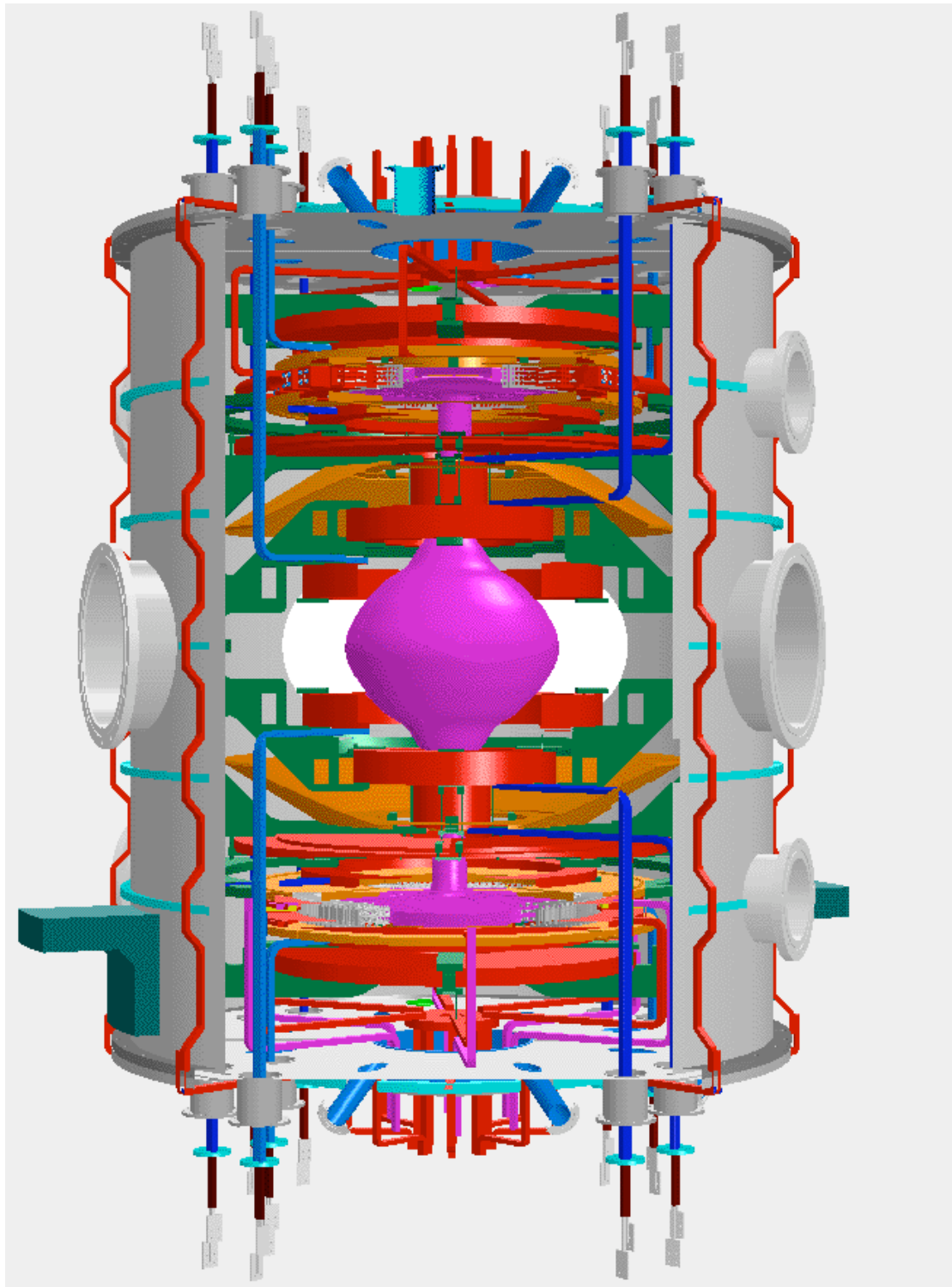
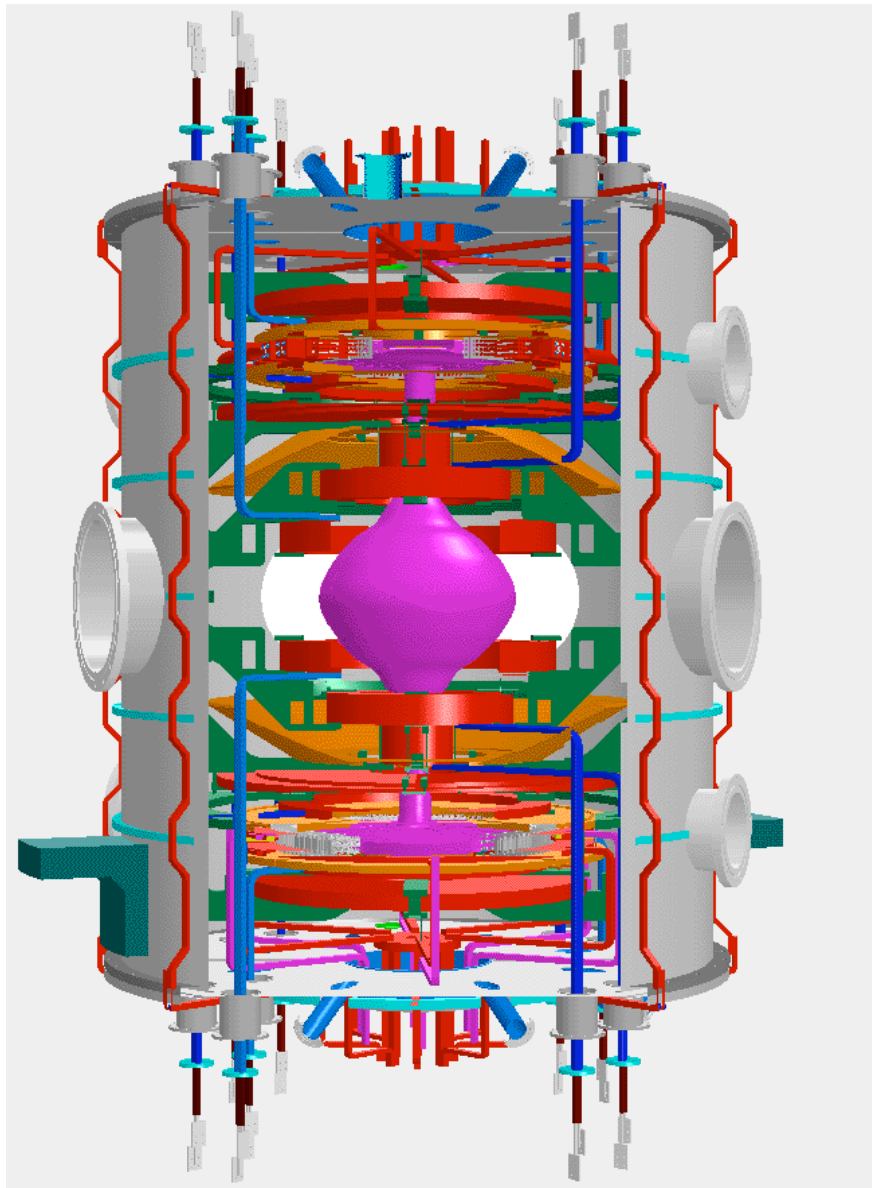


PROTO-SPHERA



CR-ENEA Frascati, July 2001

PROTO-SPHERA



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EXECUTIVE SUMMARY

The most investigated magnetic fusion configurations (tokamaks) are not simply connected: a central post, containing the inner part of the toroidal magnet and the ohmic transformer, links the plasma torus. The feasibility of simply connected, fusion relevant, magnetic configuration would strongly simplify the design of a fusion reactor. The **PROTO-SPHERA** experiment (**S**pherical **P**lasma for **H**elicity **R**elaxation **A**ssessment), proposed at CR-ENEA Frascati, will be devoted to demonstrate the feasibility of a spherical torus (ST) where a Hydrogen plasma arc, in the form of a screw pinch (SP) fed by electrodes, replaces the central conductor. The SP and the ST have a common magnetic separatrix: magnetic reconnections will occur at the X-points, injecting magnetic helicity, poloidal flux and plasma current into the ST.

The screw pinch plasma will be magnetically shaped as a disk near each electrode, which will be built as an annulus composed by ~ 100 pressed radial modules. In presence of a hot cathode, heated by AC current to 2600°C , the voltage required at the anode in order to form the screw pinch will be $V_e \sim 100\text{ V}$, with Hydrogen filling pressures $p_H \sim 1 \cdot 10^{-3} \div 1 \cdot 10^{-2}\text{ mbar}$. The screw pinch will be formed at an electrode current $I_e = 8.5\text{ kA}$, which guarantees MHD stability, as its winding number (safety factor) will be $q_{\text{pinch}} > 2$. Raising the electrode current up to $I_e = 60\text{ kA}$, the screw pinch will become unstable, as q_{pinch} will become much less than unity. During the instability the poloidal field compression coils will be pulsed and the spherical torus will be generated around the screw pinch, driven in part by the inductive flux and in part by the helicity injection. The formation of PROTO-SPHERA will parallel the scheme successfully demonstrated by the TS-3 experiment at the University of Tokyo, which in 1993 produced a small spherical torus around a screw pinch, and maintained it stable for at least $80\text{ }\mu\text{s} \sim 100\text{ }\mu\text{s}$ (100 Alfvén times).

Magnetic helicity, which, for a simply connected volume, bounded by a magnetic surface (where $\vec{B} \cdot \vec{n} = 0$), is defined as $K = \int \vec{A} \cdot \vec{B} dV$ (where $\vec{A}$ is the vector magnetic potential), measures how much the lines of force are interlined, kinked or twisted and is a slowly decaying ideal invariant. The Taylor assumption states that energy decays to the minimum value it can have, subject to the conservation of magnetic helicity. Any initial configuration will self organize in a relaxed state $\vec{B} = \vec{B}_0$, with $\vec{B}_0 = \text{constant}$ all over the plasma, after sufficient time. A more realistic physical situation of a domain containing magnetized plasma, with open field lines passing through the boundary, offers the opportunity of refurbishing the helicity content of the magnetized plasma. This happens if the magnetic helicity can be injected through the boundary (by driving current along the lines of force) more quickly than it is dissipated inside the domain by resistive processes. The origin of magnetic helicity injection is connected with the electric current forced to flow along a DC magnetic field, generating perpendicular magnetic flux and causing the magnetic field lines to kink up, with a helical pattern. Magnetic flux, plasma current and magnetic energy will be injected along with the helicity and reconnection processes will convert part of the magnetic energy into kinetic energy of the magnetized plasma. If the helicity source (the screw pinch discharge in the case of PROTO-SPHERA) is physically

separated from the helicity sink (the spherical torus of PROTO-SPHERA), a gradient in the relaxation parameter ($\bar{\eta} \neq 0$) will appear: resistive MHD instabilities will produce a helicity flow from regions of larger $\bar{\eta}$ to regions of smaller $\bar{\eta}$. A helicity diffusion coefficient will rule the helicity flux from source to sink.

The first goal of the PROTO-SPHERA experiment will be to compress the ST to the lowest possible aspect ratio ($A=1.2 \div 1.3$), in a time of about 1800 Alfvén times ($1800 \cdot \tau_A \sim 1000 \mu\text{s}$). The second goal will be to show that efficient helicity injection, through the X-points of the magnetic separatrix, can sustain the spherical torus around the screw pinch for at least one resistive time ($\tau_k \sim 70 \text{ ms}$). The magnetic configuration designed for PROTO-SPHERA will provide an elongated ($\bar{\eta}=b/a \sim 2.2 \div 2.3$) spherical plasma (aspect ratio $A=R/a \sim 1.2 \div 1.3$) with a diameter $2 \cdot R_{\text{sph}}=70 \text{ cm}$. A longitudinal current $I_e=60 \text{ kA}$ will flow inside the screw pinch and a toroidal current $I_p=120 \div 240 \text{ kA}$ inside the spherical torus, which will have an edge safety factor $q_{95} \sim 2.5 \div 3$.

A cylindrical vacuum vessel (2.5 m high and 2.0 m in diameter) will contain the load-assembly of PROTO-SPHERA. The anode will be at positive voltage $\sim 100 \text{ V}$, the poloidal field coils will be floating and the cathode will be at ground, together with the vessel and the remaining load-assembly. There will be two sets of poloidal field (PF) coils inside the vacuum vessel of PROTO-SPHERA, each composed by coils connected in series: (i) a first set of coils which shape the screw pinch and whose currents do not vary during the plasma evolution; (ii) a second set of coils which compress the ST and whose currents vary during the plasma evolution. As the formation time of the configuration will be 1 ms, the coils whose variable currents compress the ST will be shielded inside thin metal cases (time constant $\sim 200 \mu\text{s}$). On the other hand the coils with constant currents will have to be enclosed inside thick metal cases (time constant $\geq 2 \text{ ms}$), in order to stabilize the plasma disks near the electrodes during the formation of the spherical torus. It will be possible to connect each individual PF coil case either to the anode or to the cathode potential or to keep it floating.

The screw pinch power supply must deliver $I_e=60 \text{ kA}$ at $V_e \leq 300 \text{ V}$, with a rise time of $500 \mu\text{s}$ and a response time of 5 ms . The constant current PF coils must deliver $I_B=2 \text{ kA}$ at $V_B=500 \text{ V}$, with a rise time of 0.1 s . The power supply feeding the variable current PF coils must be able to deliver $I_A=0.5 \text{ kA}$, with a rise time of 1 ms at a voltage $V_A=15 \text{ kV}$; thereafter to increase the current to $I_A=1.2 \text{ kA}$, at $V_A \leq 1 \text{ kV}$; finally to maintain $I_A=1.2 \text{ kA}$, with a response time of 10 ms at a voltage $V_A \leq 100 \text{ V}$.

The computation of the ideal MHD stability of PROTO-SPHERA raises new problems: (i) the combined configuration composed by the spherical torus (ST), with closed field lines, and by the screw pinch (SP), with open field lines ending upon electrodes, must be correctly modeled; (ii) a magnetic separatrix defines the interface between the ST and the SP. An ideal MHD upper limit to the ratio between the toroidal ST current I_p and the longitudinal SP current I_e has been found. Such a limit depends upon the volume averaged beta value of the spherical torus, $\beta_{\text{ST}}=2\bar{\beta}_0 \langle p \rangle_{\text{ST}} / \langle B^2 \rangle_{\text{ST}}$. With $\beta_{\text{ST}} \sim 30\%$ the ideal stability of the configuration is limited to $I_p=I_e$, with $\beta_{\text{ST}} \sim 20\%$ I_p can reach a value of $2 \div 3 \cdot I_e$ and with $\beta_{\text{ST}} \sim 10\%$ I_p can reach a value of $4 \cdot I_e$ (the design limit of PROTO-SPHERA). The position of the conducting shells near the plasma does not seem to be critical for the ideal MHD stability results,

which do not change even removing all the conductors to infinite distances from the plasma.

During the design of PROTO-SPHERA the electrodes' benchmark experiment PROTO-PINCH has been built and operated, with the goal of testing modular units of the cathode and of the anode. PROTO-PINCH has produced, within a Pyrex vacuum vessel, Hydrogen and Helium arcs in the form of screw pinch discharges, stabilized by two poloidal field coils located outside the vacuum. PROTO-PINCH, with an anode-cathode distance of 0.75 m and a stabilizing magnetic field up to $B=0.1$ T, has a current capability of $I_c=0.67$ kA, (with a pinch winding number $q_{\text{Pinch}}>2$).

The technical solution for the 5 cm diameter electrodes are: (i) a W-Cu(5%) hollow anode, with H_2 puffed through it (a feedback system stabilizes the filling pressure p_H in the vessel); (ii) a directly AC heated W cathode. The cathode filaments are heated up to 2600°C , by a total current $I_{\text{cath}}=590$ A (rms). Pinch discharges have been obtained with $B=0.1$ T, $I_c=600\div 700$ A and $V_c=70\div 120$ V. The arc discharges have been sustained for $2\div 5$ s, limited by the heating of PF coils and Pyrex vessel. The arc current has been obtained in the filling pressure range $p_H=1\cdot 10^{-3}\div 1\cdot 10^{-2}$ mbar. Spectroscopic measurements, in the visible light, have shown that the Hydrogen and Helium plasmas display only barely perceptible signs of impurities, at a count level of about 10^{-2} of the largest H and He line counts. An exhaustive series of full performance discharges (hundreds of shots) has shown the endurance of the electrodes to current and power densities equal to those required in PROTO-SPHERA.

The extrapolation to the 100 cathode modules required for PROTO-SPHERA, indicates that the cathode will be heated by a total AC current $I_K=60$ kA (rms) at $V_K<20$ V, with a total heating power rising up to $P_K=850$ kW in 15 s. The power injected into the electrode plasma sheaths of PROTO-SPHERA will be $P_{\text{Pinch}}^{\text{el}}\leq 4.6$ MW, the ohmic input to the screw pinch of PROTO-SPHERA will be a further $P_{\text{Pinch}}^{\square}\leq 5.4$ MW and finally the power required for the helicity injection will be $P_{\text{Pinch}}^{\text{HI}}\sim 0.6$ MW, summing up to a total $P_c\leq 10.6$ MW.

The accurate study of a laboratory plasma like the one of PROTO-SPHERA could provide useful information also on some astrophysical phenomena: mainly solar and protostellar flares. As a matter of fact in a number of astrophysical (gravity-confined) systems, unstable twisted magnetic flux tubes are able to produce, through magnetic reconnection, helical twisted toroidal plasmoids. The fate of these toroids is to expand and to be expelled from the generating gravity-confined parent systems. In this process the system is able to eject helicity and to shed a relevant magnetic flux, with a negligible loss of mass. These phenomena bear a strong resemblance to the formation and sustainment of the (magnetically confined) plasma of PROTO-SPHERA, although they occur at magnetic Lundquist numbers which are much larger ($S=\eta_k/\eta_\lambda\sim 10^8\div 10^{13}$) than the magnetic Lundquist number of PROTO-SPHERA ($S\sim 10^5$). Also the range of η at which these phenomena occur, $\eta\ll 1$ in the solar corona, $\eta\leq 1$ in collapsing magnetized clumps inside giant molecular clouds and $\eta\gg 1$ in protostar magnetized accretion disks, span a larger range of values than the $\eta\leq 0.3$ of PROTO-SPHERA.

From the point of view of Magnetic Fusion Energy research, the PROTO-SPHERA project is in the framework of the research on Compact Tori (ST, spheromaks, field reversed configurations-FRC) and has the capability of exploring the connections between the three concepts. In particular it has the goal of forming and sustaining a flux-core-spheromak with a new technique. The magnetic configuration of the experiment has been designed aiming at a safety factor profile that is similar to the ones obtained in spherical tori with metal centerpost. The compression of the screw pinch, while decreasing the longitudinal pinch current I_e , could even lead to the formation of an FRC with a new technique.

Finally, with slight modifications in the load assembly, the PROTO-SPHERA experiment could host an unrelaxed Chandrasekhar-Kendall-Furth (CKF) configuration, which contains a magnetic separatrix and is composed by a "main spherical torus", two "secondary tori" on top and bottom and a "spheromak" discharge surrounding the three tori. If driving current on its closed flux surfaces can sustain the surrounding spheromak discharge, magnetic reconnections will occur at the X-points of the configuration, injecting magnetic helicity, poloidal flux and plasma current into the main spherical torus. Also the secondary tori will be a by-product of the same magnetic reconnections. An unrelaxed CKF can be viewed as a spherical torus (with winding numbers $q_0^{ST} \sim 1$ on axis and $q_{95}^{ST} \sim 2$ at the separatrix) surrounded by a spheromak endowed with high elongation and therefore with high winding number ($q_0^P \sim 3$ on the symmetry axis and $q_{95}^P \sim 5$ at the separatrix). Unrelaxed CKF configurations can be stable to all low- n ideal MHD modes, up to unity beta values $\beta = 2\mu_0 \langle p \rangle_{vol} / \langle B^2 \rangle_{vol} = 1$. This high β value opens the possibility that plasma motions (i.e. radial electric field) can sustain the magnetic field of CKF configurations. Whereas the breakdown and the inductive formation of an unrelaxed CKF in the modified PROTO-SPHERA seem feasible, a method for sustaining it, through current or torque injection, remains to be developed.

Looking at the world program on compact tori, results from PROTO-SPHERA, if obtained as early as in 2004, should be relevant and timely for this research line.

Moreover PROTO-SPHERA contains elements of general interest in plasma physics:

- To form and sustain a magnetic confinement configuration through the non-linear saturation of an instability (self-organization).
- To investigate the coexistence between the dynamo effect (reconnections and axisymmetry breaking) and magnetic confinement.
- To simulate in laboratory plasma the solar and the protostellar flares.
- To assess the fusion relevant performances of simply connected magnetic confinement configurations.

The success of PROTO-SPHERA could lead to a larger size and more fusion oriented experiment based upon an unrelaxed Chandrasekhar-Kendall-Furth configuration. Unrelaxed CKF fusion reactors with the right helicity injection, β limit and energy confinement, will allow for an unimpeded outflow of the high energy charged fusion products, easing direct energy conversion and the use of the burner as a space thruster.

The three major points that have to be demonstrated on PROTO-SPHERA are: that the formation scheme is effective and reliable, furthermore that the combined configuration can be sustained in 'steady-state' by DC helicity injection and finally that the energy confinement is not worse than the one measured on spherical tori.

Parameters of PROTO-SPHERA

Parameters of the spherical torus (ST):

Equatorial radius of the ST	R_{sph}	=	0.36 m
Major, minor radius of the ST	R	=	0.20 m, $a = 0.16$ m
Aspect ratio of the ST (R/a)	A	=	1.25
Elongation of the ST	η	=	2.17
Toroidal ST plasma current	I_p	=	180 kA
Safety factor of the ST at the edge	q_{95}	=	2.6
Greenwald density limit of the ST	$\langle n_e \rangle_G$	=	$3 \cdot 10^{20} \text{ m}^{-3}$
ST volume averaged electron density	$\langle n_e \rangle$	=	$0.5 \cdot 10^{20} \text{ m}^{-3}$
ST volume averaged electron temperature	$\langle T_e \rangle$	=	140 eV
Energy confinement time of the ST	τ_E	=	1.6 ms
Resistive time of the ST	τ_R	=	70 ms
Alfvén time of the ST	τ_A	=	0.5 μ s
Magnetic Lundquist number of the ST	S	=	$1.2 \cdot 10^5$
Total toroidal beta of the ST	β_T	=	10÷30%
Poloidal beta of the ST	β_{pol}	$\leq$	0.15

Parameters of the screw pinch (SP):

Equatorial radius of the SP	$\rho_{\text{pinch}}(0)$	=	0.04 m
Longitudinal current in the SP	I_e	=	60 kA
...corresponding to a toroidal field	B_{T0}	=	0.05 T at $R = 0.23$ m
... including paramagnetism	B_T	=	0.14 T at $R = 0.23$ m
SP electron density	n_e^{Pinch}	=	$0.15 \cdot 10^{20} \text{ m}^{-3}$
SP electron temperature	T_e^{Pinch}	=	36 eV

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1. GENERAL FRAMEWORK

The purpose of this Section is to provide the general framework of the research on compact tori. As compact tori are here designated all the magnetic confinement configurations in which a closed magnetic surface takes the shape of a torus with the minor radius a approaching the major radius R : i.e. whose aspect ratio tends to unity, $A=R/a \rightarrow 1$.

The main configurations included within this framework are spherical tori (ST), spheromaks and field reversed configurations (FRC). The spherical torus is at the moment the one explored with most success, due to its similarity to the much more investigated tokamak confinement scheme. As a matter of fact one could argue that the spherical torus is the attempt of solving many of the tokamak problems (turbulence, disruptions, beta limit, etc...) by pushing upon the configuration leverage. On the other hand, the latter two confinement schemes (spheromaks and field-reversed configurations) have been much less studied in the laboratory, although they possess in principle many attractive features.

The reason why they have, up to now, been less successful is mainly connected with the fact that they rely more heavily upon plasma self-organization, both for their formation as well as for their sustainment. Although many formation schemes have produced in the last twenty years interesting spheromaks and field reversed configurations, at the present moment (July 2001) no sustainment scheme has been soundly and fully demonstrated.

The PROTO-SPHERA experiment aims at sustaining a flux-core-spheromak, while exploring the configuration space that connects spherical tori and spheromaks. The compression of the central pinch, while decreasing the total longitudinal pinch current, would lead, if successful, to the formation of a field reversed configuration. So PROTO-SPHERA could also explore a new technique for setting up an FRC. Finally PROTO-SPHERA could also aim at exploring the novel Chandrasekhar-Kendall-Furth configuration consisting of a spherical torus enclosed within a spheromak, which will be introduced in Section 3.

1.1. Spherical Tori, Spheromaks and FRCs

After more than thirty years of development, the tokamak concept has come very close to achieving controlled thermonuclear fusion break-even conditions and a number of proposed next generation experimental devices could provide a burning plasma. However, the tokamak is very large, complex, and expensive. Even improved tokamaks may not overcome the shortcomings of low power density, high complexity, large unit size, and high development cost. It is therefore important to develop alternatives to conventional tokamaks with designs optimized for simplicity, small size and low cost. Several alternatives have been proposed with various tradeoffs between ultimate attractiveness and present feasibility.

A number of research groups world-wide have been working on three related alternate concepts: the spherical torus (ST), the spheromak, and the field-reversed configuration (FRC). These concepts are at very different stages of development.

Fig. 1.1 compares the magnetic topologies of ST, spheromak and FRC. The ST is a modification of the conventional tokamak and differs by having a much smaller aspect ratio. Spheromaks are low β toroidal confinement configurations where currents flowing in the plasma produce the magnetic field almost entirely; they have a finite internal toroidal magnetic field, which vanishes at the plasma surface; hence no external field coils link the plasma. FRCs are high β toroidal confinement configurations with zero toroidal magnetic field everywhere and so, like spheromaks, do not have coils linking the plasma. Thus, spheromaks manage to have a toroidal field without having toroidal field coils; FRCs do not have toroidal field coils, but also do not have a toroidal field.

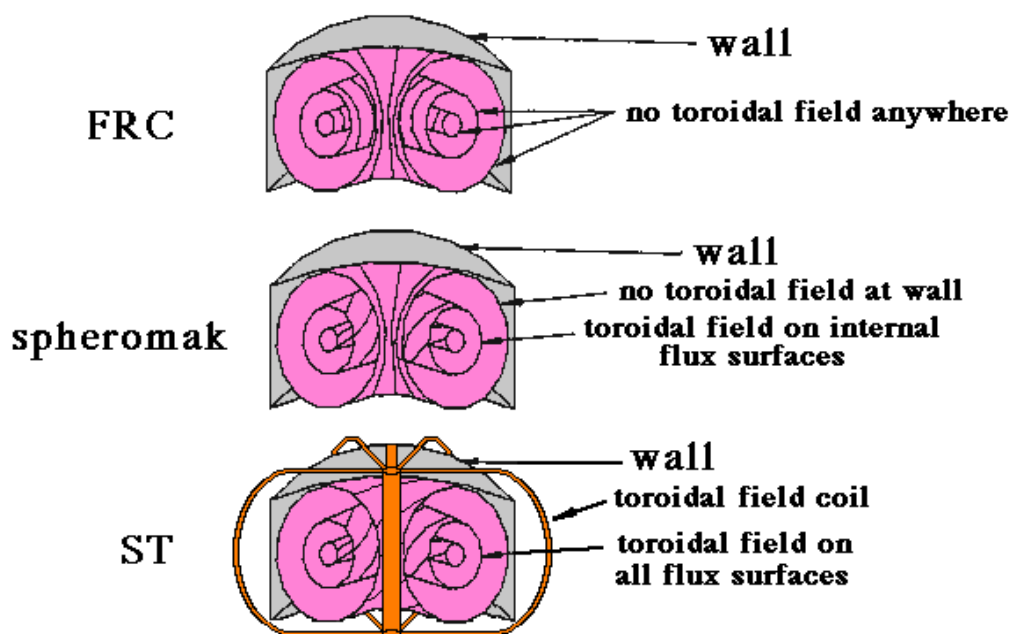


Fig. 1.1. Comparison between ST, spheromak and FRC.

1.2. Spheromaks

The spheromak [1, 2] is a compact magnetofluid configuration of simple geometry with attractive reactor attributes, including no material centerpost, high engineering beta, and sustained steady-state operation through helicity injection. It is a candidate for liquid metal walls in a high-power-density reactor and has a simple geometry for incorporating a divertor, as shown schematically in Fig. 1.2. It has toroidal and poloidal fields of comparable strengths.

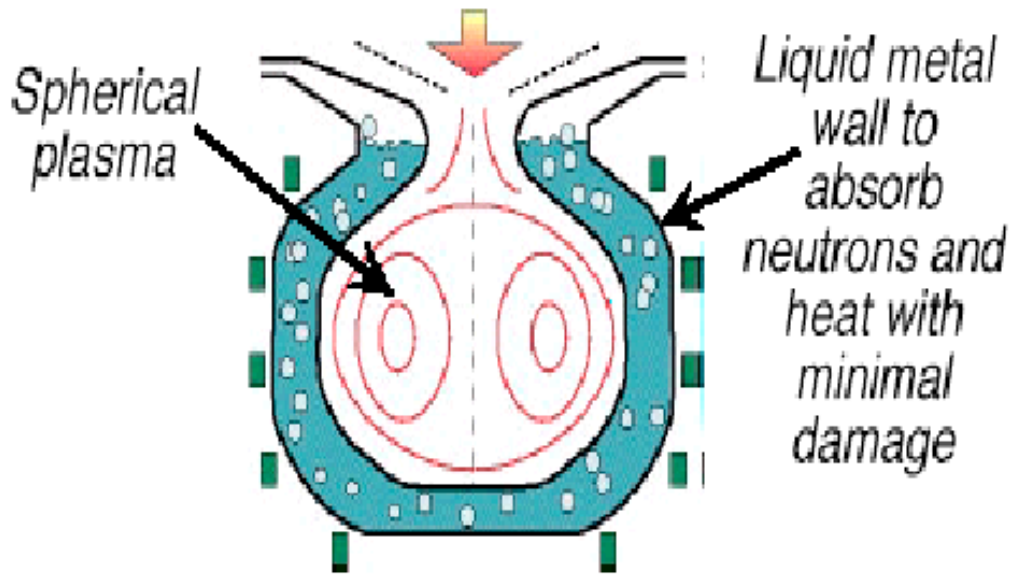


Fig. 1.2. Schematic of a self-ordered spheromak configuration, illustrating near spherical reactor geometry using liquid metal blanket and shield.

Spheromaks do not use a transformer (as in tokamaks) to produce the nested poloidal flux surfaces required for confinement. Instead spheromaks are formed by the self-organization of naturally occurring MHD instabilities. The self-organization means that there is not a unique way to make spheromaks, and indeed, several different methods have been successfully demonstrated [3, 4, 5 and 6]. Magnetic helicity (linked magnetic fluxes) plays an important role in forming and sustaining spheromaks. An initial configuration with sufficient helicity and energy will spontaneously relax to a spheromak, given appropriate boundary conditions. Figure 1.3 shows how a magnetized coaxial plasma gun creates a spheromak. A puff of gas is introduced into the annular gap between the inner and outer coaxial cylindrical electrode [7] (Fig. 1.3.a). High voltage capacitors charged to 5÷10 kV are connected to the electrodes and cause the gas to ionize and become a toroid of plasma. The current flowing in the gun and through the plasma interacts with its own magnetic field to produce a $\vec{j} \times \vec{B}$ force, which accelerates the plasma towards the open end of the gun (Fig. 1.3.b). A strong magnetic field, called the "stuffing field", is produced by an external magnetic coil and is concentrated in the center electrode with a slug of high permittivity metal. The plasma encounters this magnetic field at the opening of the gun and resists the change in field, according to Faraday's law. Because the plasma is an excellent conductor, currents flow in the toroid of plasma as it distends the stuffing field (Fig. 1.3.c). If the magnetic pressure from the gun exceeds the

magnetic tension of the stuffing field, the toroid breaks away to form a spheromak. The field lines distend and then reconnect in back as the spheromak forms. The spheromak inherits toroidal field from the gun field and poloidal field from the stuffing field (Fig. 1.3.d). The amount of gun current required to overcome the stuffing field is called the formation threshold.

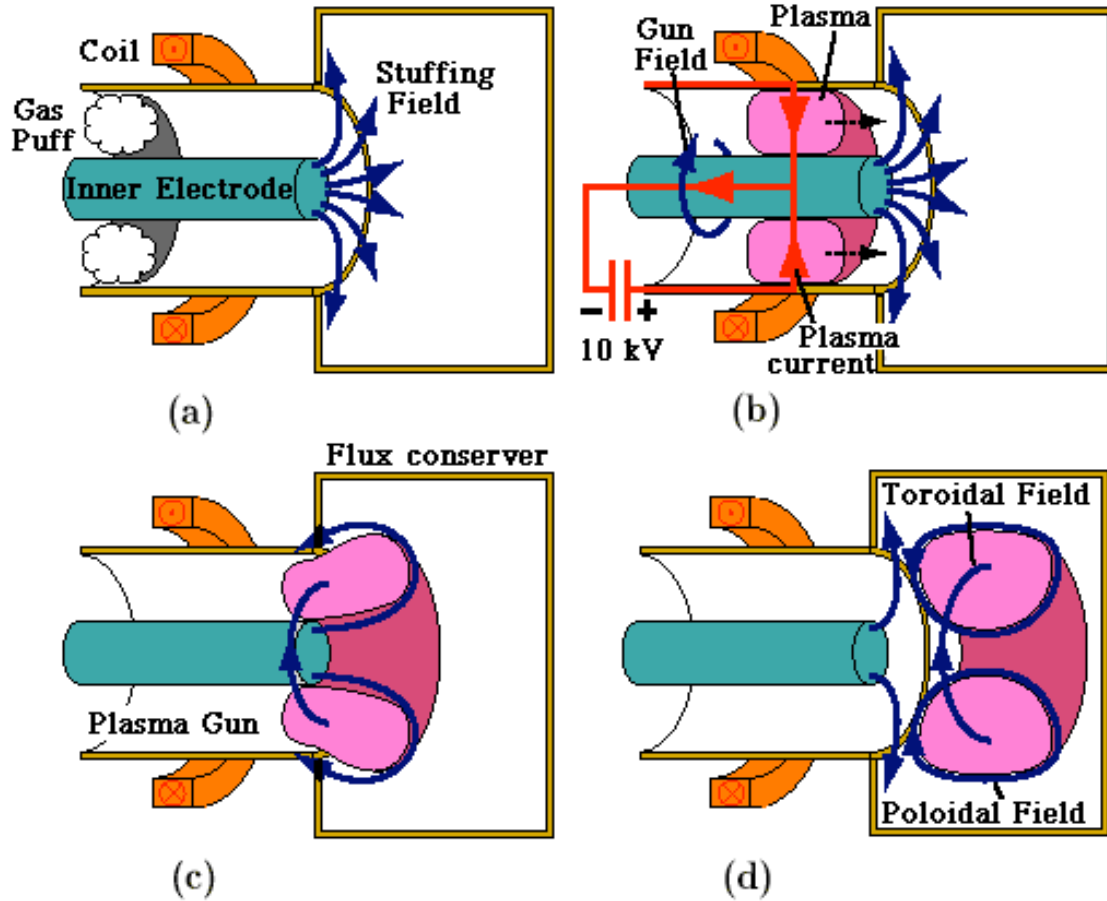


Fig. 1.3. Spheromak formation: injection by a plasma gun inside a flux conserver [7].

In the exploratory scale device CTX (Los Alamos National Laboratory) central electron temperatures $T_e(0)=400$ eV, average $\beta \sim 5\%$ and central $\beta(0) \sim 20\%$ were obtained with a 2-T magnetic field [8]. Analysis of CTX data found the energy confinement in the plasma core to be consistent with Rechester-Rosenbluth transport in a fluctuating magnetic field, potentially scaling to good confinement at higher electron temperatures.

The MHD stability against the tilt mode is an issue as well as the efficient sustainment of the plasma current. Electrostatic helicity injection has been demonstrated to sustain the spheromak current via a magnetic dynamo involving flux conversion and has been implemented, for limited duration, in several experiments. Experiments have shown that the spheromak is subject to continuous resistive MHD modes, similar to those in the reversed field pinches (RFP), which tear the magnetic fields but reduce the plasma confinement. The SPHEX group (Manchester, England) studied the dynamo in sustained spheromaks in a cold plasma [9]. A new concept exploration experiment [10], SSPX (see Fig. 1.4), has recently begun operation at Lawrence Livermore National Laboratory and is addressing the physics of a mid-sized sustained spheromak with tokamak-quality vacuum conditions and no diagnostics internal to the plasma.

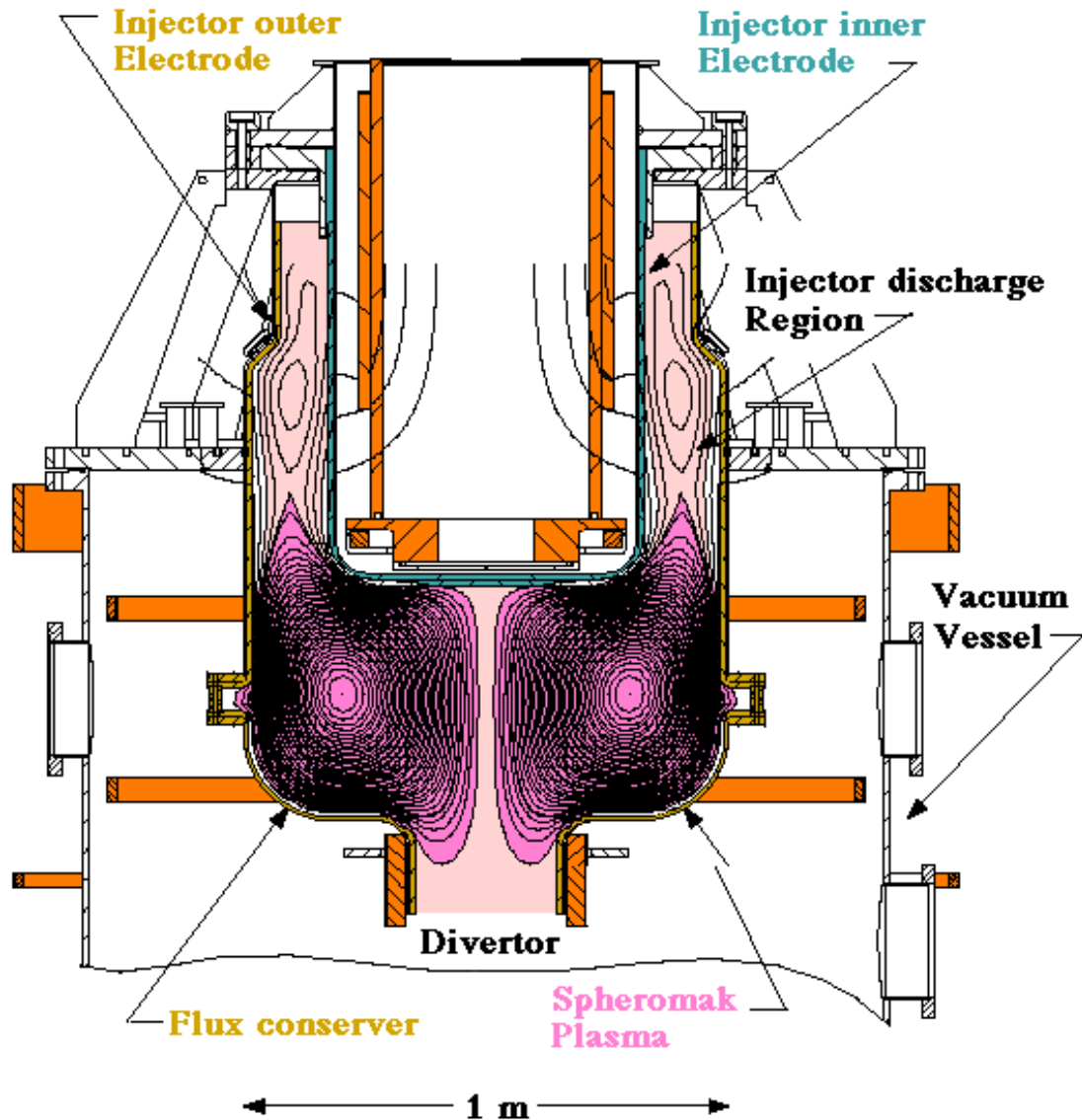


Fig. 1.4. Schematic of the new SSPX spheromak experiment at Livermore.

Operating spheromak experiments are the Swarthmore Spheromak Experiment (SSX) (reconnection) [8], the Caltech Helicity Experiment (spheromak formation issues) [11], the FACT/HIST experiment at the Himeji Institute of Technology [12] and BCTX at the University of California, Berkeley. The physics of reconnection is being studied in MRX at Princeton Plasma Physics Laboratory (PPPL) [13] and on TS-3 and TS-4 at the University of Tokyo [14].

1.3. FRCs

A field-reversed configuration (FRC) is a compact torus plasma with negligible toroidal magnetic field. It is usually fairly elongated, contained in a magnetic field produced by a cylindrical solenoid, and possesses a simple, unobstructed divertor (Fig. 1.5). The plasma beta is close to unity, and an FRC is thus both extremely compact and geometrically simple [15, 16]. The coils and divertor geometry are the simplest of any configuration.

