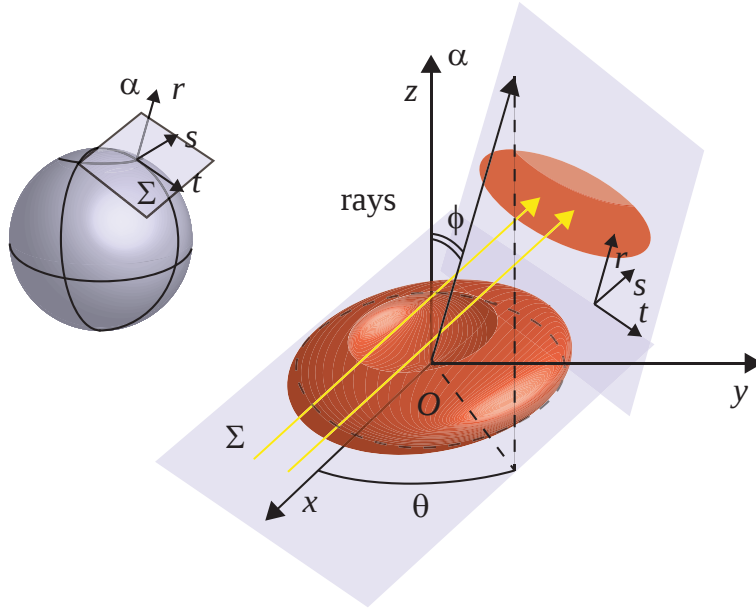


Figure 3: Ray integrals over a 3D function.

$$p(\theta, \phi, t, r) = \int_{-\infty}^{\infty} f(t, s, r) ds. \quad (9)$$

167 The integral $p(\theta, \phi, t, r)$ is in practice a line projection of the 3D function.
 168 The set of all the line projections on the plane (t, r) for given θ and ϕ will
 169 be denoted as $P_{\theta, \phi}$.

170 2.2. 3D Fourier slice theorem

171 The extension of the Fourier slice theorem to higher dimensions, enables
 172 direct reconstruction of 3D objects. The 3D Fourier Slice Theorem can be
 173 stated as follows.

174 **Theorem 1.** *A line slice of the 3D Fourier transform of the function $f(x, y, z)$*
 175 *equals the 1D Fourier Transform of the 3D Radon transform $[\mathcal{R}_3 f](\boldsymbol{\alpha}, t)$ at*
 176 *the same angle.*

177 For proof, the interested reader can consult [38, 4]. However, the theorem
 178 in this form is of little use for practical purposes. A useful corollary can be
 179 stated.

180 **Corollary 1.1.** *A plane slice of the 3D Fourier transform of the density func-*
 181 *tion $f(x, y, z)$ equals the 2D Fourier Transform of the set of line projections*
 182 *at the same angle.*

183 *Proof.* Let us consider the line projection of a 3D density function $P_{\theta, \phi}$, and
 184 take its 2D Fourier Transform ($\mathcal{F}_2$):

$$[\mathcal{F}_2 P_{\theta, \phi}](\mu, \nu) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} P_{\theta, \phi}(t, r) e^{-2\pi i(\mu t + \nu r)} dt dr, \quad (10)$$

185 where μ and ν are the wavenumbers in the t, r directions, respectively and i
 186 is the imaginary unit.

187 By substituting equation (9) into equation (10), we get:

$$[\mathcal{F}_2 P_{\theta, \phi}](\mu, \nu) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \left[\int_{-\infty}^{\infty} f(t, s, r) ds \right] e^{-2\pi i(\mu t + \nu r)} dt dr. \quad (11)$$

188 Based on the transformation (8), the rotated coordinate system (t, s, r) can
 189 be converted back into the initial (x, y, z) system:

$$[\mathcal{F}_2 P_{\theta, \phi}](\mu, \nu) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x, y, z) e^{-2\pi i k} dx dy dz \quad (12)$$

$$\text{with } k = \mu \cos \phi \cos \theta x + \mu \cos \phi \sin \theta y + \mu \sin \phi z + \nu \sin \phi \cos \theta x + \nu \sin \phi \sin \theta y + \nu \cos \phi z.$$

190 It is obvious that the right hand side of (12) represents the 3D Fourier Trans-
 191 form $\mathcal{F}_3$ of f at wavenumbers $u = \mu \cos \phi \cos \theta + \nu \sin \phi \cos \theta$, $v = \mu \cos \phi \sin \theta +$
 192 $\nu \sin \phi \sin \theta$ and $w = \mu \sin \phi + \nu \cos \phi$:

$$[\mathcal{F}_3 f](u, v, w) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x, y, z) e^{-2\pi i (ux + vy + wz)} dx dy dz, \quad (13)$$

193 which proves the corollary. The quantities u , v and w are wavenumbers in
 194 directions x , y and z , respectively. $\square$

195 The interesting consequence of this corollary is that, given the line pro-
 196 jections, on calculating their 2D Fourier Transform, and then, the inverse 3D
 197 Fourier Transform of the resulting set of 2D Fourier transforms, the original
 198 image is directly reconstructed. The following subsection provides numerical
 199 proof of the effectiveness of the procedure.

200 2.3. Numerical example of the 3D direct reconstruction

201 Let us consider an input density function modeled as a cube with a uni-
 202 form density of 0.5, side length of 0.6, and centered at the origin of the
 203 coordinate system (Figure 4a). Inside this cube, two smaller spheres are
 204 embedded: one with a density of 0.3, a radius of 0.1, and centered at coordi-
 205 nates (0.05, 0.05, -0.2); the other with a density of 0.1, a radius of 0.05, and
 206 centered at (-0.1, 0, 0.1). These variations in density are used to verify the
 207 accuracy of the reconstruction method, ensuring it can effectively distinguish
 208 between different density regions. Although the spheres are not visible from
 209 the exterior of the solid cube, Figure 4b illustrates their positions by ren-
 210 dering the cube with partial transparency. It is important to note that for
 211 this numerical example, the theoretical framework developed for continuous
 212 systems in the previous sections has been adapted for use with discrete data.
 213 In this case, we employed a total of $N = 64 \times 64 \times 64$ sampling points, with
 214 64 points along each side of the cube. Additionally, $N_{deg} = 12 \times 12 = 144$
 215 spherical projections were taken, with 12 projections for each angle θ and ϕ .
 216 The angular step size was kept constant over π , set to $\pi/12$.

