

Measurement

3D tomographic reconstruction of light emission density of plasma in the PROTO-SPHERA experiment --Manuscript Draft--

Manuscript Number:	MEAS-D-24-11112
Article Type:	Research Paper
Keywords:	3D image reconstruction; magnetically confined plasma; nuclear fusion; monitoring
Corresponding Author:	Annamaria Pau Sapienza University of Rome Rome, ITALY
First Author:	Annamaria Pau
Order of Authors:	Annamaria Pau
	Shayesteh Naghinajad
	Franco Alladio
	Paolo Micozzi
	Luca Boncagni
Abstract:	We apply a method to reconstruct 3D density functions based on a corollary of the 3D Fourier slice theorem. This corollary, proved here in its general form, links the projections of a 3D density function onto planes to the 3D Fourier transform of the density function itself. By performing a coordinate transformation and calculating the 3D inverse Fourier transform, we directly solve the inverse problem of reconstructing the density function from its projections. This method is used to represent the visible light emission density distribution of magnetically confined plasma, based on experimental data collected by ENEA for the PROTO-SPHERA project. Emissions from Helium and Hydrogen plasma are captured by cameras arranged in cylindrical symmetry. The technique allows observation of the self-organization process of a plasma torus around a centerpost, confirming theoretical predictions, and enables visualization of plasma cross-sections, revealing its internal organization and emission spectrum.

DIPARTIMENTO DI INGEGNERIA
STRUTTURALE E GEOTECNICA



SAPIENZA
UNIVERSITÀ DI ROMA

Rome, 10th of November 2024

Dear Editor

We are submitting the work titled *3D tomographic reconstruction of light emission density of plasma in the PROTO-SPHERA experiment* for consideration for publication in Measurement. The paper describes the application of a corollary of the 3D Fourier slice theorem to the 3D reconstruction of density functions. This method is used to represent the visible light emission density distribution of magnetically confined plasma, based on experimental data collected by ENEA for the PROTO-SPHERA project. The technique allows observation of the self-organization process of a plasma torus around a centerpost, confirming theoretical predictions, and enables visualization of plasma cross-sections, revealing its internal organization and emission spectrum.

We have read the journal's policy on machine learning/ neural networks papers on the page

<https://www.sciencedirect.com/journal/measurement/about/policies>

We also declare that in this paper, we made no use of generative artificial intelligence. The material presented is the authors' original work and has not been submitted for evaluation elsewhere.

We thank you in advance for your time and consideration.

Yours sincerely,

A handwritten signature in black ink, appearing to read 'Annamaria Pau'.

Annamaria Pau,
Associate professor of Structural Mechanics
Department of Structural and Geotechnical Engineering
Università di Roma La Sapienza
Via Eudossiana 18 - 00184 Roma, Italy
annamaria.pau@uniroma1.it

- A corollary of the 3D Fourier slice theorem is proved in its general form.
- A method proposed to directly reconstruct density functions from projection data.
- Six-camera cylindrical setup achieves accurate 3D plasma reconstruction over time.
- Technique reveals hidden core emissions, previously obscured by outer plasma layers.
- Plasma torus arrangement around a centerpost observed, evolving within 8 ms.

Declaration of interests

☐The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

☒The authors declare the following financial interests/personal relationships which may be considered as potential competing interests:

Annamaria Pau reports financial support was provided by National Center on HPC, Big Data and Quantum Computing CN00000013. If there are other authors, they declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Title Page

Title:

*3D tomographic reconstruction of light emission density of plasma in the
PROTO-SPHERA experiment*

Author Information

Author names:

Annamaria Pau^a, corresponding author, annamaria.pau@uniroma1.it
Shayesteh Naghinajad^a, shayesteh.naghinajad94@gmail.com
Franco Alladio^b, franco.alladio@protosphaera.it
Paolo Micozzi^c, paolo.micozzi@enea.it
Luca Boncagni^c, luca.boncagni@enea.it

Affiliations:

^a *Department of Structural and Geotechnical Engineering, Sapienza University of Rome, via
Eudossiana 18, 00184 Roma, Italy,*

^b *INAF-IAPS National Institute of Astrophysics, Via del Fosso del Cavaliere 100, 00133 Roma, Italy*

^c *Nuclear Research Department, ENEA Italian National Agency for Technology, Energy and
Sustainable Economic Development, Via Enrico Fermi 45, 00044 Frascati (RM), Italy*

Corresponding author:

Annamaria Pau, annamaria.pau@uniroma1.it

3D tomographic reconstruction of light emission density of plasma in the PROTO-SPHERA experiment

Abstract

We apply a method to reconstruct 3D density functions based on a corollary of the 3D Fourier slice theorem. This corollary, proved here in its general form, links the projections of a 3D density function onto planes to the 3D Fourier transform of the density function itself. By performing a coordinate transformation and calculating the 3D inverse Fourier transform, we directly solve the inverse problem of reconstructing the density function from its projections. This method is used to represent the visible light emission density distribution of magnetically confined plasma, based on experimental data collected by ENEA for the PROTO-SPHERA project. Emissions from Helium and Hydrogen plasma are captured by cameras arranged in cylindrical symmetry. The technique allows observation of the self-organization process of a plasma torus around a centerpost, confirming theoretical predictions, and enables visualization of plasma cross-sections, revealing its internal organization and emission spectrum.

Keywords: 3D image reconstruction, magnetically confined plasma, nuclear fusion, monitoring

1. Introduction

Tomographic computed (CT) imaging or scanning is a technique of data treatment which enables visualization of the inner part of objects instead of making real cuts. This has obvious and important applications in the medical field, as well as in the nondestructive monitoring of structures and industrial processes. CT requires the solution of an inverse problem, which can be formulated as follows: given the knowledge of some data called projections, reconstruct the spatial distribution of a density function $f(x, y, x)$ which represents some property (for instance mass density) of an object in the 3D space. Projections are obtained by illuminating the object with stim-

11 uli which produce a measurable response, which is measured on the object
 12 boundary. Projections are, in practice, a sort of shade that the function
 13 $f(x, y, x)$ projects onto lines, or planes. The mathematical demonstration
 14 that a density function could be reproduced from an infinite set of its pro-
 15 jections was provided by the mathematician J. Radon [31] at the beginning
 16 of last century. However, it is quite remarkable that reconstruction of im-
 17 ages from projections was not attempted until 1940. In 1972, Hounsfield [19]
 18 proposed the first method that allowed to image plane slices of the inter-
 19 nal structure of a human body using X-rays projections. The Nobel prize
 20 in physiology and medicine in 1979 awarded to Hounsfield and Cormack [7]
 21 for their work on the initial computed tomography device signifies the im-
 22 portance that this technique has in the medical field. Since then, the use
 23 of computed tomography was extended to a wide number of industrial ap-
 24 plications [9], among which food industry [32], materials science [26], and
 25 geosciences [6].

26 Medical and industrial computed tomography is based on the same un-
 27 derlying principles but differs in the acquisition system layout, as well as in
 28 the stimulus that is used to obtain the projections, which is X-rays in med-
 29 ical CT, but can also be another physical source. In fact, the reflected or
 30 transmitted signal of any source which produces a measurable response of the
 31 scanned object can be used. The feasibility of using ultrasounds [22, 20, 23],
 32 the photo elastic effect [37], or even ground-penetrating radar emitting mi-
 33 crowaves [1] has been proved to be effective to visualize internal patterns
 34 of objects. Different sources interact with different properties, which can
 35 include, besides mass density, electrical, acoustic, and electromagnetic prop-
 36 erties of materials and tissues. For instance, in nuclear medicine, the interest
 37 is focused on reconstructing a cross-sectional image of radioactive isotope
 38 distributions within the human body [11], while in magnetic resonance the
 39 magnetic properties of the object is reconstructed [14]. The photelastic ef-
 40 fect instead was used to determine the plane stress distribution in an optical
 41 fiber [29]. The main difference between the mentioned sources is that some of
 42 them are non-diffracting, like X-rays, which travel in straight lines [4], while
 43 some others are diffracting, like ultrasounds, that is, they are scattered in all
 44 directions, resulting in a more complex inverse problem [21].

45 To convert the measured response data into an image, appropriate pro-
 46 cessing is necessary. The most popular algorithms that are in use can be
 47 classified into two major categories, which are iterative and analytical meth-
 48 ods [18]. Iterative reconstruction algorithms employ a model to predict the

measured data from an initial estimate and then refine the image estimate to better fit the measured data. This process is repeated iteratively until the image quality meets a predetermined standard or until further iterations do not significantly change the image [33, 34]. Starting from the Algebraic Reconstruction Technique (ART) [16], research continues to evolve with refinements and developments, such as the customized Simultaneous Algebraic Reconstruction Technique (SART) [12], and the Simultaneous Iterative Reconstruction Technique (SIRT) [38]. Analytical algorithms are based on mathematical models and explicit formulas that directly compute the image from the acquired projection data. They can be further categorized into two groups: the direct or Fourier-Transform-based methods [39, 17] and the Back-Projection-based algorithms, like Simple Back Projection (SBP), Linear Back Projection (LBP) and Filtered Back Projection (FBP) [28, 38]. Fourier-Transform-based algorithms rely on the relationship between the Fourier transform of the image and the Fourier transform of its projections. More specifically, projections are obtained using Radon transforms [21], that is by computing line integrals of the density function. A more detailed discussion of the literature on the classification of CT algorithms is beyond the scope of this paper. The interested reader can consult [23, 10] for a more exhaustive presentation of the matter.