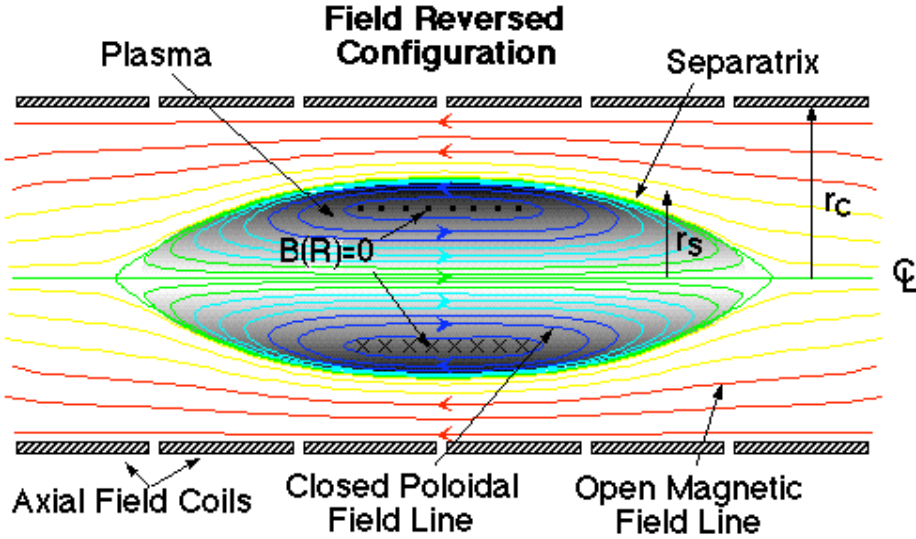


Fig. 1.5. FRC geometry.

FRCs have been formed with high plasma pressures in theta-pinch devices [17], like BN (Trinitite), FIX (Osaka University), NUCTE-3 (Nihon University), LSX, STX and TCS (University of Washington).

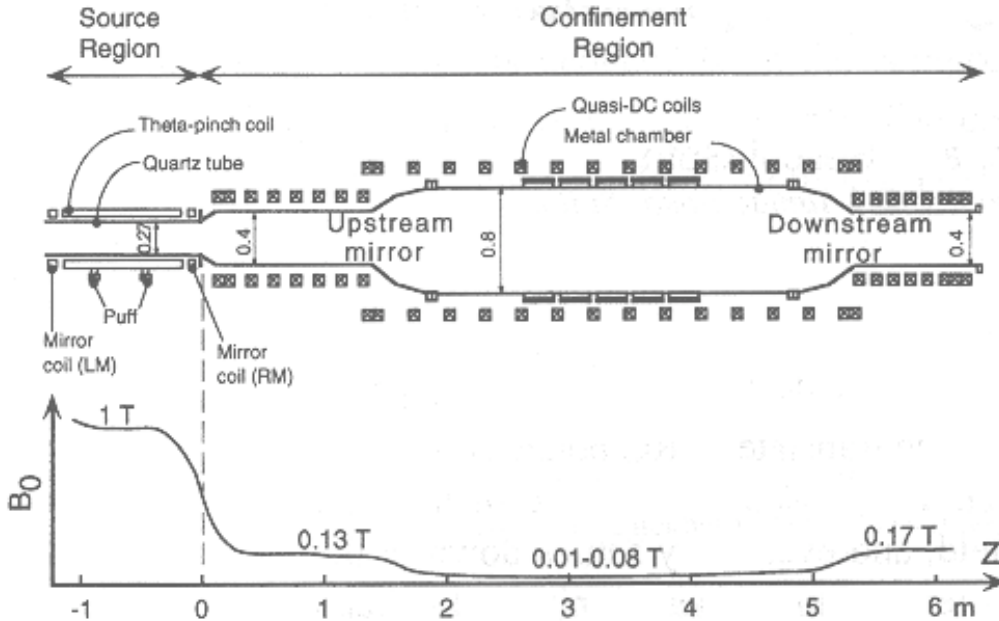


Fig. 1.6. Schematic drawing of the FIX machine, with axial B_0 profile in vacuum [18].

Outside a quartz vessel the theta-pinch coils (see Fig. 1.6) are typically pulsed to 1 T field level in a few μ s. Without an external current drive, these current rings decay on sub-millisecond L/R times. Typical electron densities of $n_e \sim 5 \cdot 10^{21} \text{ m}^{-3}$, $n_E \sim 10^{18} \text{ m}^{-3} \cdot \text{s}$ and the highest average β of 50% to 80% have been achieved in FRCs with major radii of 15 cm, at several 100-eV temperatures. Lifetime has been observed to increase with density: shorter-lived FRCs are easily produced at $n_e \sim 10^{21} \text{ m}^{-3}$, with keV temperatures.

After the impulsive formation inside a theta-pinch coil the FRC is translated along a guide field, expanded (lowering n_e by factors up to 100) inside a metal vessel with quasi-steady magnetic field and then stopped by a mirror field (see Fig. 1.7).

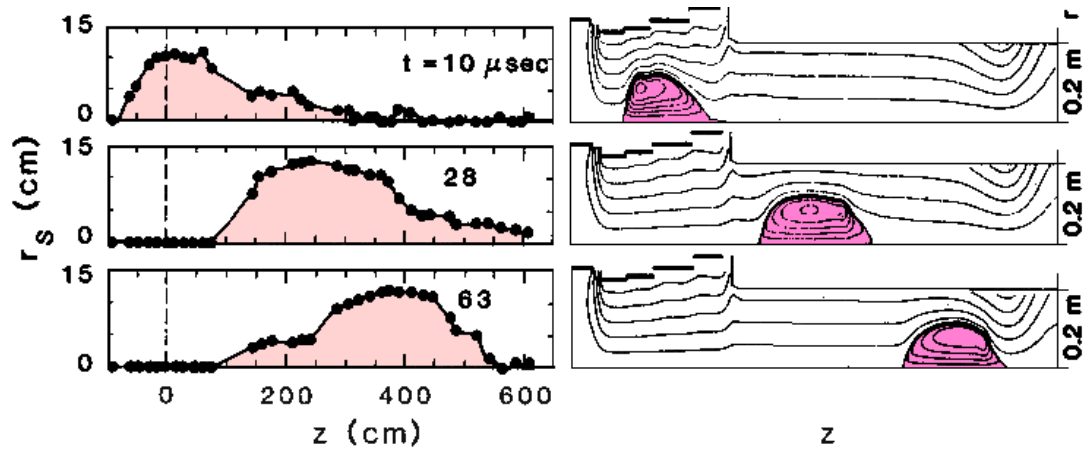


Fig. 1.7. Time evolution of separatrix radius profile r_s and of flux function in FRX [19].

The theta-pinch formation technique is limited to the tens of mWb level. Several Weber are required for a reactor, so other methods of formation are being studied. The slow formation of an FRC, using two merging spheromaks with opposite helicity [20] (Fig. 1.8) has been demonstrated [21] on spheromak merging facilities, such as TS-3 (Tokyo), MRX (Princeton) and SSX (Swarthmore), see Fig. 1.9.

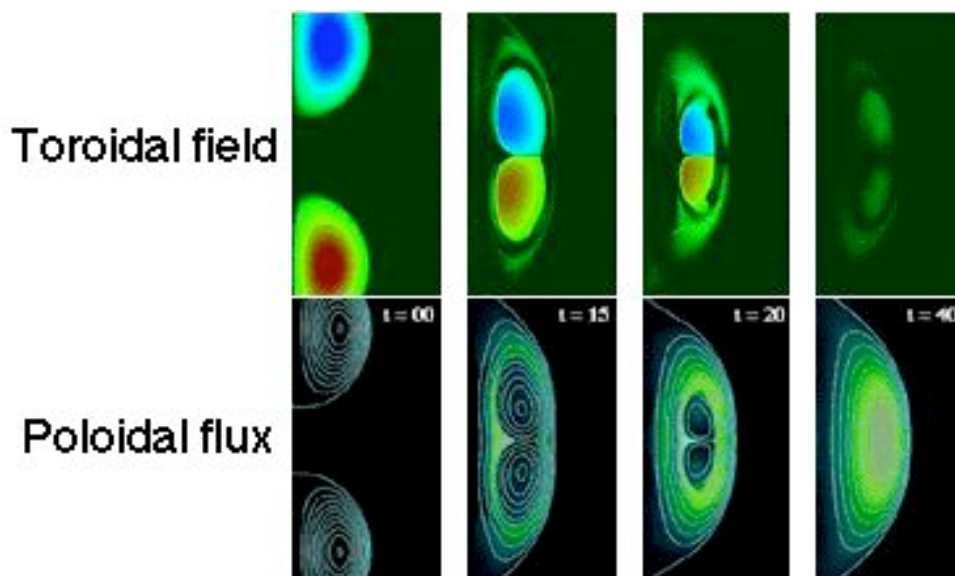


Fig. 1.8. Calculated evolution of counter-helicity spheromaks merging into an FRC [20].

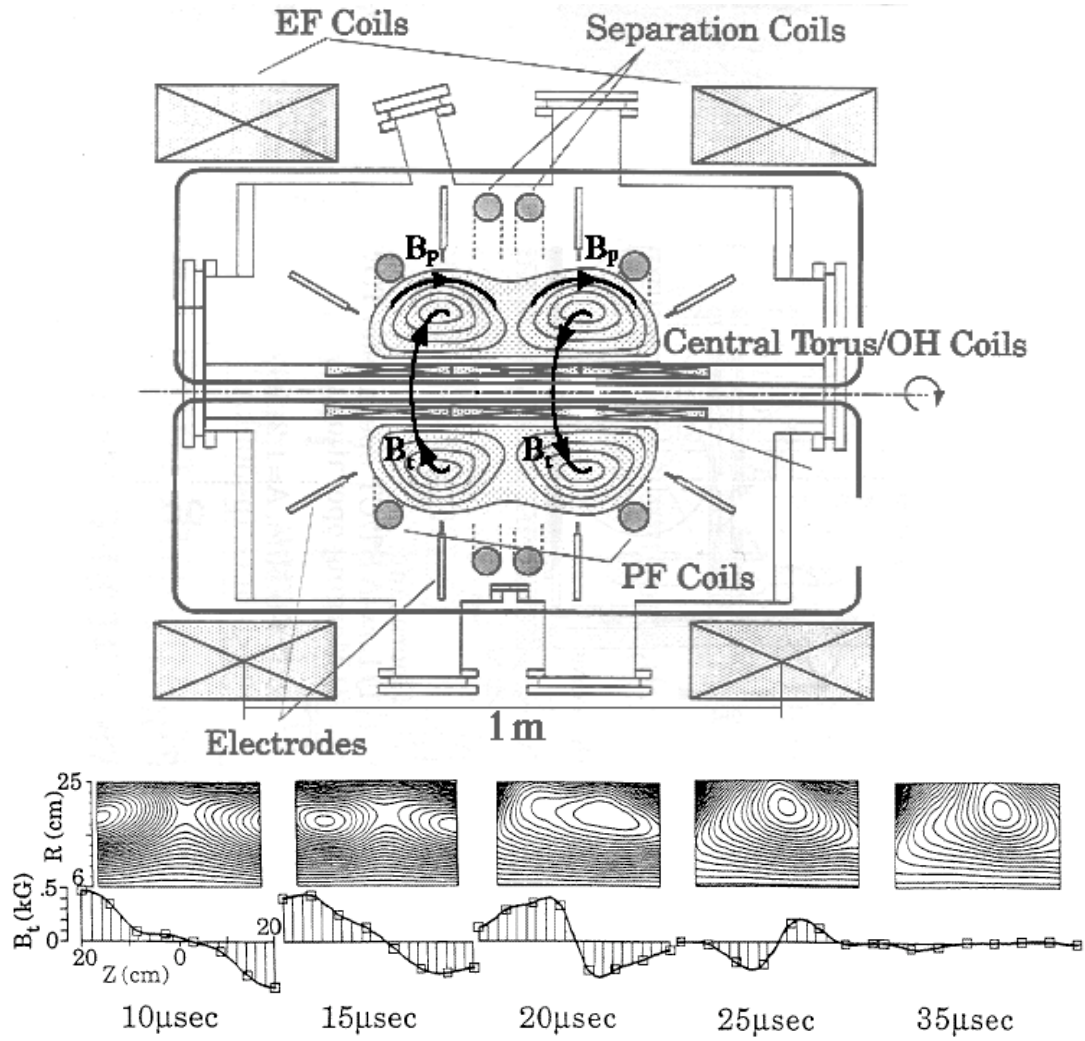


Fig. 1.9. TS-3 spheromak merging facility and experimental results from magnetics [21].

A promising approach for FRC sustenance is the application of a rotating magnetic field (RMF), using large antennas (see Fig. 1.10).

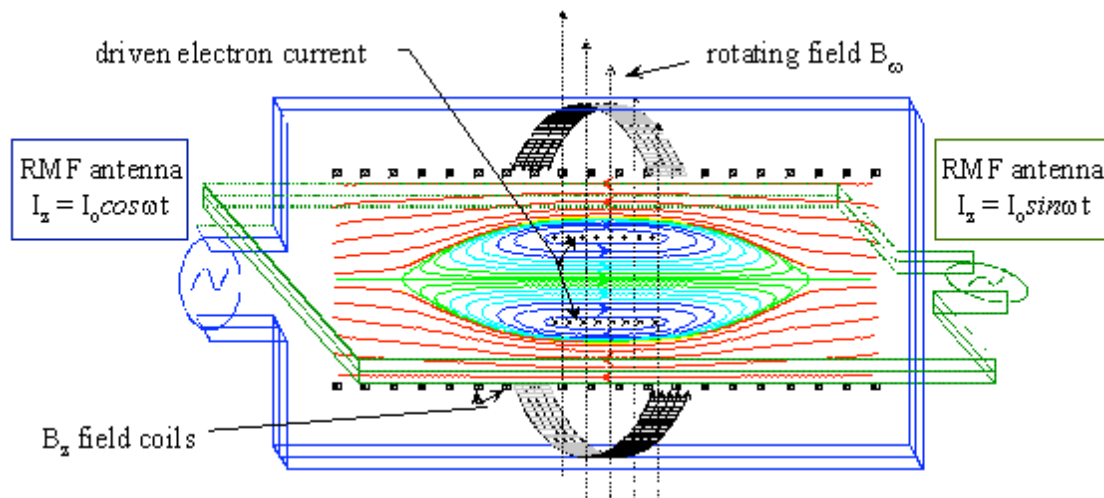


Fig. 1.10. Schematic of FRC sustenance by RMF [22].

A partially ionized spherical FRC discharge has been produced and sustained for 40 ms by (RMF) in the Rotamak device [23] (Flinders University, Australia).

An interesting observation is that the FRC plasmas produced in all the experiments are more globally stable than ideal MHD theory would predict. The observed stability in present experiments is thought to be due to kinetic effects, which have been characterized by a parameter s , equal to the number of ion gyro-radii between the field null R and the separatrix r_s (see Fig. 1.5). The utility of the concept depends upon demonstrating stability as s is increased from present values of about 4 to the level of 20÷30 thought necessary to provide reactor level confinement. The enhancement of kinetic stabilization, either through addition of energetic particles (e.g., ion ring merging or neutral beam current drive) or naturally occurring fusion reaction particles, may be an essential component of the concept. Furthermore there is some theoretical and experimental evidence that FRCs may be naturally occurring minimum energy states stabilized by sheared rotation [24], akin to spheromaks and reversed-field-pinches (RFP), when the total helicity (which includes angular momentum) is conserved.

Among alternative concepts based on low-density plasma magnetic confinement, the FRC offers arguably the best reactor potential because of high power density, simple structural and magnetic topology, simple heat exhaust handling, and potential for advanced fuels. Particle distributions driven, for example, by beams and including the effects of nuclear polarization can provide certain benefits in magnetic fusion devices. Beams of ions, colliding at energies near the peak in their fusion cross-section, lead to a higher Q than a thermal distribution of the same mean energy. This increase may be in the form of a nuclear resonance; hence, such distributions are far better than simply hotter plasmas. The benefits and needs are greatest for high-beta magnetic fusion energy devices, for example, the field-reversed configuration (FRC), spheromak, and spherical tokamak (ST). Because of its demonstrated very high beta and potential for direct electrical conversion of the exhaust, the FRC is particularly interesting as a candidate to burn aneutronic fuels.

The FRC magnetic configuration has an ideal geometry for a future fusion propulsion utilizing D^3He fuel (see Fig. 1.11). As a matter of fact the null field region and the high beta mean low synchrotron radiation and moderate field requirements even at high plasma temperatures. Furthermore the linear geometry and the unimpeded outflow are natural for obtaining direct energy conversion [25].

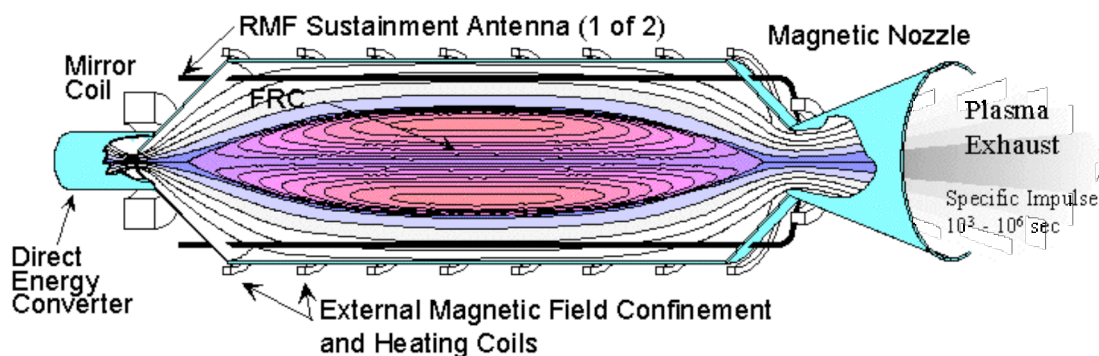


Fig. 1.11. Idealized fusion propulsion utilizing D^3He fuel [25].

1.4. Spherical Tori

The spherical torus ($A=R/a<2$) was proposed in 1986 by M. Peng and D.J. Strickler (Oak Ridge National Laboratory, USA) [26], as a modification of the conventional tokamak, from which it differs because of a much smaller aspect ratio, see Fig. 1.12.

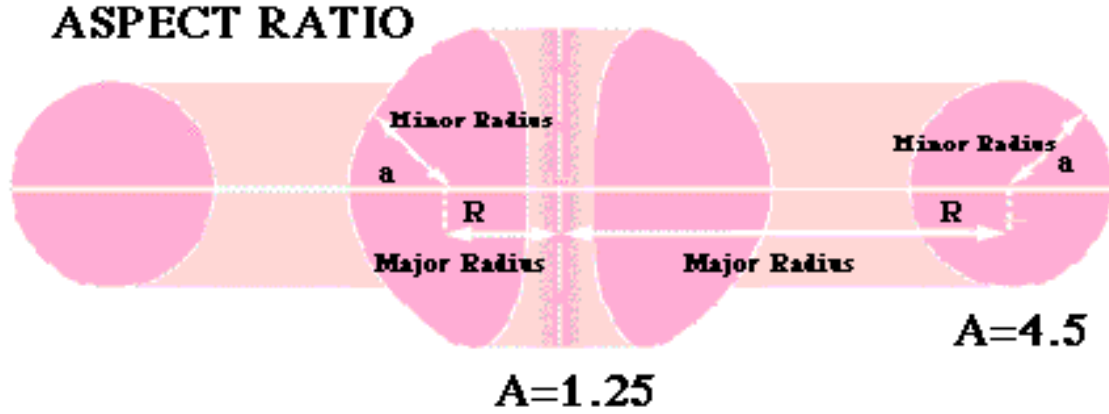


Fig. 1.12. Difference in aspect ratio between an ST and a conventional tokamak.

Peng and Strickler pointed out certain advantages connected with low A , concerning for examples the high value of the ratio between thermal energy and magnetic energy, $\beta=2\mu_0\langle p\rangle_{vol}/B^2$ that can be achieved. No dedicated experiment along this line was built until the early '90. The first explorations of low aspect ratio configuration were made by modifying spheromak experiments with the addition of a central rod, carrying a current I_{tr} , to produce a toroidal magnetic field. The objective was to control the tilting instability of the spheromak. The main results of this work was that a tokamak configuration could exist down to $A=1.1$. HSE (Heidelberg Spheromak Experiment) [27], Rotamak in Australia [28], SPHEX in UK [29], FBX II in Japan [30], were devoted to these experiments between 1987 and 1991, see Fig. 1.13. The drawbacks of the plasmas produced in these devices were the low temperature obtained ($T_e<50$ eV) and the short pulse duration ($t_{pulse}<2$ ms), which prevented a strong assessment on the feasibility and advantages of these configurations.

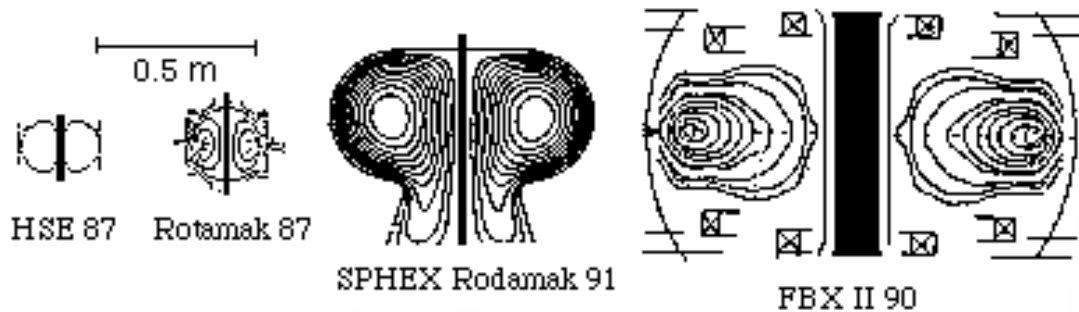


Fig. 1.13. Schemes of the first-generation ST experiments (1987-1991).

START at Culham [31, 32] began operation in 1991. Plasma currents up to $I_p=250$ kA were obtained by first inducing two current carrying plasma tori with large major radius. Thereafter they were merged and compressed down to $A\sim 1.25$. A pulse length

of ~ 40 ms, (extended by the addition of a compact central solenoid), allowed the attainment of hot ($T_e \sim 500$ eV) and dense ($n_e > 10^{20} \text{ m}^{-3}$) plasmas. Thus some characteristics of spherical tori could be, for the first time, compared with theoretical expectations with some confidence. CDX-U at Princeton [33, 34] and HIT at Seattle [35] completed the series of the experiments that produced spherical tori of sufficient duration to enable plasma properties to be evaluated. HIT is particularly relevant for PROTO-SPHERA (see Section 2.7), since it was devoted to the study of helicity injection to drive the plasma current. Indeed up to $I_p = 200$ kA has been driven by this mechanism for ~ 20 ms, with a good power efficiency. Fig. 1.14 shows the schemes of the magnetic equilibria achieved in START, CDX-U and HIT.

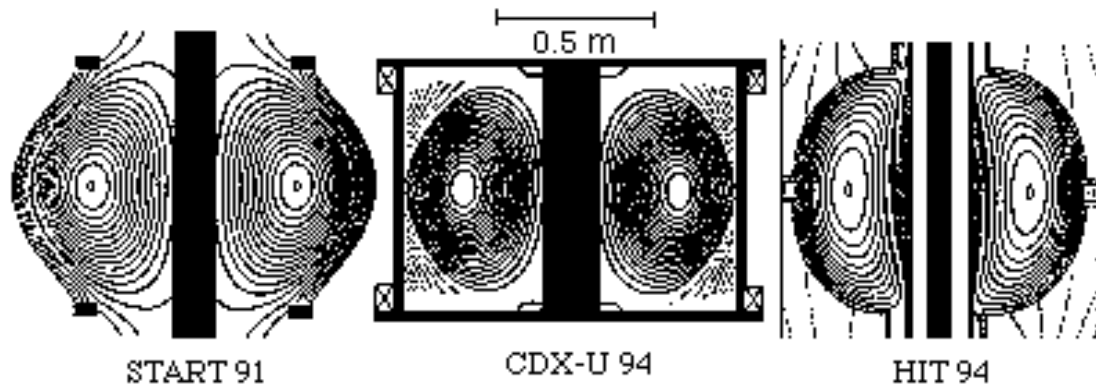


Fig. 1.14. Schemes of the second-generation ST experiments (1991-1994).

After October 1995 START was modified by the installation of additional poloidal field coils [36], allowing for the obtainment of a divertor plasma, see Fig. 1.15.

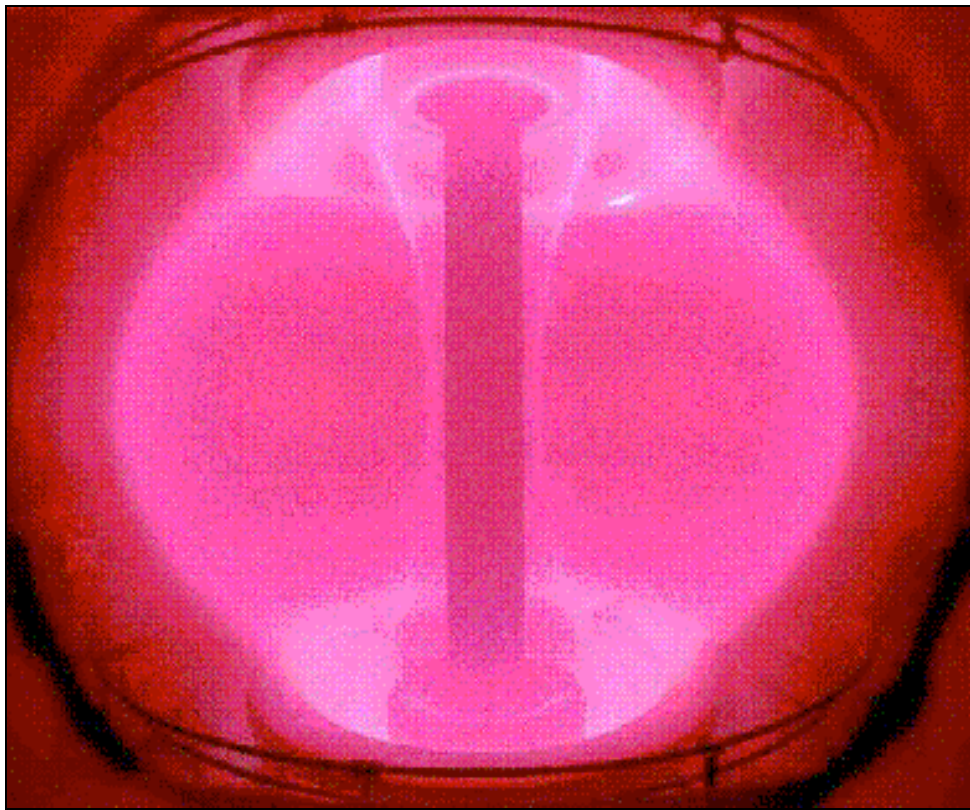
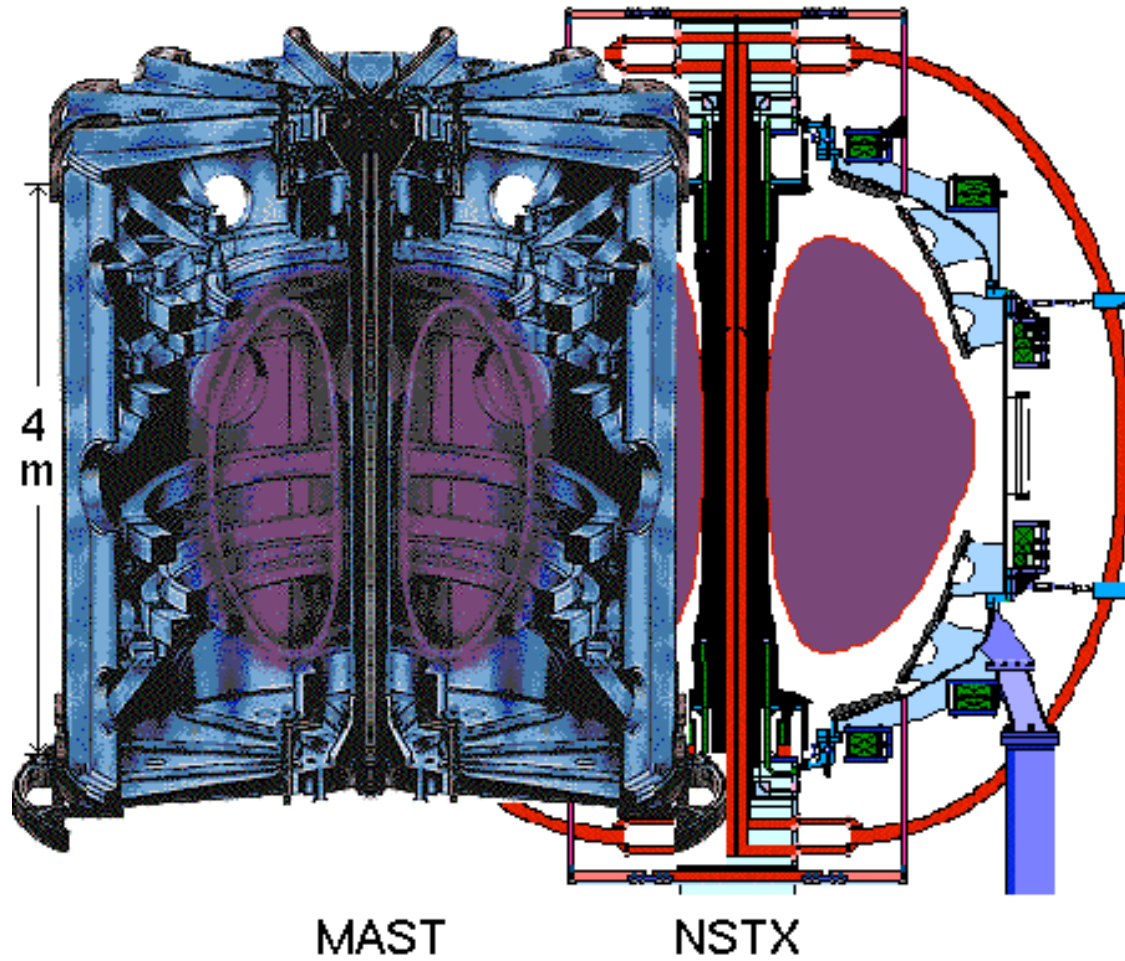


Fig. 1.15. Visible light image of an X-point plasma in START.

Using a 1 MW neutral beam injector (NBI) START has reached an average toroidal beta value $\beta_{T0} = 2\beta_0 \langle p \rangle_{vol} / B_{T0}^2 \sim 40\%$ [36], where B_{T0} is the vacuum toroidal field on the magnetic axis. The very encouraging results of START have provided a good basis for building new ST experiments, capable of carrying currents in the MA range; the two main ST of this category now in operation are MAST and NSTX. In Fig. 1.16 the parameters of the two devices are compared.



	Culham		Princeton	
Operation:	1999		1999	
Major radius:	R	= 0.7 m	R	= 0.85 m
Minor radius:	a	= 0.5 m	a	= 0.68 m
Elongation:	κ	≤ 2	κ	≥ 2
Toroidal field:	B_T	≤ 0.6 T	B_T	≥ 0.3 T
ST plasma current:	I_p	≤ 2 MA	I_p	≤ 2 MA
Plasma duration:	t_{Plasma}	≤ 2 s	t_{Plasma}	≤ 5 s
Additional power:	P_{add}	= 6.5 MW	P_{add}	= 12 MW

Fig. 1.16. Schemes and parameters of MAST and NSTX (third-generation ST).

MAST (Mega Amp Spherical Tokamak), has been built by the Culham Laboratory [37, 38], which is a scaled up version of START, has produced its first plasma in

1999 (Fig. 1.17.a) and 1 MA plasma in May 2000. MAST is endowed with a central solenoid, which can sustain an ohmic spherical tokamak. Its main aim is to extend the experimental results of START to the 1÷2 MA range of plasma current. Two broad objectives are: 1) to make a significant contribution to the understanding of tokamak physics (confinement scaling, plasma exhaust, MHD stability, etc.); 2) to test the spherical tokamak concept, in order to provide a database for a possible future Material Test Facility. In more detail, most interest is devoted to the study of exhaust in divertor configuration at high density; to the exploration of energy confinement properties, essentially to the dependence on the aspect ratio; to the study of the characteristics of the H-mode in these configurations; to the exploration of the operational limits (plasma density n_e , average toroidal beta value β_{T0} , safety factor at the edge q_0 or at 95% of the poloidal flux q_{95}) and to the MHD stability properties; to the investigation of the efficiency of current drive systems (neutral beams). Additional heating is planned after the ohmic phase, and it is based on neutral beam injection (~5 MW) and on 60 GHz electron cyclotron heating (~1.5 MW).

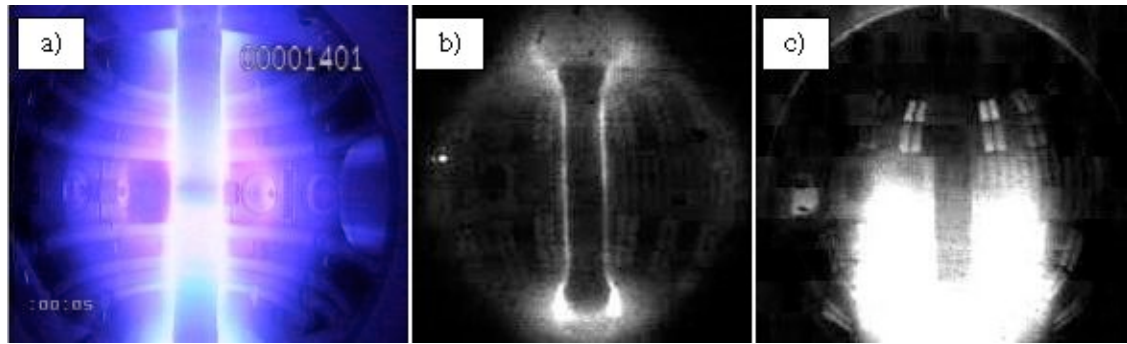


Fig. 1.17. a) Visible light image of one of the first $I_p=300$ kA discharges in MAST.
b) Fast camera image of an $I_p=500$ kA ohmic discharge in NSTX.
c) Same for helicity injection startup at $I_p=50$ kA in NSTX.

NSTX (National Spherical Tokamak Experiment) is a very low aspect ratio ($A \sim 1.25$) device built as a national facility by the Princeton Laboratory [39, 40] and has produced its first 1 MA plasma in December 1999 (Fig. 1.17.b and 1.17.c). Also NSTX is endowed with a central solenoid. The plasma current is in the same range as that of MAST, while its objectives are different. Apart from exploring confinement scaling and q limits, the main goal is to achieve and explore reactor relevant ST regimes, characterized by low collisionality, high β , high bootstrap current fraction f_{BS} [41, 42, 43], at fully relaxed current density profiles. Thus from the beginning NSTX is designed having in mind the additional heating and current drive systems, with a capability of magnet pulse up to 5 seconds. It has to be noted that NSTX relies on helicity injection for plasma start-up and for edge current drive and has a conducting shell to help the plasma formation and the high beta stability.

The aspect ratio ($A=R/a$) of the spherical torus (ST) plasma approaches unity ($A=1.1 \div 1.6$ typically), compared to $A=2.5 \div 5.0$ for the conventional tokamak. Its magnetic surfaces combine a short field line of bad curvature and high pitch angle toward the outboard plasma edge with a long field line of good curvature and low pitch angle toward the inboard plasma edge (see Fig. 1.18). Another feature of ST is that the geodesic curvature of the lines of force is almost zero on the inboard of the torus; at high β value a magnetic well (local minimum $|B|$)

appears on the outboard of the torus. Therefore the small value of the banana width (quasi omnigenity) and the time-averaged concentration of the trapped particle orbits in favorable curvature region [44] could limit the micro-instabilities related to trapped particles.

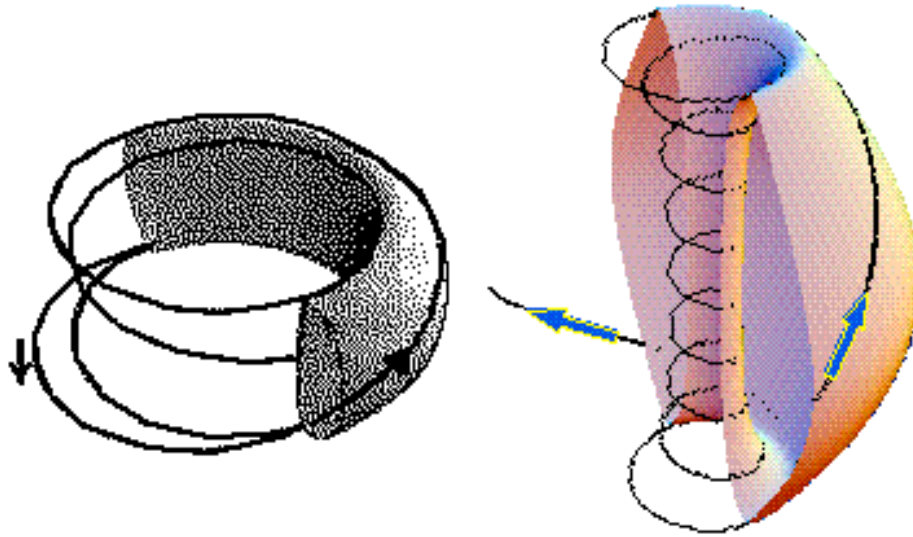


Fig. 1.18. Magnetic lines of force in a conventional tokamak and in an ST.

As the aspect ratio is reduced, for instance from $A = 2.5$ to $A = 1.2$, the elongation increases naturally, i.e. maintaining a uniform vertical field with a null field index (see Fig. 1.19). ST plasmas with cross-sections elongated up to $\kappa = 3$ can have intrinsic vertical stability.

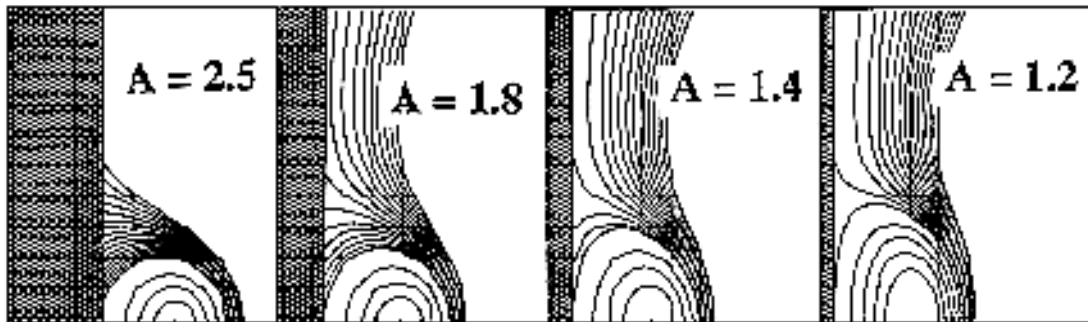


Fig. 1.19. Reducing the aspect ratio, the elongation of free boundary equilibria increases.

In an ST the poloidal field B_{pol} is comparable to the toroidal field B_T , whereas in a tokamak $B_{\text{pol}} \ll B_T$. As a result, the ST uses a modest applied toroidal field (TF), but has large values of the normalized current I_p/aB_T and of the ratio between the plasma current and the toroidal field current I_p/I_{tf} . High toroidal plasma current I_p can be driven with low toroidal field B_T and with very simple windings, compared to conventional tokamaks (see Fig. 1.20). This corresponds to a high ohmic power density and to operations at high plasma density.

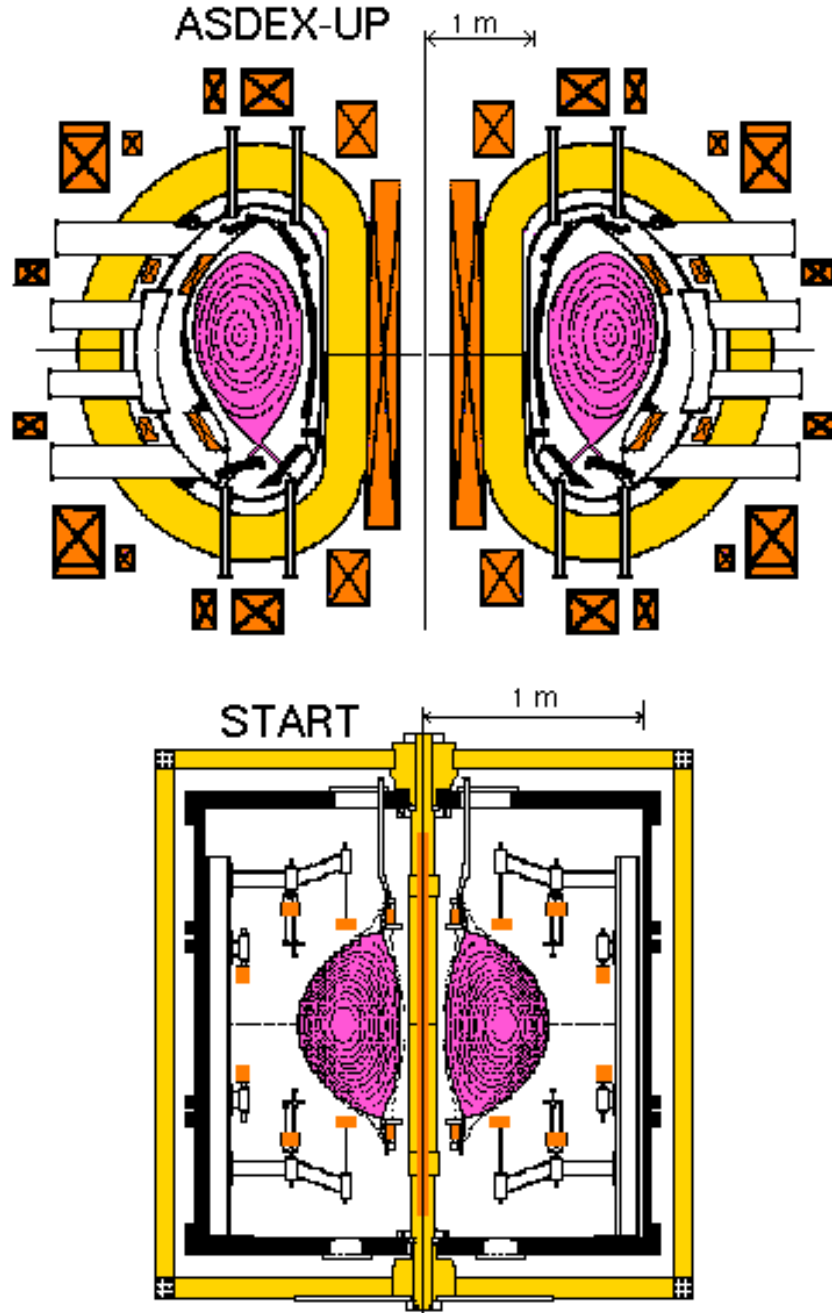


Fig. 1.20. Comparison of TF and PF field coils between ASDEX-UP and START.