217 Figures 5a and b provide samples of the spherical projections used for
 218 reconstruction, taken at angles $(\theta, \phi) = (\pi/2, 0)$ and $(2\pi/3, 0)$, respectively.
 219 Note that the color scale extends beyond 0.5 because it represents line inte-
 220 grals of the density function over the entire domain, rather than local values.
 221 Regions of lower density show in fact lower integral values. By combining
 222 a total of 144 two-dimensional Fourier transforms of the spherical projec-
 223 tions and computing their three-dimensional inverse Fourier transform, the
 224 reconstructed density function shown in Figure 6a is obtained, matching the
 225 original function from Figure 4a. Figures 6b, c, and d present cross-sections
 226 of the reconstructed density function along two vertical planes (b,c) and a
 227 horizontal plane (d) passing through the origin, with the corresponding color
 228 scale. This Figure shows that the embeddes spheres are exactly reproduced
 229 not only in terms of position, but also in terms of density. See in fact that
 230 the large sphere has high density, while the small one low.

231 The previous example was implemented using a large number of projec-
 232 tions. While reducing the number of projections results in a slightly blurred
 233 reconstructed image, even a very limited number of projections still pro-
 234 vides a sufficiently clear reconstruction. This effect is illustrated in Figure
 235 7, which shows the reconstructed density function with varying numbers of
 236 projections. The source density function is depicted in Figure 7. It is shaped
 237 as a cube with a unit density, a side length of 0.2, and is centered at the ori-
 238 gin of the coordinate system. A sphere with unit density, a diameter larger
 239 than the cube's side, and centered at the origin, was subtracted from the
 240 density function, causing its volume to intersect with the cube's faces. Al-
 241 though the image quality decreases as the number of projections is reduced,
 242 the shape remains distinctly recognizable even with a very limited number
 243 of projections, as few as 9 (Figure 7d).

244 **3. 3D Reconstruction of light emission density in PROTO-SPHERA** 245 **experiment**

246 *3.1. The PROTO-SPHERA experiment*

247 PROTO-SPHERA (Spherical Plasma for Helicity Relaxation Assessment)
 248 is an innovative magnetic confinement plasma experiment aimed at advanc-
 249 ing controlled thermonuclear fusion research. Its goal is to form plasma
 250 within a confined environment. Unlike tokamaks, where the plasma forms
 251 around a metal centerpost, in PROTO-SPHERA, the plasma forms around
 252 a plasma centerpost [23]. Tokamaks, the most extensively studied magnetic

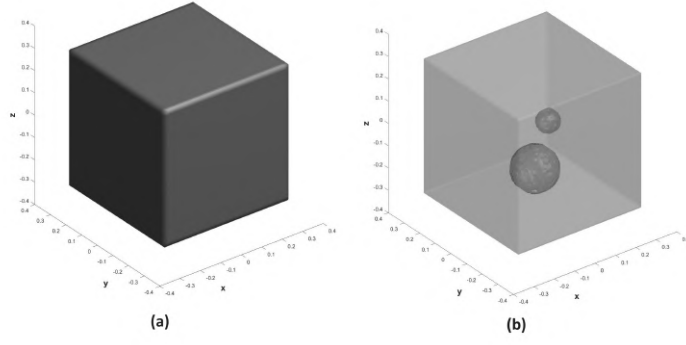


Figure 4: a) 3D density function b) density function made partially transparent to show the embedded spheres.

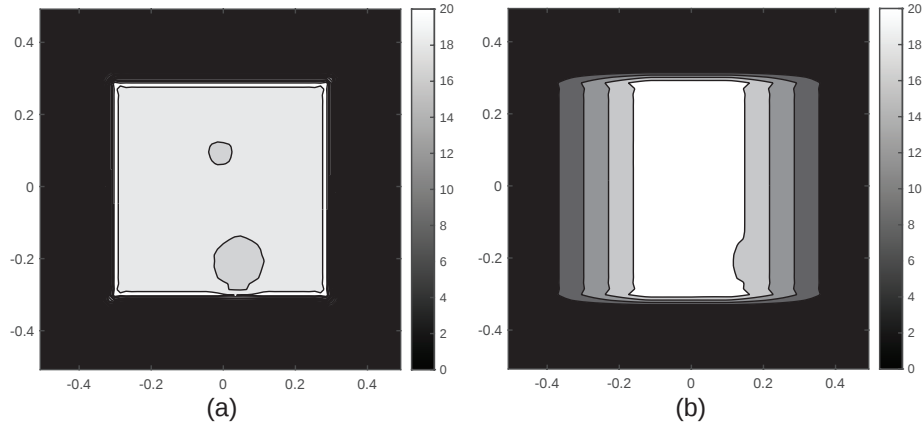


Figure 5: A couple of spherical projections pertaining to angles a) $\theta = \pi/2$, $\phi = 0$ and b) $\theta = 2\pi/3$, $\phi = 0$