Traditionally, and up to the present day [25, 13], CT 3D scans have been obtained by assembling a stack of 2D images. In the classical tomographic approach, the 3D image is reconstructed by assembling slices of images deriving from a 2D setting of the problem of image reconstruction on several parallel planes. To do so, the Fourier slice theorem in 2D is employed, which connects the Fourier transform of the Radon transform to the 2D Fourier transform of the image itself. Despite a n th-dimension formulation of the Radon transform was provided by Radon himself, to the best of our knowledge, applications of the 3D Radon transform are limited in the literature. In most of the cases, for the analytical reconstruction, the 3D Fourier slice theorem [5] is applied in the form connecting 1D Fourier transforms of line subsets of the 3D Radon transform [36, 40, 35], with applications also to image-recognition problems [8]. The sole reference that can be found in the literature employing the connection between 2D Fourier transforms of plane projections with central slices of the 3D Fourier volume, is the work by Zosso et al. [40], where the necessary corollary is formulated and the resulting method is employed to track bone movements in stereoscopic X-ray image series of a knee joint.

87 Here, we employ the imaging technique based on the corollary to Fourier
 88 slice theorem in 3D formulated by [40], but demonstrate it for the more gen-
 89 eral case of arbitrary angles. By calculating 3D inverse Fourier transforms
 90 of 2D Fourier transforms of plane projections, the 3D image of the scanned
 91 object can be reconstructed. This technique is fairly general and applies to
 92 the interpretation of the different data, ranging from structural and material
 93 inspection to biological and physical phenomena. The 3D approach is partic-
 94 ularly useful in the case under investigation, where the projections are a given
 95 by a set of digital camera pictures, which will be considered as the discrete 3D
 96 Radon transforms of the visible light emission. We develop an algorithm of
 97 image reconstruction and apply it to the 3D reconstruction of the light emis-
 98 sion density of magnetically confined Helium and Hydrogen plasma. Data
 99 was recorded during the deployment of the PROTO-SHPERA experiment
 100 [2], which took place at ENEA (Italian National Agency for New Technolo-
 101 gies, Energy and Sustainable Development) laboratories in Frascati, Italy.
 102 During the experiment, the plasma emits photons, which light is captured
 103 by six cameras arranged with cylindrical symmetry. The PROTO-SHPERA
 104 experiment is an innovative magnetic confinement plasma experiment for
 105 controlled thermonuclear fusion research, whose aim is to form a plasma
 106 torus not around a metal centerpost, as in tokamaks, but around a plasma
 107 centerpost.

108 This paper is organized into two sections: Section 2 describes the pro-
 109 posed method for image reconstruction, and Section 3 reports the experimen-
 110 tal results. In Subsection 2.1, the 3D continuous Radon transform is recalled,
 111 then in Subsection 2.2 the lemma of the Fourier slice theorem that is at the
 112 base of the image reconstruction method employed is proved. In Subsec-
 113 tion 2.3 proof of the effectiveness of the reconstruction method is provided
 114 by using numerical data. Subsection 3.1 describes the PROTO-SPHERA ex-
 115 periments, and Subsection 3.2 illustrates how the tomographic reconstruction
 116 enabled to prove the sustainment of the plasma torus around the centerpost
 117 and provided further useful information on the dynamics of confined plasma.

118 **2. Analytical direct 3D image reconstruction**

119 *2.1. 3D continuous Radon Transform*

120 For simplicity's sake, the 2D Radon transform is first introduced. Let us
 121 consider a plane object - or a slice of a 3D object - with density function
 122 $f(x, y)$ (Figure 1). Its projections onto a line inclined by an angle θ with

respect to the x axis are obtained by integrating $f(x, y)$ on the set of parallel lines (rays) that are inclined by an angle $\theta + \pi/2$ with respect to the x axis and at a distance t from the origin of the coordinate system (bold line in 1). The equation of a ray is:

$$t = x \cos\theta + y \sin\theta. \quad (1)$$

The projection of the function $f(x, y)$ is then defined as:

$$p(\theta, t) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x, y) \delta(x \cos\theta + y \sin\theta - t) dx dy, \quad (2)$$

where δ is the Dirac's delta function. Equation (2) represents the 2D continuous Radon transform $[\mathcal{R}_2 f](\theta, t)$ of the function f , which is depicted schematically in Figure 1 by the blue curve subtending the sky blue area. It is often preferred to integrate in the rotated coordinate system (t, s) , where the projection can be written as:

$$p(\theta, t) = \int_{-\infty}^{\infty} f(t, s) ds. \quad (3)$$

Geometrically, the continuous 2D Radon transform maps a function in $\mathbb{R}^2$ into the set of its line integrals in $\mathbb{R}^2$. The reconstructed image function $f(x, y)$ is formally obtained by inverting the Radon transform:

$$f(x, y) = [\mathcal{R}_2^{-1} p](\theta, t). \quad (4)$$

Let us now consider a 3D object with density function $f(x, y, z)$ (Figure 2). Its projections are obtained by integrating $f(x, y, z)$ on the plane Σ that is orthogonal to the versor $\boldsymbol{\alpha}$ and at a distance d from the origin of the coordinate system. The versor has direction cosines $\boldsymbol{\alpha} = (\cos\theta \cos\phi, \sin\theta \cos\phi, \sin\phi)^T$, where ϕ is the angle that $\boldsymbol{\alpha}$ forms with the z axis, and θ is the angle that the projection of $\boldsymbol{\alpha}$ onto the xy plane forms with the x axis. The equation of a plane orthogonal to $\boldsymbol{\alpha}$ and at a distance d from the origin is:

$$d = x \cos\theta \cos\phi + y \sin\theta \cos\phi + z \sin\phi. \quad (5)$$

In an analogy with Equation (2), the projection of the function $f(x, y, z)$ associated with the couple $(\boldsymbol{\alpha}, d)$ is then defined as:

$$p(\boldsymbol{\alpha}, d) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x, y, z) \delta(x \cos \theta \cos \phi + y \cos \theta \sin \phi + z \sin \theta - d) dx dy dz. \quad (6)$$

145 The area over which the previous integral is computed is included in the
 146 dashed line in Figure 2. Equation (6) represents the 3D continuous Radon
 147 transform $[\mathcal{R}_3 f](\boldsymbol{\alpha}, t)$ of the function f . Geometrically, the continuous 3D
 148 Radon transform maps a function in $\mathbb{R}^3$ into the set of its plane integrals
 149 in $\mathbb{R}^3$. The reconstructed image function $f(x, y, z)$ is formally obtained by
 150 inverting the Radon transform:

$$f(x, y, z) = [\mathcal{R}_3^{-1} p](\boldsymbol{\alpha}, d). \quad (7)$$

151 It is important to note that in real applications we won't be dealing with plane
 152 integrals. In fact, $\mathcal{R}_3$ can be obtained by computing the $\mathcal{R}_2$ of the intersection

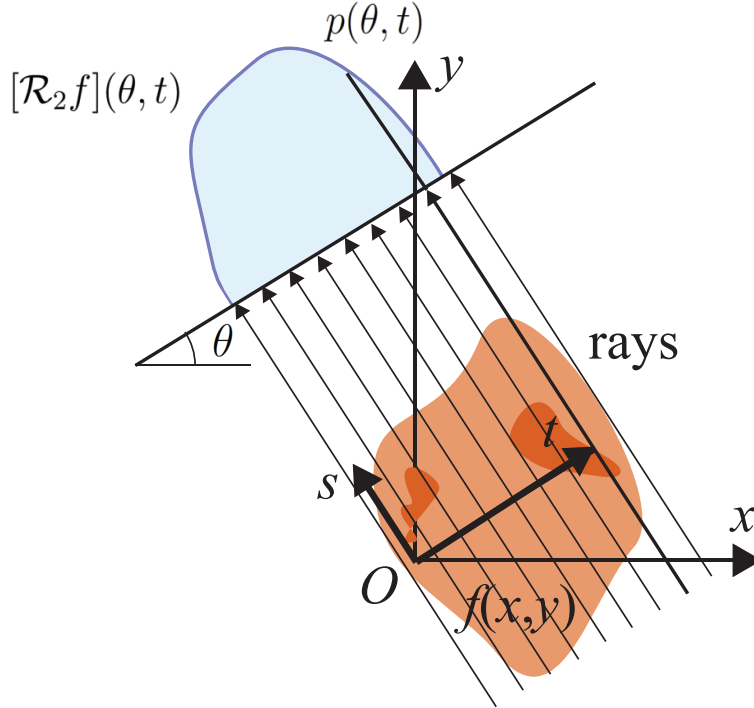


Figure 1: Projection of a plane density function onto a line.