The dominance of good field line curvature leads to magnetohydrodynamic (MHD) stability at high plasma pressure, giving the potential for order-unity average toroidal beta, $\beta_{T0} = 2\beta_0 \langle p \rangle_{vol} / B_{T0}^2$, and of order-10 normalized beta, $\beta_N = \beta_{T0} a B_T / I_p$ [% , m, T/MA]. The high beta and the magnetic configuration combine to widen the parameter domain for magnetic fusion plasmas.

Ideal MHD calculations for highly elongated ST ($\kappa = 2.5 \div 3$) show first stability beta limits in excess of $\beta_{T0} \sim 50\%$ ($\beta_N \sim 4$), in absence of any stabilizing wall near the plasma, and second stable regime at $\beta_{T0} = 100\%$ ($\beta_N \sim 8$), with a conducting shell at $r_{shell}/a < 1.2$ [45]. An advantage of spherical tori is the ability to achieve second stability with monotone q-profiles, thus avoiding instabilities associated with low shear (infernal modes) or inverted q-profiles (double tearing).

The total β_{T0} value in magnetic confinement expresses how a plasma can be well confined in an apparatus of reduced size and cost (the ideal aim being $\beta_{T0} \sim 1$). The poloidal beta value $\beta_{pol} = 2\beta_0 \langle p \rangle_{vol} / B_{pol}^2$ marks instead the distance of the configuration from a force-free state ($\vec{j} \parallel \vec{B} = 0$). In an ST, as $B_{pol} \sim B_T$, $\beta_{T0} \sim 1$ means only $\beta_{pol} \sim 1$; therefore a high β_{T0} (40%) plasma in an ST is much nearer to a force-free configuration than a low β_{T0} (4%) plasma in a conventional tokamak. The high physical beta (referred to the magnetic energy contained in the plasma) of ST is even more significant as it still corresponds to a high engineering beta (referred to the total magnetic energy contained in the assembly). This statement is not true for other high beta configurations such as the high aspect ratio advanced tokamaks (AT). The START experiment has demonstrated the high β -potential of ST achieving a toroidal volume average $\beta_{T0} = 2\beta_0 \langle p \rangle_{vol} / B_{T0}^2 = 40\%$, with a peak toroidal $\beta_{T0}(0) = 2\beta_0 p(0) / B_{T0}^2 = 70\%$, and a normalized plasma beta $\beta_N \sim 6$ (see Fig. 1.21).

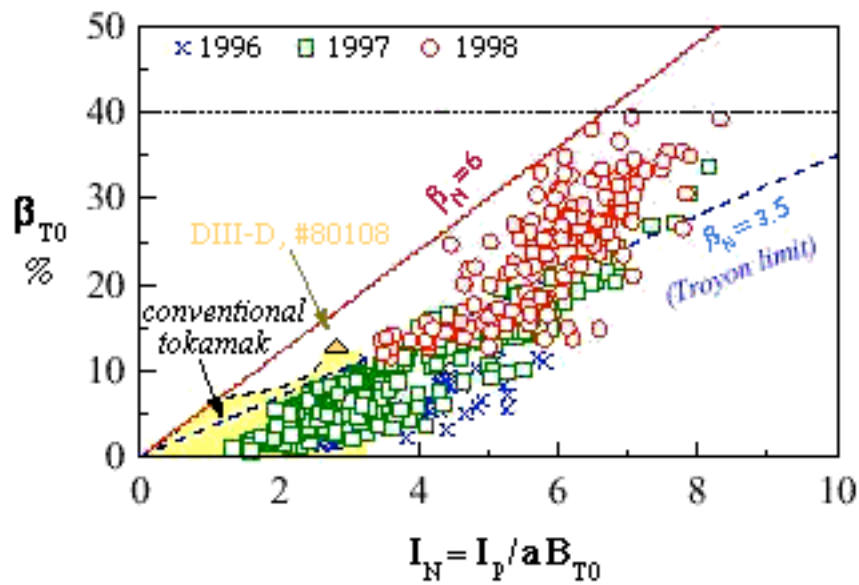


Fig. 1.21. Volume average β_{T0} versus $I_p / a B_{T0}$ for START, compared to conventional tokamaks; all the β_{T0} value in excess of 6% are due only to DIII-D, which is the lowest aspect ratio conventional tokamak ($A=2.5$) [36].

The high β_{T0} values of START are even more noteworthy as they could be sustained for several confinement times in low $q_{95} \sim 3$ discharges, characterized by a ratio $I_p / I_{if} \sim 1$ and furthermore in presence of an improved confinement regime (see Fig. 1.22) [46].

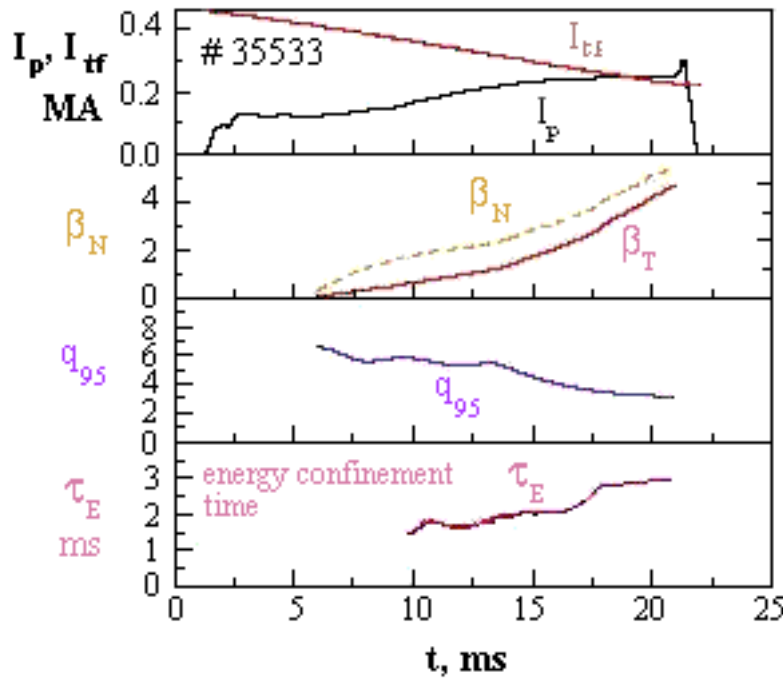


Fig. 1.22. Traces of a high- β shot (#35533) of START [46].

START has exhibited relatively high energy confinement times and density limits. H-mode like signatures have been observed in NBI heated double X-point discharges. Pellet injection has greatly extended the operating space and the density limits beyond the Greenwald limit (Fig. 1.23) [47].

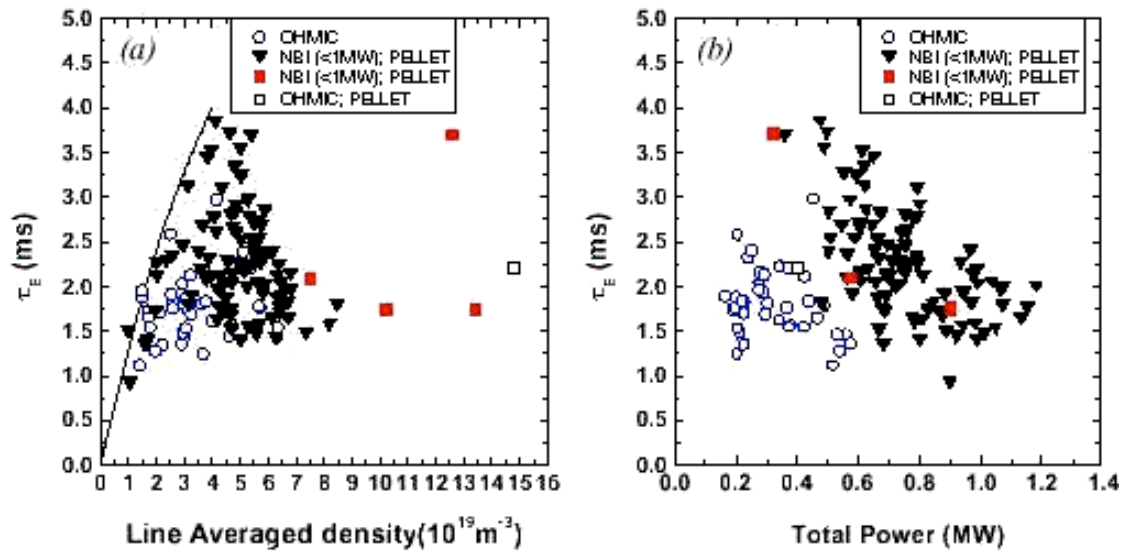
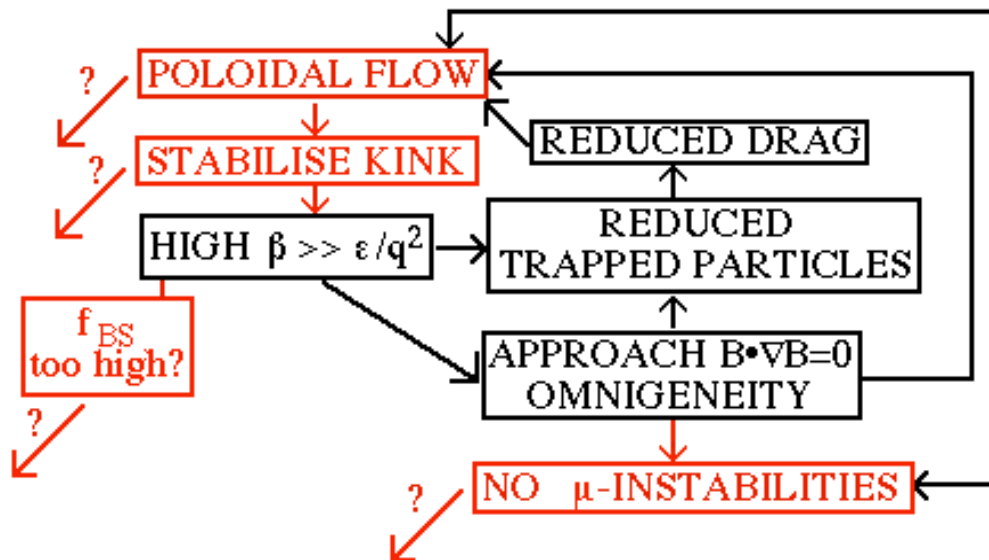


Fig. 1.23. Confinement in START with NBI [47].

From ST equilibrium calculations a very low shear is expected on the central part of the plasma cross-section, and a very high shear occurs at the plasma edge. This could lead to stabilization of MHD and micro-instabilities and eventually to a favorable energy confinement. As a matter of fact on START, no current disruption was observed for $A < 1.8$ [48, 49], at least before the installation of the X-point coils. Sawteeth and internal reconnection were still present, but did not destroy the plasma.

All the advanced features associated with ST (stabilization of MHD and micro-instabilities) are already accessible at low beta and do not depend upon the uncertain achievement of a high beta, as it is the case for the high aspect ratio advanced tokamak (see Fig. 1.24).

CONVENTIONAL ADVANCED TOKAMAK



SPHERICAL TORUS ADVANCED TOKAMAK

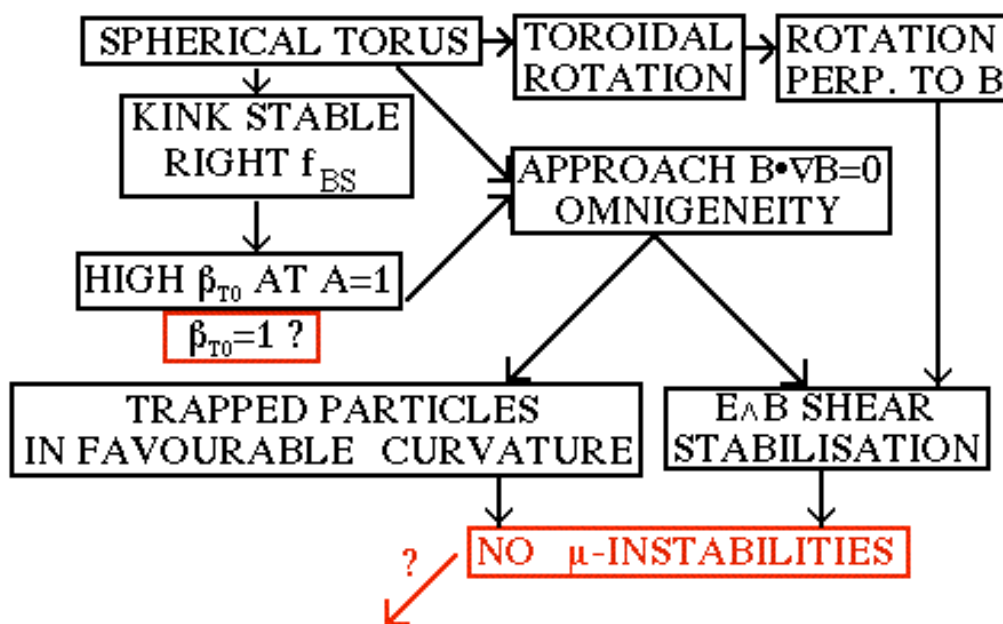


Fig. 1.24. Schematic comparison of the advanced tokamak concept, following the more conventional high aspect ratio or the more innovative spherical torus approach.

So it is conceivable that the two large ST experiments now operating, MAST and NSTX will successfully address a number of advanced tokamak issues.

The general scientific objectives that can be studied in the ST experiments are:

- Achievement of $\beta_{T0} = 1$ at aspect ratio $A \approx 1$.
- Test of resistive and neo-classical MHD at $A \approx 1$.
- Stabilization of micro-instabilities.
- Well aligned bootstrap current (the bootstrap current fraction f_{BS} can approach 100% for elongation $\kappa \approx 3$).
- Overlap of ST, spheromak and FRC.

The advantages of the ST in the path toward the development of an economically attractive fusion power source can be so summarized:

- High Q path to reactor cheaper and faster than with conventional tokamaks.
- Possibility of trying different blanket concepts and nuclear engineering components on a number of cheap experimental power sources (as it has been for fission reactors).
- Reduced waste volume.

1.5. The Ultra Low Aspect Ratio Torus (ULART)

The usual configuration of an ST is connected with a slim central rod that carries the current necessary to create the toroidal magnetic field. Thus a central solenoid coil can store only a small inductive flux. Due to the low inductance of the ST, this flux can be sufficient to bring the current to its nominal value, but then a non-inductive current drive system is needed to maintain the current during the flat top. This restriction is even more severe if one aims at $A < 1.3$, where many advantages are obtained, according to the theory. Many systems of non-inductive current drive seem of difficult or impossible applicability [50, 51]. In the RF range of frequencies, only fast wave current drive has up to now been considered [52] for proposed experiments. Neutral beams [53, 54, 55] and helicity injection [56, 57] are the systems most considered for implementation.

The main problems of the ST to be solved in the path toward the development of an economically attractive fusion power source can be so summarized:

- Achievement of reliable start-up techniques in absence of an ohmic transformer.
- Demonstration of reliable current drive (based either upon bootstrap and non-inductive methods) on ST.
- Choice of optimal aspect ratio.
- Feasibility of single turn central rod for the toroidal field coil in order to achieve an easy maintenance and substitution.

The limits to aspect ratio have been explored in the TS-3 experiment [58, 59] at the University of Tokyo, used as a spherical torus. Record of low aspect ratio $A = 1.1 \div 1.2$, with ratio of the toroidal field current and plasma current as low as $I_{TF}/I_p = 0.20$, have been achieved (see Fig. 1.25).

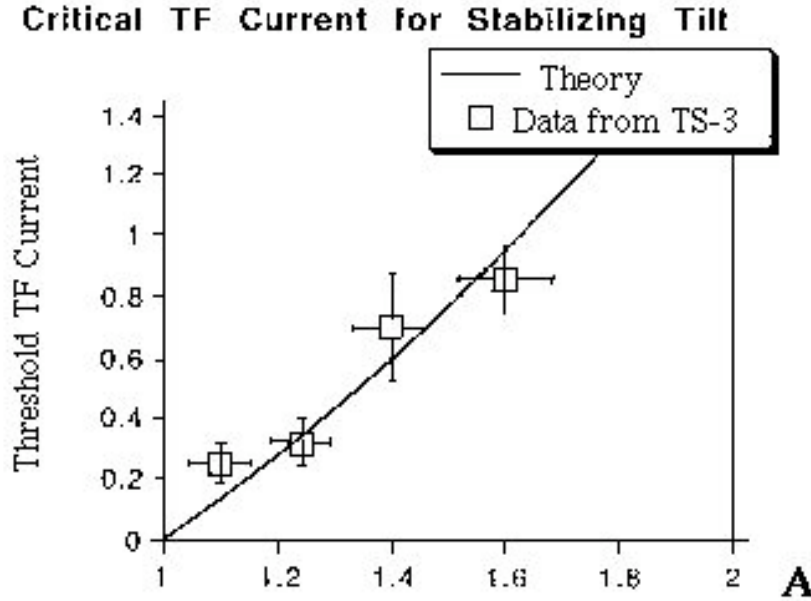


Fig. 1.25. Limit to I_{tf}/I_p at low aspect ratio in TS-3 [58].

Along with the results of TS-3, other reasons push towards the Ultra Low Aspect Ratio Tokamaks (ULART, $A < 1.3$). One of the main problems in designing an ST reactor is the central conductor that creates the toroidal magnetic field. It cannot be shielded, is bombarded by neutrons and so cannot be built by superconducting materials and can involve too large an energy dissipation with respect to the produced fusion power. The only way to avoid an excessive dissipation in the central conductor is to go down in aspect ratio until $A < 1.3$ [60]. This is allowed by the ratio I_{tf}/I_p which, for aspect ratios $A < 2$, is well described by the Katsurai formula [61]: $I_{tf}/I_p = \frac{1}{2} q_0^2 (A-1)^2 / (2 \frac{1}{2})^2$ (see Fig. 1.26).

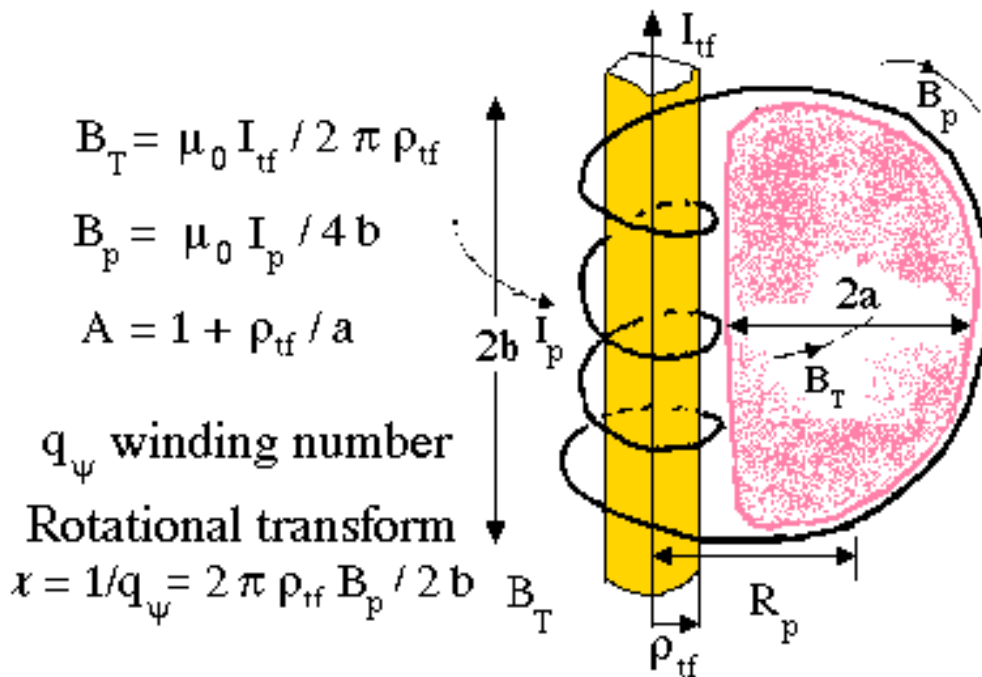


Fig. 1.26. Behavior of magnetic field lines in an ULART.

This relation is calculated from the behavior of the lines of force in the inboard region, near the central rod:

$$B_T = \mu_0 I_{tf} / 2\pi a, \quad B_{pol} = \mu_0 I_p / 4b, \quad 1/q_0 = 2\pi a B_{pol} / 2b B_T, \quad A = 1 + a/b.$$

Fixing the value of the safety factor to $q_0 = 3$ one can see that, whereas in an ST with aspect ratio $A = 1.5$ and elongation $\kappa = 2.0$, $I_{tf}/I_p = 1.2$, in an ULART with aspect ratio $A = 1.2$ and $\kappa = 3$, $I_{tf}/I_p = 0.1$.

Another limit to A is given by an analytic treatment of the rigid tilt instability, which is calculated [61] as:

$$I_{tf}/I_p \geq [2\kappa(1-n^*)C_L A]^{1/2} (A-1)/\kappa^{3/2},$$

with C_L (of order of unity), determined by the vertical field $B_z = C_L \mu_0 I_p / (4\pi R_p)$, and n^* (very near to zero), determined by $n^* = -(R/B_z) dB_z/dR$.

The two limits (see Fig. 1.27), with $C_L = 0.645$ and $n^* = 0.07$ are:

$$I_{tf}/I_p \geq 4.93 q_0 (A-1)^2 / \kappa^2 \quad \text{for the } q_0 \text{ limit}$$

$$I_{tf}/I_p \geq 1.94 A^{1/2} (A-1) / \kappa^{3/2} \quad \text{for the tilt limit}$$

With $\kappa = 2.3$ and $q_0 = 3$ the two limits coincide at $A = 1.2$; at $A > 1.2$ the q_0 sets the current limit but at $A < 1.2$ it is the tilt which limits the plasma current.

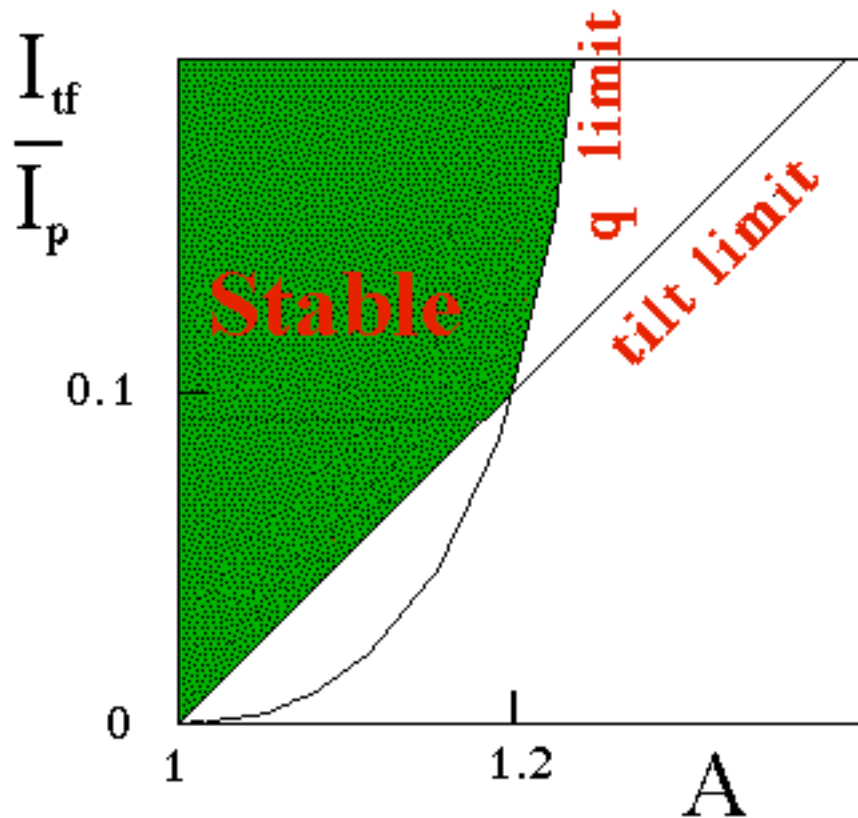


Fig. 1.27. I_{tf}/I_p limit at low aspect ratio with $\kappa = 2.3$ and $q_0 = 3$.

The ULART configuration however does not leave enough space for a central ohmic transformer and so requires non-inductive current drive methods. Furthermore the ULART configuration does not solve the problem of the neutron damaging of the central conductor in a D-T reactor.

2. PHYSICAL BASIS

The purpose of this Section is to elucidate the physical basis of the PROTO-SPHERA proposal, which can be summarized as the "injection of magnetic helicity". Magnetic helicity is a slowly decaying ideal invariant that controls to some extent the formation of a relaxed MHD state in laboratory plasmas. The origin of magnetic helicity is clear: if the electric current is forced to flow along a DC magnetic field, it generates perpendicular magnetic flux and causes the magnetic field lines to kink up, with a helical pattern. In simple geometrical circumstances (closed field line structures), the magnetic helicity can be interpreted as being the total linked magnetic flux. When dissipation is included, the magnetic energy decays on a much faster time-scale than magnetic helicity, provided that the scale-length of the dissipative phenomena is much shorter than the dimensions of the system. The Taylor assumption states that energy decays to the minimum value it can have, subject to the conservation of magnetic helicity. Any initial configuration will self-organize in a relaxed state $\vec{\nabla} \times \vec{B} = \alpha \vec{B}$, with $\alpha = \text{constant}$ all over the plasma, after sufficient time. The eigenvalue α is determined by the boundary conditions: since $\vec{\nabla} \times \vec{B} = 0$ in a relaxed state, the relaxed state can be considered to be the termination of a kink instability. The non-linear saturation of a kink instability is the process by which the PROTO-SPHERA configuration will be formed.

A more realistic physical situation of a domain containing a magnetized plasma, with open field lines passing through the boundary, compels to define a relative helicity that is gauge invariant and physically meaningful, because it is independent of the properties external to the domain. But this more realistic situation offers also the opportunity of refurbishing the helicity content of the magnetized plasma, if the magnetic helicity can be injected through the boundary (by driving current along the lines of force) more quickly than it is dissipated inside the domain by resistive processes. Magnetic energy will be injected along with the helicity and reconnection processes will convert part of the magnetic energy into kinetic energy of the magnetized plasma. If the helicity source (the screw pinch discharge in the case of PROTO-SPHERA) is physically separated from the helicity sink (the spherical torus of PROTO-SPHERA), a gradient in the relaxation parameter ($\alpha \neq 0$) will appear: resistive MHD instabilities will produce a helicity flow from regions of larger α to regions of smaller α . A helicity diffusion coefficient will rule the helicity flux from source to sink.

2.1. Aim of PROTO-SPHERA

The idea of the PROTO-SPHERA (Spherical **P**lasma for **H**elicity **R**elaxation **A**ssessment) experiment is to test a spherical torus where the central conductor current I_{tf} is substituted by the current I_e driven by a plasma discharge. This central plasma discharge takes the form of a screw pinch, fed by two electrodes placed upon the polar caps of the plasma sphere (see Fig. 2.1). Such a configuration has been devised theoretically under the name "bumpy Z-pinch" [62] or "flux-core-spheromak" [63, 64] and then studied experimentally in the FACT [65, 66] and TS-3 experiments [67, 68].

The advantages of this configuration are quite clear. The problem of damaging the central conductor just disappears. Furthermore the current injected through the electrodes, along the lines of force of the central screw pinch, allows the sustainment, through DC helicity injection, of the spherical torus configuration, also in absence of an ohmic transformer. The weak points of this solution are however also quite clear: the need of containing, as much as possible, the power injected through the electrodes and to avoid an excessive damaging of the electrodes themselves.

From the equilibrium point of view the configuration interlaces a spherical torus with a screw pinch: the screw pinch provides the stabilizing toroidal field to the ST and the ST, on its turn, provides the stabilizing longitudinal field to the screw pinch.

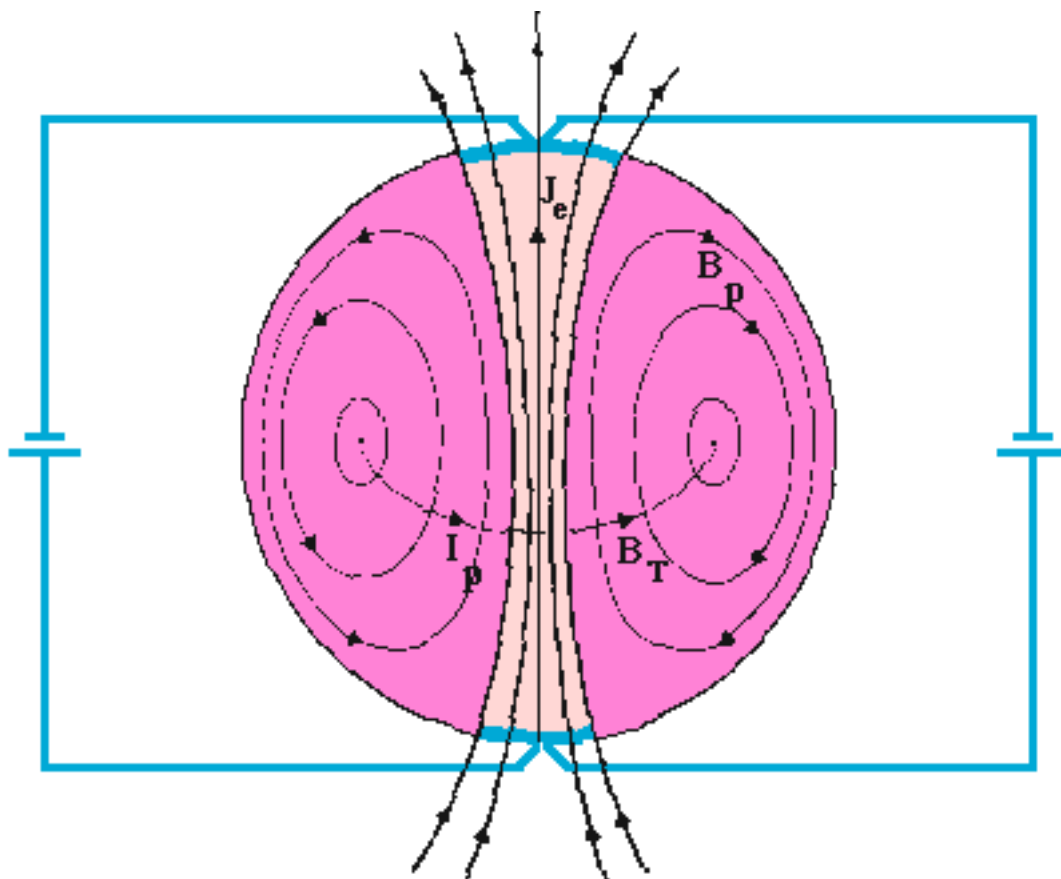


Fig. 2.1. Scheme of the spherical torus + screw pinch configuration [64].

2.2. Formation of the Toroidal Plasma and the TS-3 Experiment

The formation of the ST is obtained by the kink destabilization of a screw pinch, through an increase of the longitudinal arc current, as demonstrated on the TS-3 experiment (University of Tokyo). Figure 2.2 sketches the linear and non-linear phase of a kink unstable screw pinch, with longitudinal field B_z and 'toroidal' field B_ϕ , that means a pinch winding number $q_{\text{Pinch}} = 2\pi \rho_{\text{Pinch}} B_z / L_{\text{Pinch}} B_\phi$.

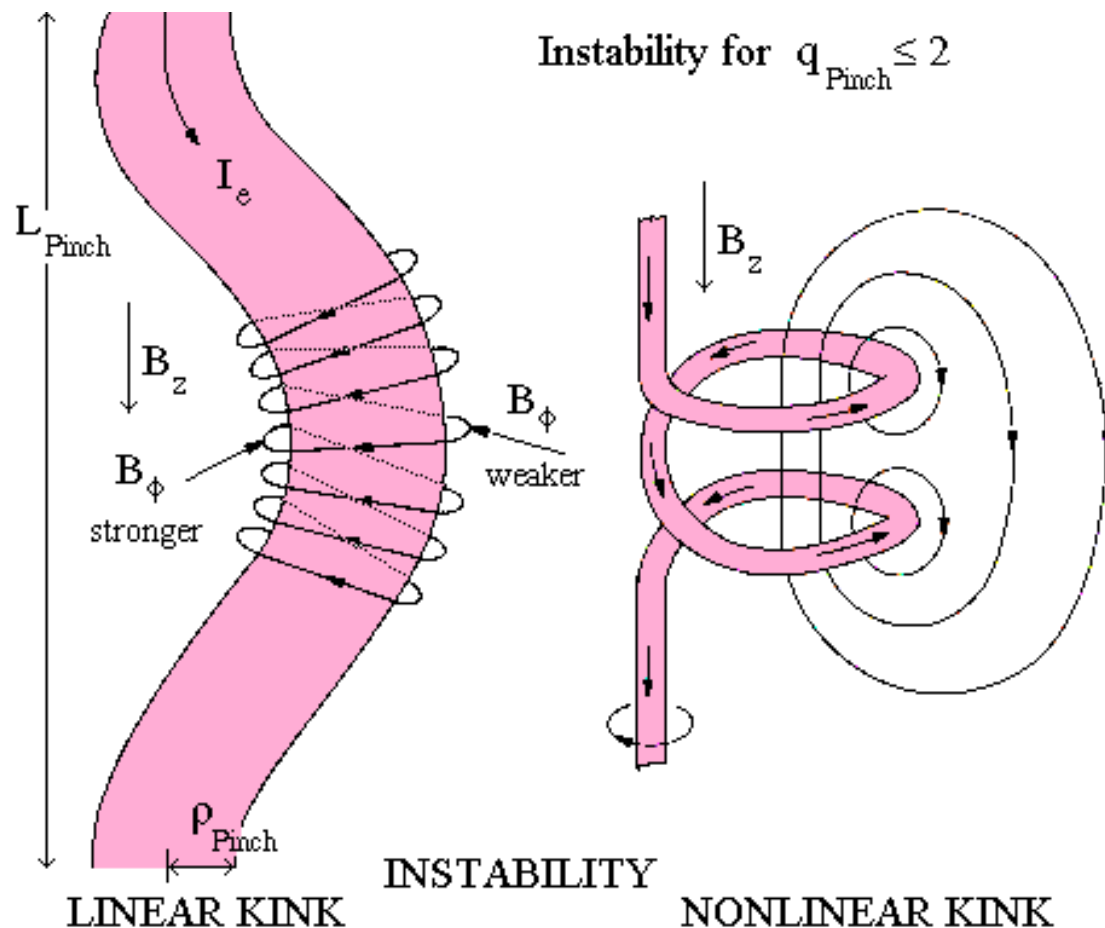


Fig. 2.2. Schematic of a kink instability converting toroidal flux into poloidal flux [69].

A configuration very similar to PROTO-SPHERA was obtained on the TS-3 experiment (University of Tokyo) in 1991-1993 [67, 68]. The configuration set-up on TS-3 was smaller than PROTO-SPHERA by a factor of about 1.6 in linear dimensions.

The screw pinch discharge was initiated breaking down ($V_e \sim 1$ kV) the filling gas ($p_H \sim 2 \cdot 10^{-2}$ mbar) between two plasma guns used as electrodes. The toroidal plasma was formed increasing the arc current (see Fig. 2.3), up to the non-linear kink instability threshold: $q_{\text{Pinch}} \ll 1 \div 2$. At this time the current of the compression coils was pulsed. The magnetic field measurements (Fig. 2.4) confirmed the establishment of a flux-core-spheromak, whose formation, from the longitudinal electrode current, was attributed to the $n=1$ kink mode, which was able to convert the magnetic flux from poloidal to toroidal.

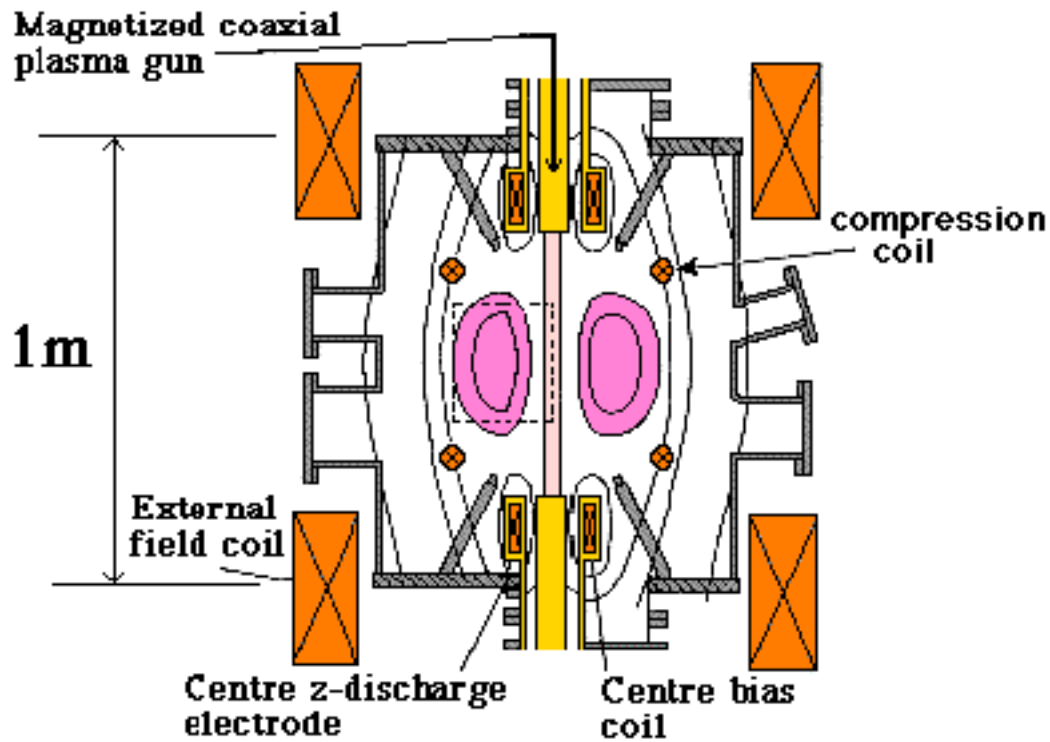


Fig. 2.3. "Flux-core-spheromak" configuration of TS-3 [68].

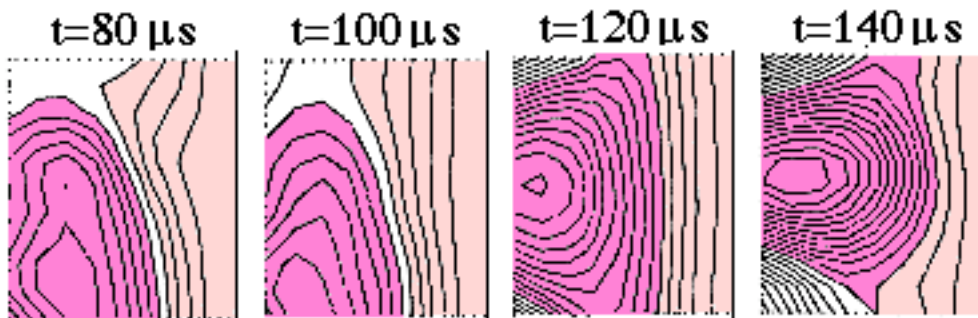


Fig. 2.4. Magnetic reconstruction of the TS-3 flux-core compression [68].

Also a compression experiment [68] was successfully undertaken. After the formation phase lasting $\sim 60 \mu\text{s}$, the compressed TS-3 configuration lasted for $20 \mu\text{sec}$, i.e. $30 \cdot \tau_A$ (thirty Alfvén times). It must be pointed out that the flux swing associated with the increase of current in the compression coils (see Fig. 2.3) has certainly contributed substantially to the formation of the toroidal plasma current loop.