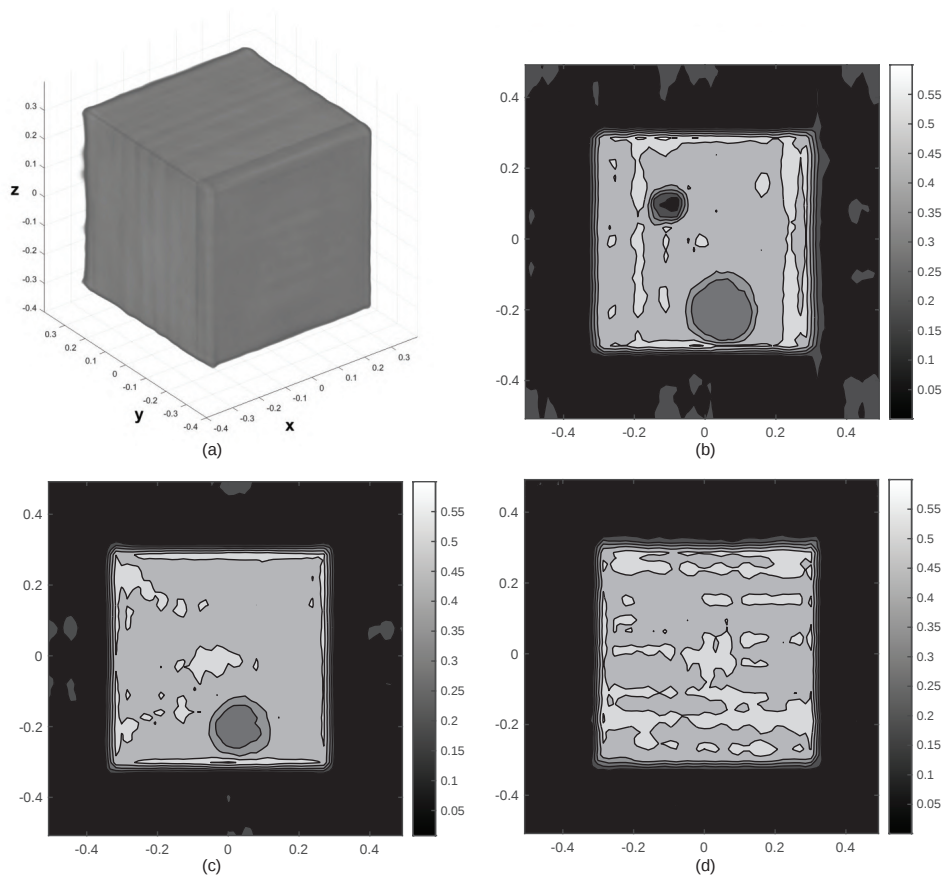


Figure 6: a) Reconstructed image, b) vertical cross section on the xz plane, c) vertical cross section on the yz plane, d) horizontal cross section on the xy plane.

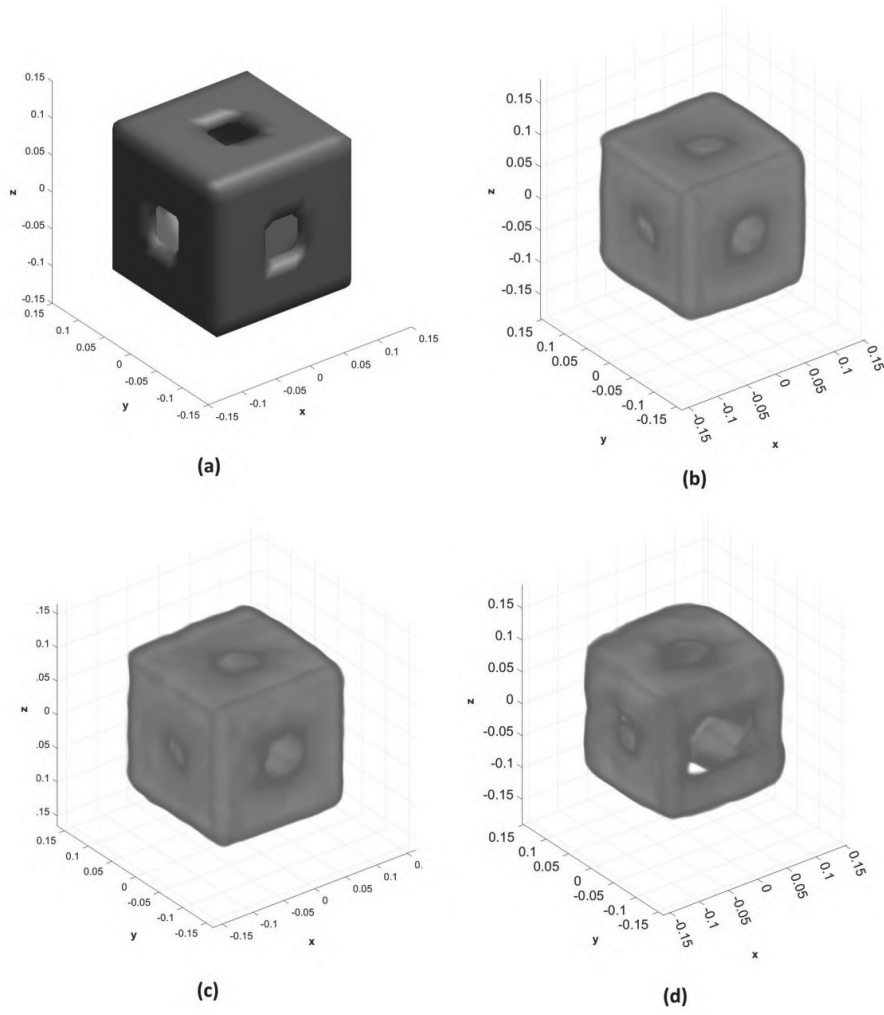


Figure 7: a) Source image and Reconstructed image with b) $12 \times$, c) 6×6 and d) 3×3 projections.

fusion configurations, have a non-simply connected geometry, where a central post—containing the inner part of the toroidal magnet and the ohmic transformer—connects the plasma torus. The development of simply connected magnetic configurations relevant for fusion would significantly simplify the design of fusion reactors. Additionally, the PROTO-SPHERA experiment seeks to emulate astrophysical plasmas, such as jet and torus structures, in a laboratory setting [2].