153 between f and the plane Σ , and then integrating over lines orthogonal to the
 154 rays, which are displayed in Figure 3 as yellow lines. The line integrals are
 155 much easier to be obtained in practical measurements. They correspond
 156 to X-ray projections taken from different angles, or, in the present case, to
 157 pictures taken from different cameras.

158 Let us assume that the integration is carried out in the rotated coordinate
 159 system (t, s, r) , whose axes t and s coincide with the tangent lines to the
 160 parallels and meridians, respectively, of an imaginary sphere centered in the
 161 origin of the system of coordinates, at the point of intersection between the
 162 sphere and the versor α , with the orientation of the r axis coinciding with
 163 that of α (Figure 3, upper left). The transformation of coordinates from
 164 (x, y, z) to (t, s, r) can be written as:

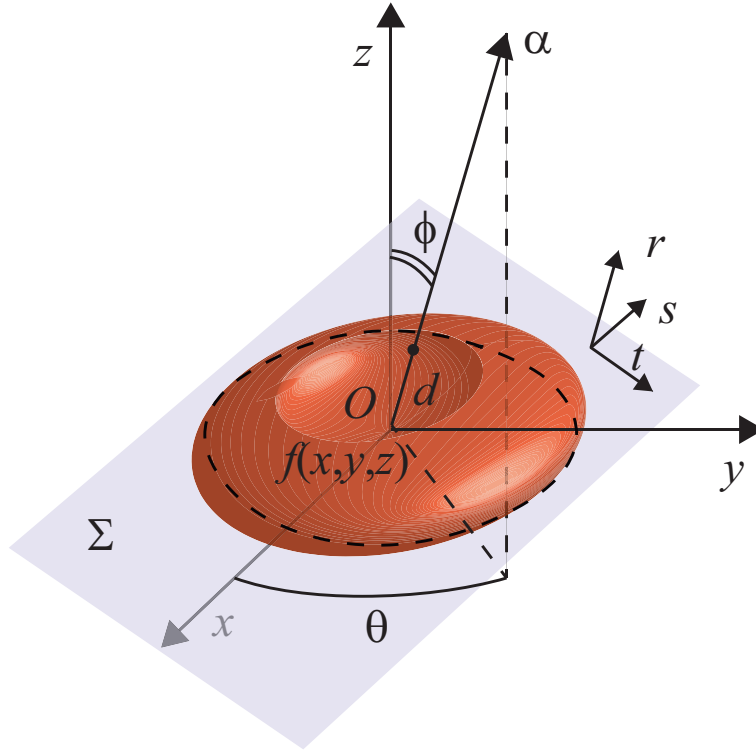


Figure 2: Projection of a 3D density function onto a plane.

$$\begin{bmatrix} t \\ s \\ r \end{bmatrix} = \begin{bmatrix} \cos\phi & 0 & \sin\phi \\ 0 & 1 & 0 \\ \sin\phi & 0 & \cos\phi \end{bmatrix} \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} \quad (8)$$

Moreover, the distance d can be expressed in the (t, s, r) system of coordinates as $d = \sqrt{t^2 + r^2}$. The ray integral is then expressed as follows:

$$p(\theta, \phi, t, r) = \int_{-\infty}^{\infty} f(t, s, r) ds. \quad (9)$$

The integral $p(\theta, \phi, t, r)$ is in practice a line projection of the 3D function. The set of all the line projections on the plane (t, r) for given θ and ϕ will be denoted as $P_{\theta, \phi}$.

2.2. 3D Fourier slice theorem

The extension of the Fourier slice theorem to higher dimensions, enables direct reconstruction of 3D objects. The 3D Fourier Slice Theorem can be stated as follows.

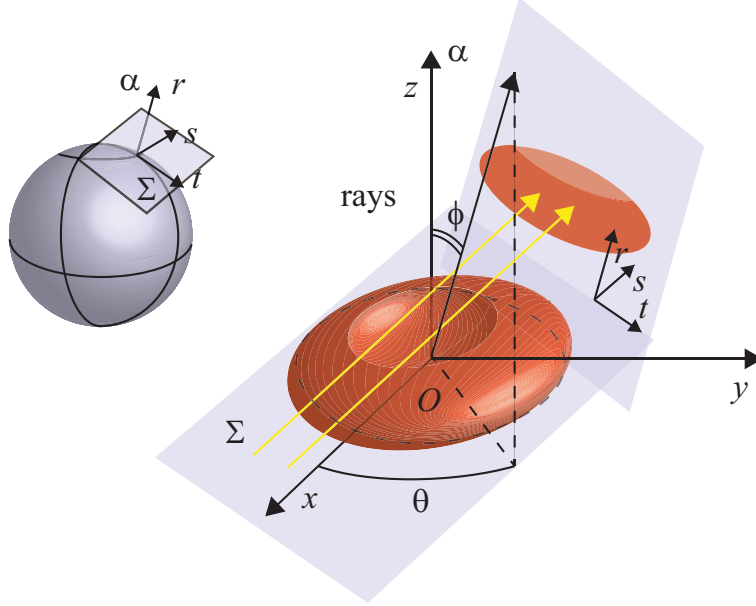


Figure 3: Ray integrals over a 3D function.

174 **Theorem 1.** *A line slice of the 3D Fourier transform of the function $f(x, y, z)$*
 175 *equals the 1D Fourier Transform of the 3D Radon transform $[\mathcal{R}_3 f](\boldsymbol{\alpha}, t)$ at*
 176 *the same angle.*

177 For proof, the interested reader can consult [40, 5]. However, the theorem
 178 in this form is of little use for practical purposes. A useful corollary can be
 179 stated.

180 **Corollary 1.1.** *A plane slice of the 3D Fourier transform of the density func-*
 181 *tion $f(x, y, z)$ equals the 2D Fourier Transform of the set of line projections*
 182 *at the same angle.*

183 *Proof.* Let us consider the line projection of a 3D density function $P_{\theta, \phi}$, and
 184 take its 2D Fourier Transform ($\mathcal{F}_2$):

$$[\mathcal{F}_2 P_{\theta, \phi}](\mu, \nu) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} P_{\theta, \phi}(t, r) e^{-2\pi i(\mu t + \nu r)} dt dr, \quad (10)$$

185 where μ and ν are the wavenumbers in the t, r directions, respectively and i
 186 is the imaginary unit.

187 By substituting equation (9) into equation (10), we get:

$$[\mathcal{F}_2 P_{\theta, \phi}](\mu, \nu) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \left[\int_{-\infty}^{\infty} f(t, s, r) ds \right] e^{-2\pi i(\mu t + \nu r)} dt dr. \quad (11)$$

188 Based on the transformation (8), the rotated coordinate system (t, s, r) can
 189 be converted back into the initial (x, y, z) system:

$$[\mathcal{F}_2 P_{\theta, \phi}](\mu, \nu) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x, y, z) e^{-2\pi i k} dx dy dz \quad (12)$$

$$\text{with } k = \mu \cos \phi \cos \theta x + \mu \cos \phi \sin \theta y + \mu \sin \phi z + \nu \sin \phi \cos \theta x + \\ + \nu \sin \phi \sin \theta y + \nu \cos \phi z.$$

190 It is obvious that the right hand side of (12) represents the 3D Fourier Trans-
 191 form $\mathcal{F}_3$ of f at wavenumbers $u = \mu \cos \phi \cos \theta + \nu \sin \phi \cos \theta$, $v = \mu \cos \phi \sin \theta +$
 192 $\nu \sin \phi \sin \theta$ and $w = \mu \sin \phi + \nu \cos \phi$:

$$[\mathcal{F}_3 f](u, v, w) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x, y, z) e^{-2\pi i(ux + vy + wz)} dx dy dz, \quad (13)$$

193 which proves the corollary. The quantities u, v and w are wavenumbers in
 194 directions x, y and z , respectively. $\square$

195 The interesting consequence of this corollary is that, given the line pro-
 196 jections, on calculating their 2D Fourier Transform, and then, the inverse 3D
 197 Fourier Transform of the resulting set of 2D Fourier transforms, the original
 198 image is directly reconstructed. The following subsection provides numerical
 199 proof of the effectiveness of the procedure.