2.3. Sustainment of the Toroidal Plasma

The PROTO-SPHERA experiment aims at sustaining the toroidal plasma, after the formation, through DC helicity injection. The physical scheme of DC helicity injection is shown in Fig. 2.5, and can be summarized as follows:

- The plasma with open field lines (intersecting the electrodes) has $\alpha \sim 0$, therefore $\vec{j} \parallel \vec{B}$.
- Because of the twist of the open field lines, the current between the electrodes also winds in the toroidal direction near the closed magnetic flux surfaces.
- Resistive MHD instabilities convert, through magnetic reconnections, open current/field lines into closed current/field lines, winding on the closed magnetic flux surfaces.
- Magnetic reconnections necessarily break, through helical perturbations, the axial symmetry, as per Cowling's anti-dynamo theorem [70].

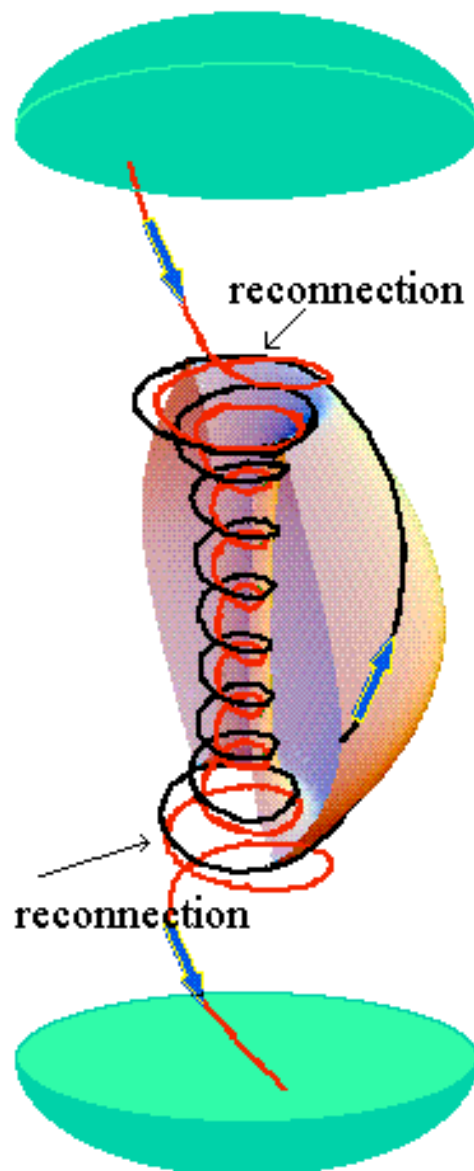


Fig. 2.5. Physical scheme of DC helicity injection.

2.4. Magnetic Helicity

The origin of the idea of applying the helicity injection to magnetic configurations of fusion interest can be traced back to J.B. Taylor [71, 72].

In a perfectly conducting plasma (i.e. with resistivity $\eta = 0$):

$$\partial \vec{A} / \partial t = \vec{\nabla} \times \vec{B} + \vec{\nabla} \chi, \quad \text{where } \chi \text{ is an arbitrary gauge.}$$

The parallel component of $\vec{A}$ satisfies the magnetic differential equation:

$$\vec{B} \cdot \vec{\nabla} \chi = \vec{B} \cdot \partial \vec{A} / \partial t$$

To make χ single valued it is necessary that

$$\oint \frac{d\ell}{B} \cdot \frac{\partial \vec{A}}{\partial t} \quad \text{and} \quad \oint \frac{dS}{|\vec{\nabla} \chi|} \cdot \frac{\partial \vec{A}}{\partial t}, \quad \text{are respectively zero upon any}$$

closed field line and upon any magnetic surface, the latter being described either by the enclosed poloidal (Φ_p) or toroidal (Φ_T) flux. So, for every flux tube labeled (Klebsch representation) by constant values of the two variables (χ, Φ),

$$K(\chi, \Phi) = \oint \vec{A} \cdot \vec{B} \, dV \text{ is an invariant, called magnetic helicity.}$$

Minimizing the magnetic energy $W = (1/2\mu_0) \oint (\vec{\nabla} \chi \times \vec{A})^2 \, dV$, under the constraint that $K = \text{constant}$, the Euler's equation of motion is obtained:

$$\vec{\nabla} \chi \times \vec{B} = \nabla(\chi, \Phi) \vec{B}; \quad \vec{B} \cdot \vec{\nabla} \chi = 0, \quad \text{which describes a force-free magnetic field.}$$

The physical meaning of the magnetic helicity for closed field structures has been elucidated in a number of papers [73, 74]. It is a measure of how much the lines of force are interlinked, kinked or twisted.

For two singly linked flux tubes with fluxes Φ_1 and Φ_2 , see Fig. 2.6, integrating by Stokes theorem, over the two volumes, one obtains $K = 2\Phi_1\Phi_2$.

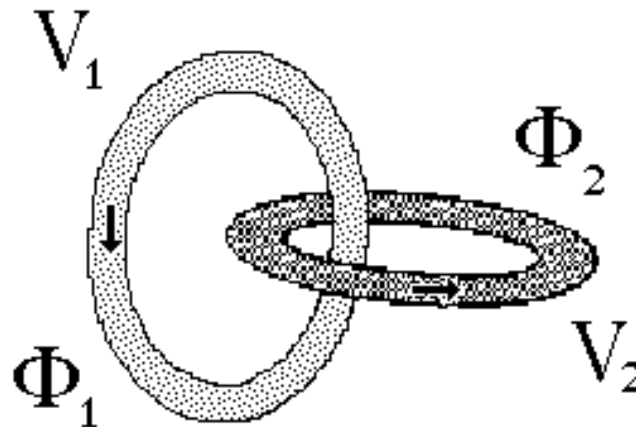


Fig. 2.6. Helicity of two singly linked flux tubes [74].

For the case of a torus with magnetic surfaces and rotational transform $\ell = \ell(\psi_T) = 1/q_0$, see Fig. 2.7, $K = \int \vec{A} \cdot \vec{B} dV = \int \psi_p d\psi_T - \int \psi_T d\psi_p = 2 \int_0^{2\pi} \psi_T d\psi_T$. For lines of force uniformly wound over the torus with twist $\ell = T = \text{constant}$: $K = T \int \psi_T^2$.

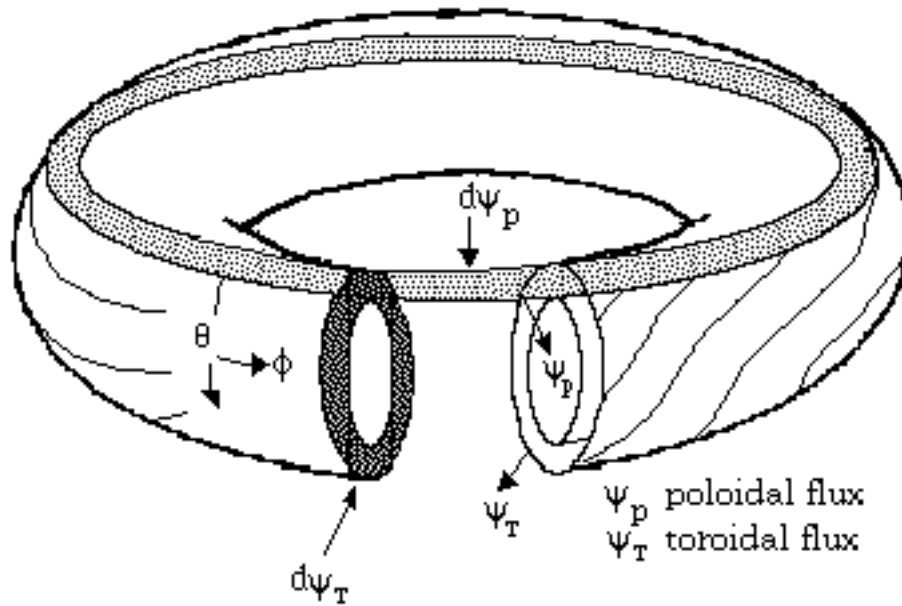


Fig. 2.7. Torus with magnetic surfaces and rotational transform [74].

A torus with $T = \pm 1$ can be distorted into a figure-of-8 in which the lines of force appear not wound, see Fig. 2.8.

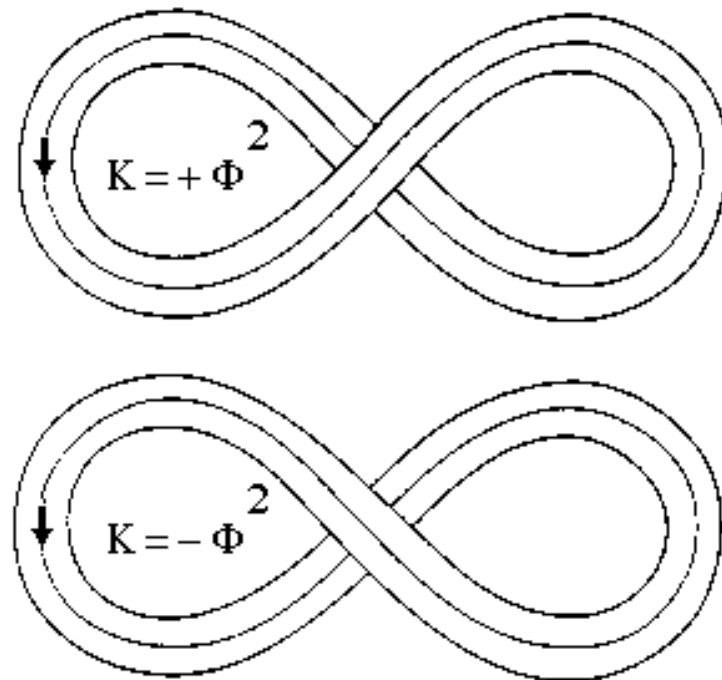


Fig. 2.8. Figure-of-8 shapes [73].

These examples clarify that the magnetic helicity is not localized in some points of the flux tubes, but can be thought of as a distributed property.

2.5. Magnetic Helicity and Reconnection

In presence of finite resistivity ($\eta \neq 0$) magnetic reconnections redistribute $\vec{A} \cdot \vec{B}$ over the plasma volume. A number of integral quantities are preserved by the magnetic reconnections [75], they can be expressed as: $K_\square = \int \square (\vec{A} \cdot \vec{B}) dV$, where $\square = q_s \square_T - \square_p$ is the helicoidal flux of the resonant surface upon which the magnetic reconnection occurs.

However the Taylor helicity invariant $K_0 = \int \vec{A} \cdot \vec{B} dV$ is the only common invariant to all winding numbers and so to all resonant surfaces. It provides the Euler's equation:

$$\square \square \vec{B} = \square \vec{B},$$

with $\square = \text{constant}$ all over the plasma. The solutions to this equation are called relaxed states and $\square$ is called relaxation parameter.

Comparing the decay time of the magnetic energy and the decay time of the Taylor's invariant can see the reason why the magnetic helicity is a robust invariant:

$$dW/dt = - \int \eta \vec{j}^2 dV ; \quad dK_0/dt = - 2 \int \eta \vec{j} \cdot \vec{B} dV,$$

where η is the plasma resistivity.

As the wavelength of the tearing mode is $k \propto \eta^{1/2}$, the following ordering applies:

$$\begin{aligned} dW/dt &= O(1) \\ dK_0/dt &= O(\eta^{1/2}). \end{aligned}$$

The magnetic helicity dissipation is then $\eta^{1/2}$ less strong than the magnetic energy dissipation.

The invariance of Taylor's helicity under reconnection can be simply understood (see Fig. 2.9) from the "reconnection" of two ribbons, both with winding number $T=1$:

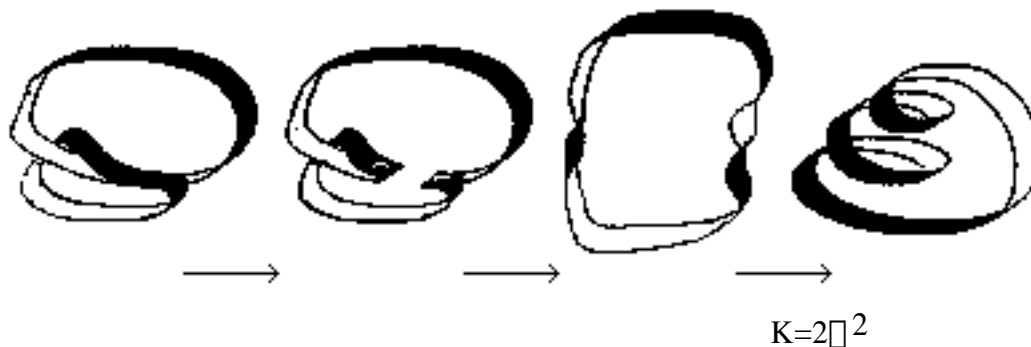


Fig. 2.9. The reconnection of two ribbons, both with winding number $T=1$, preserves the "magnetic" helicity [74].

2.6. Relative Magnetic Helicity

In a simply connected volume bounded by a magnetic surface (where $\vec{B} \cdot \vec{n} = 0$), the integral $\int_V \vec{A} \cdot \vec{B} dV$ is invariant to gauge transformations $\vec{A} = \vec{A} + \nabla \chi$. However in a multiply connected volume like a torus, there exist special gauge transformations which correspond to changing the magnetic flux through the hole. Furthermore, if the volume of interest is not bounded by a magnetic surface, then field lines will have endpoints on the boundary, and linking numbers will no longer be defined. The definition of the magnetic helicity becomes more complicated in these cases. The most simple and used definition is the relative magnetic helicity [73], although more general definitions have been introduced in the literature [76].

The definition of relative helicity in the case of two simply connected regions V_a and V_b , separated by a surface S , see Fig. 2.10, runs as follows:

If $\vec{B}_a$ and $\vec{B}'_a$ are fields with the same boundary conditions and differing only in V_a :

$\vec{B} = (\vec{B}_a, \vec{B}_b)$; $\vec{B}' = (\vec{B}'_a, \vec{B}_b)$, then it can be shown that:

$\int_K = \int_{V_a+V_b} \vec{A} \cdot \vec{B} dV - \int_{V_a+V_b} \vec{A}' \cdot \vec{B}' dV$ is independent of the field in V_b .

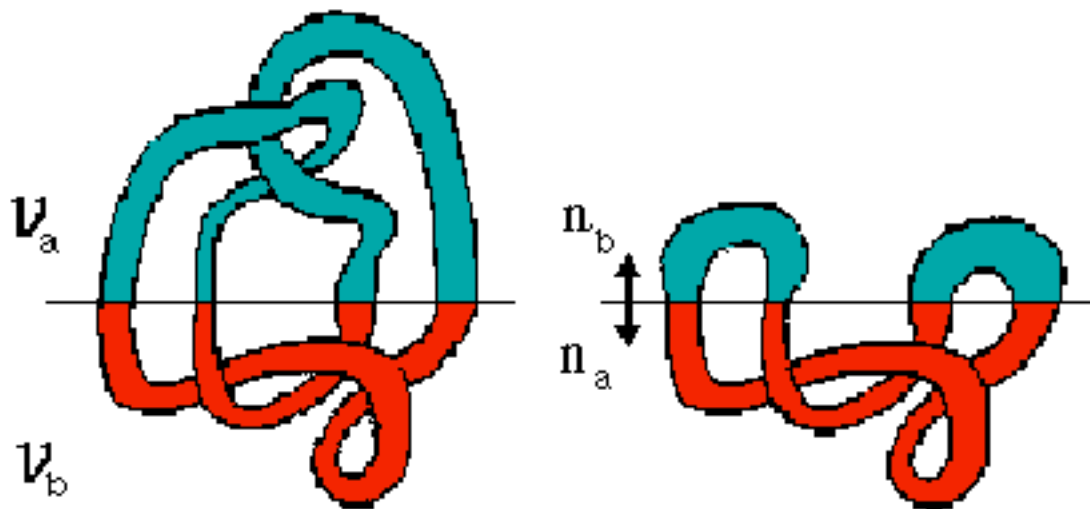


Fig. 2.10. The difference in total magnetic helicity of the two configurations is independent of the field in V_b [73].

It is then possible to define a relative magnetic helicity $\int_K$ in V_a :

$$\int_K = \int_{V_a} \vec{A} \cdot \vec{B} dV - \int_{V_a} \vec{A}' \cdot \vec{B}' dV.$$

A particularly simple choice is the vacuum potential field in V_a , $\vec{B}'_a = \vec{B}_v$. The vacuum potential field in V_a is determined by $\nabla \times \vec{B}_v = 0$, with boundary conditions $\vec{B}_v \cdot \vec{n}_a = \vec{B}_a \cdot \vec{n}_a$, it is assigned zero helicity and gives as a definition of relative magnetic helicity:

$$\int_K = \int_{V_a} (\vec{A} + \vec{A}_v) \cdot (\vec{B} - \vec{B}_v) dV.$$

This generalized magnetic helicity can be checked to be gauge invariant in almost any situation, the exceptions being magnetic monopole fields and periodic geometry with a mean field.

2.7. DC Helicity Injection

The dynamics of the relative magnetic helicity in a domain, ($\vec{n}$ points outside the domain), can be expressed through a "Poynting's theorem" [77]:

$$d(\mathcal{K})/dt = -2 \oint_E \vec{B} \cdot \vec{n} dS - 2 \oint \vec{A} \cdot \partial \vec{A} / \partial t \cdot \vec{n} dS - 2 \int (\vec{E} \cdot \vec{B}) dV, \text{ where}$$

- $2 \oint_E \vec{B} \cdot \vec{n} dS$ represents the DC helicity injection and $\oint_E$ is the electrostatic potential on the boundary;
- $2 \oint \vec{A} \cdot \partial \vec{A} / \partial t \cdot \vec{n} dS$ represents the AC helicity injection and includes the inductive helicity injection (ohmic drive) $2V_{\text{loop}} \oint_T$ (V_{loop} = loop voltage);
- $2 \int (\vec{E} \cdot \vec{B}) dV$ represents the total helicity dissipation.

The DC helicity injection is obtained by driving current along the lines of force which cross the boundary of the domain. This is performed through electrodes placed where $\vec{B} \cdot \vec{n} \neq 0$; electrodes must be electrically insulated from the rest of the boundary, upon which $\vec{B} \cdot \vec{n} = 0$. If an MHD equilibrium with beta much less than one is obtained, then the current density $\vec{j}$ is approximately parallel to the magnetic field $\vec{B}$. Therefore, current enters and leaves the electrodes at the locations where the magnetic field enters and leaves the electrodes.

The injection rate is $ld(\mathcal{K})/dt = 2V_e \oint_e$, where V_e is the electrostatic potential difference between the two electrodes and $\oint_e = 0.5 \oint |\vec{B} \cdot \vec{n}| dS$ is the magnetic flux which enters and exits both electrodes, see Fig. 2.11.

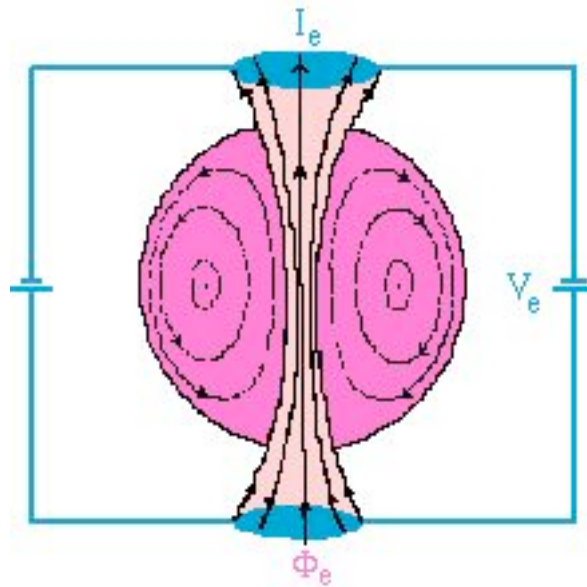


Fig. 2.11. Scheme of DC helicity injection in PROTO-SPHERA.

To inject $d(\mathcal{K})/dt > 0$:

- the electrostatic potential must be < 0 , where $\vec{B} \cdot \vec{n} dS > 0$;
- the electrostatic potential must be > 0 , where $\vec{B} \cdot \vec{n} dS < 0$.

The total current I_e which flows through the electrodes is, in the case of a relaxed state with $\alpha = \alpha_0 \vec{j} \cdot \vec{B} / B^2 = \text{constant}$, $\alpha_0 I_e = \oint_e$.

The clearest demonstration of DC helicity injection for toroidal plasma current sustainment has been obtained on the HIT spherical torus, at the Washington University (Seattle) [35, 78, and 79]. A coaxial helicity injector applies the voltage $V_{inj} \sim 0.5$ kV between the (insulated) outboard and the inboard of the vacuum vessel and injects into the lower divertor 'private region' the current $I_{inj} \sim 20$ kA. In HIT the path of the current along the toroidal plasma has been lengthened at most by using a SN (single null) configuration in which the injectors coincide with the divertor plates (coaxial helicity injector, contained within the toroidal field coils, see Fig. 2.12).

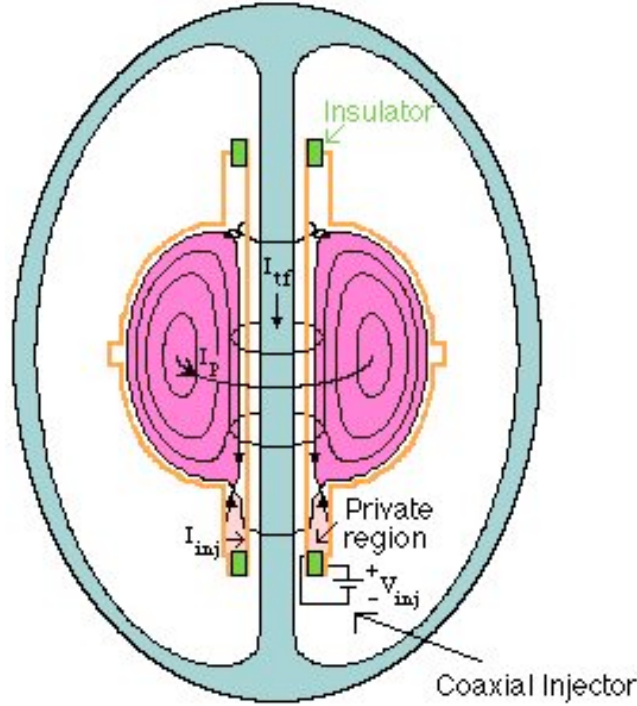


Fig. 2.12. Scheme of coaxial helicity injector on HIT [79].

The helicity dissipation inside the torus, which contains a toroidal flux Φ_T , can be described through an equivalent loop voltage $V_{loop} = V_{eff}$. The dissipated helicity within the torus is balanced by the helicity provided by the injector, which is characterized by a current I_{inj} , a voltage V_{inj} and a poloidal flux Φ_{inj} : $2V_{inj}\Phi_{inj} = 2V_{eff}\Phi_T$. So the injector is able to provide an equivalent $V_{loop} = V_{inj}\Phi_{inj}/\Phi_T$.

The injected current is transformed into a toroidal current $I_p \sim 150 \div 200$ kA in the ST (which has been sustained for a time ≥ 20 ms). Assuming helicity conservation and considering the ST and the injector as relaxed states with different relaxation parameters, $\tau = \tau_{ST}/\tau_{inj}$, the energy efficiency is $\eta = I_p V_{loop}/I_{inj} V_{inj}$. Therefore to get high helicity injection efficiency it is necessary that Φ_{inj} is not much larger than Φ_{ST} . The experimental results so far obtained on HIT are quite encouraging, with efficiency values of about $\eta = 0.25$ (see Fig. 2.13) and toroidal plasma currents over to 200 kA [79].

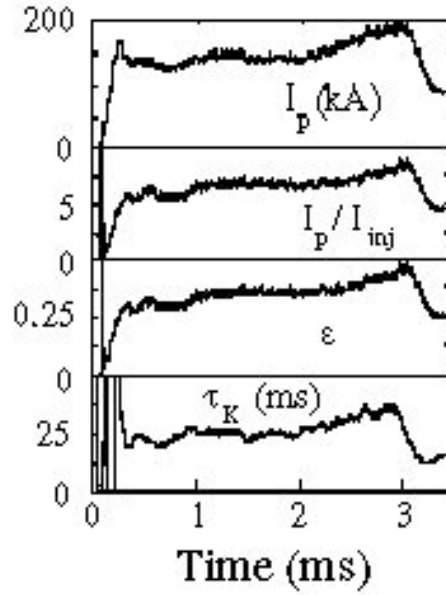


Fig. 2.13. Helicity injection formation and sustainment of toroidal plasma current and its energy efficiency ϵ in HIT [78].

2.8. Relaxed States and Compression

The literature has considered the equilibrium of a completely relaxed state, $\vec{\nabla} \times \vec{B} = \nabla \vec{B}$, with ∇ constant all over the plasma, enclosed within a perfectly conducting portion of sphere, with radius R_{sph} , fed by two electrodes upon the polar caps:

- If $\nabla R_{\text{sph}} = 4.49$ the well-known spheromak solution [80] (see Fig. 2.14.b), with $\vec{B} \cdot \vec{n} = 0$ all over the sphere, is obtained. However the spheromak configuration is very unstable, mainly due to the low value of the safety factor ($q_{\text{eq}} \sim 0.6$, $q_{\text{axis}} \sim 0.9$) and it is prone to rotations and translations of the symmetry axis.
- For values $\nabla R_{\text{sph}} > 4.49$ a plasma current flows around the spherical torus. The result of an equilibrium calculation by Taylor and Turner with $\nabla R_{\text{sph}} = 4.82$ is shown in Fig. 2.14.c. The Taylor's helicity injection theory [64] predicts that the configurations with $\nabla R_{\text{sph}} > 4.49$ are ideal MHD unstable.
- When $\nabla R_{\text{sph}} < 4.49$ a solution similar to PROTO-SPHERA is obtained. A plasma current flows within the hole of the spherical torus (screw pinch). The result of an equilibrium calculation by Taylor and Turner with $\nabla R_{\text{sph}} = 4.28$ is shown in Fig. 2.14.a. The Taylor's helicity injection theory [64] predicts that the configurations with $\nabla R_{\text{sph}} < 4.49$ are ideal MHD stable. In order to obtain configurations of fusion interest it is important to compress the screw pinch as much as possible. At fixed longitudinal current density (see Section 4.9) the power dissipated by Joule effect in the central screw pinch goes down like the square of the radius of the central pinch on the equatorial plane $\nabla_{\text{pinch}}^2(0)$ and reaches acceptable limits only at very low aspect ratios ($A < 1.2$). However below these very low aspect ratios, when $\nabla R_{\text{sph}} \ll 4.49$, the power injected into the central screw pinch becomes independent from $\nabla_{\text{pinch}}^2(0)$ and reduces to the power required to sustain the configuration by helicity injection.

The overall plasma of PROTO-SPHERA could show a tendency toward a force-free quasi-relaxed state, i.e. with $\alpha = \mu_0 \vec{j} \cdot \vec{B} / B^2$ almost constant everywhere. The idea of the PROTO-SPHERA experiment is to drive the plasma, through a formation and compression scheme, toward a stable state with $\mu R_{\text{sph}} < 4.49$, but maintaining in the spherical torus safety factor values ($q_0 \sim 1$, $q_{95} \sim 3$) typical of spherical tori with metal centerpost. The aim is that of controlling the magnetic helicity flow toward the magnetic axis and of avoiding the complete relaxation of the system.

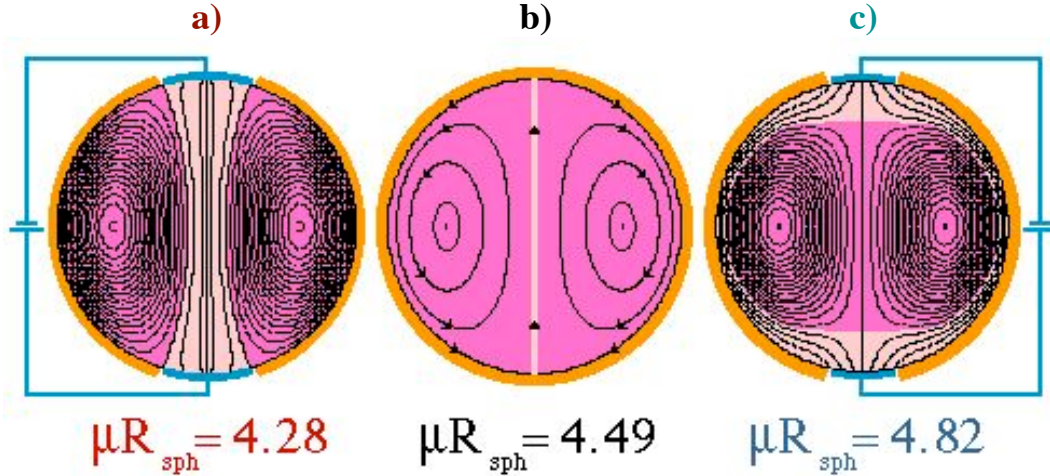


Fig. 2.14. Flux-core-spheromak configurations with various relaxation parameters μR_{sph} .

After the formation and compression of the spherical torus, the screw pinch discharge will be controlled by the current injected through the electrodes, by the gas injected through the anode and by the field shaping through the poloidal field coils. These tools will allow for maintaining the torus of PROTO-SPHERA at the highest possible level of μR_{sph} , without making the transition into a fully relaxed spheromak configuration.

The compression experiment of TS-3 (see Fig. 2.15) was optimized varying the delay between the waveforms of the pinch current and of the compression coil current.

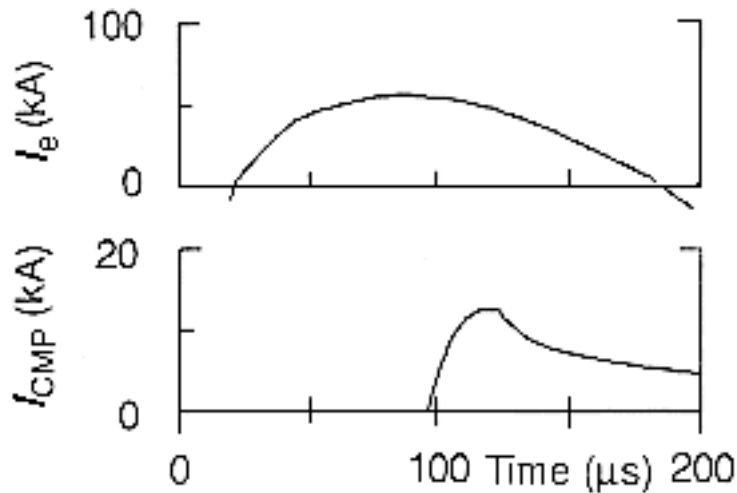


Fig. 2.15. Waveforms of the electrode and of the compression currents in TS-3 [68].

The configuration obtained in the TS-3 compression experiment has been modeled [68] as a relaxed state oblate flux-core-spheromak, see Fig. 2.16.

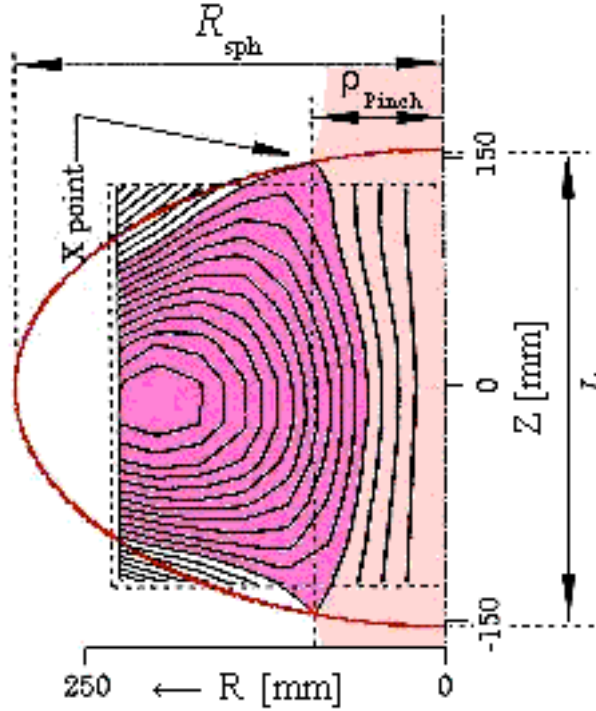


Fig. 2.16. Scheme of the compressed configuration of the flux-core of TS-3 [68].

In the framework of this modeling, TS-3 has shown that the relaxation parameter can approach the spheromak eigenvalue ($\kappa R_{sph}=4.49$, Fig. 2.17) maintaining the configuration stable for 80 μ sec, i.e. $110 \cdot \tau_A$ (110 Alfvén times).

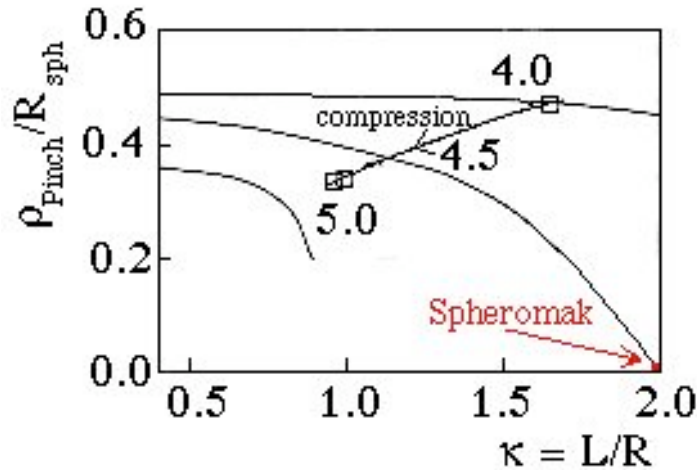


Fig. 2.17. Values of κR_{sph} obtained by the compression of TS-3 [68].

During the short duration of the TS-3 flux-core-spheromak experiment, the flux swing associated with the compression coil current has contributed substantially to the achievement of toroidal current $I_p \sim 50$ kA. So it is very difficult to derive an accurate information about the efficiency of the helicity injection in the TS-3 experiment.

2.9. Helicity Diffusion

Whereas the screw pinch of PROTO-SPHERA will certainly be a relaxed state, $\alpha_{\text{Pinch}} = \alpha_0 \vec{j} \cdot \vec{B} / B^2 = \text{constant}$, the ST will be only a quasi-relaxed state, $\alpha_0 \langle \vec{j} \cdot \vec{B} / B^2 \rangle = \langle \alpha \rangle_{\text{ST}}(\alpha)$, i.e. a state with the surface averaged relaxation parameter $\langle \alpha \rangle$ slowly varying across the plasma. The main question about quasi-relaxed states is the following one: how does the helicity injected from the boundary $\alpha = \alpha_{\text{edge}}$ diffuse toward the center of the plasma $\alpha = \alpha_{\text{axis}}$? A guess can be found calculating the ratio between the magnetic energy and the relative helicity in fully relaxed states: $d(W)/d(K) = \alpha/2\alpha_0$. This shows that the helicity flows from higher to lower α values. As a matter of fact a helicity transfer from a flux tube with larger α to a flux tube with smaller α lowers the overall magnetic energy of the two flux tubes. Therefore in PROTO-SPHERA $\alpha_{\text{Pinch}} \geq \langle \alpha \rangle_{\text{ST}}(\alpha)$. The quasi-relaxed states are the most relevant ones both in fusion research as well as in astrophysics. The development of techniques able to control the ratio $\alpha_{\text{Pinch}} / \langle \alpha \rangle_{\text{ST}}(\alpha)$, would make possible to control the helicity flow toward the axis and to avoid the complete relaxation of the plasma. Boozer has introduced an additional term in the fluctuation-averaged Ohm's law:

$$\vec{E} + \vec{v} \times \vec{B} = \eta \vec{j} - (\vec{B}/B^2) \nabla \cdot (\eta \vec{j} \cdot \vec{B} / B^2),$$

where η is a "viscous" coefficient [81], which accounts for the magnetic helicity diffusion, by preventing the current to change over scales shorter than $\lambda^2 = \eta / (\nabla^2 B^2)$; $\eta \nabla \cdot (\vec{j} \cdot \vec{B} / B^2)$ corresponds to a fully relaxed state.

With this Ohm's law there is an additional helicity flux: $\vec{Q}_\alpha = -\eta \nabla (\vec{j} \cdot \vec{B} / B^2) = -\eta \nabla \alpha$. Therefore the dynamics ("Poynting's theorem") of the relative helicity becomes:

$$d(K)/dt = - \int \nabla \cdot (\eta \nabla (\vec{j} \cdot \vec{B} / B^2)) + 2 \int \vec{E} \cdot \vec{B} + \vec{A} \cdot \partial \vec{A} / \partial t \cdot \vec{n} dS - 2 \int \eta \vec{j} \cdot \vec{B} dV.$$

The SPHEX experiment at UMIST (Manchester) has explored α in a flux-core-spheromak, created by a magnetized coaxial plasma gun (Fig. 2.18).

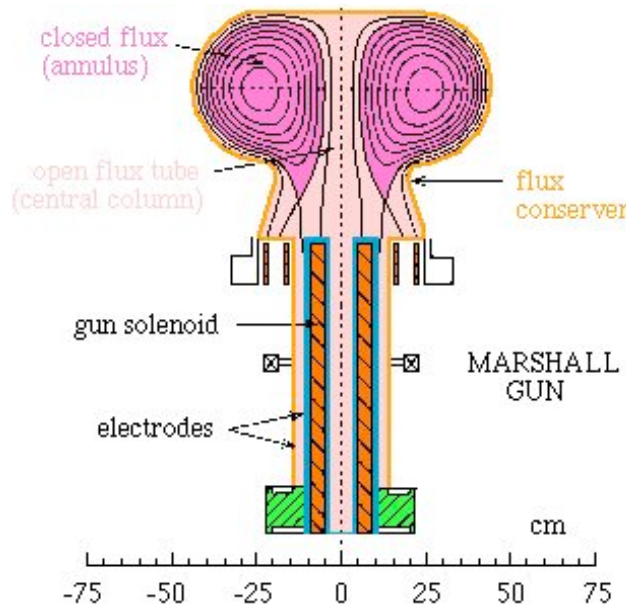


Fig. 2.18. Scheme of the SPHEX experiment at UMIST.

In agreement with the results of SPHEX, shown in Fig. 2.19, the PROTO-SPHERA equilibria have been calculated in Section 4.7, assuming values of the ratio $2.4 \leq \rho_{\text{Pinch}} / \langle \rho_{\text{ST}} \rangle (\rho_{\text{axis}}) \leq 3.3$.

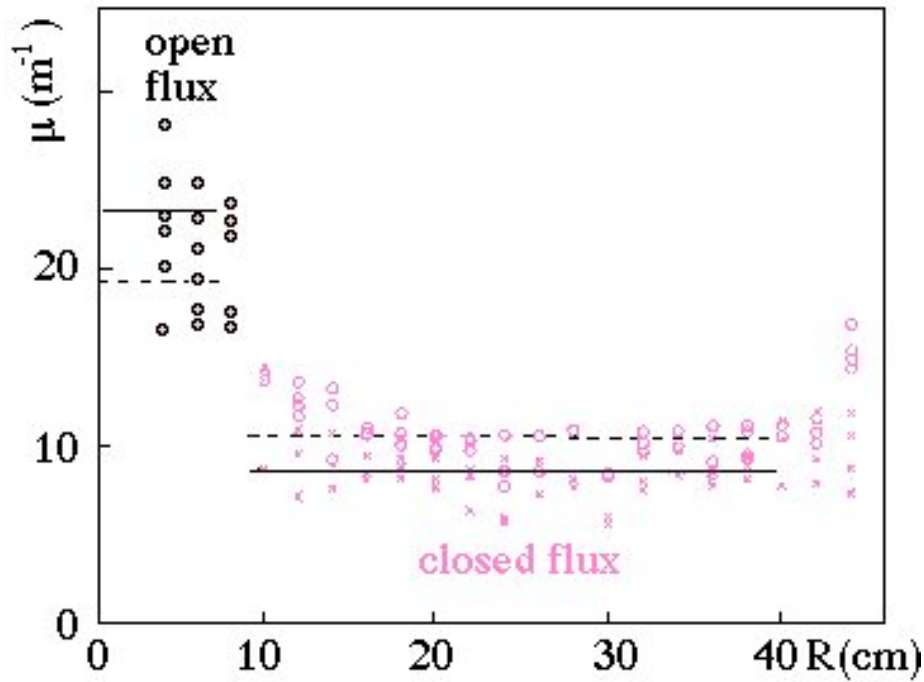


Fig. 2.19. Values of the relaxation parameter measured in SPHEX [82].

Moreover in PROTO-SPHERA ($R_{\text{sph}}=0.35$ m) the structure of the fields has been designed (see Section 4.5) in order to be as far as possible from the spheromak solution (Fig. 2.20), by increasing the ST elongation and by assuming $\rho_{\text{ST}} R_{\text{sph}} \leq 4.2$, in order to get profiles with $q_0 \sim 1$ and $q_{95} \sim 2.5 \div 3$.