Figure 8 shows a schematic of the PROTO-SPHERA experiment, which consists of a transparent cylindrical vacuum chamber measuring 2.5 meters in height and 2.0 meters in diameter. The chamber is equipped with electrodes at the ends and internal poloidal field coils. The operating sequence proceeds as follows: the tungsten filaments of the cathode are heated to 2600°C over 20 seconds, while the current in the poloidal field coils is ramped up to 1875 A. Gas is then introduced through a hollow section in the anode, reaching a pressure of 10^{-2} to 10^{-1} mbar, and a total voltage of 100-350 V is applied between the electrodes. Under these conditions, plasma forms and emits photons.

To study the plasma formation process and the magneto-hydrodynamic phenomena occurring in the experiment, six high-speed cameras are positioned on the equatorial plane of the chamber, spaced 60 degrees apart (Figure 9). Each camera has a resolution of 640x480 pixels and can capture 600 frames per second. Sample images recorded by the cameras at a specific time instant are shown in Figures 10a and 11a for shots with Helium and Hydrogen, respectively.

Combining the data from the cameras with measurements from a Langmuir probe has provided valuable insights into the dynamics of the plasma torus formation, including its size and emission spectrum. Numerous discharges were produced and observed throughout the experimental campaign.

The images captured by the six cameras are in fact projections onto planes of the visible-light emission density, which plays the role of the density function f (Figures 10a and 11a). For each time instant, exploiting the corollary of the Fourier Slice Theorem of Section 2, these source images are 2D-Fourier transformed, then 3D-Fourier inverted to obtain the density of the visible light emitted by the plasma. The only change with respect to the procedure described in Section 2.2 is that, given the positioning of the cameras, the inversion has to be based on a cylindrical set of parallel projections instead of a spherical set, with ϕ fixed. Figures 10b and 11b show the reconstructed plasma light emission distribution derived from the

291 images on the left. It is important to note that the original images contain
 292 three layers of information, corresponding to the RGB color channels. The
 293 reconstructed images in Figures 10b and 11b are false-color representations,
 294 created by combining the RGB channels linearly with the following weighting:
 295 100% Red, 86% Green, and 25% Blue [2].

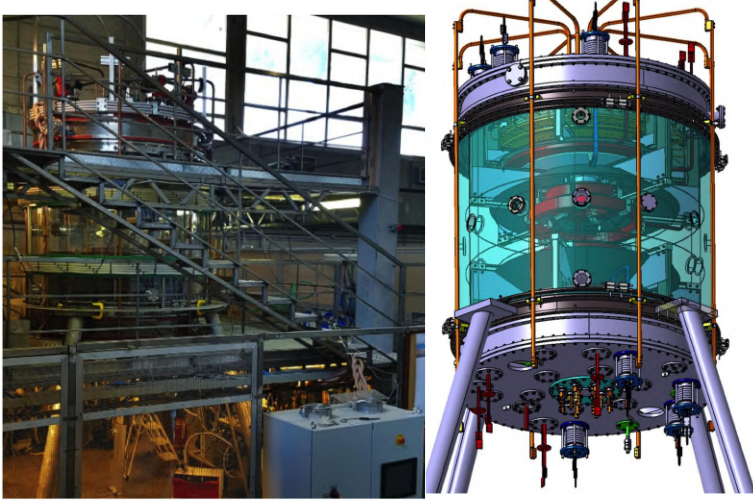


Figure 8: Schematic of the PROTO-SPHERA experiment

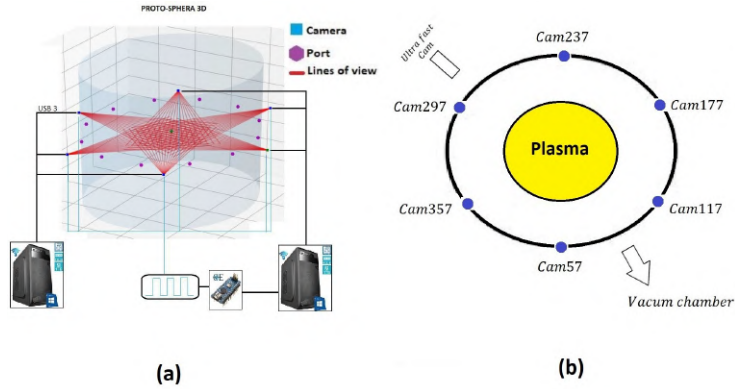


Figure 9: a) Position of the cameras in axonometry b) and plan

296 3.2. Results and discussion

297 The proposed 3D tomography technique enabled detailed observation of
 298 the plasma torus formation process. According to magneto-hydrodynamic

models [14], the sequence begins with the formation of a plasma column (pinch or centerpost) at low current. As the current increases, the magnetic field lines bend, causing the pinch to become unstable, which leads to the formation of a torus around the centerpost. This progression is visualized in Figure 12 a-h, showing the reconstructed light emission density at eight sequential time steps (each 1/600 s apart) during a Helium plasma discharge. The same sequence predicted by the work of García-Martínez et al. [14] is observed. The entire torus formation occurs within approximately 8 ms, five time steps intercurring from the arrangements of Figure 12 b to g. The presence of both jets and the torus strongly parallels astrophysical phenomena, as discussed in [2].

Once the spatial distribution of light emission density is known for each time instant, cross-sections can be derived, revealing further insights into plasma dynamics. Figure 13 shows an image of the Helium plasma captured by one of the six cameras (a) and a vertical cross-section (b) of the reconstructed light emission density in true color, taken at the same moment. To create this image, each of the three RGB channels was processed independently to generate a corresponding density function for each color. This cross-sectional view reveals the green color, which was obscured in the camera pictures by the intense golden glow of the 'ionization peel'—the layer between the plasma and the surrounding gas cushion. The correctness of the reconstruction is proved by the appearance of the Langmuir probe and of the copper bars on the right hand side of the picture.