200 2.3. Numerical example of the 3D direct reconstruction

201 Let us consider an input density function modeled as a cube with a uni-
 202 form density of 0.5, side length of 0.6, and centered at the origin of the
 203 coordinate system (Figure 4a). Inside this cube, two smaller spheres are
 204 embedded: one with a density of 0.3, a radius of 0.1, and centered at coordi-
 205 nates (0.05, 0.05, -0.2); the other with a density of 0.1, a radius of 0.05, and
 206 centered at (-0.1, 0, 0.1). These variations in density are used to verify the
 207 accuracy of the reconstruction method, ensuring it can effectively distinguish
 208 between different density regions. Although the spheres are not visible from
 209 the exterior of the solid cube, Figure 4b illustrates their positions by ren-
 210 dering the cube with partial transparency. It is important to note that for
 211 this numerical example, the theoretical framework developed for continuous
 212 systems in the previous sections has been adapted for use with discrete data.
 213 In this case, we employed a total of $N = 64 \times 64 \times 64$ sampling points, with
 214 64 points along each side of the cube. Additionally, $N_{deg} = 12 \times 12 = 144$
 215 spherical projections were taken, with 12 projections for each angle θ and ϕ .
 216 The angular step size was kept constant over π , set to $\pi/12$.

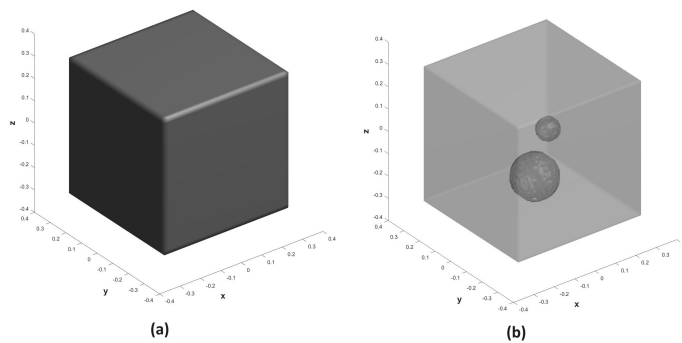


Figure 4: a) 3D density function b) density function made partially transparent to show the embedded spheres.

217 Figures 5a and b provide samples of the spherical projections used for
 218 reconstruction, taken at angles $(\theta, \phi) = (\pi/2, 0)$ and $(2\pi/3, 0)$, respectively.

219 Note that the color scale extends beyond 0.5 because it represents line inte-
 220 grals of the density function over the entire domain, rather than local values.
 221 Regions of lower density show in fact lower integral values. By combining
 222 a total of 144 two-dimensional Fourier transforms of the spherical projec-
 223 tions and computing their three-dimensional inverse Fourier transform, the
 224 reconstructed density function shown in Figure 6a is obtained, matching the
 225 original function from Figure 4a. Figures 6b, c, and d present cross-sections
 226 of the reconstructed density function along two vertical planes (b,c) and a
 227 horizontal plane (d) passing through the origin, with the corresponding color
 228 scale. This Figure shows that the embeddes spheres are exactly reproduced
 229 not only in terms of position, but also in terms of density. See in fact that
 230 the large sphere has high density, while the small one low.

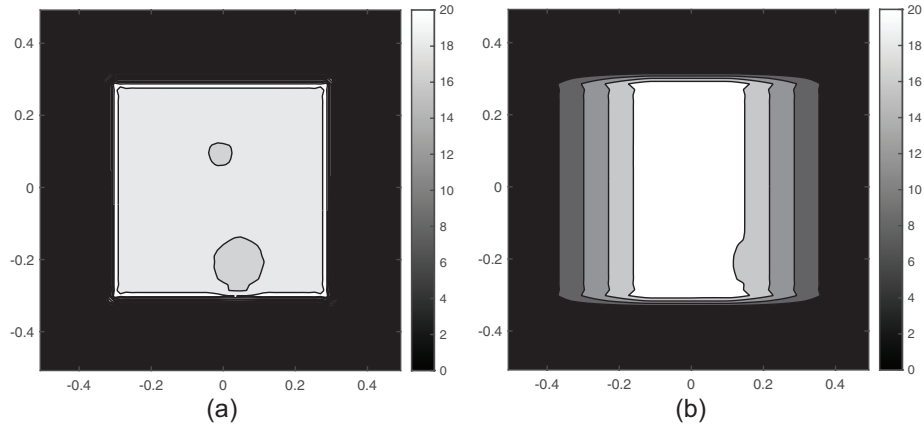


Figure 5: A couple of spherical projections pertaining to angles a) $\theta = \pi/2$, $\phi = 0$ and b) $\theta = 2\pi/3$, $\phi = 0$

231 The previous example was implemented using a large number of projec-
 232 tions. While reducing the number of projections results in a slightly blurred
 233 reconstructed image, even a very limited number of projections still pro-
 234 vides a sufficiently clear reconstruction. This effect is illustrated in Figure
 235 7, which shows the reconstructed density function with varying numbers of
 236 projections. The source density function is depicted in Figure 7. It is shaped
 237 as a cube with a unit density, a side length of 0.2, and is centered at the ori-
 238 gin of the coordinate system. A sphere with unit density, a diameter larger
 239 than the cube's side, and centered at the origin, was subtracted from the
 240 density function, causing its volume to intersect with the cube's faces. Al-

241 though the image quality decreases as the number of projections is reduced,
 242 the shape remains distinctly recognizable even with a very limited number
 243 of projections, as few as 9 (Figure 7d).

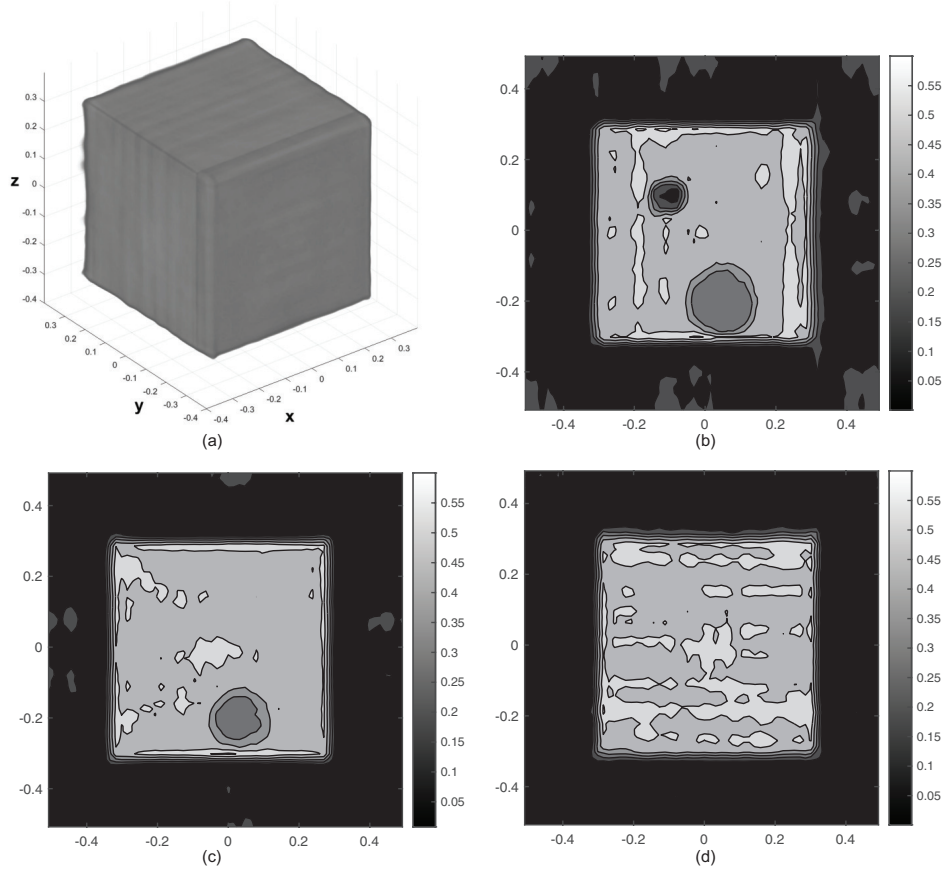


Figure 6: a) Reconstructed image, b) vertical cross section on the xz plane, c) vertical cross section on the yz plane, d) horizontal cross section on the xy plane.