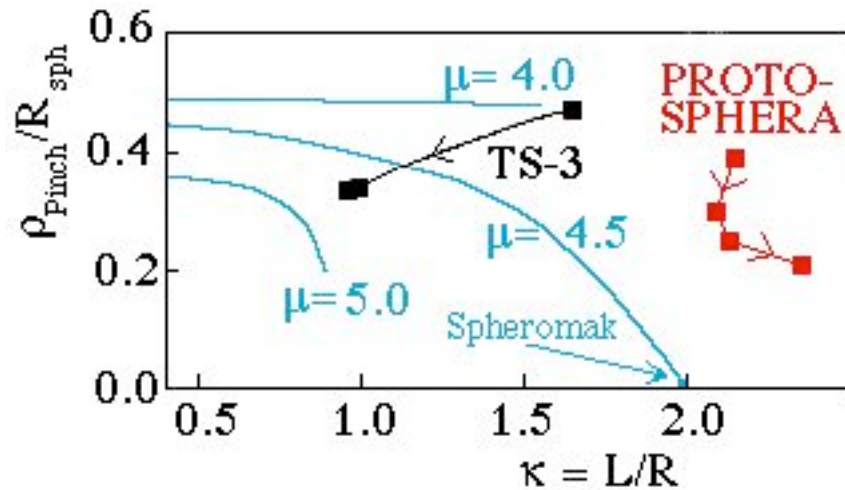


Fig. 2.20. Values of $\rho_{\text{Pinch}} R_{\text{sph}}$ for the formation and compression of PROTO-SPHERA, along with the values obtained in the TS-3 compression experiment.

2.10. Dynamo and Confinement Issues

In a plasma the magnetic helicity is not conserved exactly, but is only dissipated at a lower rate than the magnetic energy: MHD plasma spontaneously relaxes to the lowest magnetic energy state consistent with the initial helicity inventory.

The relaxation process typically involves magnetic reconnection, as flux tube linkages are broken on the microscopic scale and then re-established in a manner consistent with helicity conservation. This corresponds to a form of current drive because a configuration that initially had zero toroidal current relaxes to a state with a finite toroidal current.

The effective electric field driving this current is called a dynamo field and results from the non-linear interaction of fluctuating velocities and magnetic fields. At the present time these processes are reasonably understood in an average global sense, but there is very little understanding of the microscopic dynamics. It would be useful to demonstrate why helicity is conserved during small-scale reconnection processes. The dynamo model shows how fluctuating velocities and currents provide an effective electric field. A magnetic reconnection model can prescribe these fluctuations. The actual dynamics of reconnection are very complex and bring together many of the most difficult concepts in plasma physics. Furthermore, this process involves parallel electric fields, precisely the area where MHD is most suspect. Moreover non-axisymmetric magnetic fluctuations tend to degrade confinement. How strong a dynamo effect is compatible with confinement? The dimensionless Lundquist number $S = \tau_r / \tau_\Lambda \gg 1$ is the most relevant parameter in resistive MHD instabilities. All relevant instabilities grow on a time scale intermediate between the Alfvén time τ_Λ and the resistive diffusion time τ_r . It is therefore necessary to produce plasma pulses that are longer than the resistive time $\tau_r = \mu_0 a^2 / \eta$.

Standard reconnection theories treat reconnection as an exponentially growing process, whereas in the experiments the magnetic relaxation is observed [66] to involve cyclic or periodic oscillations (see Fig. 2.21, showing the data of the flux-core-spheromak FACT).

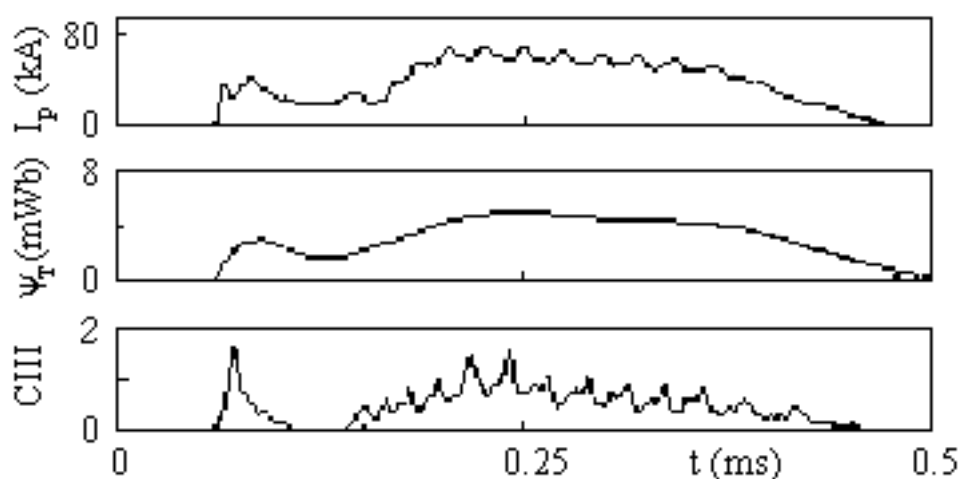


Fig. 2.21. Waveforms of the FACT flux-core-spheromak showing cyclic relaxations [66].

3. CHANDRASEKHAR-KENDALL-FURTH CONFIGURATIONS

Chandrasekhar-Kendall-Furth (CKF) fields are simply connected plasma equilibria containing a magnetic separatrix, which divides a "main spherical torus", two "secondary tori" on top and bottom of the main torus and a "spheromak" discharge surrounding the three tori.

Whereas CKF force-free fields have no pressure gradient ($\vec{\nabla}p=0$) and a relaxation parameter $\alpha=\alpha_0 \vec{j} \cdot \vec{B}/B^2$ constant all over the plasma, unrelaxed ($\vec{\nabla} \alpha \neq 0$, $\vec{\nabla} p \neq 0$) CKF equilibria can be calculated with the boundary condition that $\alpha=\alpha_0 \vec{j} \cdot \vec{B}/B^2$ is constant only at the edge of the plasma. Unrelaxed CKF equilibria can be defined as spherical tori (with winding numbers $q_0^{ST} \sim 1.0$ on axis and $q_{95}^{ST} \sim 2.0$ at the separatrix) enclosed within spheromaks endowed with high elongation and therefore with high winding number ($q_0^P \sim 3$ on the symmetry axis and $q_{95}^P \sim 5$ at the separatrix). They have the advantage of being stable to all low- n ideal MHD modes, up to order-unity beta values $\beta=2\alpha_0 \langle p \rangle_{vol} / \langle B^2 \rangle_{vol} = 1$. This high α value opens the possibility that plasma motions (i.e. radial electric field) can sustain the magnetic field of CKF configurations. Unrelaxed CKF fusion reactors with the right helicity injection, α limit and energy confinement, will allow for an unimpeded outflow of the high energy charged fusion products, easing direct energy conversion and the use of the burner as a space thruster.

However a method for injecting current (or torque) into a CKF configuration has still to be developed. PROTO-SPHERA can be viewed as a preliminary experiment that will study the properties of an unrelaxed CKF configuration, where a Hydrogen force-free screw pinch, fed by electrodes, replaces the innermost part of the surrounding spheromak discharge, while poloidal field coils replace the secondary tori.

3.1. Chandrasekhar-Kendall-Furth (CKF) Force-Free Fields

A simply connected magnetic confinement scheme can be obtained superposing two axisymmetric homogeneous force-free fields, each with $\vec{\nabla} \times \vec{B} = \alpha \vec{B}$, both having the same value of the relaxation parameter $\alpha = \alpha_0 \vec{j} \cdot \vec{B} / B^2$. The first is the Chandrasekhar-Kendall force-free field [83] of order-1, which in spherical geometry (r, ϑ, φ) admits the poloidal flux function: $\psi_{\text{CK},1}(r, \vartheta) = \alpha (\alpha r) j_1(\alpha r) \sin \vartheta P_1^1(\cos \vartheta)$ (see Fig. 3.1.a), where $j_1(\alpha r)$ is the spherical Bessel function of order 1, having its m -th radial zero at $(\alpha r) = x_{1,m}$ and $P_1^1(\cos \vartheta)$ is the Legendre polynomial. The second is the Furth square-toroid force-free field [84], which can be written as: $\psi_{\text{F},\alpha}^F(r, \vartheta) = \sqrt{\alpha^2 - \alpha_0^2} r \sin \vartheta J_1\left(\sqrt{\alpha^2 - \alpha_0^2} r \sin \vartheta\right) \cos(\alpha r \cos \vartheta)$ (see Fig. 3.1.b), for any value of $\alpha < \alpha_0$, where J_1 is the cylindrical Bessel function.

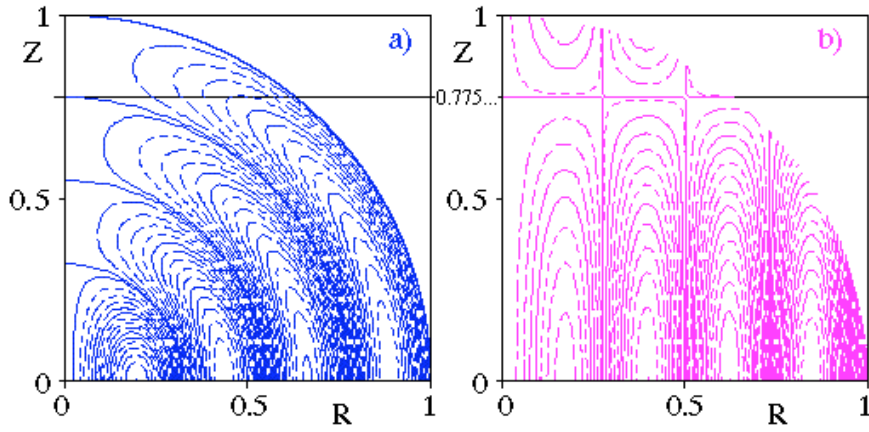


Fig. 3.1. a) Chandrasekhar-Kendall force-free field $\psi_{\text{CK},1}^{\text{CK}}$ with $\alpha = 14.066$.
b) Furth square-toroid force-free field $\psi_{\text{F},\alpha}^F$ with the same α and $\alpha_0 = 2.026$.

The relaxation parameter is chosen as $\alpha = x_{1,4} = 14.066\dots$, so that within a unitary circle four zeroes of $\psi_{\text{CK},1}^{\text{CK}}$ are present in the Chandrasekhar-Kendall field. The parameter α_0 of the Furth square-toroid field is chosen as: $\alpha_0 = x_{1,4} \alpha / 2x_{1,3} = 2.026\dots$, so that at $R=0$, $Z = x_{1,3}/x_{1,4} = 0.775\dots$ the third zero of $\psi_{\text{CK},1}^{\text{CK}}$ and the first zero of $\psi_{\text{F},\alpha}^F$ coincide (see Fig. 3.1). The superposition of the two force-free fields is written: $\psi(r, \vartheta) = \psi_{\text{CK},1}^{\text{CK}} + \alpha \psi_{\text{F},\alpha}^F$ (Fig. 3.2). For values of the superposition constant $\alpha \geq 0.402\dots$, it contains, in a simply connected region near the origin, a toroidal current density j_φ of the same sign and can be called a Chandrasekhar-Kendall-Furth force-free field (CKF).

Figure 3.3 shows the cross-section of the CKF force-free field with superposition constant $\alpha = 0.55$ and details its composition in terms of different plasma regions, divided by a magnetic separatrix. The "main spherical torus" (ST) has a safety factor (inverse of the rotational transform, $q = 1/\ell$), which is $q_0^{\text{ST}} \sim 1.0$ on the magnetic axis and $q_{95}^{\text{ST}} \sim 1.5$ at the edge (95% of the poloidal flux of the magnetic separatrix). The two "secondary tori" (SC), present on top and bottom of the main torus, also have $q_0^{\text{SC}} \sim 1.0$ on their magnetic axes and $q_{95}^{\text{SC}} \sim 1.5$ at their edges. The discharge surrounding the three tori, which will be dubbed as "spheromak" (P) has a larger safety factor,

respectively $q_0^P \sim 1.5$ on the symmetry axis and $q_{95}^P \sim 3.7$ at the separatrix. When the superposition constant exceeds $\beta = 0.69 \dots$ the secondary tori disappear (see Fig. 3.2).

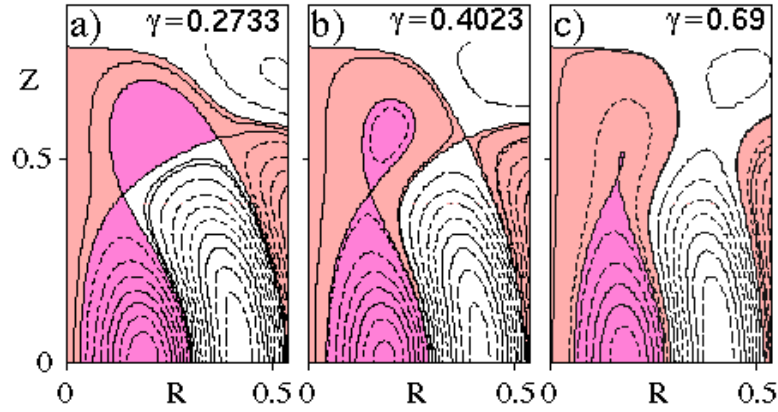


Fig. 3.2. Poloidal flux function contours of CKF force-free fields $\psi = \psi_{0,1}^{CK} + \beta \psi_{0,0}^F$, with relaxation parameter $\beta = 14.066$ and square toroid parameter $\beta = 2.026$. The superposition constants are: (a) $\beta = 0.273$; (b) $\beta = 0.402$; (c) $\beta = 0.690$. Colored areas mean toroidal current density $j_{\parallel} > 0$; white areas mean $j_{\parallel} < 0$.

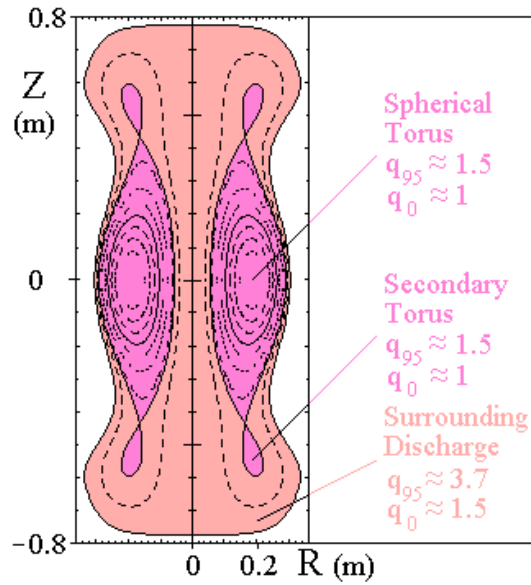


Fig. 3.3. Contours of the poloidal flux function of the Chandrasekhar-Kendall-Furth force-free field $\psi = \psi_{0,1}^{CK} + \beta \psi_{0,0}^F$, with superposition constant $\beta = 0.55$.

Once a CKF force-free field is formed, if the surrounding spheromak discharge can be sustained by driving current on its closed flux surfaces, magnetic reconnections will occur at the X-points of the configuration, injecting magnetic helicity, poloidal flux and plasma current into the main spherical torus. Also the secondary tori will be a by-product of the same magnetic reconnections.

3.2. Ideal MHD Stability of CKF Force-Free Fields

The ideal MHD stability of the Chandrasekhar-Kendall-Furth (CKF) force-free fields has been studied by solving the eigenvalue problem [85]: $\vec{W} \cdot \vec{\eta} = \eta^2 \vec{K} \cdot \vec{\eta}$, where $\vec{W}$ is the plasma perturbed potential energy and $\vec{K}$ the plasma perturbed kinetic energy, associated with the perturbed plasma displacement $\vec{\eta}$. The expressions for the perturbed energies become simpler [86] if the equilibrium is analyzed in non-orthogonal periodical Boozer coordinates (φ_T -radial, ϑ -poloidal, φ -toroidal) [87], with Jacobian $\sqrt{g} \mu 1/B^2$. More details on these coordinates can be found in Section 5.3. The radial variable φ_T is the toroidal flux divided by 2π , with $\varphi_T=0$ on the magnetic axis of the main torus, $\varphi_T=\varphi_T^X$ on the separatrix and $\varphi_T=\varphi_T^{\max}$ at the edge of the surrounding spheromak and on the symmetry axis. The continuity of the rotational transform $\bar{I}(\varphi_T)$ and of the toroidal and poloidal plasma currents $I(\varphi_T)$ and $f(\varphi_T)$ joins the Boozer coordinates of these equilibria at the ST-SP interface, $\varphi_T=\varphi_T^X$ (see Fig. 3.4).

The energy principle (see Section 5.5) for a compressible plasma [88] is used, Fourier analyzing the normal $\eta^\perp = \vec{\eta} \cdot \vec{\eta}_T$, binormal $\eta^\parallel = \vec{\eta} \cdot (\vec{\eta}_T \times \vec{\eta}_\vartheta)$ and parallel $\eta^\parallel = \sqrt{g} \vec{\eta} \cdot \vec{\eta}_\vartheta$ components of the displacement away from the (up/down symmetric) equilibrium as: $\eta^\perp = \eta_\perp(\varphi_T) \sin(m_\vartheta \vartheta - n\varphi)$, $\eta^\parallel = \eta_\parallel(\varphi_T) \cos(m_\vartheta \vartheta - n\varphi)$, $\eta^\parallel = \eta_\parallel(\varphi_T) \cos(m_\vartheta \vartheta - n\varphi)$, in terms of a toroidal mode number n and of a superposition of poloidal mode numbers m_ϑ . Only the normal displacement $\eta^\perp$ must be continuous [88] at the separatrix between the three tori and the surrounding spheromak, whereas the binormal $\eta^\parallel$ and the tangential $\eta^\parallel$ components can make jumps at $\varphi_T=\varphi_T^X$.

The poloidal angle ϑ makes an excursion $[0, \vartheta_X]$ on the lower outboard of the main spherical torus and then runs in the range $[\vartheta_X, 2\pi+\vartheta_X]$ in the lower secondary torus. It continues to the angles $[2\pi+\vartheta_X, 4\pi-\vartheta_X]$ on the inboard of the main spherical torus, runs in the range $[4\pi-\vartheta_X, 6\pi-\vartheta_X]$ in the upper secondary torus and finishes its run in the range $[6\pi-\vartheta_X, 6\pi]$ on the upper outboard of the main spherical torus. Inside the surrounding spheromak it simply covers the range $[0, 2\pi]$. So a correspondence is established between the poloidal angle ϑ inside the tori and the poloidal angle $3\cdot\vartheta$ inside the surrounding spheromak.

MHD "global" modes, which exist over the whole plasma, must have periodical perturbed displacements in terms of the Boozer poloidal angle ϑ . For these global modes the allowed poloidal mode numbers can be all the integers $m_\vartheta = \dots, -2, -1, 0, 1, 2, 3, \dots$ inside the main and the secondary tori. Therefore the continuity of $\eta^\perp$ forces the allowed poloidal mode numbers of the global perturbations to be limited to multiples of 3: $m_\vartheta = \dots, -6, -3, 0, 3, 6, 9, \dots$ inside the surrounding spheromak. However other MHD "internal" modes can still exist if their radial extent is limited to the surrounding spheromak, $\varphi_T^X \leq \varphi_T \leq \varphi_T^{\max}$. These internal perturbations have as allowed poloidal mode numbers all the integers $m_\vartheta = \dots, -2, -1, 0, 1, 2, 3, \dots$, also inside the surrounding spheromak: their radial $\eta^\perp$ component will go to zero at the magnetic separatrix, $\eta^\perp(\varphi_T^X)=0$, and will not have to match any requirement of periodicity on the three tori.

The radial behavior is discretized by a one dimensional finite hybrid element method [88]: $\eta(\varphi_T)$ by hat functions and $\eta_\parallel(\varphi_T)$, $\eta_\perp(\varphi_T)$, $\partial\eta/\partial\varphi_T$ by piecewise constant functions, covering the radial range $0 \leq \varphi_T \leq \varphi_T^{\max}$ with a mesh φ_T^i , where $i=1, 2, \dots, N_\varphi$.

The result of the ideal MHD stability calculations for low toroidal mode numbers ($n=1,2,3$), assuming fixed boundary conditions at the edge of the plasma: $\psi(\psi_T^{\max})=0$, is that the Chandrasekhar-Kendall-Furth force-free fields are stable when the value of the superposition parameter is greater than $\gamma=0.5$ (see Fig. 3.5).

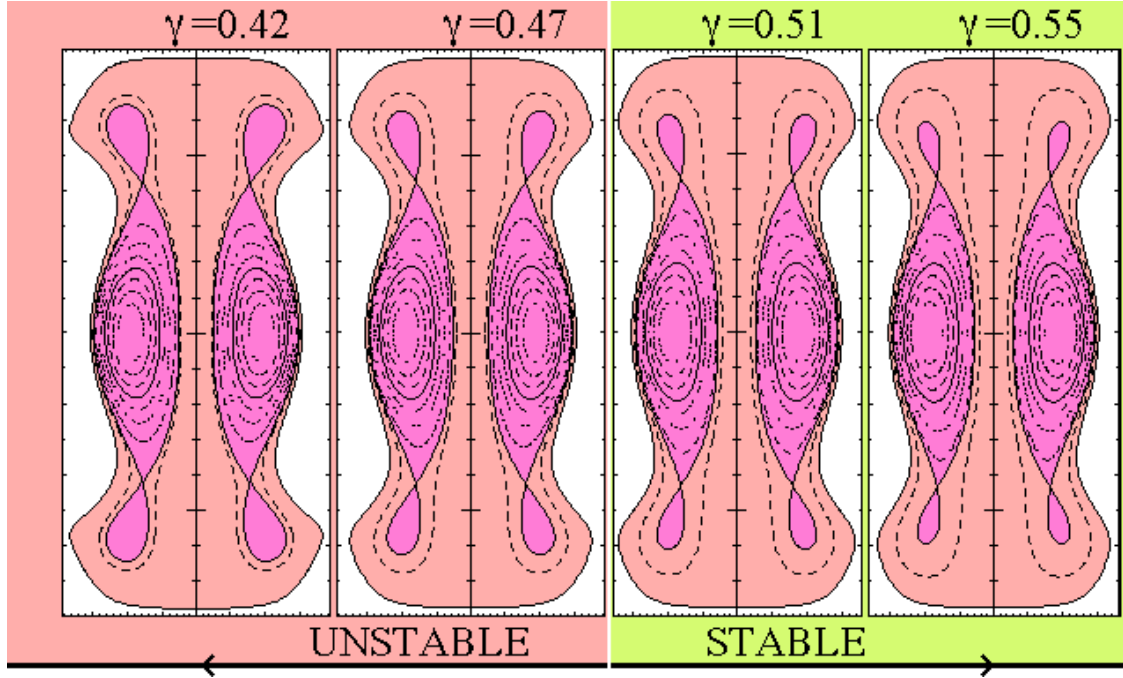


Fig. 3.5. Sequence of CKF force-free fields with ideal MHD stability boundary as a function of the superposition parameter γ

3.3 Unrelaxed CKF Configurations

However force-free fields have $\bar{\rho}p=0$ and are so unable to confine plasmas of fusion interest. Nevertheless a variety of unrelaxed ($\bar{\rho}\neq 0$, $\bar{\rho}p\neq 0$) MHD fixed boundary equilibria, similar in shape and topology to the Chandrasekhar-Kendall-Furth force-free fields, can be calculated (see Fig. 3.6).

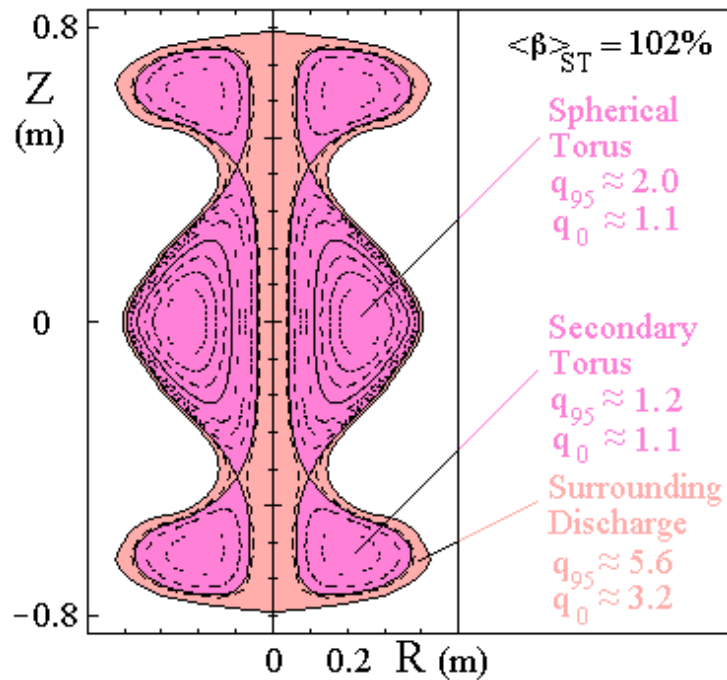


Fig. 3.6. Flux function contours of an unrelaxed CKF equilibrium, with $\langle \beta \rangle_{ST} = 102\%$.

They have $\bar{\rho} = \bar{\rho}_0 \cdot \bar{j} \cdot \bar{B} / B^2 = \text{constant}$ only at the edge of the plasma ($\bar{\rho}_T = \bar{\rho}_T^{\max}$), as a boundary condition for the MHD equilibrium. In the example shown in Fig. 3.7 the surface averaged value $\langle \bar{\rho} \rangle = \bar{\rho}_0 \langle \bar{j} \cdot \bar{B} / B^2 \rangle$ drops from $\langle \bar{\rho} \rangle = 25$, at the edge of the surrounding spheromak, down to $\langle \bar{\rho} \rangle = 8$, on the axis of the main spherical torus. Such $\langle \bar{\rho} \rangle$ profile corresponds to a sustainment obtained by driving current on the flux surfaces of the surrounding spheromak. Magnetic helicity, flowing down the $\langle \bar{\rho} \rangle$ gradient, will be injected into the main spherical torus, through magnetic reconnections at the X-points. The gradient of the pressure profile will presumably be concentrated in the same region where the gradient of $\langle \bar{\rho} \rangle$ has the largest variation (see Fig. 3.7).

Equilibrium calculations, based on the poloidal flux function $\bar{\rho} = 2\bar{\rho}RA_{\bar{\rho}}$, have been performed, using the following choices for the pressure and the diamagnetic current functions in the Grad-Shafranov equation:

$$\begin{aligned}
 p(\bar{\rho}) &= C_p \cdot \left\{ 1 + \bar{\rho} p \left(1 - \cos \left(\bar{\rho} / \bar{\rho}_c \right) / 2 \right) \right\} & \text{for } \bar{\rho} < \bar{\rho}_c & \text{and} \\
 p(\bar{\rho}) &= C_p \cdot \{ 1 + \bar{\rho} p \} & \text{for } \bar{\rho} \geq \bar{\rho}_c & ; \\
 I_{\text{dia}}(\bar{\rho}) &= C_I \cdot \left\{ \bar{\rho}_{\text{edge}} \bar{\rho}_c + (2 \bar{\rho} \bar{\rho}_c / \bar{\rho}) \cdot \left[\cos(\bar{\rho} / 2 \bar{\rho}_c) - 1 \right] \right\} / \bar{\rho}_0 & \text{for } \bar{\rho} < \bar{\rho}_c & \text{and} \\
 I_{\text{dia}}(\bar{\rho}) &= C_I \cdot \left\{ \bar{\rho}_{\text{edge}} \bar{\rho}_c - (2 \bar{\rho} \bar{\rho}_c / \bar{\rho}) + (\bar{\rho}_{\text{edge}} - \bar{\rho}) (\bar{\rho} - \bar{\rho}_c) \right\} / \bar{\rho}_0 & \text{for } \bar{\rho} \geq \bar{\rho}_c & .
 \end{aligned}$$

The total toroidal current I_p flowing inside the ST is an input, along with the total

$$\beta_p^{ST} = \frac{2 \int_0^{R_p} \beta_p dV}{\mu_0 (I_p)^2 / V_p^{ST}} \cdot \frac{1}{C_p^{ST} \left(\frac{R_p}{R_T} \right)^2} \cdot \frac{1}{\int_0^{R_p} \beta_p \cdot d\vec{I}_p}.$$

The two inputs I_p and β_p^{ST} determine, by iteration, the values of C_p , C_1 and the $\langle \beta \rangle$ profile. As a matter of fact a much more adequate choice would be to impose directly $\langle \beta \rangle(R)$ instead than $I_{dia}(R)$, but such a choice would make the equilibrium calculation much more cumbersome. The case shown in Figs. 3.6, 3.7 and 3.8 corresponds to the choices $\beta_p=5$, $\beta_c=4 \cdot \beta_x$ (where β_x is the poloidal flux function at the separatrix), $\beta=2$, $\beta_{edge}=14.066$, $\beta=9.5$ and $\beta_p^{ST}=0.115$. It has a very high beta in the spherical torus $\beta_{ST}=2\beta_0 \langle \beta \rangle_{ST} / \langle B^2 \rangle_{ST} = 102\%$. It refers to an unrelaxed CKF equilibrium which has roughly the same geometrical dimension as PROTO-SPHERA. The total poloidal spheromak current is $I_c=60$ kA (the same as the longitudinal pinch current of PROTO-SPHERA), whereas the total toroidal current in the main spherical torus is $I_p=451$ kA (much larger than in the case of PROTO-SPHERA, which has $I_p=120 \div 240$ kA).

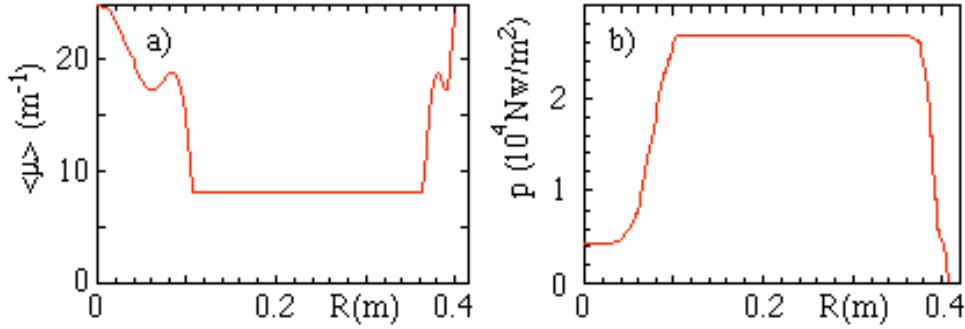


Fig. 3.7. Profiles of: a) surface averaged $\langle \beta \rangle = \beta_0 \langle j/B \rangle$; b) pressure, on the equator of the unrelaxed $\beta_{ST}=102\%$ CKF of Fig. 3.6.

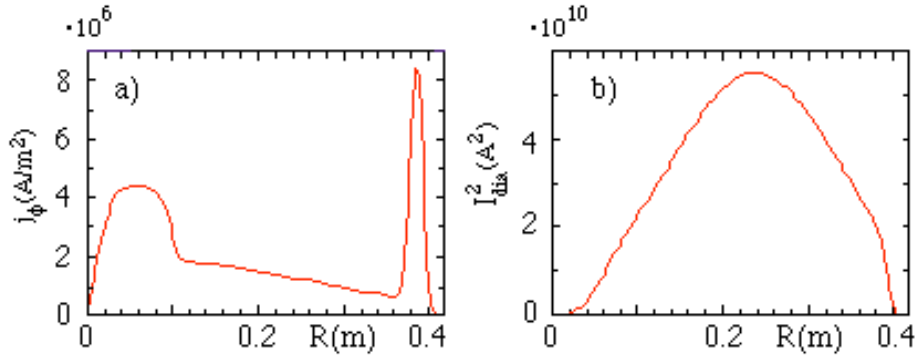


Fig. 3.8. Profiles of: a) toroidal current density j_ϕ ; b) diamagnetic current I_{dia}^2 , on the equator of the unrelaxed $\beta_{ST}=102\%$ CKF of Fig. 3.6.

The $\langle \beta \rangle$ profile in Fig. 3.7 is locally not monotonous, as the choice of fixing the pressure and diamagnetic current is not fully adequate at $\beta_{ST} \sim 100\%$. At lower values of beta the $\langle \beta \rangle$ profile is monotonous, as Fig. 4.27 shows for $\beta_{ST} \sim 50\%$ and Fig. 12.3 shows for $\beta_{ST} \sim 78\%$.

The fixed boundary equilibrium is calculated iteratively in spherical coordinates $(r, \vartheta, \varphi_G)$, where the poloidal flux function can be expanded, in terms of index-1, order- n spherical harmonics $\sin \vartheta P_n^1(\cos \vartheta)$ and of the internal $M_n^i(r)$ and external $M_n^e(r)$ spherical multipolar moments, as $\varphi = \sum_{n=1}^{N_{\max}} \left[M_n^i(r) r^{-n} + M_n^e(r) r^{n+1} \right] \sin \vartheta P_n^1(\cos \vartheta)$. It is necessary to use a large number of spherical multipolar moments ($N_{\max}=40$) for obtaining a correct description of the configuration. The main spherical torus has a safety factor (inverse of the rotational transform, $q=1/\varphi$), which is shown in Fig. 3.9. It has the values: $q_0^{ST} \sim 1.1$ on the magnetic axis and $q_{95}^{ST} \sim 2.0$ at the edge (95% of the poloidal flux of the magnetic separatrix).

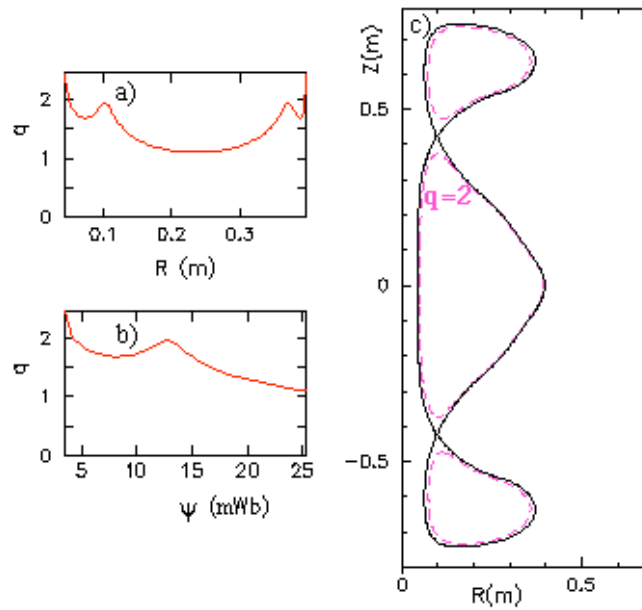


Fig. 3.9. a) Winding number q on the equatorial plane; b) q versus the poloidal flux function ψ , inside the main spherical torus; c) contour plot of q of the unrelaxed $\langle \varphi \rangle_{ST}=102\%$ CKF of Fig. 3.6.

The two secondary tori, present on top and bottom of the main torus, have $q_0^{SC} \sim 1.1$ on their magnetic axes and $q_{95}^{SC} \sim 1.2$ at their edges. The spheromak discharge, which surrounds the three tori, has a larger safety factor: respectively $q_0^P \sim 3.2$ on the symmetry axis and $q_{95}^P \sim 5.6$ at the separatrix. An unrelaxed CKF equilibrium can be defined as a spherical torus ($q_0^{ST} \sim 1, q_{95}^{ST} \sim 2$) enclosed within a spheromak endowed with high elongation and therefore with high winding number ($q_0^P \sim 3, q_{95}^P \sim 5$).

The ideal MHD stability of the unrelaxed CKF configurations has been calculated with the same numerical code used for the CKF force-free fields (see Section 3.2). Figure 3.10 shows the Boozer coordinates for the equilibrium shown in Fig. 3.6. Strong distortions of the curves representing constant values of the poloidal angle ϑ are present at the edge of the main spherical torus. In this region the gradients of $\langle \varphi \rangle(\vartheta_T)$ and $p(\vartheta_T)$ have their largest variations.

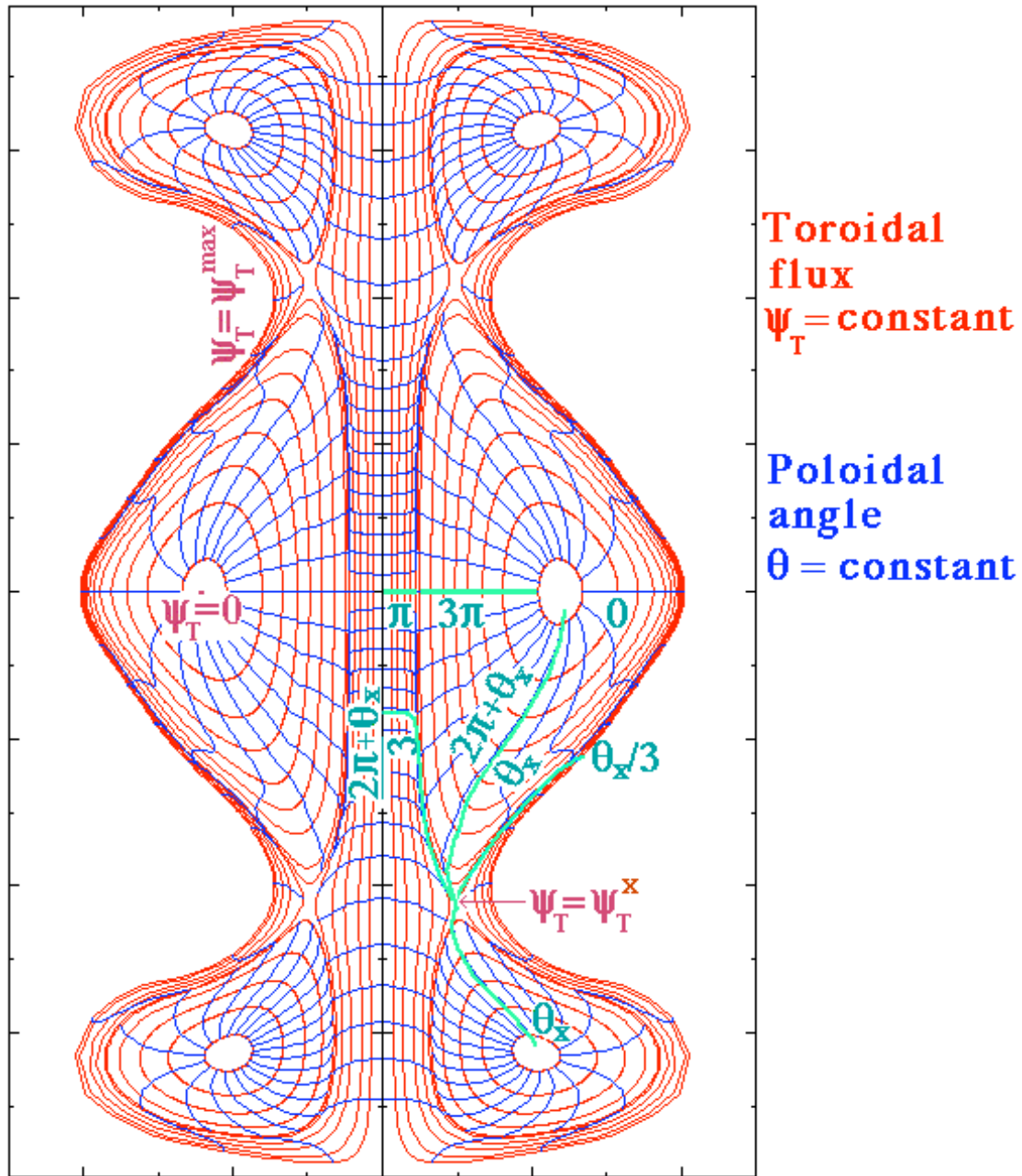


Fig. 3.10. Boozer coordinates for the $\langle \beta \rangle_{ST}=102\%$ unrelaxed CKF of Fig. 3.6.

The ideal MHD stability of unrelaxed CKF equilibria, with profiles of $\langle \beta \rangle(\psi_T)$ and $p(\psi_T)$ similar the ones shown in Fig. 3.7, has been calculated with fixed boundary conditions at the edge: $\psi_T(\psi_T^{\max})=0$. Cases, like the one shown in Fig. 3.6, with a beta of the main spherical torus β_{ST} even exceeding unity, remain stable to all the ideal MHD perturbations with low toroidal mode numbers ($n=1,2,3$).

The unrelaxed CKF configurations can be contained in an almost cylindrical solenoid by simple external poloidal coils, see Fig. 3.11. The total current flowing inside the poloidal field coils is $I_{PF} = \sum_{i=1}^{N_{PF}} |I_{PF}^i| = 500$ kA, which is almost equal to the total toroidal current inside the main spherical torus $I_p=451$ kA.