Figure 14 presents vertical cross-sections of the plasma light emission density at the same time instants as Figure 12. These images further confirm the sequence of events leading to the formation of the torus, as described in the previous paragraph. Specifically, Figure 14 illustrates the emergence of a stable pinch (a-b), its progression towards instability (c-f), and ultimately the arrangement of the torus around the centerpost (d-h). Similar patterns, with comparable duration, were observed in other plasma discharges [2, 28]. Figure 15 presents a horizontal cross-section at the equatorial level, enabling observation of the same phenomena from a different perspective. In Figures 15d and 15e, the pinch's instability is particularly evident, as shown by the loss of axial symmetry and the presence of green regions on the right side of the images. Additionally, the horizontal cross-section analysis provides insights into plasma stability and torus thickness. Figure 16 illustrates selected horizontal cross-sections of Helium plasma at the equatorial level across sequential times, clearly depicting the torus encircling the centerpost. Notably,

the light emission intensity varies over time, suggesting potential correlations with plasma density and stability. These images also allowed for the measurement of torus thickness, defined as the difference between the outer and inner circumferences' radii, which was observed to be between 0.25 and 0.35 m. Similar behaviors were noted for Hydrogen plasma discharges, with an equatorial horizontal cross-section provided in Figure 17 for reference.

4. Conclusions

An imaging technique based on the formulation of a corollary the Fourier slice theorem was applied to reconstruct the light emission distribution of magnetically confined plasma in three dimensions. The algorithm was used on data capturing the visible light emission from Helium and Hydrogen plasmas, recorded by six cameras as part of the PROTO-SPHERA experiment at the ENEA Frascati laboratories. The results demonstrate that six images taken by cameras arranged in cylindrical symmetry enable an accurate 3D reconstruction of the plasma's spatial distribution at different time instants during the experiment. We observed that the plasma torus forms in approximately 8 ms, evolving from a kinked filament caused by the destabilization of the plasma centerpost. This 3D reconstruction allows for cross-sectional analysis, revealing previously hidden green light emission from the core, which had been obscured by the outer golden ionized layer. Additionally, equatorial cross-sections enabled the measurement of the torus thickness, which was found to range between 0.25 and 0.35 meters.

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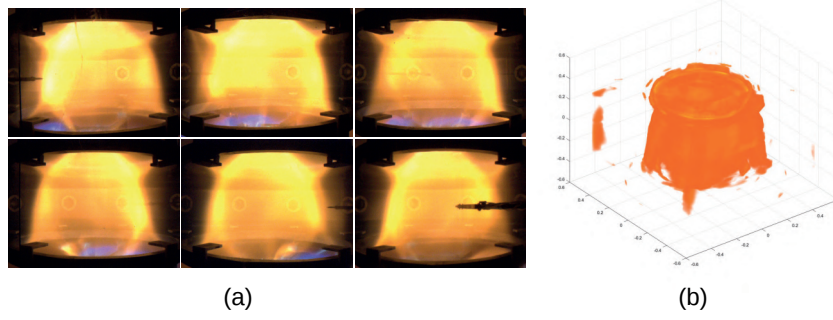


Figure 10: a) Pictures recorded by the six cameras at a given time instant (267 msec) of the PROTO-SPHERA experiment from shot 2505 of Helium b) 3D reconstruction of plasma light emission density

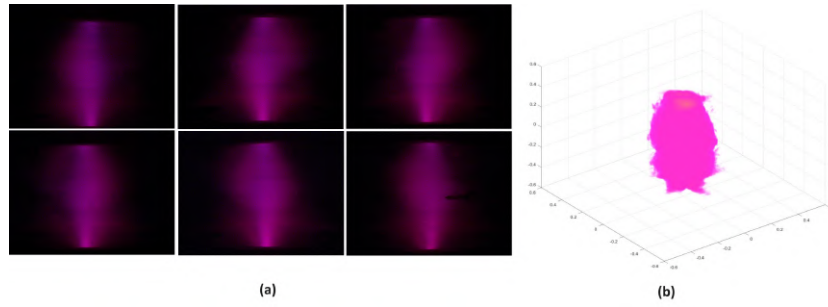


Figure 11: a) Pictures recorded by the six cameras at a given time instant (233 msec) of the PROTO-SPHERA experiment from shot 2154 of Hydrogen b) 3D reconstruction of plasma light emission density

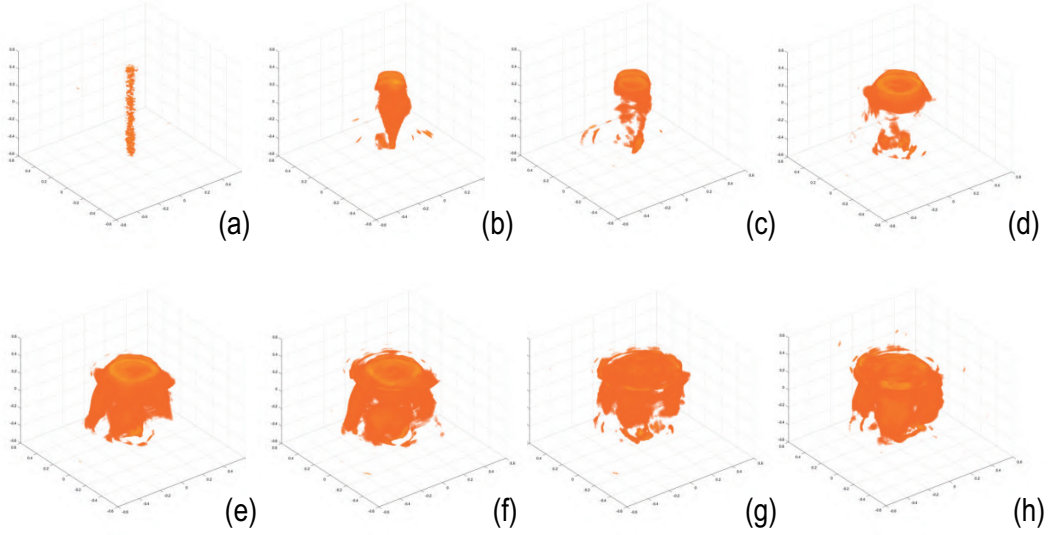


Figure 12: Reconstructed light emission density of Helium discharge 2505 at eight sequential time steps: stable pinch (a-b), pinch instability (c-f), formation of the plasma torus around the pinch (g-h)