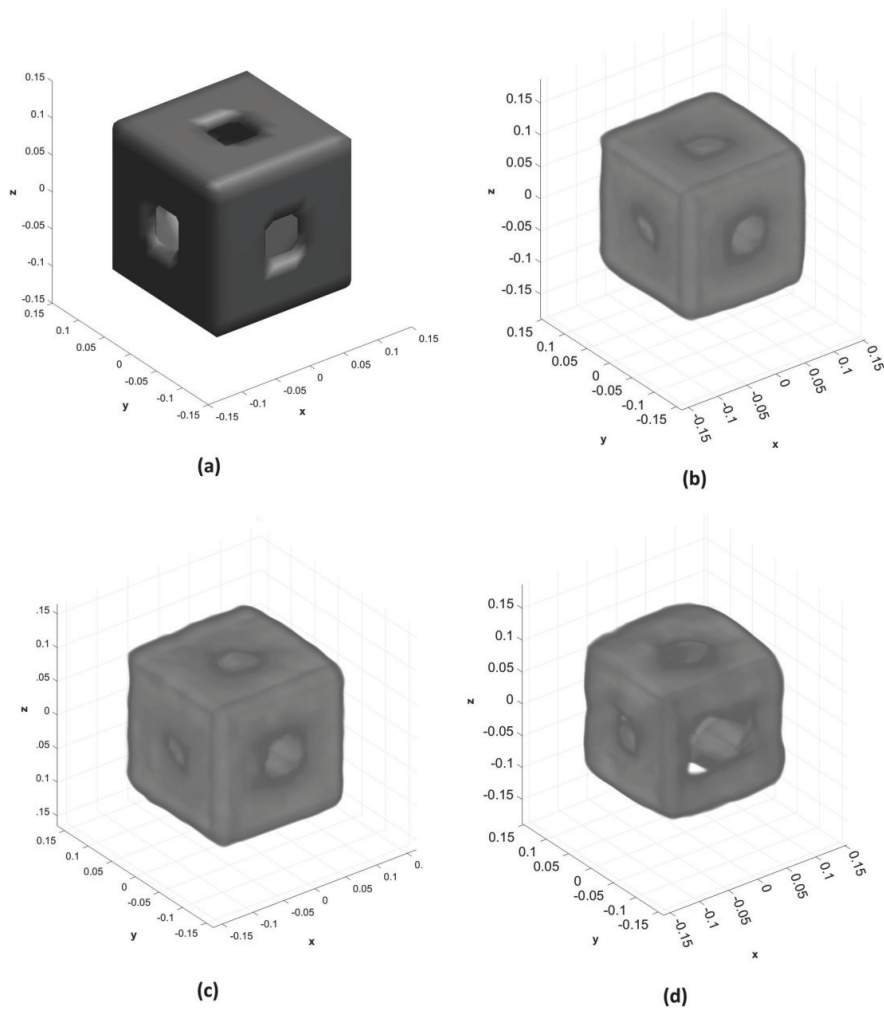


Figure 7: a) Source image and Reconstructed image with b) $12\times$, c) 6×6 and d) 3×3 projections.

3. 3D Reconstruction of light emission density in PROTO-SPHERA experiment

3.1. The PROTO-SPHERA experiment

PROTO-SPHERA (Spherical Plasma for Helicity Relaxation Assessment) is an innovative magnetic confinement plasma experiment aimed at advancing controlled thermonuclear fusion research. Its goal is to form plasma within a confined environment. Unlike tokamaks, where the plasma forms around a metal centerpost, in PROTO-SPHERA, the plasma forms around a plasma centerpost [3, 24, 27]. Tokamaks, the most extensively studied magnetic fusion configurations, have a non-simply connected geometry, where a central post—containing the inner part of the toroidal magnet and the ohmic transformer—connects the plasma torus. The development of simply connected magnetic configurations relevant for fusion would significantly simplify the design of fusion reactors. Additionally, the PROTO-SPHERA experiment seeks to emulate astrophysical plasmas, such as jet and torus structures, in a laboratory setting [2].

Figure 8 shows a schematic of the PROTO-SPHERA experiment, which consists of a transparent cylindrical vacuum chamber measuring 2.5 meters in height and 2.0 meters in diameter. The chamber is equipped with electrodes at the ends and internal poloidal field coils. The operating sequence proceeds as follows: the tungsten filaments of the cathode are heated to 2600°C over 20 seconds, while the current in the poloidal field coils is ramped up to 1875 A. Gas is then introduced through a hollow section in the anode, reaching a pressure of 10^{-2} to 10^{-1} mbar, and a total voltage of 100-350 V is applied between the electrodes. Under these conditions, plasma forms and emits photons.

To study the plasma formation process and the magneto-hydrodynamic phenomena occurring in the experiment, six high-speed cameras are positioned on the equatorial plane of the chamber, spaced 60 degrees apart (Figure 9). Each camera has a resolution of 640x480 pixels and can capture 600 frames per second. Sample images recorded by the cameras at a specific time instant are shown in Figures 10a and 11a for shots with Helium and Hydrogen, respectively.

Combining the data from the cameras with measurements from a Langmuir probe has provided valuable insights into the dynamics of the plasma torus formation, including its size and emission spectrum. Numerous discharges were produced and observed throughout the experimental campaign.

281 The images captured by the six cameras are in fact projections onto
 282 planes of the visible-light emission density, which plays the role of the density
 283 function f (Figures 10a and 11a). For each time instant, exploiting the
 284 corollary of the Fourier Slice Theorem of Section 2, these source images
 285 are 2D-Fourier transformed, then 3D-Fourier inverted to obtain the density
 286 of the visible light emitted by the plasma. The only change with respect
 287 to the procedure described in Section 2.2 is that, given the positioning of
 288 the cameras, the inversion has to be based on a cylindrical set of parallel
 289 projections instead of a spherical set, with ϕ fixed. Figures 10b and 11b
 290 show the reconstructed plasma light emission distribution derived from the
 291 images on the left. It is important to note that the original images contain
 292 three layers of information, corresponding to the RGB color channels. The
 293 reconstructed images in Figures 10b and 11b are false-color representations,
 294 created by combining the RGB channels linearly with the following weighting:
 295 100% Red, 86% Green, and 25% Blue [2].

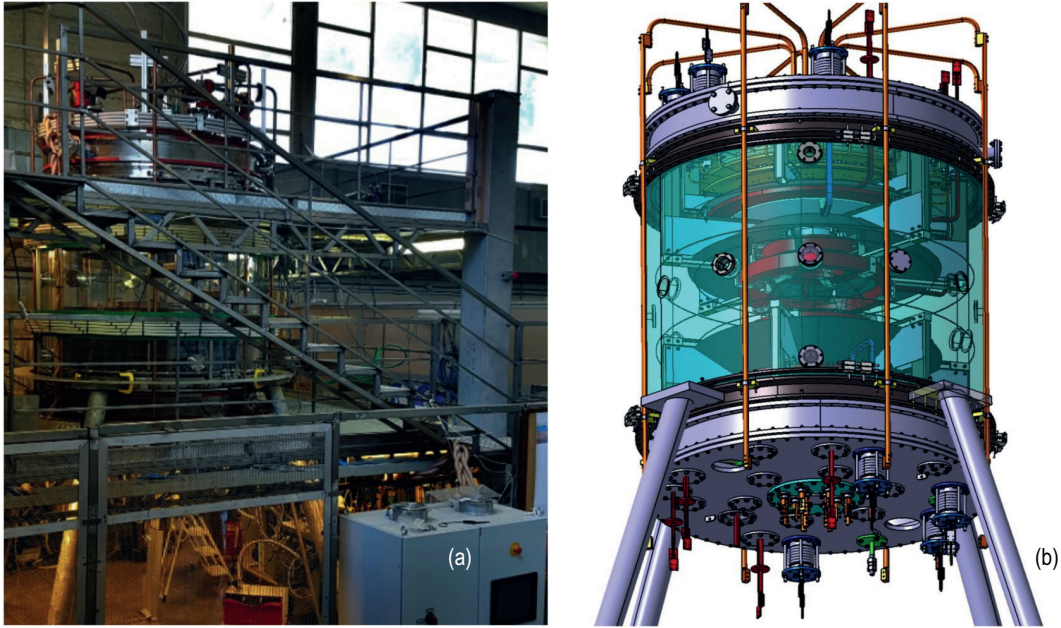


Figure 8: Picture of PROTO-SPHERA setup and schematic of the machine b)

296 3.2. Results and discussion

297 The proposed 3D tomography technique enabled detailed observation of
 298 the plasma torus formation process. According to magneto-hydrodynamic

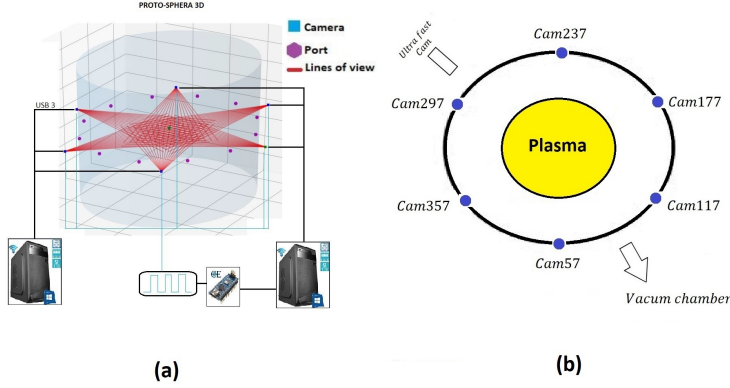


Figure 9: a) Position of the cameras in axonometry b) and plan

models [15], the sequence begins with the formation of a plasma column (pinch or centerpost) at low current. As the current increases, the magnetic field lines bend, causing the pinch to become unstable, which leads to the formation of a torus around the centerpost. This progression is visualized in Figure 12 a-h, showing the reconstructed light emission density at eight sequential time steps (each 1/600 s apart) during a Helium plasma discharge. The same sequence predicted by the work of García-Martínez et al. [15] is observed. The entire torus formation occurs within approximately 8 ms, five time steps intercurring from the arrangements of Figure 12 b to g. The presence of both jets and the torus strongly parallels astrophysical phenomena, as discussed in [2].