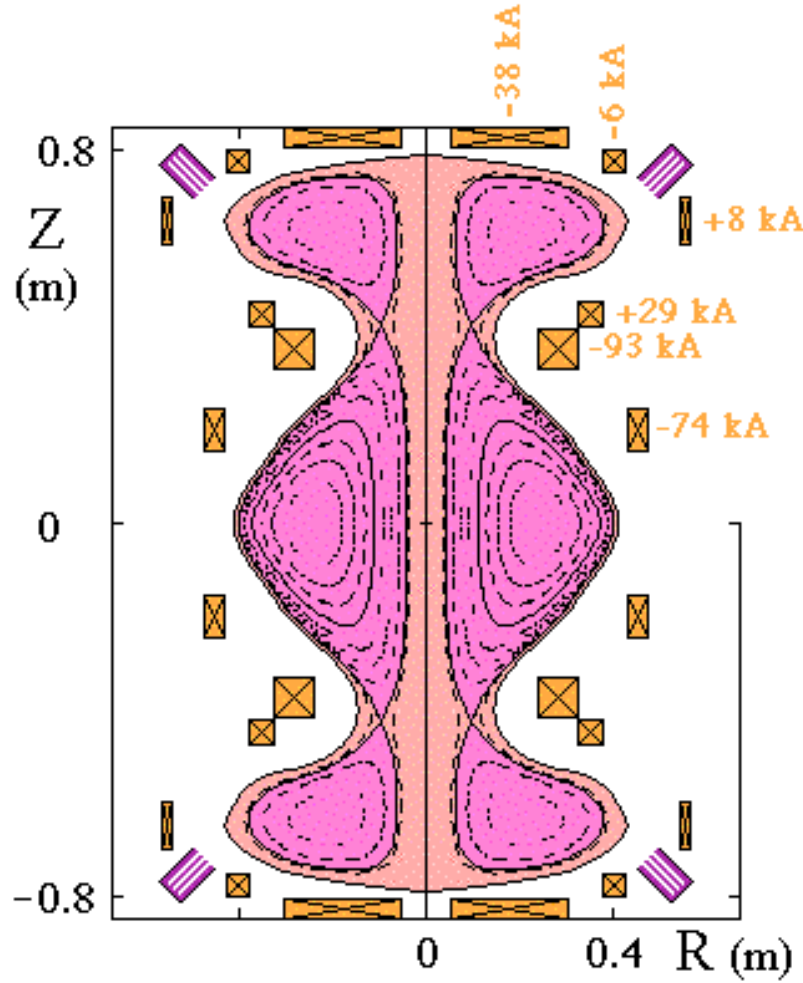


Fig. 3.11. PF coils and currents for the $\langle \beta \rangle_{ST}=102\%$ unrelaxed CKF of Fig. 3.6.

The breakdown of the plasma, before the formation of the spherical torus, can be produced by electrodes (see Fig. 3.12) and takes the appearance of a screw pinch discharge. The case shown in Fig. 3.12 refers to the start-up of the unrelaxed CKF equilibrium shown in Fig. 3.11. The total longitudinal screw pinch current is $I_c=30$ kA. The total current flowing at the start-up inside the poloidal field coils is $I_{PF}=\sum_{i=1}^{N_{PF}} |I_{PF}^i|=234$ kA. The signs and magnitudes of the currents flowing inside the PF coils at the start-up (Fig. 3.12) are compared with those of the final configuration (Fig. 3.11). Such a comparison shows that the sudden variation of the PF coils currents can provide approximately enough magnetic flux to the screw pinch plasma as to induce the full toroidal plasma current of the final configuration. It is however completely uncertain whether the longitudinal screw pinch current can be rerouted on the closed flux surfaces during the formation and by which method the plasma current can be maintained flowing along the surrounding spheromak, during the sustainment of the CKF configuration.

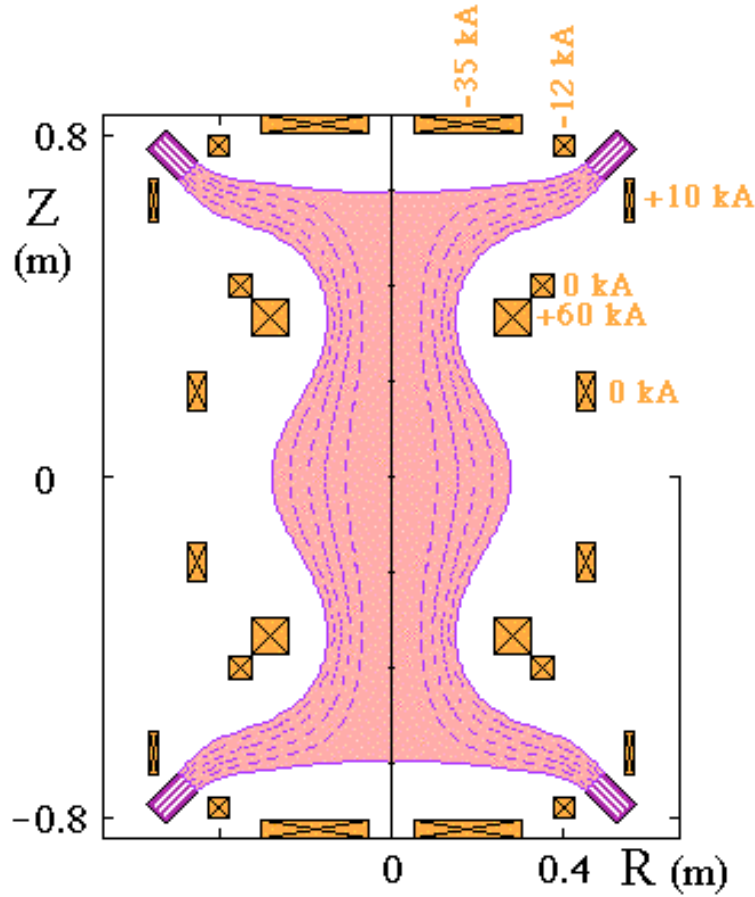


Fig. 3.12. Electrode breakdown for the unrelaxed CKF configuration of Fig. 3.11. The electrodes are sketched in the corners of the figure.

Another major uncertainty is the efficiency of the helicity injection, i.e. the gradient of the surface averaged value $\langle \chi \rangle = \oint_0 \langle \vec{j} \cdot \vec{B} / B^2 \rangle$. It will determine the ratio I_p / I_e between the toroidal current inside the main ST and the poloidal current inside the surrounding spheromak. An incorrect value of the ratio I_p / I_e could produce ideal and resistive MHD χ limits. Furthermore, as the CKF configurations will be sustained by helicity injection, magnetic reconnections will necessarily break, through helical perturbations, the axial symmetry, as per Cowling's anti-dynamo theorem [70]. The breaking of the axial symmetry could degrade the energy confinement in the main ST.

However, reactor extrapolations (see Section 12) of unrelaxed CKF magnetic configurations, endowed with the right helicity injection, χ limit and energy confinement, will allow for an unimpeded outflow of the high energy charged fusion products. The charged fusion products will drift across the magnetic separatrix, to the degenerate X-points ($B=0$) on top/bottom of the configuration (Fig. 3.13), easing direct energy conversion and the use of the burner as a space thruster.

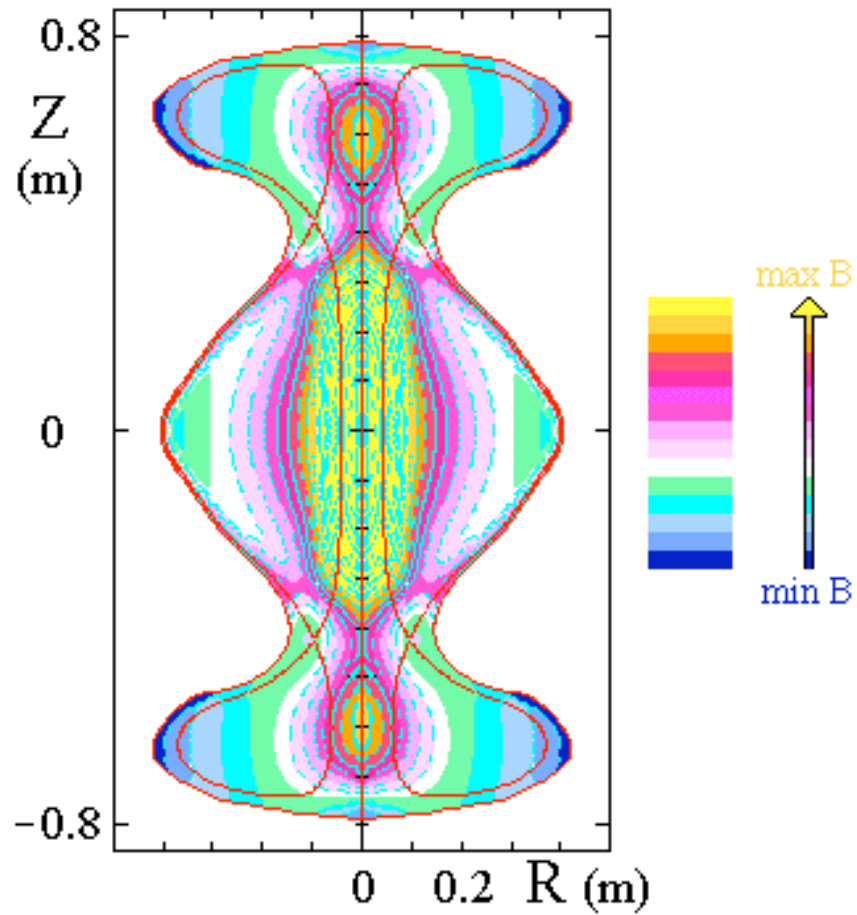


Figure 3.13. Contours of constant $|B|$ for the configuration of Fig. 3.6, superposed to flux surfaces.

In Fig. 3.13 the maximum field on symmetry axis is $B_m^i=0.67$ T, the maximum total field at the edge is $B_m^e=0.27$ T, the averaged total field inside the main spherical torus is $\sqrt{\langle B^2 \rangle}=0.24$ T and obviously the minimum field is $B=0$ on the degenerate X-points.

4. PHYSICAL DESIGN

While a method for injecting current (or torque) into a sustained Chandrasekhar-Kendall-Furth (CKF) configuration remains to be developed, a preliminary experiment is proposed in order to begin the study of these configurations. The PROTO-SPHERA experiment will study the properties of a CKF configuration where a Hydrogen force-free screw pinch, fed by electrodes, replaces in part the surrounding spheromak discharge, while poloidal field coils replace the secondary tori. PROTO-SPHERA, with a longitudinal pinch current $I_c=60$ kA, will produce a spherical torus of equatorial diameter $2\cdot R_{sph}=70$ cm, aspect ratio $A=1.2\div 1.3$, carrying a toroidal current $I_p=120\div 240$ kA. The main parameters of the plasma are illustrated and compared with the corresponding ones obtained in the TS-3 experiment at the University of Tokyo, which in 1993 produced a small spherical torus around a screw pinch, and maintained it stable for about 100 Alfvén times.

The main constraint used in the physical design of PROTO-SPHERA has been the current density at the plasma-electrode interface. A secondary constraint has been the choice of limiting the versatility of the poloidal field coils by grouping them in two sets, each composed of coils connected in series; a further constraint has been to limit the current density inside the poloidal field coils. These two last constraints serve the purpose of simplifying the design of a concept exploration experiment like PROTO-SPHERA.

A new predictive MHD equilibrium code has been developed in order to calculate the combined screw pinch + spherical torus configurations of PROTO-SPHERA, as no existing equilibrium code was able to deal with this problem. The plasma formation sequence has been calculated as a sequence of equilibria by this code. The assumptions used in these calculations are illustrated and connected to the physical basis developed in Section 2. The plasma performances (mainly the electron temperature T_e and magnetic Lundquist number $S=\eta_r/\eta_k$ of the spherical torus) are evaluated through semi-empirical tokamak scaling laws, which describe successfully the experimental energy confinement of spherical tori with metal centerpost.

A number of comparisons between PROTO-SPHERA and existing experiments are undertaken. First the equilibrium is compared with the one of TS-3, in order to remark the features that distinguish PROTO-SPHERA. Then the formation scheme and the equilibrium characteristics are compared with the ones of past and existing spheromak experiments. The rationale for the choice of the size of PROTO-SPHERA is given and can be so summarized: the size of PROTO-SPHERA is dictated by the need of allowing for a clear-cut comparison between the results of PROTO-SPHERA with "plasma centerpost" and those of the spherical tori with metal centerpost.

Later, with slight modifications in the load assembly, the PROTO-SPHERA experiment could host an unrelaxed Chandrasekhar-Kendall-Furth (CKF) configuration. Whereas the breakdown and the inductive formation of the CKF should be feasible, a method for sustaining it, through current or torque injection, remains to be developed.

4.1. PROTO-SPHERA Parameters

The PROTO-SPHERA cross-section is shown in Fig. 4.1. The toroidal plasma current $I_p=240$ kA (which is the design limit), flows in a tight torus with diameter $2\cdot R_{sph}=0.7$ m, aspect ratio $A=1.2$ and elongation $\kappa=2.35$. All these parameters correspond to the ones that characterized the START spherical torus experiment [31, 32, 36]. However the longitudinal screw pinch current $I_e=60$ kA is much smaller than the toroidal field current $I_{tf}\geq 200$ kA flowing into the START metal centerpost (see Fig. 1.15 and 1.22).

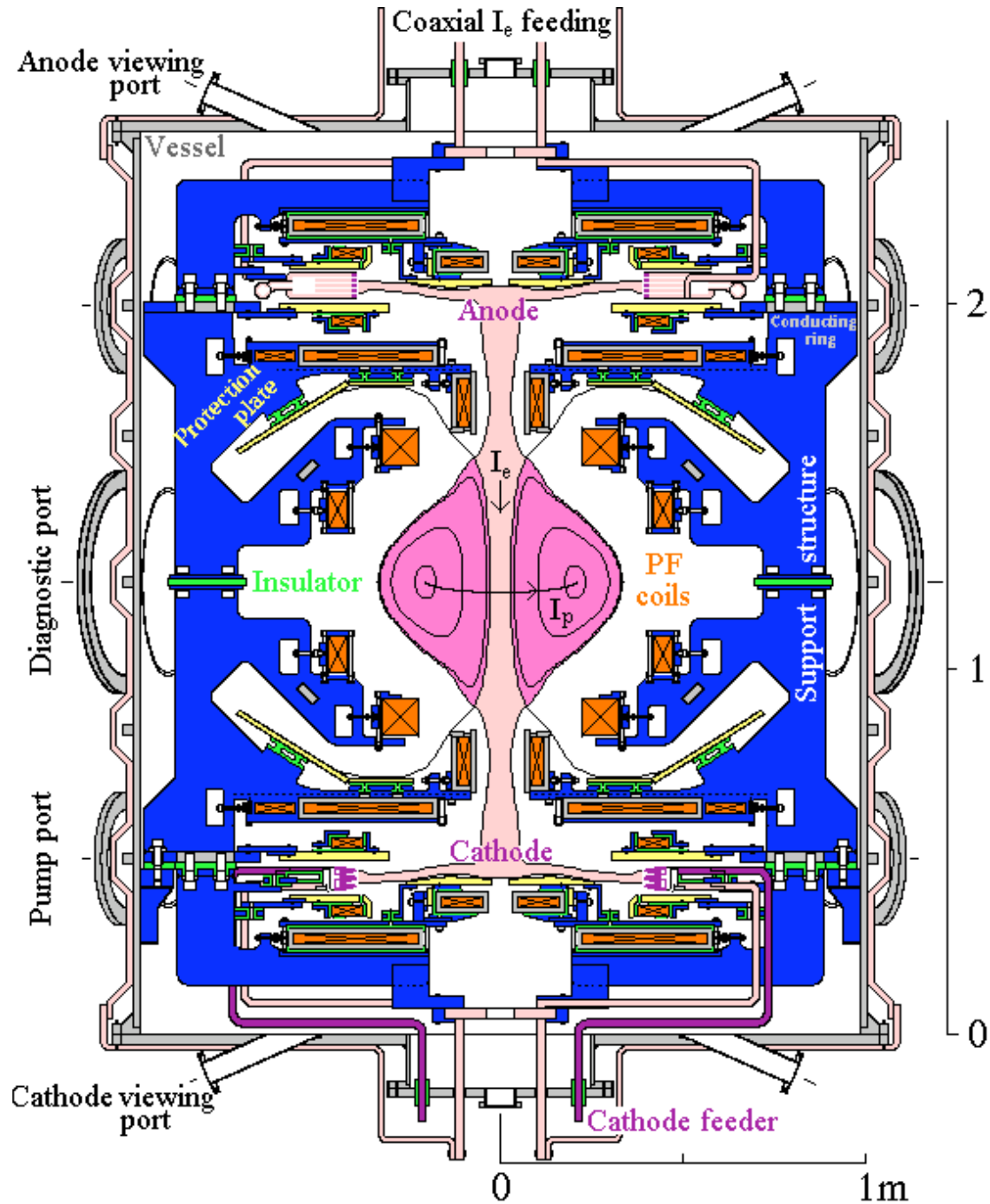


Fig. 4.1. Cross-section view of PROTO-SPHERA plasma and load assembly.

4.2. Comparison between PROTO-SPHERA and TS-3

The comparison between PROTO-SPHERA and the TS-3 flux-core-spheromak experiment [67, 68], described in more detail in Section 2.2, is shown in Fig. 4.2.

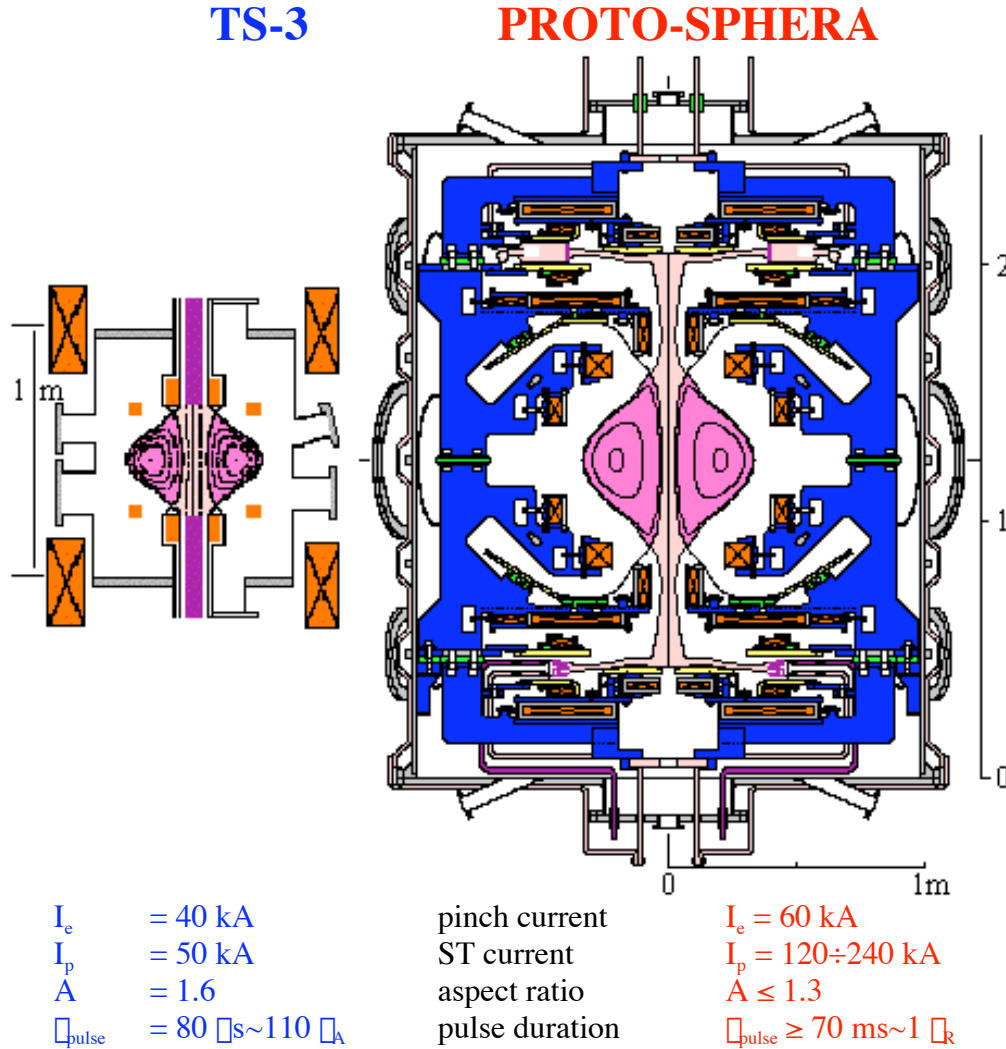


Fig. 4.2. Comparison between TS-3 flux-core experiment and PROTO-SPHERA.

The main differences with TS-3 can be so summarized:

- Each of the two PROTO-SPHERA electrodes is contained inside a separate chamber and surrounds a disk shaped plasma, whereas TS-3 did not have separate electrode chambers, but produced an almost straight screw pinch between cylindrical electrodes.
- PROTO-SPHERA aims at a prolate ST with elongation $\lambda \sim 2.3$, to get $q_0 \sim 1$ and $q_{95} \sim 2.5 \div 3$, whereas TS-3 obtained an oblate ST with elongation $\lambda \sim 1.6$ and got $q_{95} \leq 1$.
- PROTO-SPHERA aims at sustaining the configuration for more than $\tau_r = \tau_0 a^2 / \lambda$ (one resistive time), whereas TS-3 achieved $\leq 110 \cdot \tau_A$ (110 Alfvén times, $\tau_A = R_{\text{axis}} / v_A$).
- PROTO-SPHERA aims at obtaining a dimensionless Lundquist number $S = \tau_r / \tau_A \sim 1.2 \cdot 10^5$, whereas the TS-3 flux-core-spheromak experiment had $S = \tau_r / \tau_A \sim 9 \cdot 10^3$.

4.3. Current Densities as Constraints

The main constraints in the physical design of PROTO-SPHERA have been the current densities, both in the plasma as well as in the poloidal field coils:

- At the plasma-cathode interface: the maximum current density experimentally demonstrated on the electrode's testbench PROTO-PINCH has been 100 A/cm^2 (Section 6.2), upon the cross-section of a single emitting filament. However, due to the space left in the vertical direction between the emitting filaments, the maximum current density will be considered $j_c \sim 80 \text{ A/cm}^2$. This figure limits the total pinch current I_c , emerging from the plasma-cathode interface, which has the shape of a ribbon with radius $R_{ci} = 0.4 \text{ m}$ and a width $\Delta Z \sim 3.0 \text{ cm}$, to $I_c = 2\pi R_{ci} \Delta Z j_c \sim 60 \text{ kA}$. The electrodes are the most unconventional items of PROTO-SPHERA. They are designed as modular (~ 100 modules) and are composed by a large number of elementary tubes and wires (~ 500). The electrodes are made out of refractory metal (directly heated cathodes and hollow gas puffed anodes) and pressed radially in a disk, as shown in Fig. 4.3.

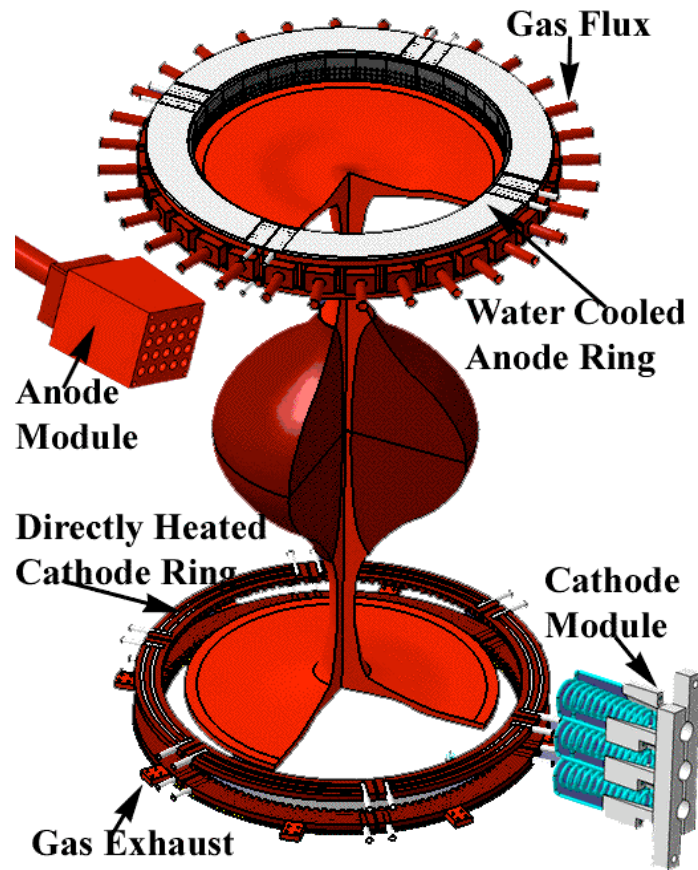


Fig. 4.3. Scheme of the PROTO-SPHERA electrodes.

- Within the poloidal field coils: the maximum current density allowing for a simple water cooling system and for a high duty-cycle experiment (a plasma shot every 5') has been chosen as $j_{pf} \leq 2 \text{ kA/cm}^2$. This figure implies a temperature increase due to the ohmic self-heating $dT/dt \leq 2 \text{ }^\circ\text{C/sec}$ for all the poloidal field coils. Two sets of coils exist (Fig. 4.4). To minimize the number of electrical power supplies all the coils of each set are connected in series.

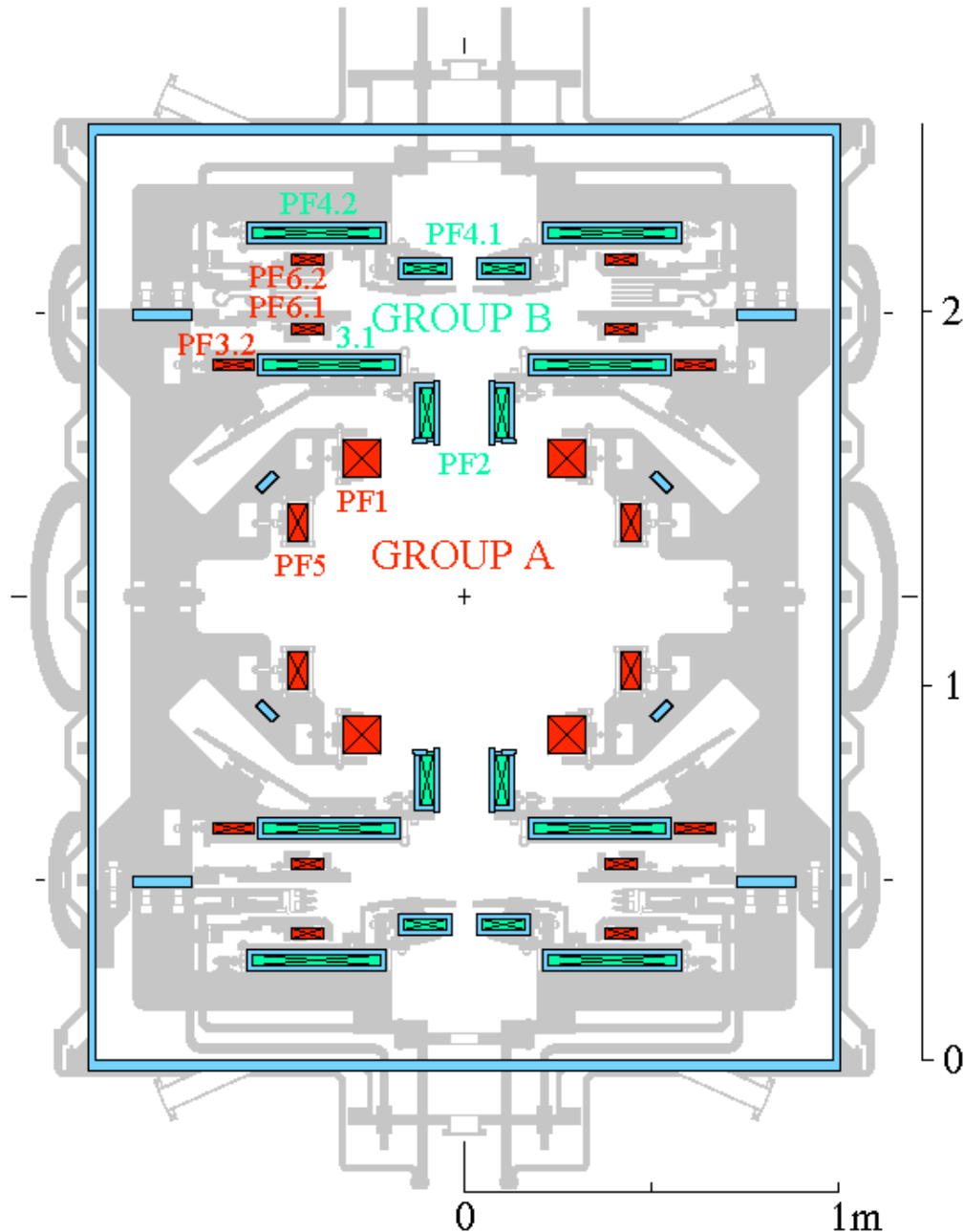


Fig. 4.4. Scheme of the poloidal field coils of PROTO-SPHERA, divided in set 'A' and 'B'. Also all the axisymmetric passive conductors are evidenced.

- Group 'B': coils that have the task of shaping the screw pinch; their current is fixed during the plasma shot. These coils are enclosed inside thick vacuum tight metal cases (10-mm thick Stainless Steel or 6 mm-thick W-Cu). The time constant of these coils is $>2\text{ms}$ and serves the purpose of stabilizing the plasma disks near the electrodes during the formation of the spherical torus.
- Group 'A': coils that have the task of compressing the torus; their current is variable during the plasma shot. These coils are enclosed inside thin vacuum tight metal cases (1.5 mm-thick Inconel). The time constant of these coils is $<200\text{ }\mu\text{s}$ and serves the purpose of allowing a fast response of the field compressing the plasma torus.

4.4. Screw Pinch Formation

The screw pinch (SP) is formed by a hot cathode breakdown. First the cathode filaments are heated to 2600 °C and the current in the poloidal field coils of group 'B' reaches its nominal value $I_B=1875$ A. At this point a voltage $V_e\sim 100$ V, applied on the anode, is sufficient to breakdown the Hydrogen gas, filling the vessel at a pressure $p_H\sim 10^{-3}\div 10^{-2}$ mbar. The longitudinal arc current is limited to less than $I_e\sim 8.5$ kA (Fig. 4.5) and the total toroidal current inside the force-free screw pinch to $I_\phi^{\text{Pinch}}\sim 3$ kA. The pinch discharge is kink stable ($q_{\text{Pinch}}\geq 2$, $\beta_{\text{Pinch}}\sim 2.6$ m⁻¹). No current flows in the PF coils of group 'A'.

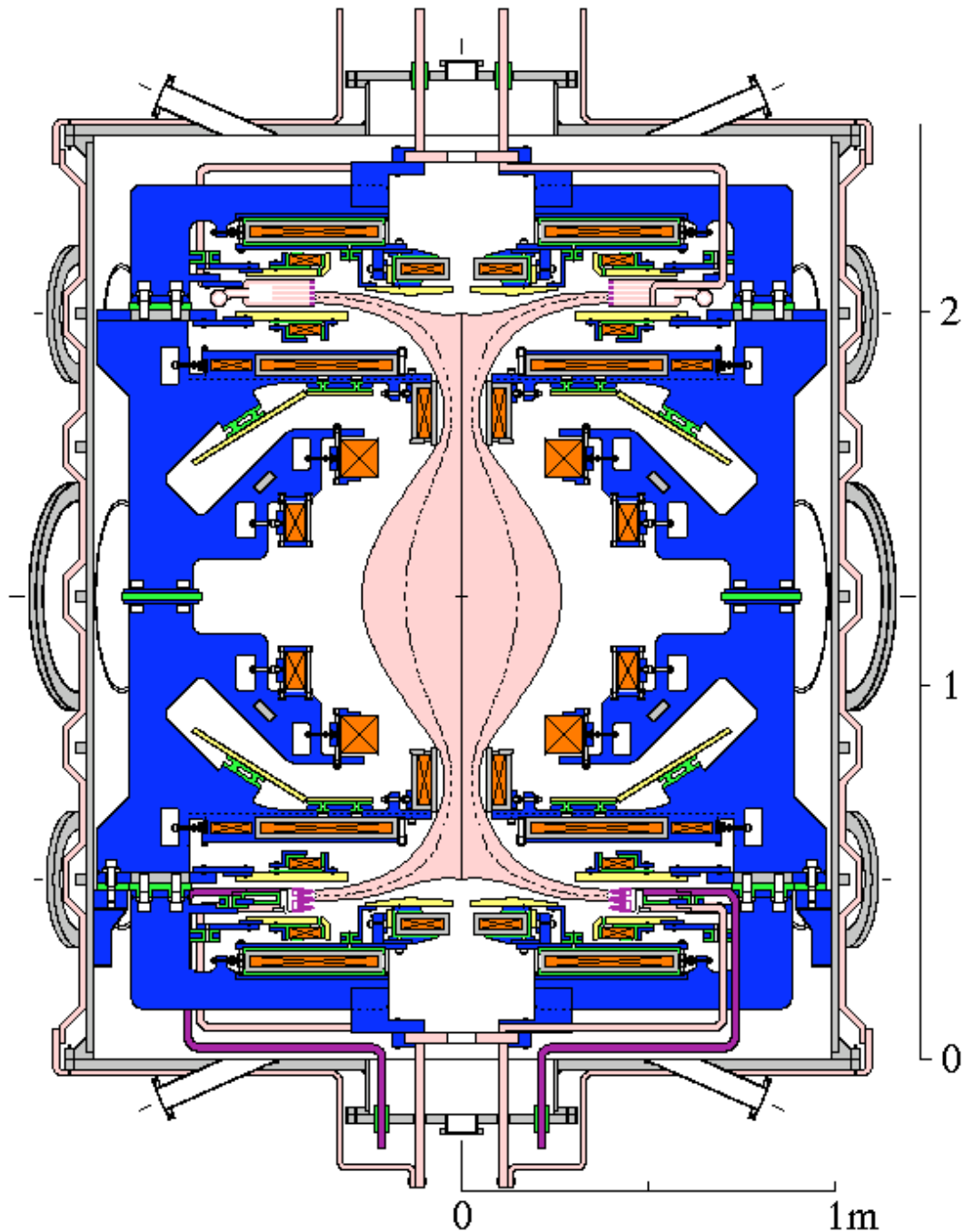


Fig. 4.5. Cross-section view of PROTO-SPHERA plasma at the screw pinch formation.

As the power required to maintain the kink-stable screw pinch will be quite low (< 1 MW, see Section 6.5), this plasma discharge can be sustained easily for 1 s. In the scenario which leads to the toroidal plasma formation (see Section 8.4), the stable screw pinch is maintained for only 0.1 s. During this time interval (0.1 extensible to 1 s) it should be possible, through control of the gas puffing system, to increase or decrease the screw pinch electron density. This control tool would be quite useful, should the best choice for the filling pressure at the breakdown turn out to be incompatible with the best choice for the screw pinch electron density at the formation of the spherical torus.

Pushing the pinch current up to $I_e=60$ kA ($\beta_{\text{Pinch}}\sim 18$ m⁻¹), on a time scale of about 500 μ s, the screw pinch goes kink unstable ($q_{\text{Pinch}}<2$). With a delay of about 100 μ s also the current in the poloidal field coils of group 'A' starts to increase, reaching $I_A=0.7$ kA on a time scale of ~ 1 ms. After a further delay of 100 μ s the spherical torus starts to form, as in the TS-3 flux-core-spheromak experiment, reaching $I_p=120$ kA on a time scale of ~ 1 ms. Figure 4.6 shows the comparison between the vacuum field of PROTO-SPHERA and the corresponding vacuum field of TS-3, just before the formation of the spherical torus. The geometry of the field at the torus formation is the same as in TS-3, where the ST formation was 100% successful [68] (i.e. the spherical torus was formed in every shot).

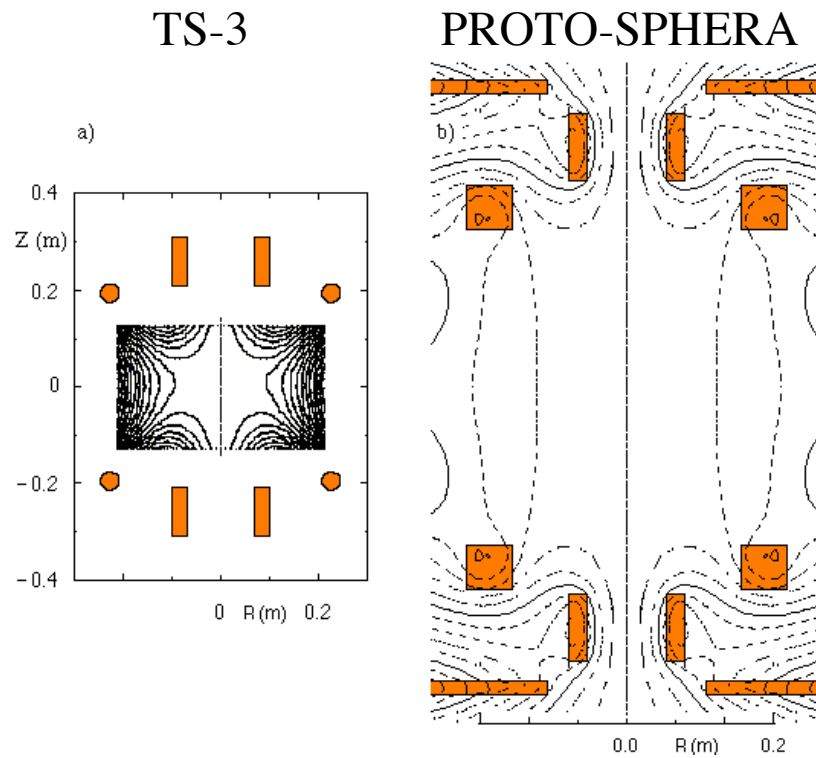


Fig. 4.6. Cross-section view of the vacuum field flux surfaces just before the formation of the spherical torus in:
a) TS-3 [68] and b) PROTO-SPHERA.

The success of the TS-3 formation scheme can be due to the flux swing induced by the increase of the current in the compression coils. As a matter of fact, the flux swing available in PROTO-SPHERA provides a loop voltage $V_{\text{loop}}\sim 10$ V, for about 1 ms. This flux swing alone should be able to push up I_p to 120 kA in about 1 ms (see Section 8.5).

4.5. Equilibrium Code for Combined Spherical Torus + Screw Pinch Configurations

The equilibrium scenario has been calculated by a predictive equilibrium code based on spherical coordinates [89], which solves the Grad-Shafranov equation for the combined equilibrium of a spherical torus and of a force-free screw pinch. The configuration of PROTO-SPHERA is composed by a screw pinch (SP), with open field lines ending upon the electrodes: the plasma-electrode contact surfaces are cylindrical ribbons placed at $R=R_{el}$. The SP is surrounded by a spherical torus (ST), with closed field lines: the SP and the ST have a common magnetic separatrix (see Fig. 4.1).

Coupled equilibrium calculations, based on the poloidal flux function $\psi = 2\pi R A_\phi$ (Fig. 4.7.a), have been performed, using the following assumptions:

- The plasma pressure $p(\psi)$ and the normalized diamagnetic current $f(\psi) = (\mu_0 I_{dia}/2\pi)$ are continuous at the ST-SP interface ($\psi = \psi_X$), whereas the current density $\vec{j}$ may have jumps at the interface (Fig. 4.7.d).
- The plasma of the SP is force-free ($\vec{j} \parallel \vec{B}$), with $p(\psi) = p_e = \text{constant}$ and $f(\psi) = (\mu_0 I_e/2\pi)(\psi/\psi_X)$ inside the SP ($0 < \psi < \psi_X$). The poloidal flux function is $\psi = 0$ on the symmetry axis and the total longitudinal (electrode) current I_e is an input.
- The ST-SP interface is defined by the magnetic separatrix (which has X-points with non-orthogonal cross-section); $\vec{j} \neq 0$ only within the separatrix for the ST; $\vec{j} \neq 0$ within the area bounded by the electrodes for the pinch (Fig. 4.8).
- $p(\psi) = p_e + C_p(\psi/\psi_X)^{1.1}$ and $f^2(\psi) = (\mu_0 I_e/2\pi)^2 + C_f(\psi/\psi_X)^{1.1}$ inside the ST ($\psi_X < \psi < \psi_{max}$) (see Fig. 4.7.b and 4.7.c).
- The total toroidal current $I_\phi^{ST} = I_p$ flowing inside the contour $C^{ST}(\psi_T^X)$ of the ST is an input, along with the total poloidal beta inside the volume V_p^{ST} of the ST,

$$\beta_p^{ST} = \frac{2}{\mu_0 (I_p)^2} \frac{\int_{V_p^{ST}} p dV}{\int_{C^{ST}(\psi_T^X)} \vec{B} \cdot d\vec{l}_p}.$$

The two inputs I_p and β_p^{ST} determine, by iteration, the values of C_p and C_f . The total toroidal current I_ϕ^{Pinch} , flowing inside the force-free screw pinch, is instead an output.

The iterative equilibrium calculation is performed in spherical coordinates (r, θ, ϕ) , where the poloidal flux function can be expanded, in terms of index-1, order-n spherical harmonics $\sin\theta P_n^1(\cos\theta)$ and of the internal $M_n^i(r)$ and external $M_n^e(r)$

spherical multipolar moments, as $\psi = \sum_{n=1}^{N_{max}} [M_n^i(r)r^{-n} + M_n^e(r)r^{n+1}] \sin\theta P_n^1(\cos\theta)$. It is

necessary to use a large number of spherical multipolar moments ($N_{max}=40 \div 50$) for obtaining a correct description of the narrowest part of the screw pinch. The solution for the combined equilibrium is shown in Fig. 4.8: it exhibits two X-points at the SP-ST magnetic separatrix ($\psi = \psi_X$) and two degenerate X-points ($B=0$) on the symmetry axis, where the surface $\psi=0$ branches towards the electrodes.