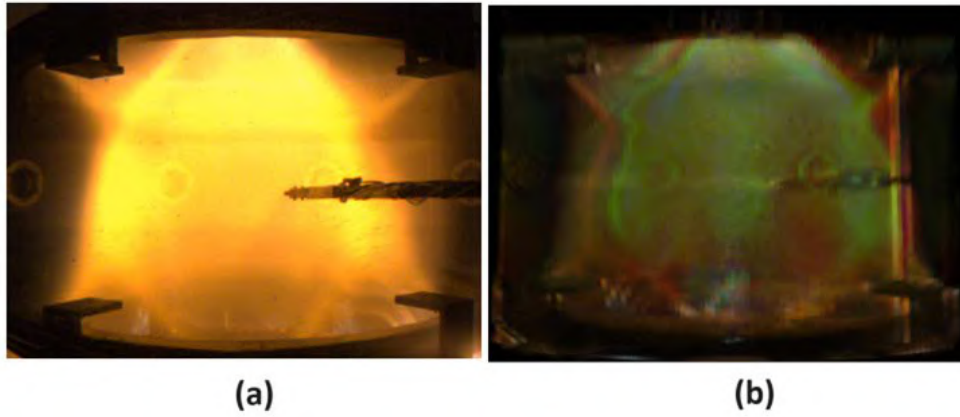


Figure 13: A projection of Helium plasma visible light emission captured by a camera during shot 2141 (a) and cross-section of the reconstructed light emission density in true colors (b).

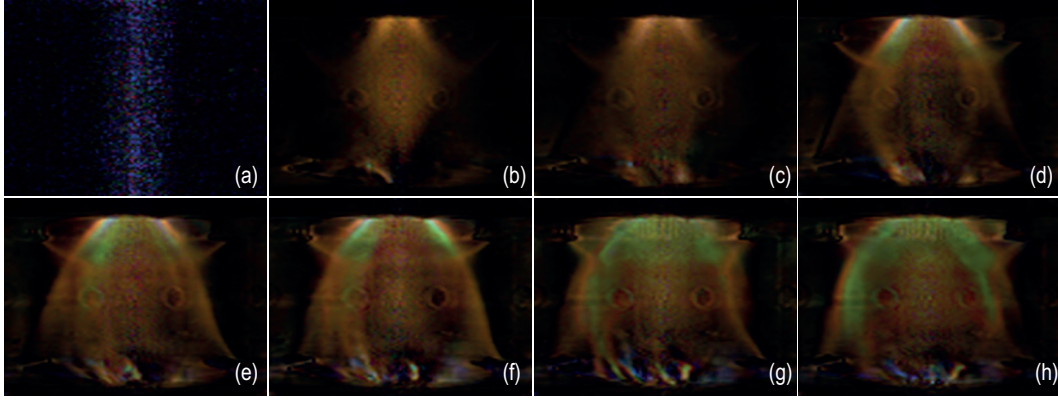


Figure 14: Vertical cross-sections of the reconstructed light emission density of Helium discharge 2025 at the same eight sequential time instants as Figure 12: stable pinch (a-b), pinch instability (c-f), formation of the plasma torus around the pinch (g-h)

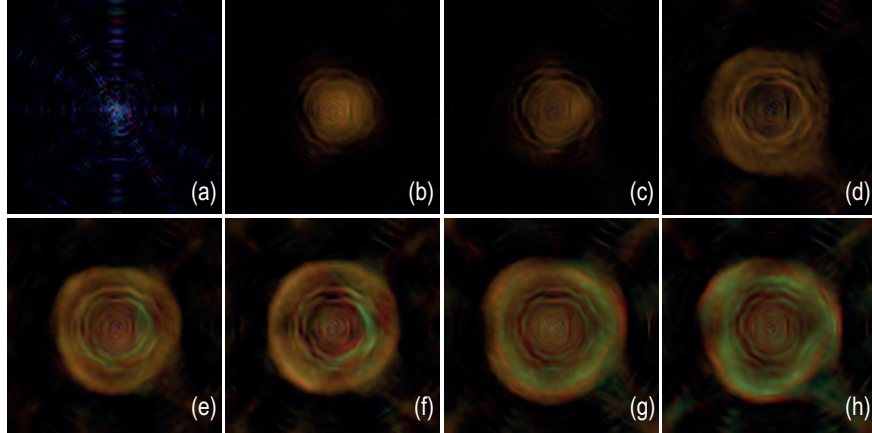


Figure 15: Horizontal cross-sections of the reconstructed light emission density of Helium discharge 2025 at the same eight sequential time instants as Figure 12: stable pinch (a-b), pinch instability (c-f), formation of the plasma torus around the pinch (g-h)