Once the spatial distribution of light emission density is known for each time instant, cross-sections can be derived, revealing further insights into plasma dynamics. Figure 13 shows an image of the Helium plasma captured by one of the six cameras (a) and a vertical cross-section (b) of the reconstructed light emission density in true color, taken at the same moment. To create this image, each of the three RGB channels was processed independently to generate a corresponding density function for each color. This cross-sectional view reveals the green color, which was obscured in the camera pictures by the intense golden glow of the 'ionization peel'—the layer between the plasma and the surrounding gas cushion. The correctness of the reconstruction is proved by the appearance of the Langmuir probe and of the copper bars on the right hand side of the picture.

Figure 14 presents vertical cross-sections of the plasma light emission den-

323 sity at the same time instants as Figure 12. These images further confirm
 324 the sequence of events leading to the formation of the torus, as described in
 325 the previous paragraph. Specifically, Figure 14 illustrates the emergence of
 326 a stable pinch (a-b), its progression towards instability (c-f), and ultimately
 327 the arrangement of the torus around the centerpost (d-h). Similar patterns,
 328 with comparable duration, were observed in other plasma discharges [2, 30].
 329 Figure 15 presents a horizontal cross-section at the equatorial level, enabling
 330 observation of the same phenomena from a different perspective. In Figures
 331 15d and 15e, the pinch’s instability is particularly evident, as shown by the
 332 loss of axial symmetry and the presence of green regions on the right side of
 333 the images. Additionally, the horizontal cross-section analysis provides in-
 334 sights into plasma stability and torus thickness. Figure 16 illustrates selected
 335 horizontal cross-sections of Helium plasma at the equatorial level across se-
 336 quential times, clearly depicting the torus encircling the centerpost. Notably,
 337 the light emission intensity varies over time, suggesting potential correlations
 338 with plasma density and stability. These images also allowed for the mea-
 339 surement of torus thickness, defined as the difference between the outer and
 340 inner circumferences’ radii, which was observed to be between 0.25 and 0.35
 341 m. Similar behaviors were noted for Hydrogen plasma discharges, with an
 342 equatorial horizontal cross-section provided in Figure 17 for reference.

343 4. Conclusions

344 An imaging technique based on the formulation of a corollary the Fourier
 345 slice theorem was applied to reconstruct the light emission distribution of
 346 magnetically confined plasma in three dimensions. The algorithm was used
 347 on data capturing the visible light emission from Helium and Hydrogen plas-
 348 mas, recorded by six cameras as part of the PROTO-SPHERA experiment
 349 at the ENEA Frascati laboratories. The results demonstrate that six images
 350 taken by cameras arranged in cylindrical symmetry enable an accurate 3D
 351 reconstruction of the plasma’s spatial distribution at different time instants
 352 during the experiment. We observed that the plasma torus forms in approxi-
 353 mately 8 ms, evolving from a kinked filament caused by the destabilization of
 354 the plasma centerpost. This 3D reconstruction allows for cross-sectional anal-
 355 ysis, revealing previously hidden green light emission from the core, which
 356 had been obscured by the outer golden ionized layer. Additionally, equato-
 357 rial cross-sections enabled the measurement of the torus thickness, which was
 358 found to range between 0.25 and 0.35 meters.

359 **5. Acknowledgement**

360 Annamaria Pau gratefully acknowledges the funding contribution from
361 National Center on HPC, Big Data and Quantum Computing CN00000013.
362 This manuscript reflects only the authors' views and opinions, neither the
363 European Union nor the European Commission can be considered responsible
364 for them.

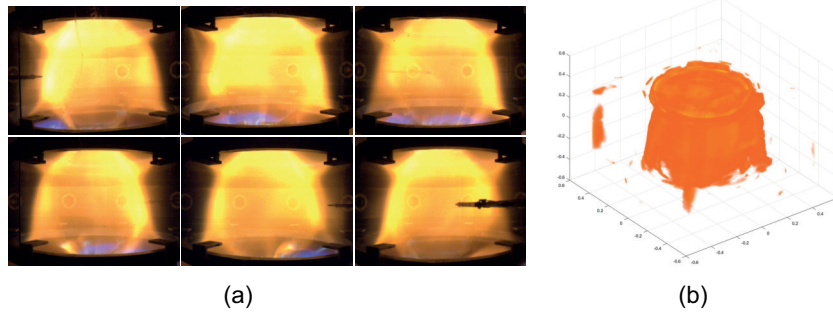


Figure 10: a) Pictures recorded by the six cameras at a given time instant (267 msec) of the PROTO-SPHERA experiment from shot 2505 of Helium b) 3D reconstruction of plasma light emission density

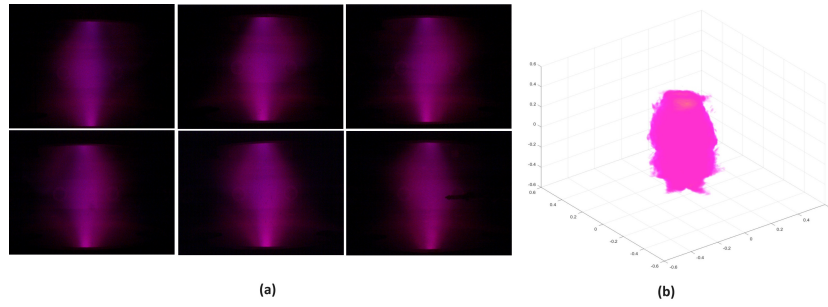


Figure 11: a) Pictures recorded by the six cameras at a given time instant (233msec) of the PROTO-SPHERA experiment from shot 2154 of Helium b) 3D reconstruction of plasma light emission density

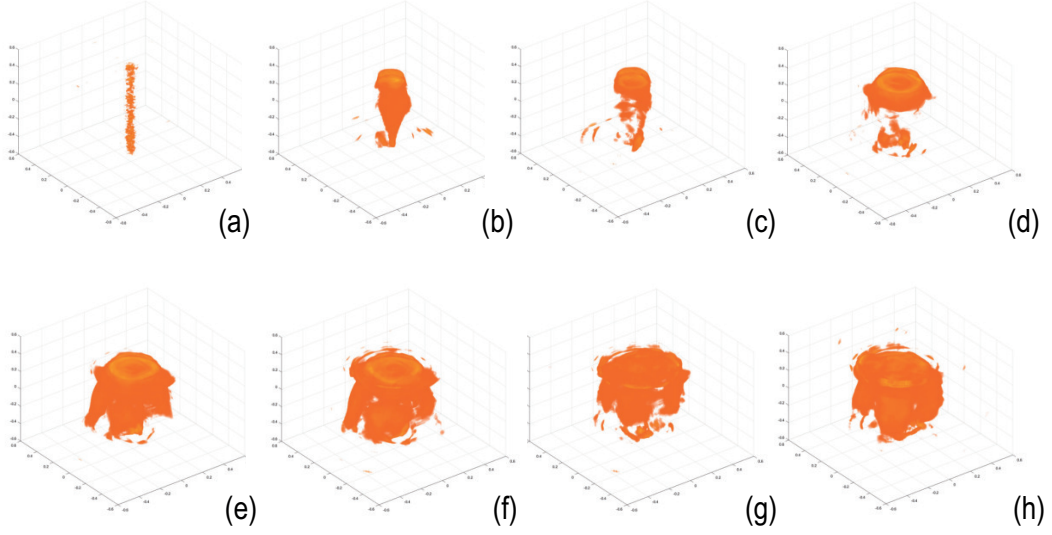


Figure 12: Reconstructed light emission density of Helium discharge 2505 at eight sequential time steps: stable pinch (a-b), pinch instability (c-f), formation of the plasma torus around the pinch (g-h)

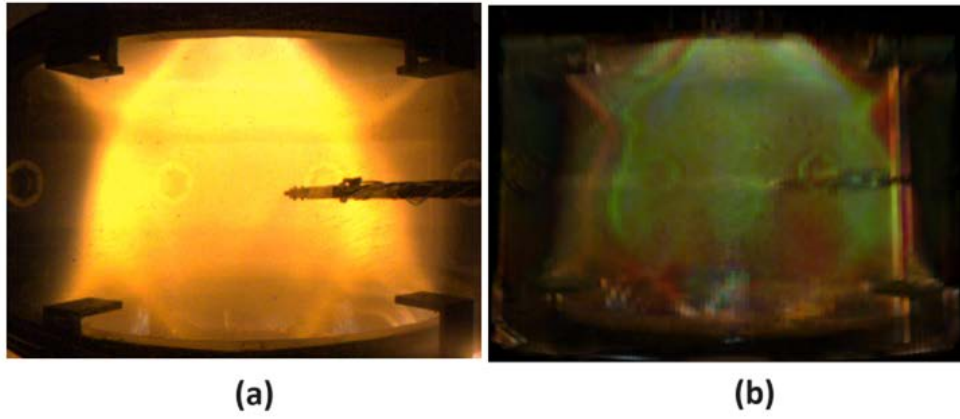


Figure 13: A projection of Helium plasma visible light emission captured by a camera during shot 2141 (a) and cross-section of the reconstructed light emission density in true colors (b).