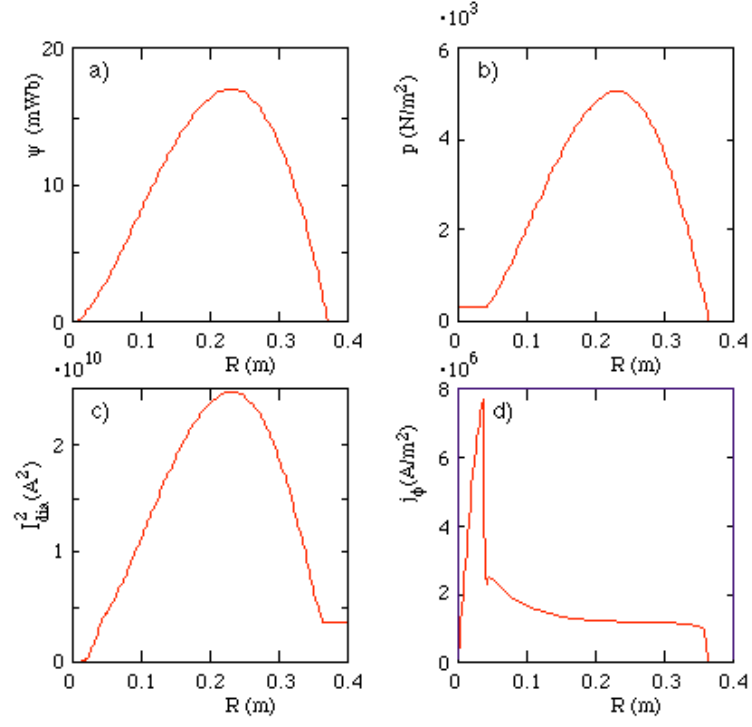


Fig. 4.7. Equilibrium calculation for PROTO-SPHERA: $I_p=180$ kA, $I_c=60$ kA, $\beta_p^{ST}=0.22$. Profiles on the equatorial plane of: a) Flux function ψ ; b) plasma pressure $p(\psi)$; c) diamagnetic current $I_{dia}^2(\psi)$ and d) toroidal current density j_ϕ

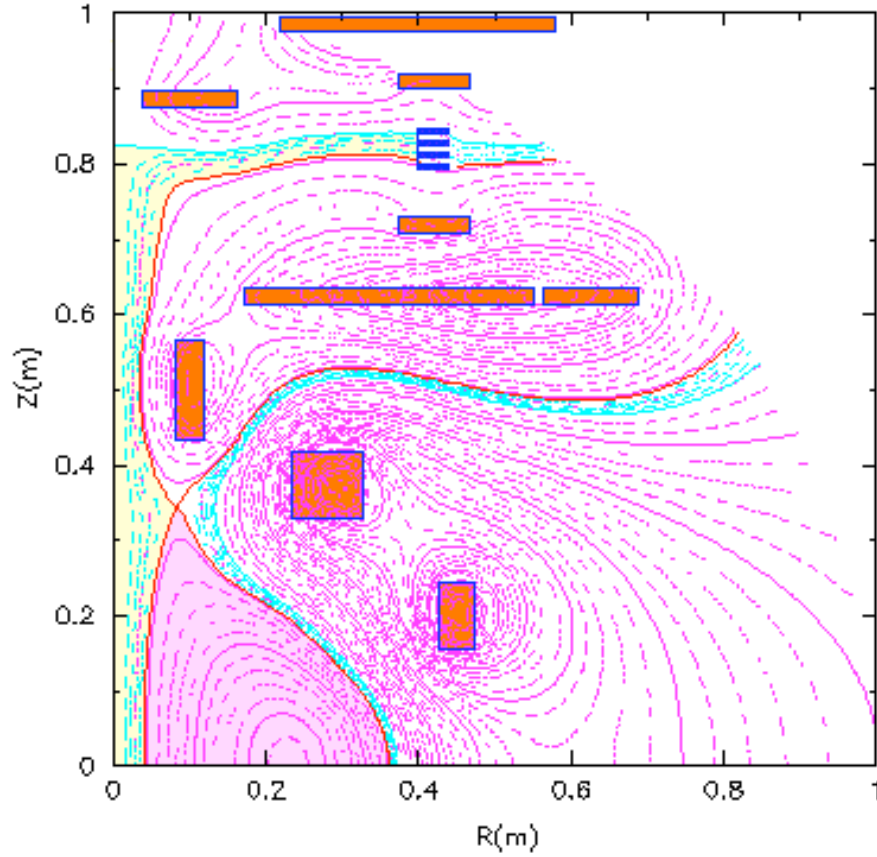


Fig. 4.8. Equilibrium calculation for PROTO-SPHERA: $I_p=180$ kA, $I_c=60$ kA, $\beta_p^{ST}=0.22$. Contour plot of the poloidal flux function ψ .

The values of the winding number (also called safety factor) are $q_0 \sim 0.94$ at the magnetic axis (due to the very strong paramagnetism), with $q_{\min} \sim 0.88$, and $q_{\square} \sim 6$ at the edge. However the region of strong shear at the edge starts at $q_{95} \sim 2.6$ (where q_{95} is the value of q at $\square = 0.95 \cdot \square_x$) and is extremely narrow, as shown in Fig. 4.9.

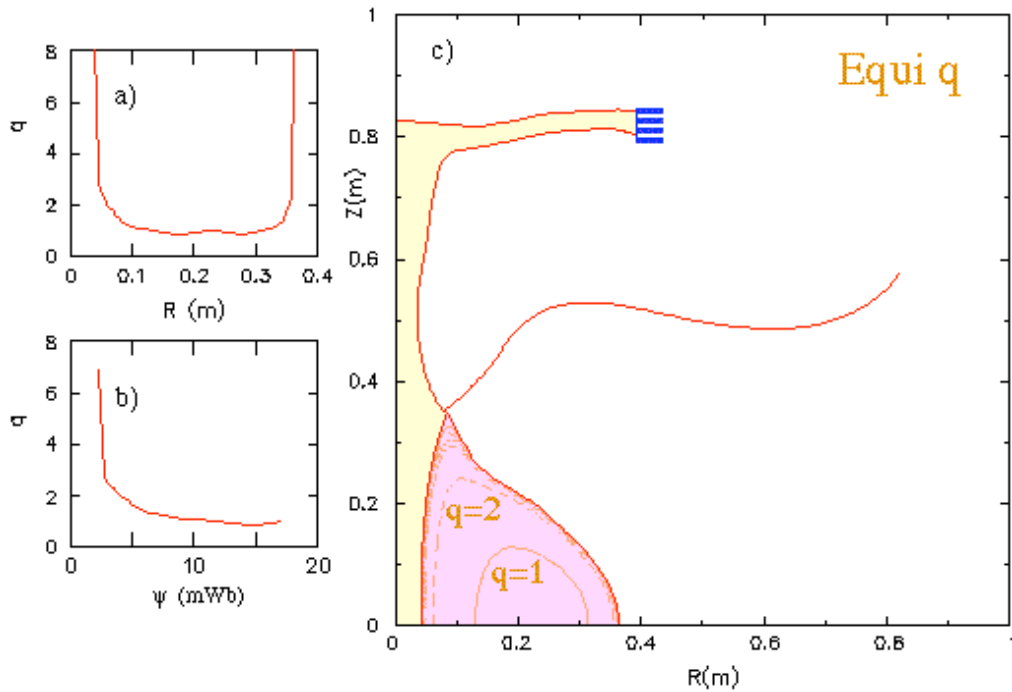


Fig. 4.9. Equilibrium calculation for PROTO-SPHERA: $I_p=180$ kA, $I_e=60$ kA, $\square_p^{ST}=0.22$.
a) Winding number q on the equatorial plane; b) q versus the poloidal flux function $\square$, inside the main spherical torus; c) contour plot of q .

The surface averaged relaxation parameter $\langle \mu \rangle = \square_0 \langle \vec{j} \cdot \vec{B} / B^2 \rangle$ (Fig. 4.10) is constant $\mu \sim 35$ m⁻¹ inside the screw pinch. It is slowly varying, starting from values around 9 m⁻¹ near the plasma magnetic axis, and increases up to 35 m⁻¹ at the edge of the ST, where it matches the value in the screw pinch. The volume-averaged value of μR_{sph} in the ST is about 4.

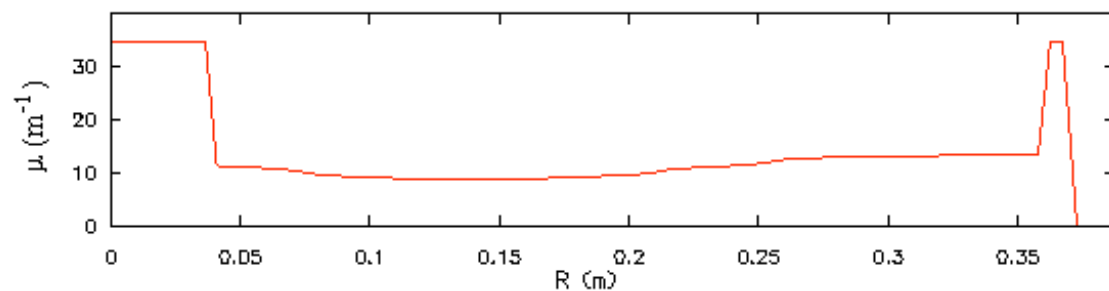


Fig. 4.10. Calculated equilibrium of PROTO-SPHERA: $I_p=180$ kA, $I_e=60$ kA, $\square_p^{ST}=0.22$.
Profile of the relaxation parameter $\langle \mu \rangle$ on the equatorial plane.

The equilibrium code has also been used to simulate the experimental results of TS-3 flux core spheromak, using the published PF coil currents waveforms. The screw pinch, which in TS-3 simply terminated on cylindrical electrodes, has been assumed to be force free: $p(\rho)=p_e=\text{constant}$ and $f(\rho)=(\rho_0 I_e/2\rho)(\rho/\rho_x)$ (for $0<\rho<\rho_x$), like in PROTO-SPHERA. The assumptions used for the profiles inside the spherical torus: $p(\rho)=p_e+C_p(\rho/\rho_x)^{1.1}$ and $f^2(\rho)=(\rho_0 I_e/2\rho)^2 + C_f(\rho/\rho_x)^{1.1}$ (for $\rho_x<\rho<\rho_{\text{max}}$) are the same as those used for PROTO-SPHERA. The longitudinal screw pinch current is $I_e=40$ kA and the toroidal current inside the ST is $I_p=50$ kA. A much larger poloidal beta value ($\beta_p^{\text{ST}}=0.60$, $\beta_{\text{ST}}=29\%$ for TS-3, versus $\beta_p^{\text{ST}}=0.22$, $\beta_{\text{ST}}=22\%$ for PROTO-SPHERA) is required in order to match in the equilibrium calculation the measured shape of the spherical torus. The results of the equilibrium simulations of TS-3 are shown in Fig. 4.11 and 4.12.

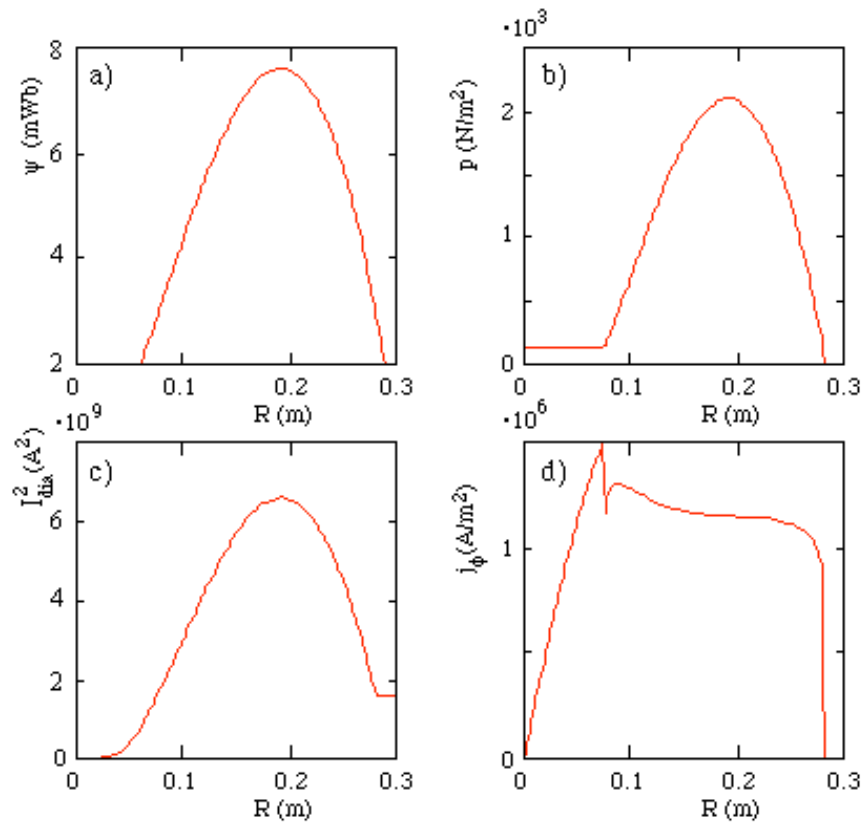


Fig. 4.11. Equilibrium calculation for TS-3: $I_p=50$ kA, $I_e=40$ kA, $\beta_{\text{ST}}=29\%$. Profiles on the equator of: a) Flux function ψ ; b) plasma pressure $p(\rho)$; c) diamagnetic current $I_{\text{dia}}^2(\rho)$ and d) toroidal current density j_ϕ .

The values of the winding number (safety factor) are, at the magnetic axis $q_0 \sim 0.62$ (due to the very strong paramagnetism), and at the edge $q_\rho \sim 2$. However the region of strong shear at the edge starts at $q_{0.95} \sim 0.97$ (where $q_{0.95}$ is the value of q at $\rho = 0.95 \cdot \rho_x$) and is extremely narrow.

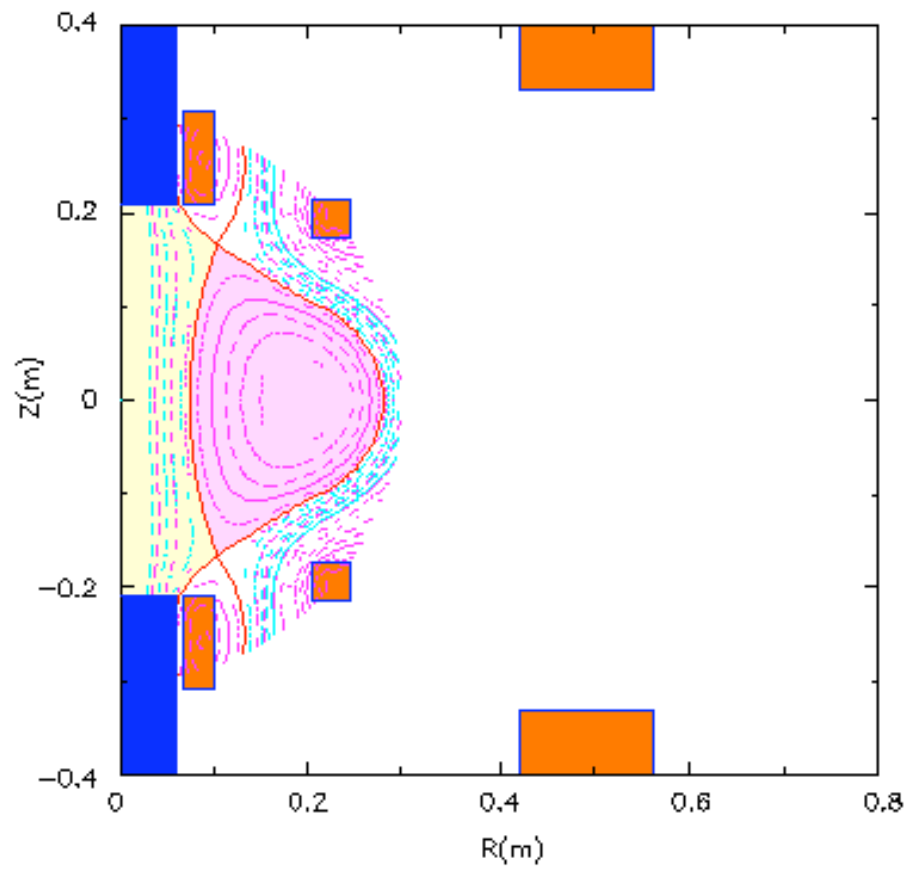


Fig. 4.12. Equilibrium calculation for TS-3: $I_p=50$ kA, $I_e=40$ kA, $\beta_{ST}=29\%$.
Contour plot of the poloidal flux function Ψ .

4.6. Equilibrium Comparison with TS-3

The geometry of the PF coils in PROTO-SPHERA has been designed in order to produce a magnetic configuration with well-defined features. Among the main differences of the equilibrium between PROTO-SPHERA and TS-3, the two worth mentioning are:

- The q profile in the spherical torus is different. Comparing the TS-3 equilibrium simulation with the PROTO-SPHERA equilibrium calculation (see Section 4.5) it turns out that TS-3 was characterized by a $q \sim 1$ all over the spherical torus, whereas PROTO-SPHERA aims at a $q_{95} \sim 3$ (see Fig. 4.13).
- The disk shape of the plasma near the electrodes plays a major role in stabilizing the rigid shift/tilt modes in PROTO-SPHERA (see Section 5.1).

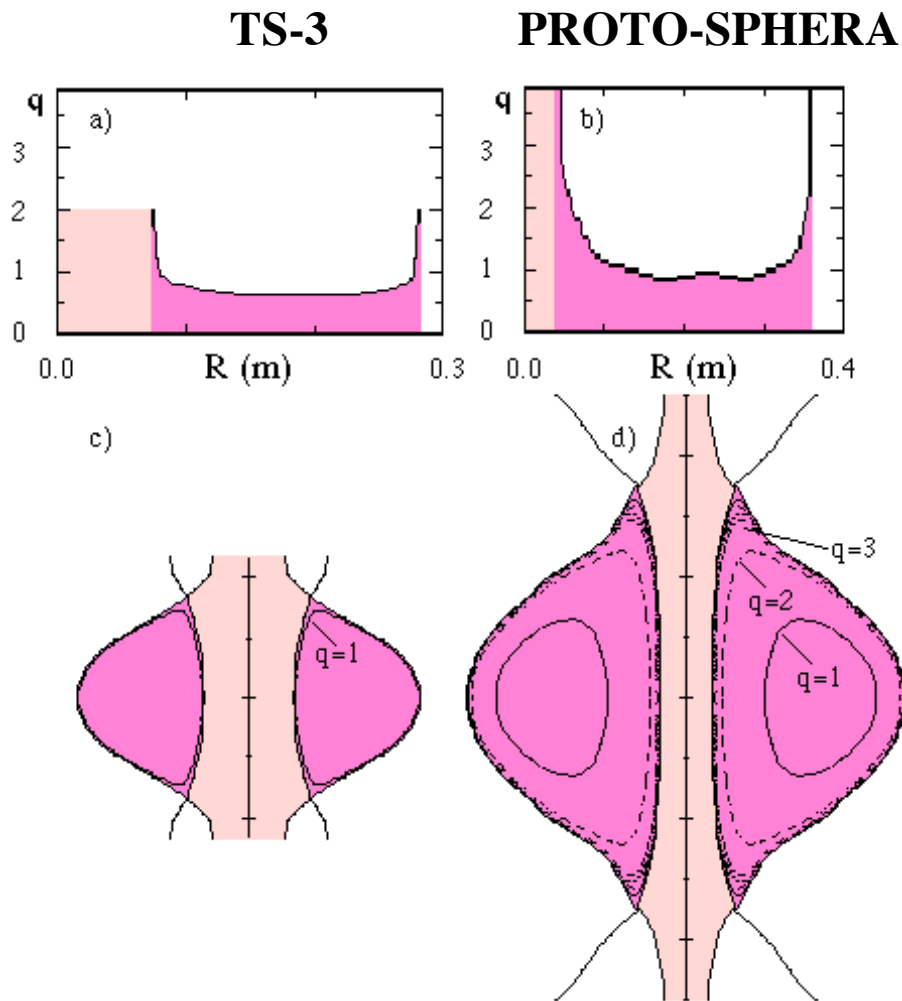


Fig. 4.13. Equilibrium calculated for TS-3 with $I_p=50$ kA, $I_e=40$ kA and for PROTO-SPHERA with $I_p=180$ kA, $I_e=60$ kA. Profiles on the equator of $q(\square)$ for: a) TS-3; b) PROTO-SPHERA. Contour plot of $q(\square)$ for: c) TS-3; d) PROTO-SPHERA.

4.7. Formation Sequence

The formation sequence starts 250 μ s after the torus formation with $I_p=30$ kA= $I_c/2$: the equilibrium has an aspect ratio $A=1.80$, an elongation $\kappa=2.17$, a safety factor at the edge $q_{95}=3.4$, a paramagnetic effect $B_T/B_{T0}=1.20$ and a toroidal pinch current $I_{\phi}^{\text{Pinch}}=179$ kA.

The relaxation parameter in the pinch is $\kappa_{\text{pinch}}R_{\text{sph}}=6$ and its volume average in the torus is $\langle\kappa_{\text{ST}}R_{\text{sph}}\rangle_{\text{Vol}}\sim 2.45$ (with $R_{\text{sph}}=0.35$ m). The equilibrium is shown in Fig. 4.14.

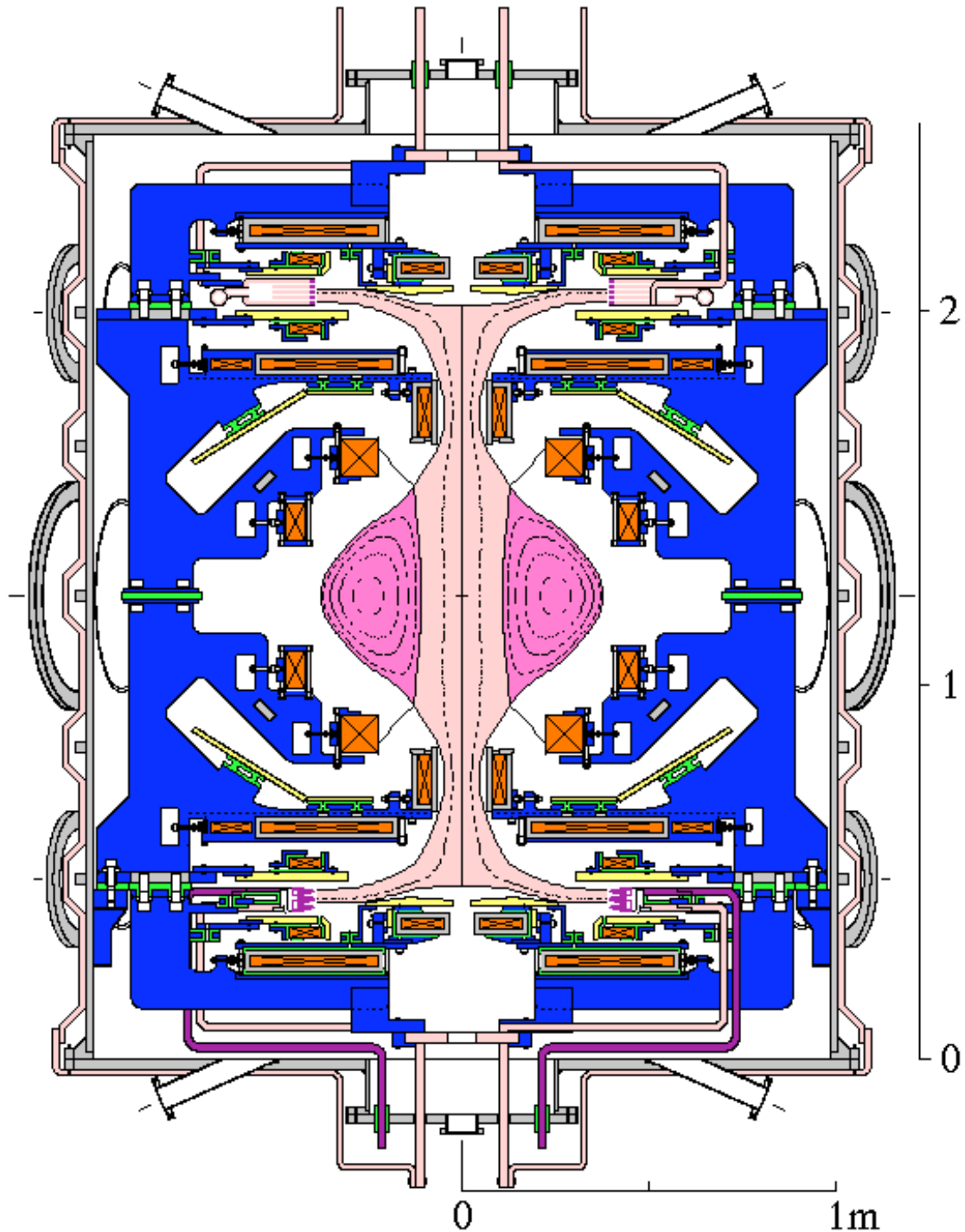


Fig. 4.14. Equilibrium of PROTO-SPHERA at $I_p=30$ kA, 250 μ s after torus formation.

The formation sequence continues 500 μ s after the torus formation with $I_p=60$ kA= I_c : the equilibrium has an aspect ratio $A=1.51$, an elongation $\kappa=2.15$, a safety factor at the edge $q_{95}=3.1$, a paramagnetic effect $B_T/B_{T0}=1.47$ and a toroidal pinch current $I_{\square}^{\text{Pinch}}=247$ kA.

The relaxation parameter in the pinch is $\square_{\text{Pinch}}R_{\text{sph}}=8$ and its volume average in the torus is $\langle \square_{\text{ST}}R_{\text{sph}} \rangle_{\text{Vol}} \sim 3.15$ (with $R_{\text{sph}}=0.35$ m). The equilibrium is shown in Fig. 4.15.

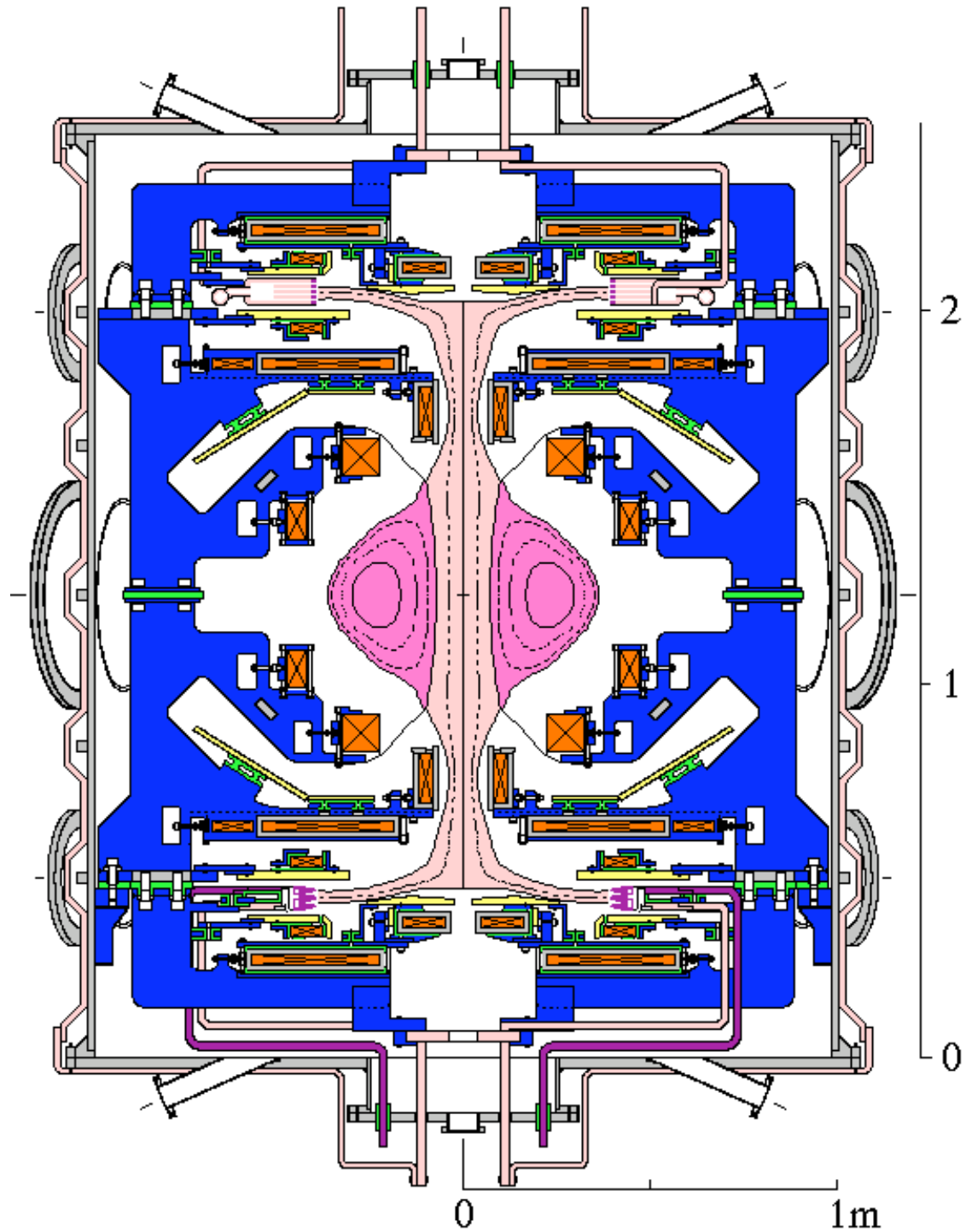


Fig. 4.15. Equilibrium of PROTO-SPHERA at $I_p=60$ kA, 500 μ s after torus formation.

The formation continues 1000 μ s after the torus formation with $I_p = 120 \text{ kA} = 2 \cdot I_c$: the equilibrium has an aspect ratio $A = 1.32$, an elongation $\kappa = 2.16$, a safety factor at the edge $q_{95} = 2.8$, a paramagnetic effect $B_T/B_{T0} = 2.1$ and a toroidal pinch current $I_{\text{pinch}} = 310 \text{ kA}$.

The relaxation parameter in the pinch is $\kappa_{\text{pinch}} R_{\text{sph}} = 10.5$ and its volume average in the torus is $\langle \kappa_{\text{ST}} R_{\text{sph}} \rangle_{\text{Vol}} \sim 3.85$ (with $R_{\text{sph}} = 0.35 \text{ m}$). The equilibrium is shown in Fig. 4.16.

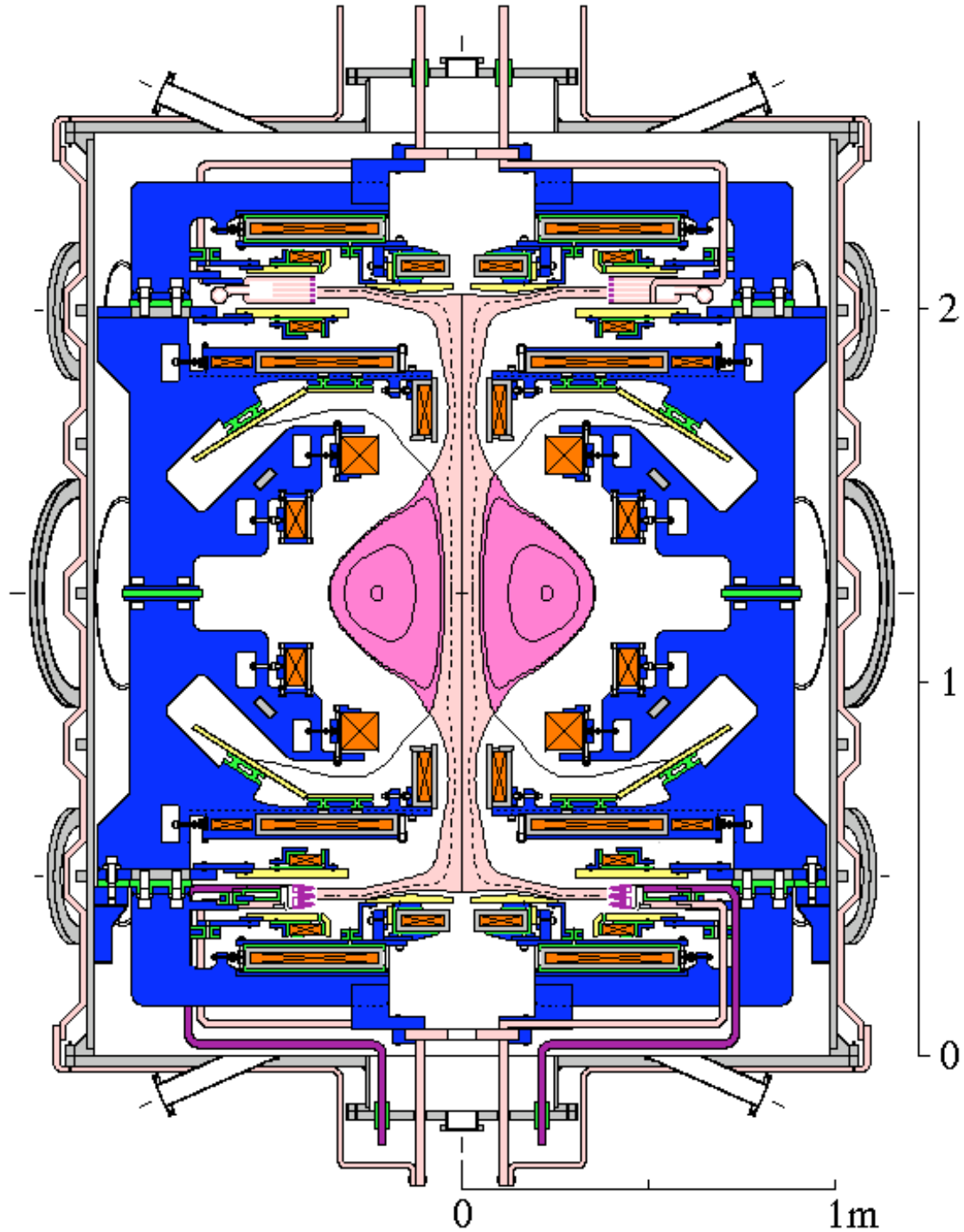


Fig. 4.16. Equilibrium of PROTO-SPHERA at $I_p = 120 \text{ kA}$, 1000 μ s after torus formation.

The formation ends 10 ms after the torus formation with $I_p = 240 \text{ kA} = 4 \cdot I_c$: the equilibrium has an aspect ratio $A = 1.21$, an elongation $\kappa = 2.35$, a safety factor at the edge $q_{95} = 2.8$, a paramagnetic effect $B_T/B_{T0} = 3.1$ and a toroidal pinch current $I_{\square}^{\text{Pinch}} = 407 \text{ kA}$.

The relaxation parameter in the pinch is $\kappa_{\text{Pinch}} R_{\text{sph}} = 14$ and its volume average in the torus is $\langle \kappa_{\text{ST}} R_{\text{sph}} \rangle_{\text{Vol}} \sim 4.2$ (with $R_{\text{sph}} = 0.35 \text{ m}$). The equilibrium is shown in Fig. 4.17.

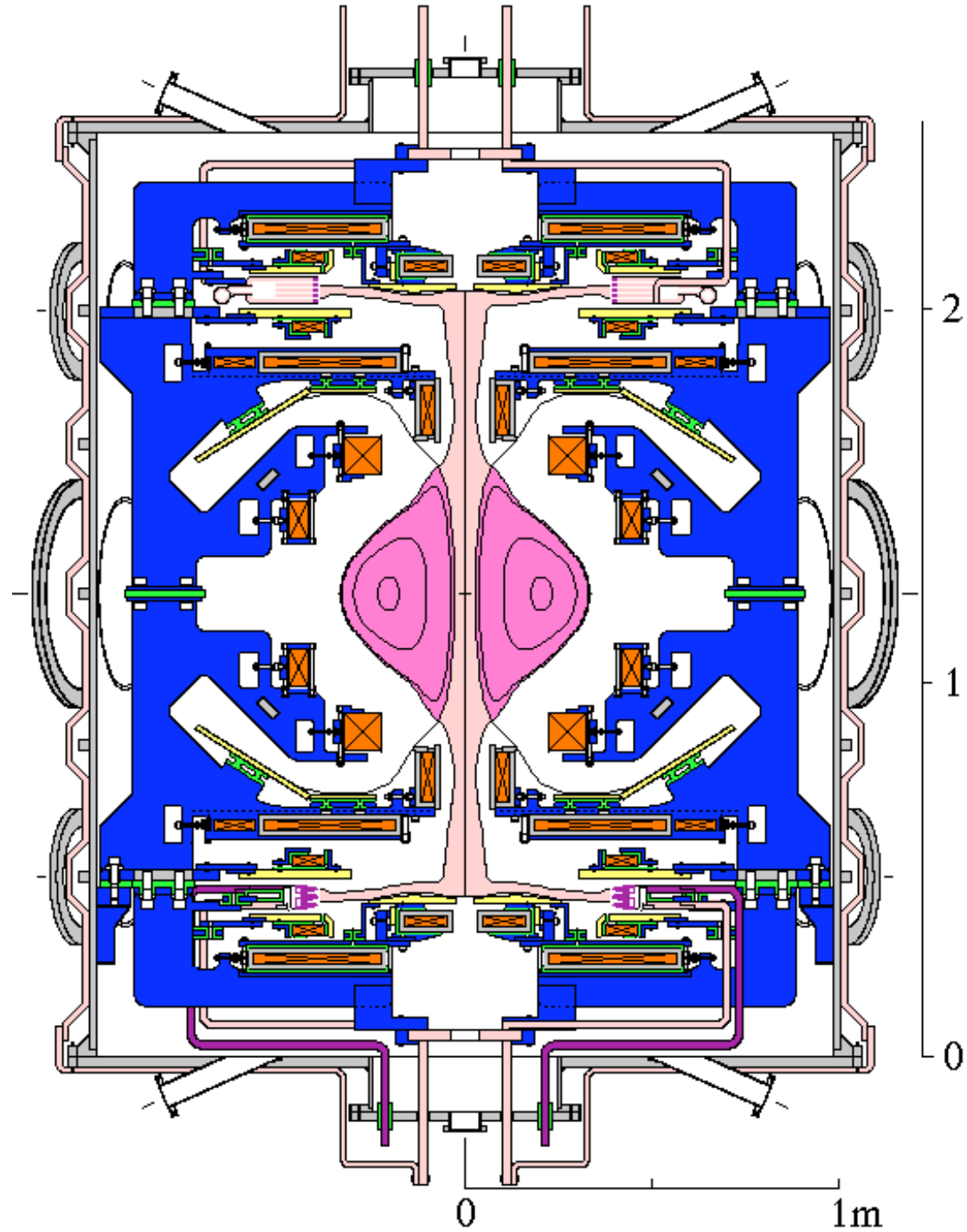


Fig. 4.17. Equilibrium of PROTO-SPHERA at $I_p = 240 \text{ kA}$, 10 ms after torus formation.

4.8. Plasma Performances at $I_p=180$ kA

The following parameters, calculated from the equilibrium code, will be used for evaluating the plasma performances of the spherical torus of PROTO-SPHERA. The case chosen is intermediate between the time slice at 1.0 ms (Fig. 4.16) with $I_p=120$ kA and the time slice at 10 ms (Fig. 4.17) with $I_p=240$ kA. This choice is due to the fact that it is a high current case ($I_p=180$ kA), but still ideal MHD stable with a large beta value ($\beta_{ST}=22\%$, $\beta_p^{ST}=0.22$), see Section 5.9. It is the same case whose MHD equilibrium is shown in Section 4.5 (see Fig. 4.7, 4.8, 4.9 and 4.10).

Its parameters are:

Major radius $R=0.20$ m, minor radius $a=0.16$ m, elongation $\kappa=2.17$.

Radius of the magnetic axis $R_{axis}=0.23$ m, poloidal cross-section surface $S_{pol}=0.13$ m².

Total ST plasma volume $V_p=0.14$ m³, radius of the X-point $\kappa_{pinch}=0.081$ m.

Toroidal field: on axis $B_{\kappa_{axis}}\sim 1360$ G (paramagnetism=2.6), at X-point $B_{\kappa_{pinch}}\sim 1480$ G.

Toroidal ST plasma current $I_p=0.18$ MA, $\langle j_{\kappa} \rangle = I_p/S_{pol}=1.4$ MA/m², $q_{95}=2.6$, $M=1$ (H₂).

The density limit of the ST of PROTO-SPHERA is evaluated by using the Greenwald density limit [90], which fits well the START data [47]: $\langle n_G \rangle = \kappa \langle j_{\kappa} \rangle$, where $\langle n_G \rangle$ is in units of 10^{20} m⁻³, $\langle j_{\kappa} \rangle$ is in MA/m², the averages are on the ST poloidal cross-section area and κ is the elongation.

Therefore the density limit for PROTO-SPHERA is: $\langle n_G \rangle = 3.0 \cdot 10^{20}$ m⁻³.

The total energy confinement time is evaluated from the semi-empirical Lackner-Gottardi L-mode plateau-scaling [91]:

$$\tau_E^{LG} = 120 I_p^{0.8} R^{1.8} a^{0.4} \langle n_e \rangle^{0.4} q_{95}^{0.4} M^{0.5} P^{0.6} \kappa / (1 + \kappa)^{0.8} \text{ [ms; MA, m, } 10^{20} \text{ m}^{-3}, \text{ a.m.u., MW]}.$$

The total power required for the helicity injection through the X-points of the configuration is taken as $P_{HI}=4 \cdot P_{ST}^{oh}$ [78], where P_{ST}^{oh} the equivalent resistive power required for sustaining the spherical torus. It is assumed that half of this power is dissipated inside the spherical torus, therefore $P=P_{ST}^{HI}=2 \cdot P_{ST}^{oh}$ is used in the confinement scaling. The estimation of the resistive power $P_{ST}^{oh}=I_p V_{loop}$ comes from the Spitzer conductivity [92]:

$$1/\kappa = 2 \cdot 10^2 \ln \kappa T_e^{3/2} / Z_{eff} \text{ [Siemens; Coulomb logarithm, eV, ion-effective/proton charge]}.$$

The energy confinement is calculated by an iterative procedure, starting from a guessed volume averaged electron temperature $\langle T_e \rangle$, evaluating a provisional resistive input power and a provisional energy confinement, which give a revised

$$\langle T_e \rangle = \tau_E^{LG} P_{ST}^{HI} / (3 \cdot 1.6 \cdot 10^{-19} \langle n_e \rangle V_p) \text{ [eV; s, W, m}^{-3}, \text{ m}^3],$$

and so on until the convergence of $\langle T_e \rangle$ is obtained.