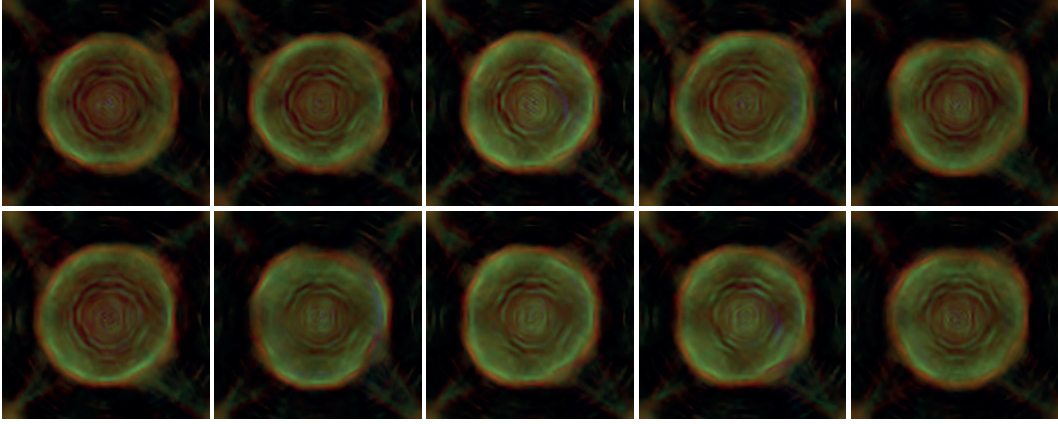


Figure 16: Sequence of reconstructed equatorial cross-sections of light emission density of Helium discharge 2025 at 10 time instants

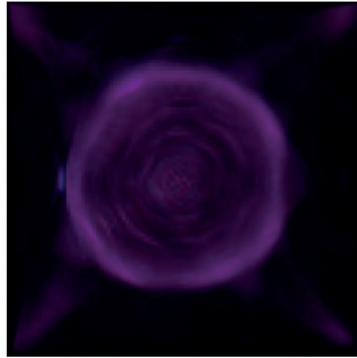


Figure 17: Equatorial cross-section of light emission density of Helium discharge 2154

365 References

- 366 [1] Amir M. Alani, Francesco Soldovieri, Ilaria Catapano, Iraklis Giannakis,
 367 Gianluca Gennarelli, Livia Lantini, Giovanni Ludeno, and Fabio Tosti.
 368 The use of ground penetrating radar and microwave tomography for the
 369 detection of decay and cavities in tree trunks. *Remote Sensing*, 11(18),
 370 2019.
- 371 [2] F. Alladio, P. Micozzi, L. Boncagni, A. Pau, S. Naghinajad, S. Macera,
 372 Y. Damizia, P. Buratti, F. Filippi, G. Galatola Teka, F. Giammanco,
 373 E. Giovannozzi, M. Iafrati, A. Lampasi, and P. Marsili. PROTO-
 374 SPHERA: a magnetic confinement experiment which emulates the jet
 375 + torus astrophysical plasmas. *Plasma Physics and Controlled Fusion*,
 376 66(3), 2024.
- 377 [3] Habib Ammari. Tomographic imaging with non-diffracting sources.
 378 an introduction. *Mathematics of Emerging Biomedical Imaging*,
 379 28(10):95–106, 2008.
- 380 [4] Amir Averbuch and Yoel Shkolnisky. 3d Fourier based discrete Radon
 381 transform. *Applied and Computational Harmonic Analysis*, 15(1):33 –
 382 69, 2003.
- 383 [5] V. Cnudde and M.N. Boone. High-resolution X-ray computed tomogra-
 384 phy in geosciences: A review of the current technology and applications.
 385 *Earth-Science Reviews*, 123:1 – 17, 2013.
- 386 [6] A. M. Cormack. Early two-dimensional reconstruction and recent topics
 387 stemming from it. *Science*, 209:1482–1486, 1980.
- 388 [7] Petras Daras, Dimitrios Zarpalas, Dimitrios Tzovaras, and Michael G.
 389 Strintzis. Shape matching using the 3d Radon transform. page 953 –
 390 960, 2004.
- 391 [8] L. De Chiffre, S. Carmignato, J.-P. Kruth, R. Schmitt, and A. Wecken-
 392 mann. Industrial applications of computed tomography. *CIRP Annals*
 393 - *Manufacturing Technology*, 63(2):655 – 677, 2014.
- 394 [9] S.R. Deans. *The Radon Transform and some of its applications*. Wiley:
 395 New York, 1983.

- 396 [10] Musa Sani Musa Niyazi Şentürk Fatih Veysel Nurçin Dilber
397 Uzun Ozsahin, Berna Uzun and Ilker Ozsahin. Evaluating nuclear
398 medicine imaging devices using fuzzy PROMETHEE method. *Proce-*
399 *dia computer science*, 120:699–705, 2017.
- 400 [11] S. Donato, L. Brombal, L.M. Arana Peña, F. Arfelli, A. Contillo, P. De-
401 logu, F. Di Lillo, V. Di Trapani, V. Fanti, R. Longo, P. Oliva, L. Rigon,
402 L. Stori, G. Tromba, and B. Golosio. Optimization of a customized
403 simultaneous algebraic reconstruction technique algorithm for phase-
404 contrast breast computed tomography. *Physics in Medicine and Biology*,
405 67(9), 2022.
- 406 [12] Mario D’Acunto, Antonio Benassi, Davide Moroni, and Ovidio Salvetti.
407 3D image reconstruction using Radon transform. *Signal, Image and*
408 *Video Processing*, 10(1):1 – 8, 2016.
- 409 [13] Jeffrey Fessler. Optimization methods for magnetic resonance image
410 reconstruction: Key models and optimization algorithms. *IEEE signal*
411 *processing magazine*, 37(1):33–40, 2020.
- 412 [14] Pablo Luis García-Martínez, Leandro Gabriel Lampugnani, and Ricardo
413 Farengo. Effect of the helicity injection rate and the Lundquist number
414 on spheromak sustainment. *Physics of Plasmas*, 21(12), 2014.
- 415 [15] Richard Gordon, Robert Bender, and Gabor T. Herman. Alge-
416 braic reconstruction techniques (ART) for three-dimensional electron
417 microscopy and x-ray photography. *Journal of Theoretical Biology*,
418 29(3):471–476,IN1–IN2,477–481, 1970.
- 419 [16] David Gottlieb, Bertil Gustaffson, and Patrick Forssen. On the direct
420 Fourier method for computed tomography. *IEEE Transactions on Med-*
421 *ical Imaging*, 19(3):223–232, 2000.
- 422 [17] B Gustaffson. Mathematics for computed tomography. *Physica Scripta*,
423 T61:38– 43, 1996.
- 424 [18] G. N. Hounsfield. Computerized transverse axial scanning (tomog-
425 raphy): Part I. description of system. *British Journal of Radiology*,
426 46(3):1016–1022, 1973.