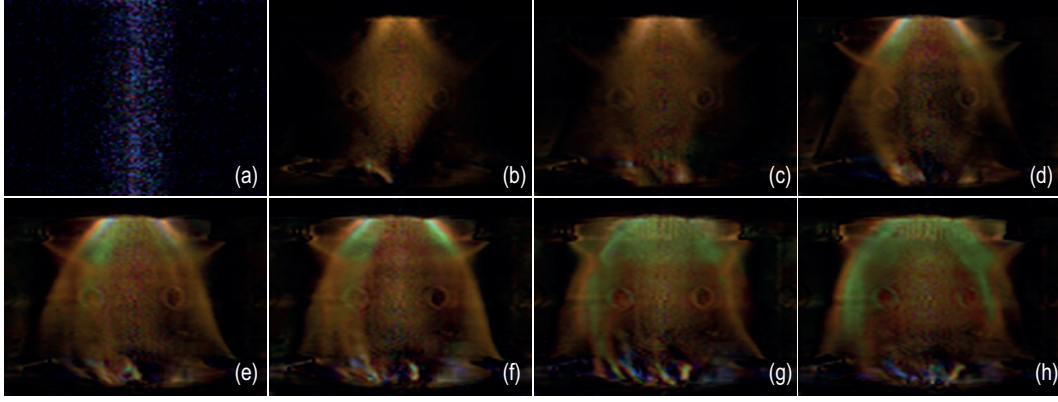


Figure 14: Vertical cross-sections of the reconstructed light emission density of Helium discharge 2505 at the same eight sequential time instants as Figure 12: stable pinch (a-b), pinch instability (c-f), formation of the plasma torus around the pinch (g-h)

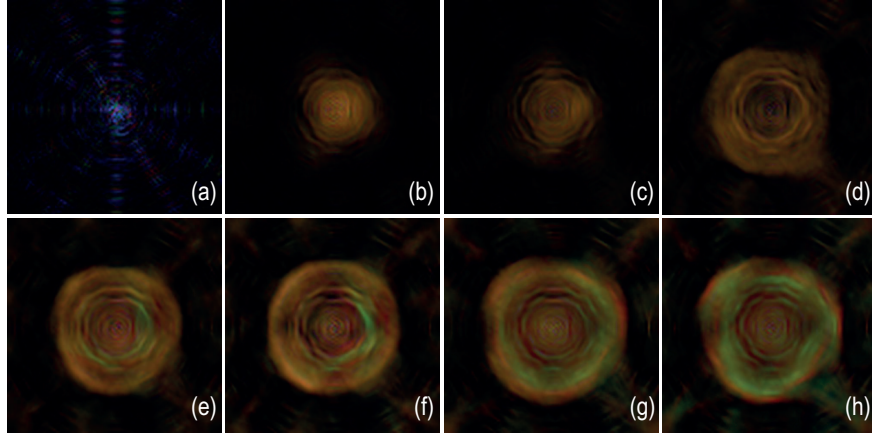


Figure 15: Horizontal cross-sections of the reconstructed light emission density of Helium discharge 2505 at the same eight sequential time instants as Figure 12: stable pinch (a-b), pinch instability (c-f), formation of the plasma torus around the pinch (g-h)

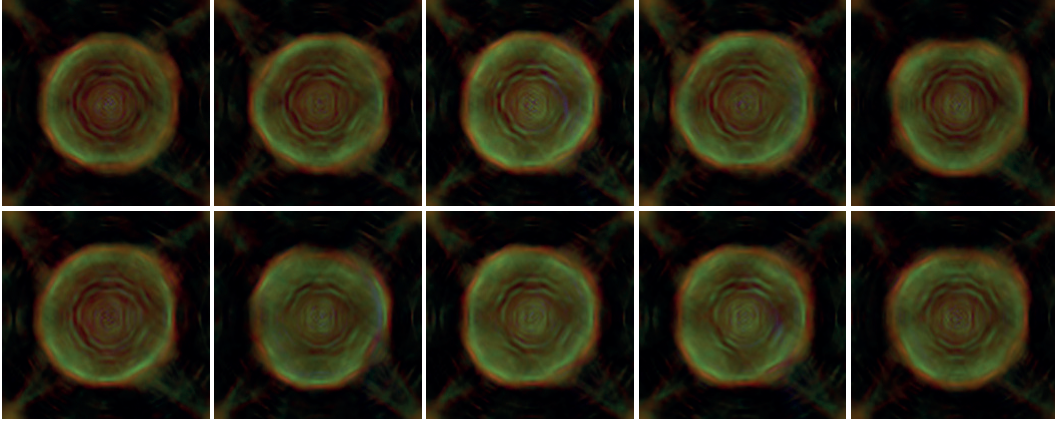


Figure 16: Sequence of reconstructed equatorial cross-sections of light emission density of Helium discharge 2505 at 10 time instants

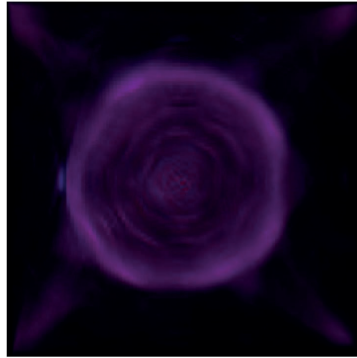


Figure 17: Equatorial cross-section of light emission density of Helium discharge 2154

References

- [1] Amir M. Alani, Francesco Soldovieri, Ilaria Catapano, Iraklis Giannakis, Gianluca Gennarelli, Livia Lantini, Giovanni Ludeno, and Fabio Tosti. The use of ground penetrating radar and microwave tomography for the detection of decay and cavities in tree trunks. *Remote Sensing*, 11(18), 2019.
- [2] F. Alladio, P. Micozzi, L. Boncagni, A. Pau, S. Naghinajad, S. Macera, Y. Damizia, P. Buratti, F. Filippi, G. Galatola Teka, F. Giammanco, E. Giovannozzi, M. Iafrati, A. Lampasi, and P. Marsili. PROTO-SPHERA: a magnetic confinement experiment which emulates the jet + torus astrophysical plasmas. *Plasma Physics and Controlled Fusion*, 66(3), 2024.
- [3] Franco Alladio, Paolo Costa, Alessandro Mancuso, Paolo Micozzi, Stamos Papastergiou, and Francois Rogier. Design of the PROTO-SPHERA experiment and of its first step (MULTI-PINCH). *Nuclear Fusion*, 46:S613–S624, 2006.
- [4] Habib Ammari. Tomographic imaging with non-diffracting sources. an introduction. *Mathematics of Emerging Biomedical Imaging*, 28(10):95–106, 2008.
- [5] Amir Averbuch and Yoel Shkolnisky. 3d Fourier based discrete Radon transform. *Applied and Computational Harmonic Analysis*, 15(1):33 – 69, 2003.
- [6] V. Cnudde and M.N. Boone. High-resolution X-ray computed tomography in geosciences: A review of the current technology and applications. *Earth-Science Reviews*, 123:1 – 17, 2013.
- [7] A. M. Cormack. Early two-dimensional reconstruction and recent topics stemming from it. *Science*, 209:1482–1486, 1980.
- [8] Petras Daras, Dimitrios Zarpalas, Dimitrios Tzovaras, and Michael G. Strintzis. Shape matching using the 3d Radon transform. page 953 – 960, 2004.
- [9] L. De Chiffre, S. Carmignato, J.-P. Kruth, R. Schmitt, and A. Weckenmann. Industrial applications of computed tomography. *CIRP Annals - Manufacturing Technology*, 63(2):655 – 677, 2014.

- 398 [10] S.R. Deans. *The Radon Transform and some of its applications*. Wiley:
399 New York, 1983.
- 400 [11] Musa Sani Musa Niyazi Şentürk Fatih Veysel Nurçin Dilber
401 Uzun Ozsahin, Berna Uzun and Ilker Ozsahin. Evaluating nuclear
402 medicine imaging devices using fuzzy PROMETHEE method. *Proce-*
403 *dia computer science*, 120:699–705, 2017.
- 404 [12] S. Donato, L. Broubal, L.M. Arana Peña, F. Arfelli, A. Contillo, P. De-
405 logu, F. Di Lillo, V. Di Trapani, V. Fanti, R. Longo, P. Oliva, L. Rigon,
406 L. Stori, G. Tromba, and B. Golosio. Optimization of a customized
407 simultaneous algebraic reconstruction technique algorithm for phase-
408 contrast breast computed tomography. *Physics in Medicine and Biology*,
409 67(9), 2022.
- 410 [13] Mario D’Acunzio, Antonio Benassi, Davide Moroni, and Ovidio Salvetti.
411 3D image reconstruction using Radon transform. *Signal, Image and*
412 *Video Processing*, 10(1):1 – 8, 2016.
- 413 [14] Jeffrey Fessler. Optimization methods for magnetic resonance image
414 reconstruction: Key models and optimization algorithms. *IEEE signal*
415 *processing magazine*, 37(1):33–40, 2020.
- 416 [15] Pablo Luis García-Martínez, Leandro Gabriel Lampugnani, and Ricardo
417 Farengo. Effect of the helicity injection rate and the Lundquist number
418 on spheromak sustainment. *Physics of Plasmas*, 21(12), 2014.
- 419 [16] Richard Gordon, Robert Bender, and Gabor T. Herman. Alge-
420 braic reconstruction techniques (ART) for three-dimensional electron
421 microscopy and x-ray photography. *Journal of Theoretical Biology*,
422 29(3):471–476,IN1–IN2,477–481, 1970.
- 423 [17] David Gottlieb, Bertil Gustaffson, and Patrick Forssen. On the direct
424 Fourier method for computed tomography. *IEEE Transactions on Med-*
425 *ical Imaging*, 19(3):223–232, 2000.
- 426 [18] B Gustaffson. Mathematics for computed tomography. *Physica Scripta*,
427 T61:38– 43, 1996.

- 428 [19] G. N. Hounsfield. Computerized transverse axial scanning (tomog-
429 raphy): Part I. description of system. *British Journal of Radiology*,
430 46(3):1016–1022, 1973.
- 431 [20] P. Huthwaite. Guided wave tomography with an improved scattering
432 model. *Proceedings of the Royal Society A: Mathematical, Physical and*
433 *Eng. Sciences*, 472(2195), 2016.
- 434 [21] A. C. Kak and M. Slaney. *Principles of Computerized Tomographic*
435 *Imaging*. IEEE press, 1988.
- 436 [22] Avinash C. Kak. Computerized tomography with X-ray, emission, and
437 ultrasound sources. *Proceedings of the IEEE*, 67(9):1245 – 1272, 1979.
- 438 [23] Mohd Taufiq Mohd Khairi, Sallehuddin Ibrahim, Mohd Amri Md Yunus,
439 M. Faramarzi, Goh Pei Sean, Jaysuman Pusppanathan, and A. Abid.
440 Ultrasound computed tomography for material inspection: Principles,
441 design and applications. *Measurement: Journal of the International*
442 *Measurement Confederation*, 146:490 – 523, 2019.
- 443 [24] Alessandro Lampasi, Giuseppe Maffia, Franco Alladio, Luca Boncagni,
444 Federica Causa, Edmondo Giovannozzi, Luigi Andrea Grosso, Alessan-
445 dro Mancuso, Paolo Micozzi, Valerio Piergotti, Giuliano Rocchi,
446 Alessandro Sibio, Benedetto Tilia, and Vincenzo Zanza. Progress of the
447 plasma centerpost for the PROTO-SPHERA spherical tokamak. *Ener-*
448 *gies*, 9(7), 2016.
- 449 [25] Xin Li, Yudong Lu, Xiaozhou Zhang, Wen Fan, Yangchun Lu, and
450 Wangsheng Pan. Quantification of macropores of malan loess and the
451 hydraulic significance on slope stability by x-ray computed tomography.
452 *Environmental Earth Sciences*, 78(16), 2019.
- 453 [26] E. Maire and P.J. Withers. Quantitative X-ray tomography. *Int. Mate-*
454 *rials Reviews*, 59(1):1 – 43, 2014.
- 455 [27] Paolo Micozzi, Franco Alladio, Alessandro Mancuso, Vincenzo Zanza,
456 Gerarda Apruzzese, Francesca Bpmbarda, Luca Boncagni, Paolo Bu-
457 ratti, Francesco Filippi, Giuselle Galatola Teka, Francesco Giammarco,
458 Edmondo Giovannozzi, Andrea Grosso, Matteo Iafrati, Alessandro Lam-
459 pasi, Violeta Lazic, Simone Magagnino, Simone Mannori, Paolo Marsili,

- 460 Paolo Piergotti, Giuliano Rocchi, Alessandro Sibio, Benedetto Tilia, and
 461 Onofrio Tudisco. Final results of the first phase of the PROTO-SPHERA
 462 experiment: obtainment of the full current stable screw pinch and first
 463 evidences of the jet + torus combined plasma configuration. *Plasma*
 464 *Science and Technology*, 26:025103, 2024.
- 465 [28] Tugba Ozge Onur. An application of filtered back projection method for
 466 computed tomography images. *International Review of Applied Sciences*
 467 *and Engineering*, 12(2):194 – 200, 2021.
- 468 [29] Yongwoo Park, Un-Chul Paek, and Dug Young Kim. Characterization
 469 of a stress-applied polarization-maintaining (PM) fiberthrough photoe-
 470 lastic tomography. *Journal of lightwave technology*, 21(4):997 – 1004,
 471 2003.
- 472 [30] A. Pau, S. Naghinajad, F. Alladio, P. Micozzi, and L. Boncagni. 3D
 473 tomography of Hydrogen and Helium plasma produced in th PROTO-
 474 SPHERA experiment. In *Proceedings of the ASME 2024 51st Annual*
 475 *Review Progress in Quantitative Nondestructive Evaluation QNDE2024*,
 476 pages 1–7, Denver, Colorado, July 2024.
- 477 [31] Johann Radon. On the determination of functions from their integral
 478 values along certain manifolds (translation from the original German
 479 text published in Berichte der Sächsischen Akademie der Wissenschaft,
 480 vol. 69, pp. 262-277, 1917). *IEEE Transactions on Medical Imaging*,
 481 MI-5(4):170–176, 1986.
- 482 [32] L. Schoeman, P. Williams, A. du Plessis, and M. Manley. X-ray micro-
 483 computed tomography (μ CT) for non-destructive characterisation of
 484 food microstructure. *Trends in Food Science and Technology*, 47:10 –
 485 24, 2016.
- 486 [33] James Anthony Seibert. Iterative reconstruction: how it works, how to
 487 apply it. *Pediatric Radiology*, 44(3):431 – 439, 2014.
- 488 [34] Wolfram Stiller. Basics of iterative reconstruction methods in computed
 489 tomography: A vendor-independent overview. *European Journal of Ra-*
 490 *diology*, 109:147 – 154, 2018.
- 491 [35] Weijie Tao, Damien Rohmer, Grant T. Gullberg, Youngho Seo, and Qiu
 492 Huang. An analytical algorithm for tensor tomography from projections

- 493 acquired about three axes. *IEEE Transactions on Medical Imaging*,
494 41(11):3454 – 3472, 2022.
- 495 [36] M. Vassholz, B. Koberstein-Schwarz, A. Ruhlandt, M. Krenkel, and
496 T. Salditt. New x-ray tomography method based on the 3d Radon
497 transform compatible with anisotropic sources. *Physical Review Letters*,
498 116(8), 2016.
- 499 [37] M.L.L. Wijerathne, Kenji Oguni, and Muneo Hori. Tensor field tomog-
500 raphy based on 3D photoelasticity. *Mechanics of Materials*, 34(9):533 –
501 545, 2002.
- 502 [38] Tao Zhang, Yuxiang Xing, Li Zhang, Xin Jin, Hewei Gao, and Zhiqiang
503 Chen. Stationary computed tomography with source and detector in lin-
504 ear symmetric geometry: Direct filtered backprojection reconstruction.
505 *Medical physics*, 47(5):2222–2236, 2020.
- 506 [39] Yingying Zhang-O’Connor and Jeffrey A. Fessler. Fourier-based for-
507 ward and back-projectors in iterative fan-beam tomographic image re-
508 construction. *IEEE Transactions on Medical Imaging*, 25(5):582 – 589,
509 2006.
- 510 [40] Dominique Zosso, Benoît Le Calennec, Meritxell Bach Cuadra, Kamiar
511 Aminian, Brigitte M. Jolles, and Jean-Philippe Thiran. Bi-planar 2d-to-
512 3d registration in fourier domain for stereoscopic x-ray motion tracking.
513 volume 6914, 2008.