- 427 [19] P. Huthwaite. Guided wave tomography with an improved scattering
428 model. *Proceedings of the Royal Society A: Mathematical, Physical and*
429 *Eng. Sciences*, 472(2195), 2016.
- 430 [20] A. C. Kak and M. Slaney. *Principles of Computerized Tomographic*
431 *Imaging*. IEEE press, 1988.
- 432 [21] Avinash C. Kak. Computerized tomography with X-ray, emission, and
433 ultrasound sources. *Proceedings of the IEEE*, 67(9):1245 – 1272, 1979.
- 434 [22] Mohd Taufiq Mohd Khairi, Sallehuddin Ibrahim, Mohd Amri Md Yunus,
435 M. Faramarzi, Goh Pei Sean, Jaysuman Puspanathan, and A. Abid.
436 Ultrasound computed tomography for material inspection: Principles,
437 design and applications. *Measurement: Journal of the International*
438 *Measurement Confederation*, 146:490 – 523, 2019.
- 439 [23] Alessandro Lampasi, Giuseppe Maffia, Franco Alladio, Luca Boncagni,
440 Federica Causa, Edmondo Giovannozzi, Luigi Andrea Grosso, Alessan-
441 dro Mancuso, Paolo Micozzi, Valerio Piergotti, Giuliano Rocchi,
442 Alessandro Sibio, Benedetto Tilia, and Vincenzo Zanza. Progress of the
443 plasma centerpost for the PROTO-SPHERA spherical tokamak. *Ener-*
444 *gies*, 9(7), 2016.
- 445 [24] Xin Li, Yudong Lu, Xiaozhou Zhang, Wen Fan, Yangchun Lu, and
446 Wangsheng Pan. Quantification of macropores of malan loess and the
447 hydraulic significance on slope stability by x-ray computed tomography.
448 *Environmental Earth Sciences*, 78(16), 2019.
- 449 [25] E. Maire and P.J. Withers. Quantitative X-ray tomography. *Int. Mate-*
450 *rials Reviews*, 59(1):1 – 43, 2014.
- 451 [26] Tugba Ozge Onur. An application of filtered back projection method for
452 computed tomography images. *International Review of Applied Sciences*
453 *and Engineering*, 12(2):194 – 200, 2021.
- 454 [27] Yongwoo Park, Un-Chul Paek, and Dug Young Kim. Characterization
455 of a stress-applied polarization-maintaining (PM) fiberthrough photoe-
456 lastic tomography. *Journal of lightwave technology*, 21(4):997 – 1004,
457 2003.

- [28] A. Pau, S. Naghinajad, F. Alladio, P. Micozzi, and L. Boncagni. 3D tomography of Hydrogen and Helium plasma produced in the PROTO-SPHERA experiment. In *Proceedings of the ASME 2024 51st Annual Review Progress in Quantitative Nondestructive Evaluation QNDE2024*, pages 1–7, Denver, Colorado, July 2024.
- [29] Johann Radon. On the determination of functions from their integral values along certain manifolds (translation from the original German text published in *Berichte der Sächsischen Akademie der Wissenschaft*, vol. 69, pp. 262-277, 1917). *IEEE Transactions on Medical Imaging*, MI-5(4):170–176, 1986.
- [30] L. Schoeman, P. Williams, A. du Plessis, and M. Manley. X-ray micro-computed tomography (μ CT) for non-destructive characterisation of food microstructure. *Trends in Food Science and Technology*, 47:10 – 24, 2016.
- [31] James Anthony Seibert. Iterative reconstruction: how it works, how to apply it. *Pediatric Radiology*, 44(3):431 – 439, 2014.
- [32] Wolfram Stiller. Basics of iterative reconstruction methods in computed tomography: A vendor-independent overview. *European Journal of Radiology*, 109:147 – 154, 2018.
- [33] Weijie Tao, Damien Rohmer, Grant T. Gullberg, Youngho Seo, and Qiu Huang. An analytical algorithm for tensor tomography from projections acquired about three axes. *IEEE Transactions on Medical Imaging*, 41(11):3454 – 3472, 2022.
- [34] M. Vassholz, B. Koberstein-Schwarz, A. Ruhlandt, M. Krenkel, and T. Salditt. New x-ray tomography method based on the 3d Radon transform compatible with anisotropic sources. *Physical Review Letters*, 116(8), 2016.
- [35] M.L.L. Wijerathne, Kenji Oguni, and Muneo Hori. Tensor field tomography based on 3D photoelasticity. *Mechanics of Materials*, 34(9):533 – 545, 2002.
- [36] Tao Zhang, Yuxiang Xing, Li Zhang, Xin Jin, Hwei Gao, and Zhiqiang Chen. Stationary computed tomography with source and detector in lin-

- ear symmetric geometry: Direct filtered backprojection reconstruction.
Medical physics, 47(5):2222–2236, 2020.
- [37] Yingying Zhang-O’Connor and Jeffrey A. Fessler. Fourier-based forward and back-projectors in iterative fan-beam tomographic image reconstruction. *IEEE Transactions on Medical Imaging*, 25(5):582 – 589, 2006.
- [38] Dominique Zosso, Benoît Le Calennec, Meritxell Bach Cuadra, Kamiar Aminian, Brigitte M. Jolles, and Jean-Philippe Thiran. Bi-planar 2d-to-3d registration in fourier domain for stereoscopic x-ray motion tracking. volume 6914, 2008.