Choosing for PROTO-SPHERA $\langle n_e \rangle = 0.5 \cdot 10^{20}$ m⁻³, $Z_{eff}=2$ and $\ln \kappa=13$,

the procedure converges to $\langle T_e \rangle = 140$ eV, which means:

$$V_{loop}=0.8 \text{ V, } P_{ST}^{HI}=290 \text{ kW, } P_{HI}=580 \text{ kW, } \tau_E^{LG} \sim 1.6 \text{ ms.}$$

The choice of the density $\langle n_e \rangle$ corresponds to the maximum density that makes compatible the ideal MHD β limit with the Lackner-Gottardi L-mode plateau scaling. Such MHD/confinement density limit will increase (decrease) in presence of a degraded (improved) energy confinement.

The profiles of electron density, temperature and total kinetic pressure of the equilibrium of Fig. 4.8 are shown in Fig. 4.18, as a function of the normalized equivalent cylindrical radius ρ/a .

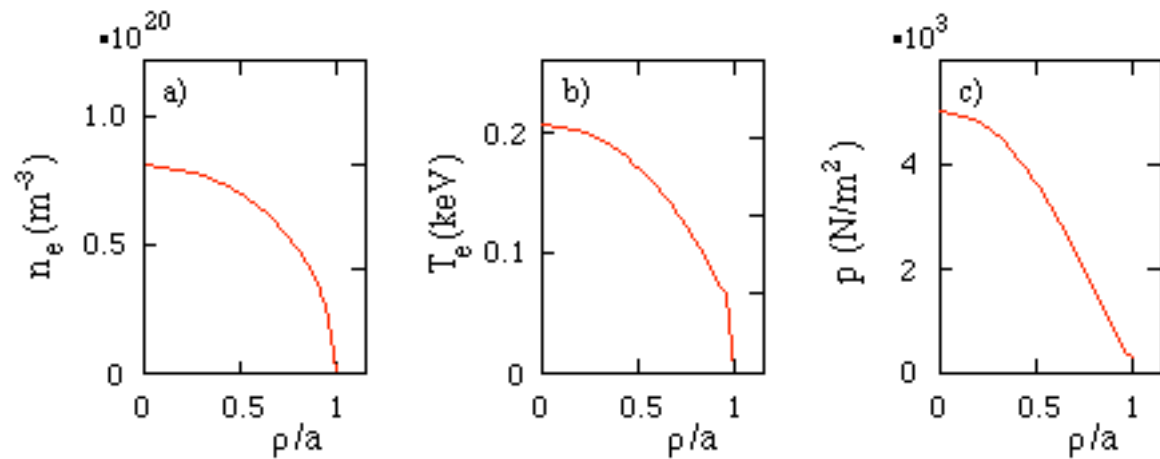


Fig. 4.18. Calculated equilibrium of PROTO-SPHERA: $I_p=180$ kA, $I_e=60$ kA, $\beta_p^{ST}=0.22$. Profiles inside the ST of: a) electron density n_e ; b) electron temperature T_e ; c) kinetic pressure p , versus the normalized equivalent cylindrical radius ρ/a .

The Alfvén time is calculated as $\tau_{Aaxis}=R_{axis}/V_A$ on axis, $\tau_{AX}=q_{95}\tau_{Pinch}/V_A$ at X-points:
 $\tau_{Aaxis}=23[\text{cm}] \cdot (m_i/m_p)^{1/2} n[\text{cm}^{-3}]^{1/2} / (2.18 \cdot 10^{11} B_{\text{axis}}[\text{G}]) \sim 0.55 \mu\text{s}$,
 $\tau_{AX}=2.6 \cdot 8.1[\text{cm}] \cdot (m_i/m_p)^{1/2} n[\text{cm}^{-3}]^{1/2} / (2.18 \cdot 10^{11} B_{\text{Pinch}}[\text{G}]) \sim 0.46 \mu\text{s}$.
The resistive time is calculated as $\tau_r = \tau_0 a^2 / \eta \sim 69$ ms.

The representative Lundquist number of PROTO-SPHERA is $S = \tau_r / \tau_A \sim 1.2 \cdot 10^5$.

4.9. Screw Pinch Power Balance

The screw pinch is modeled as a straight cylinder of radius $\langle r_{\text{Pinch}} \rangle$ and length L_{Pinch} (~ 2.0 m). Since the magnetic field lines are helices and the pinch winding number is $q_{\text{Pinch}} \sim 1/6$, the connection length (i.e. the length of a field line from one electrode to the other) is taken as $2L_{\text{Pinch}}$ (~ 4.0 m).

The Joule dissipation in the main body of the screw pinch plasma is:

$$P_{\text{Pinch}}^{\square} = \int dV j^2 / \square = \int j_{\text{Pinch}}^2(0) \square_{\text{Pinch}}^4(0) 2 \int_{-L/2}^{L/2} dz / (\square \square_{\text{Pinch}}^2(z)) .$$

As $\square_{\text{Pinch}}^2(z) j_{\text{Pinch}}(z) = \square_{\text{Pinch}}^2(0) j_{\text{Pinch}}(0)$, and taking from the equilibrium calculation the estimate $\langle 1/\square_{\text{Pinch}}^2(z) \rangle = 1/(2.5 \square_{\text{Pinch}}^2(0))$, one obtains, under the assumption of constant electrical conductivity in the screw pinch, $\square = \text{constant}$:

$$P_{\text{Pinch}}^{\square} = \int j_{\text{Pinch}}^2(0) \square_{\text{Pinch}}^2(0) 2 L_{\text{Pinch}} / (2.5 \square) .$$

Inserting the Spitzer conductivity [92] $\square = 2 \cdot 10^2 \ln \square T_e^{3/2} / Z_{\text{eff}}$, one finds that:

$$P_{\text{Pinch}}^{\square} = \int j_{\text{Pinch}}^2(0) \square_{\text{Pinch}}^2(0) 2 L_{\text{Pinch}} Z_{\text{eff}} / (5 \cdot 10^2 \ln \square T_e^{3/2}), \text{ where } T_e \text{ is the electron temperature in the main body of the screw pinch.}$$

In order to derive the plasma parameters of the screw pinch, a power balance is made modeling the pinch and the electrodes as a flux tube similar to the scrape off layer (SOL) of a limiter tokamak, with the main difference of the large current density carried through this plasma.

Thus, similarly to the power losses through the sheath of a limiter:

- The power incident on the electrodes due to the particle flux and localized on the electrode sheaths can be written as: $P_{\text{Pinch}}^{\text{el}} = 1.6 \cdot 10^{-19} S_{\text{el}} \square k_s n_{\text{el}} T_{\text{el}}^{3/2}$ Watt, where S_{el} is the total effective area of the electrodes, $\square$ is the energy transmission factor through the sheath ($\square \sim 8$), k_s the numerical coefficient for the sound velocity ($k_s = 9.78 \cdot 10^3 / A^{1/2}$ m/s, A being the mass number of the incident ion) and T_{el} , n_{el} [eV, m^{-3}] are the electron temperature and density at the electrodes.
- The power lost by diffusion through the pinch surface is:
 $P_{\text{Pinch}}^{\text{Diff}} = 1.6 \cdot 10^{-19} k_B 2 \square L_{\text{Pinch}} (n_e T_e^2 / B)$ Watt, where a thermal conductivity of the Bohm type ($\mu T_e / B$; $k_B = 0.06$ when T_e in eV and B in T) is assumed.
- The power lost by radiation is expressed by:
 $P_{\text{Pinch}}^{\text{rad}} = f_{\text{imp}} n_e^2 R(T_e) \square \langle \square_{\text{Pinch}} \rangle L_{\text{Pinch}}$, where f_{imp} is the impurity fraction and $R(T_e)$ the cooling rate.
- The power loss connected with the convected flux due to the current at the anode is: $P_{\text{Pinch}}^{\text{an}} = (5/2) \langle j \rangle T_e \square \langle \square_{\text{Pinch}}^2 \rangle$.

The weight of the various loss terms is evaluated. The pinch radius at which the radial power losses dominate with respect to the longitudinal power lost at the electrodes, is determined by the most strict between the two conditions:

$$\langle \square_{\text{Pinch}} \rangle < [2 L_{\text{Pinch}} k_B T_e^{1/2} / (\square k_s B)]^{1/2}; \quad \langle \square_{\text{Pinch}} \rangle < [1.28 \cdot 10^{-19} 2 L_{\text{Pinch}} k_B n_e T_e / (\langle j \rangle_{\text{Pinch}} B)]^{1/2}.$$

Here and in the following the electron temperature is assumed to be constant along the field lines in the main body of the screw pinch plasma. At the end it will be verified that this condition is satisfied. By substituting numerical values, it appears that only when $\langle \square_{\text{Pinch}} \rangle$ is less than a few mm (even accounting for a reasonable enhancement over Bohm diffusion), the radial loss term is significant. The radial loss term is therefore neglected in the power balance. Also the radiation term is neglected for the moment.

Far from the electrodes the power balance is determined equating the ohmic input to the current convected loss. Thus in the main body of the plasma:

$$(5/2) j_{\text{Pinch}}(0) T_e \pi^2 \rho_{\text{Pinch}}(0) = (\pi j_{\text{Pinch}}^2(0) \rho_{\text{Pinch}}^2(0) 2L_{\text{Pinch}} Z_{\text{eff}}) / (5 \cdot 10^2 \ln \pi T_e^{3/2}).$$

For the plasma of Fig. 4.8, with $I_p=180$ kA, $I_e=60$ kA and $\rho_{\text{Pinch}}(0)=0.04$ m, substituting the values $L_{\text{Pinch}}=2.0$ m, $j_{\text{Pinch}}(0)=1.2 \cdot 10^7$ A/m², $Z_{\text{eff}}=2$, $\ln \pi=10$, the temperature of the main body of the pinch discharge is obtained: $T_e=36$ eV.

The ohmic input power is, for $T_e=36$ eV: $P_{\text{Pinch}}^{\text{ohm}} = 5.4$ MW.

Assuming an electron density of the order of 10^{19} m⁻³ (which will be justified in Section 6.5), it is easy to check that the only other power loss which is not negligible, with respect to that due to the pinch current, is the power loss localized to the electrode sheaths, which is estimated, also in Section 6.5, to be at most $P_{\text{Pinch}}^{\text{el}}=4.6$ MW. The power lost through impurity radiation is calculated assuming a 1% impurity concentration and a cooling rate $R(T_e) \sim 10^{-31}$ W•m³, which correspond to the maximal cooling rate of Oxygen. The result is $P_{\text{Pinch}}^{\text{rad}} \sim 0.1$ MW, which indeed can be neglected with respect to the total power, taking also into account that this evaluation is somewhat overestimated.

The condition for avoiding strong longitudinal temperature gradient is:

$$2L_{\text{Pinch}} \cdot n_e \leq 10^{17} T_e^2 / Z_{\text{eff}} \text{ [m}^{-2}\text{]}.$$

This condition is strictly verified for the pinch if $T_e \sim 20$ eV. At lower temperatures, the plasma temperature toward the electrodes should be lower than that at the pinch core, if (like in the SOL) the plasma pressure is constant.

During the toroidal plasma formation at $I_e=60$ kA, the evolution of the main body screw pinch electron temperature and of the ohmic input power will be:

at $I_p=30$ kA, $\rho_{\text{Pinch}}(0)=0.11$ m:	$T_e=16$ eV and $P_{\text{Pinch}}^{\text{ohm}} = 2.4$ MW;
at $I_p=60$ kA, $\rho_{\text{Pinch}}(0)=0.075$ m:	$T_e=22$ eV and $P_{\text{Pinch}}^{\text{ohm}} = 3.3$ MW;
at $I_p=120$ kA, $\rho_{\text{Pinch}}(0)=0.05$ m:	$T_e=33$ eV and $P_{\text{Pinch}}^{\text{ohm}} = 5.0$ MW;
at $I_p=180$ kA, $\rho_{\text{Pinch}}(0)=0.04$ m:	$T_e=36$ eV and $P_{\text{Pinch}}^{\text{ohm}} = 5.4$ MW;

Instead the power loss localized at the electrode sheaths should remain at most at the constant value $P_{\text{Pinch}}^{\text{el}}=4.6$ MW, during the toroidal plasma formation and sustainment at $I_e=60$ kA.

From the Katsurai formula [61] (Section 1.5) the current I_p in the spherical torus is proportional to the current density $j_{\text{Pinch}}(0)$ on the equator of the screw pinch: $I_p = j_{\text{Pinch}}(0) \cdot 2\pi^2 a^2 / (\pi q_{95})$. Then reducing the equatorial radius of the screw pinch $\rho_{\text{Pinch}}(0)$, keeping $j_{\text{Pinch}}(0)$ unchanged, one can maintain the same plasma current I_p , unless a tilt mode is destabilized. The advantage of reducing $\rho_{\text{Pinch}}(0)$ is clear as it reduces the Joule dissipation in the main body of the screw pinch plasma. The compression of the screw pinch, while decreasing the longitudinal pinch current I_e , could even lead to the formation of an FRC with a new technique.

4.10. Comparison between Spheromaks and PROTO-SPHERA

Spheromaks are usually formed (Fig. 4.19) by magnetized coaxial plasma guns used as helicity injectors, in presence of a close conducting shell (see also Section 1.2).

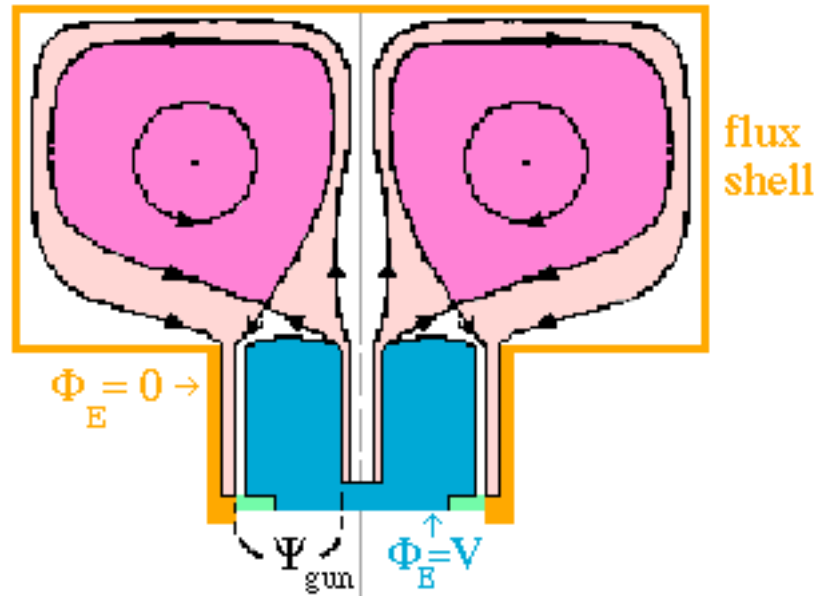


Fig. 4.19. Scheme of a spheromak formed by a coaxial plasma gun.

The main difference with PROTO-SPHERA, as far as the formation scheme is concerned, is:

- The magnetized coaxial plasma gun formation requires breakdown in small spaces, with extremely high filling pressures and kV voltages.
- This means that big amount of neutrals and impurities are released from the gun.
- After the formation, the spheromak is accelerated and expanded into a flux conserver.
- This means that the field errors already present in the gun are amplified.

PROTO-SPHERA will form instead at tokamak-like densities, with very low voltages (~100 V) and will not undergo any expansion.

The main difference with PROTO-SPHERA, as far as the q-value is concerned, is:

- Spheromak experiments have $q \sim 1$ everywhere.
- This means that too high MHD turbulence can be present.

PROTO-SPHERA has instead tokamak-like poloidal field coils, suited for $q_{95} \sim 3$.

Clearly the effective achievement of $q_{95} \sim 3$ depends upon the ratio $\beta_{\text{pinch}} / \langle \beta \rangle_{\text{ST}} \sim 3$ between the constant $\beta_{\text{pinch}} = \beta_0 \bar{\mathbf{j}} \cdot \bar{\mathbf{B}} / B^2$ in the screw pinch and the surface averaged $\langle \beta \rangle_{\text{ST}} = \beta_0 \langle \bar{\mathbf{j}} \cdot \bar{\mathbf{B}} / B^2 \rangle_{\text{ST}}$ in the spherical torus. The value of this ratio will be determined in the experiment by the efficiency of the helicity injection and by the relaxation of the whole plasma.

4.11. Rationale for the Size of PROTO-SPHERA

The first observation is that PROTO-SPHERA has as almost the same size of START. As a matter of fact the predicted performances: $\langle T_e \rangle = 140$ eV and $\tau_E^{LG} \sim 1.6$ ms at $\langle n_e \rangle = 0.5 \cdot 10^{20} \text{ m}^{-3}$ and $Z_{\text{eff}} = 2$, should be very similar to the ohmic results of START. This feature will allow for a clear-cut comparison between the results of PROTO-SPHERA with "plasma centerpost" and those of the spherical tori with metal centerpost.

The Lundquist number that PROTO-SPHERA will achieve, $S = \mu_r / \mu_a \geq 1 \cdot 10^5$, will be significant for the extrapolation to a larger and more fusion relevant Chandrasekhar-Kendall-Furth configuration experiment (Section 3).

The effect of a possible reduction of the size of PROTO-SPHERA must however be assessed. The reduction should be operated while keeping constant all the current densities (in the plasma as well as in the conductors). On the cathode cross-section the current density is kept to the level $j_c = I_c / (2\pi R_{cl} Z) \leq 80 \text{ A/cm}^2$ and within the poloidal field coils to the level $j_{PF} \leq 2 \text{ kA/cm}^2$.

A size reduction of PROTO-SPHERA by a factor 1.66 would reduce the size of the torus to that of TS-3, but would not yet yield a table top experiment, due to the additional size of the two separate electrodes chambers present in PROTO-SPHERA. This size reduction would then leave the costs almost unaffected, while it would reduce the space available inside the vacuum vessel, making very difficult to allocate all the required mechanical components. It would however reduce $\langle T_e \rangle$ to 100 eV and the MHD/confinement density limit to $\langle n_e \rangle = 2.6 \cdot 10^{19} \text{ m}^{-3}$, τ_E^{LG} to 0.31 ms and the Lundquist number to $S = 3.6 \cdot 10^4$, quite similar to TS-3.

A size reduction of PROTO-SPHERA by a factor 2.5 would yield a table top experiment, but with the size of the spherical torus smaller than that of TS-3. It could reduce the costs at the price of reducing $\langle T_e \rangle$ to 65 eV (with possible risks of radiation barriers), the MHD/confinement density limit to $\langle n_e \rangle = 1.8 \cdot 10^{19} \text{ m}^{-3}$, τ_E^{LG} to 80 μ s and the Lundquist number to $S = 9 \cdot 10^3$. The reduction by a factor of 2.5 could perhaps slash the costs by a factor of two, but the design and the construction would become pointlessly challenging. The machine in that case would become a miniature project, with a lot of difficulties in the cooling of the PF coils and of the electrodes. The power supply would be complicated in order to deal with formation times shorter than 50 μ s.

All these considerations show that the START size is the right size for PROTO-SPHERA.

4.13. A CKF Configuration inside PROTO-SPHERA?

If a method for injecting current (or torque) into a CKF unrelaxed configuration were developed, the load assembly of PROTO-SPHERA could host such a configuration, as shown in Fig. 4.20.

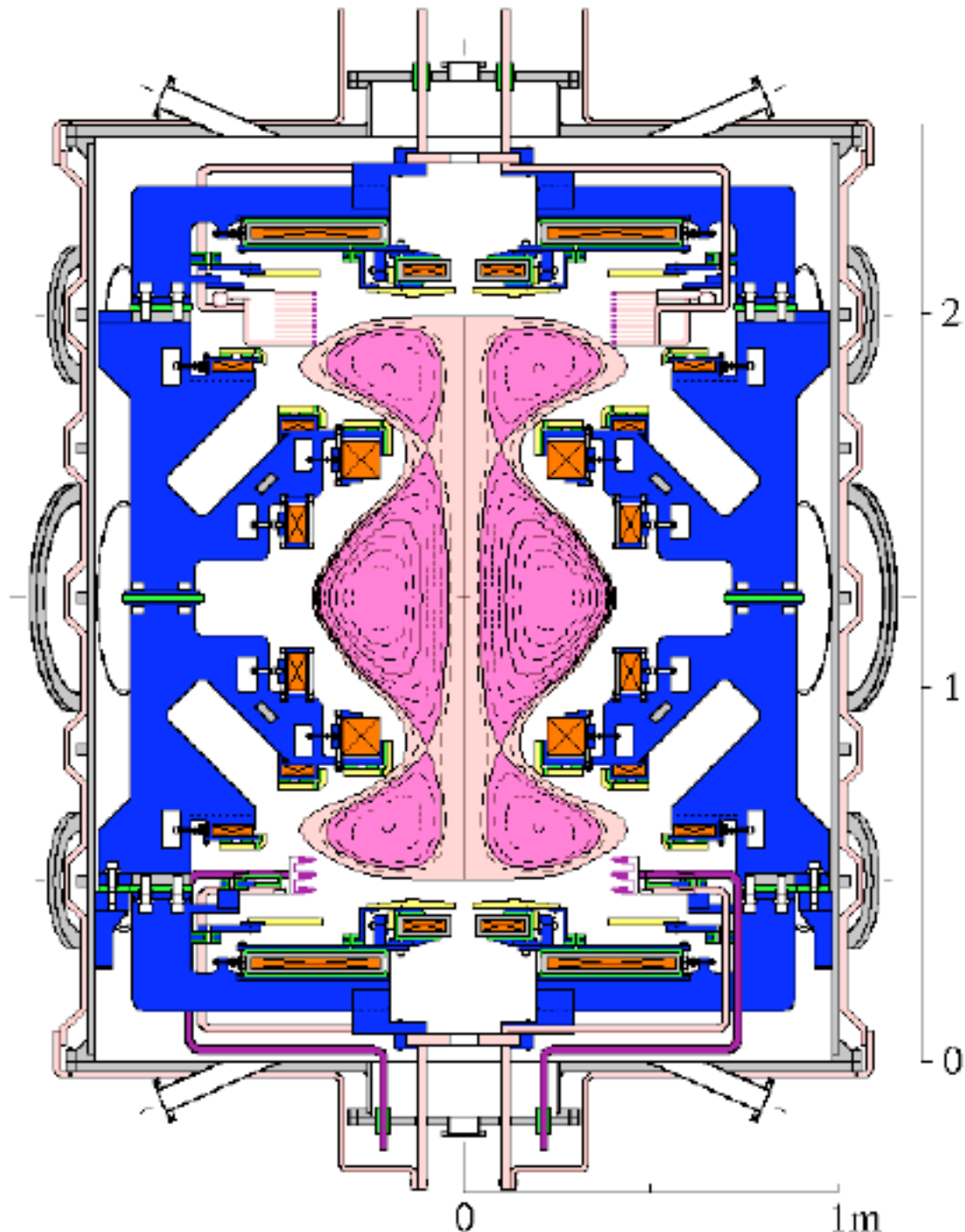


Fig. 4.20. Cross-section view of PROTO-SPHERA containing a CKF configuration inside a slightly modified load assembly.

The required modifications of the PF coils positions would be:

- The removal of the PF2 and PF3.1 "divertor poloidal field coils".
 - The removal of the PF6.2 focalizing coils.
 - The displacement of the PF6.1 focalizing coils near the PF1 compression coils.
 - Each poloidal field coil fed independently (6 power supplies instead of 2).
- The modified position of the PF coils is shown in Fig. 4.21 (compare with Fig. 4.4).

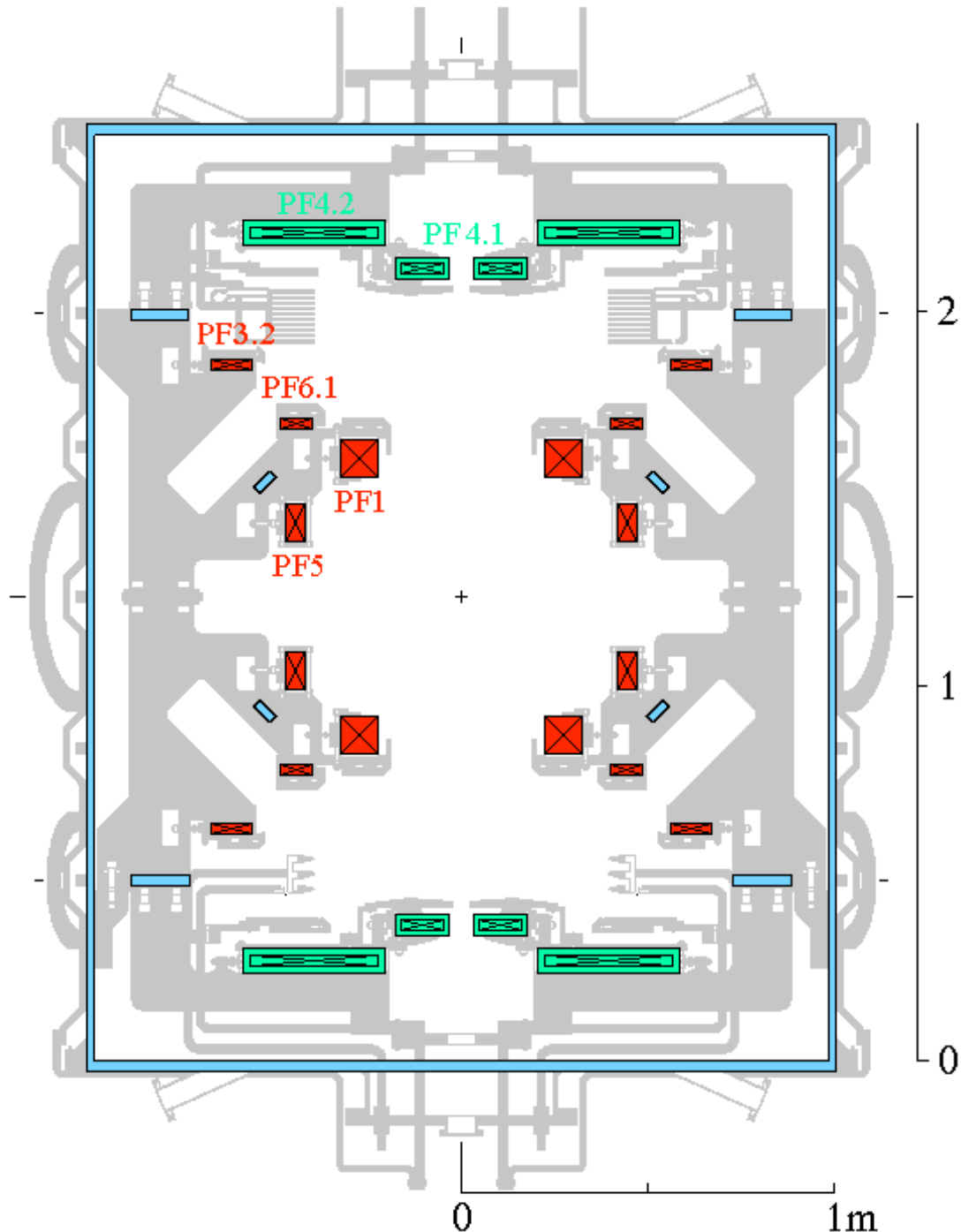


Fig. 4.21. Scheme of the poloidal field coils of modified PROTO-SPHERA.

Free-boundary equilibrium calculations, based on the poloidal flux function $\psi = 2\psi_{RA}\psi$, have been performed, using the following choices for the pressure and the diamagnetic current functions in the Grad-Shafranov equation:

$$\begin{aligned}
p(\psi) &= C_p \cdot \left\{ 1 + \frac{1}{2} \left(1 - \cos \left(\frac{\psi - \psi_c}{\psi_c} \right) \right) \right\} & \text{for } \psi < \psi_c & \text{ and} \\
p(\psi) &= C_p \cdot \{ 1 + \frac{1}{2} \} & \text{for } \psi \geq \psi_c & ; \\
I_{dia}(\psi) &= C_1 \cdot \left\{ \psi_{edge} - \psi_c + \left(2 \frac{\psi - \psi_c}{\psi_c} \right) \cdot \left[\cos \left(\frac{\psi - \psi_c}{2\psi_c} \right) - 1 \right] \right\} / \psi_0 & \text{for } \psi < \psi_c & \text{ and} \\
I_{dia}(\psi) &= C_1 \cdot \left\{ \psi_{edge} - \psi_c - \left(2 \frac{\psi - \psi_c}{\psi_c} \right) + (\psi_{edge} - \psi_c)(\psi - \psi_c) \right\} / \psi_0 & \text{for } \psi \geq \psi_c & .
\end{aligned}$$

The total toroidal current I_p flowing inside the ST is an input, along with the total

$$\text{poloidal beta of the ST } \beta_p^{ST} = \frac{2}{\psi_0 (I_p)^2} \frac{\int_V p dV}{V_p^{ST}} \frac{\oint_C \vec{e}_p \cdot d\vec{l}}{C^{ST}(\psi_x^T)^2} .$$

The two inputs I_p and β_p^{ST} determine, by iteration, the values of C_p , C_1 and the $\langle \psi \rangle$ profile. The case shown in Fig. 4.20 corresponds to the choices $\psi_p=5$, $\psi_c=4 \cdot \psi_x$ (where ψ_x is the poloidal flux function at the separatrix), $\psi=2$, $\psi_{edge}=14.066$, $\psi_0=9.5$ and $\beta_p^{ST}=0.06$. Assuming for the poloidal spheromak current the value $I_{se}=60$ kA (just equal to the longitudinal screw pinch current of PROTO-SPHERA), the total toroidal current in the main spherical torus would be $I_p=327$ kA (larger than in the case of PROTO-SPHERA, which has $I_p=120 \div 240$ kA flowing inside the ST).

The total current flowing inside the poloidal field coils would be $I_{PF} = \sum_{i=1}^{N_{PF}} |I_{PF}^i| = 296$ kA (see Fig. 4.22), almost equal to the total toroidal current inside the main spherical torus $I_p=327$ kA. The configuration is calculated to be ideal MHD stable at $\beta_{ST} = 2 \psi_0 \langle p \rangle_{ST} / \langle B^2 \rangle_{ST} = 50\%$.

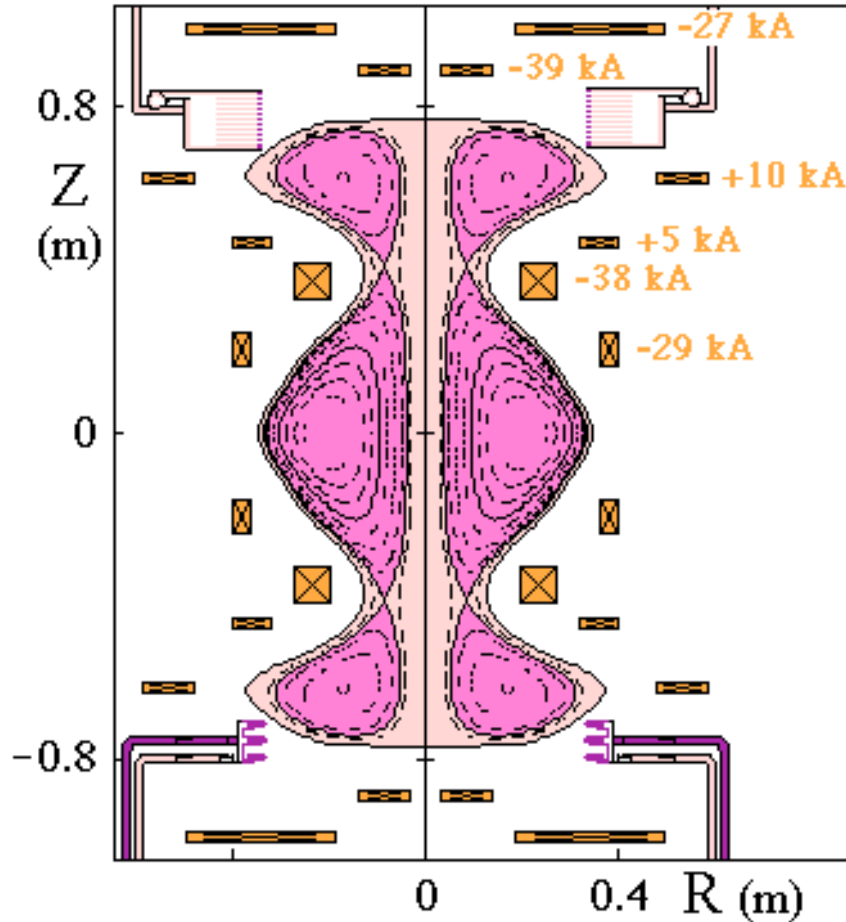


Fig. 4.22. PF coils currents required in the modified PROTO-SPHERA.

Electrodes (see Fig. 4.23) could produce the breakdown of the plasma, before the formation of the spherical torus. The total longitudinal pinch current is assumed to be, $I_c=8.5$ kA (like in the PROTO-SPHERA breakdown).

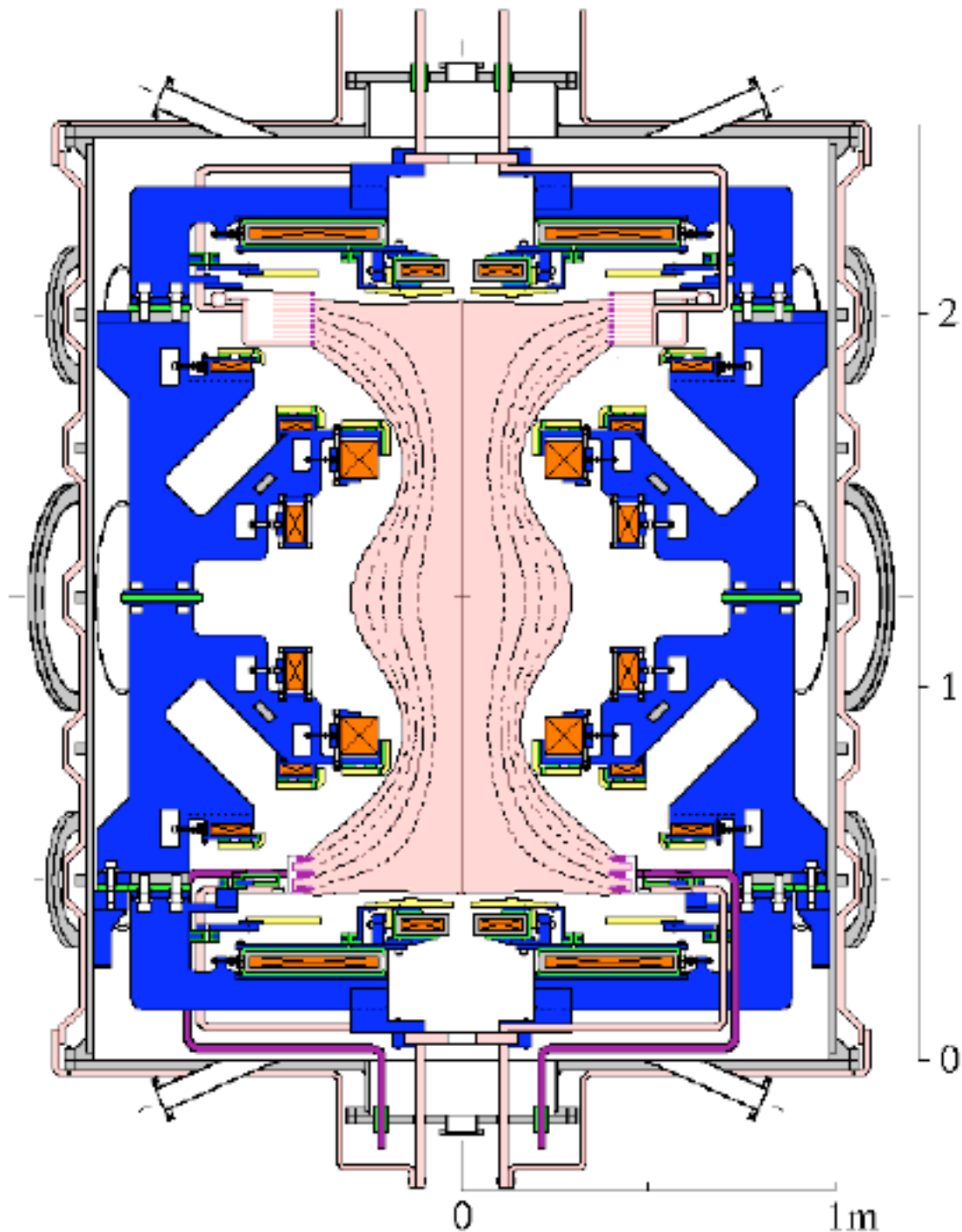


Fig. 4.23. Cross-section view of PROTO-SPHERA containing a CKF configuration inside a slightly modified load assembly at the plasma breakdown.

The breakdown would require some modifications of the electrodes:

- A slightly shorter distance between the electrodes: 1.54 m instead of 1.63 m.
- A larger vertical thickness of the electrodes: $\Delta Z=12$ cm instead of $\Delta Z=5.4$ cm, which can be obtained displacing the same emitting cathode filaments farther away from each other in the vertical direction and increasing the vertical width of the anode modules.

The total current flowing at the start-up inside the poloidal field coils would be $I_{PF} = \sum_{i=1}^{N_{PF}} |I_{PF}^i| = 234$ kA. Comparing the signs and magnitudes of the currents flowing inside the PF coils at the start-up (Fig. 4.24) with those of the final configuration (Fig. 4.22), it turns out that the sudden variation of these currents could provide approximately enough magnetic flux to the plasma as to induce the full toroidal plasma current of the final configuration.

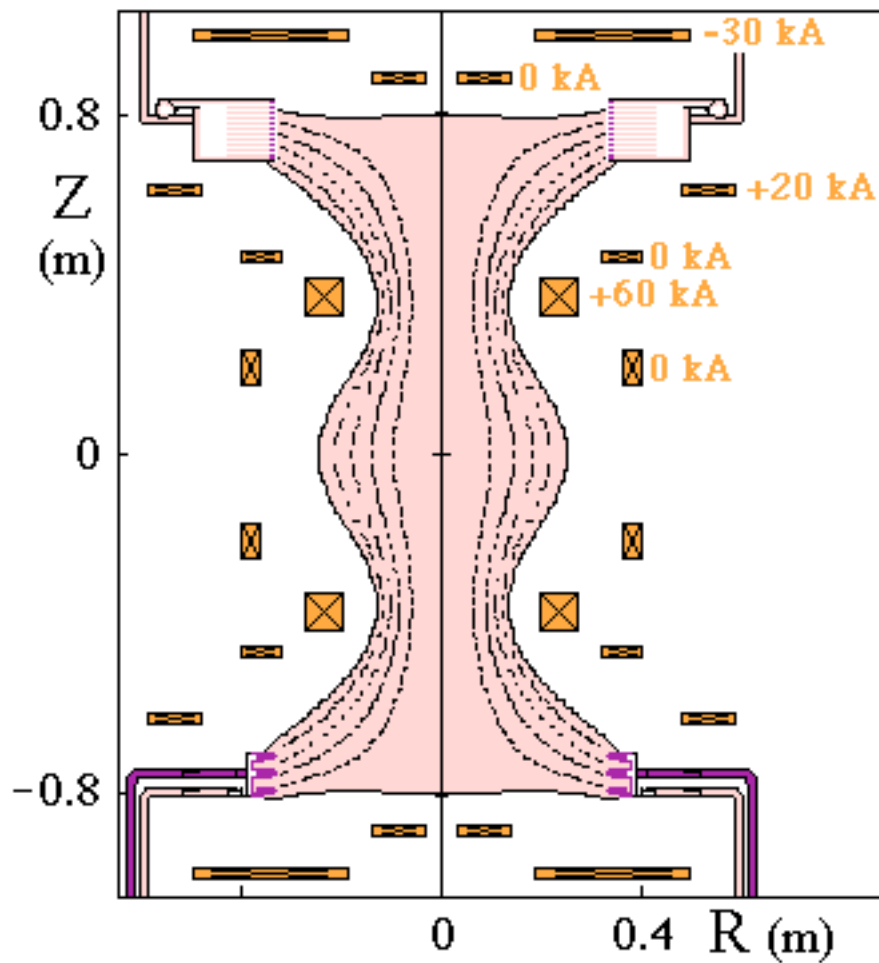


Fig. 4.24. PF coils currents required in the breakdown of the modified PROTO-SPHERA.

4.13. Characteristics of a CKF inside PROTO-SPHERA

Fig. 4.25 shows the profiles of flux function, pressure, diamagnetic current and toroidal current density inside the CKF unrelaxed configuration contained in the modified PROTO-SPHERA.

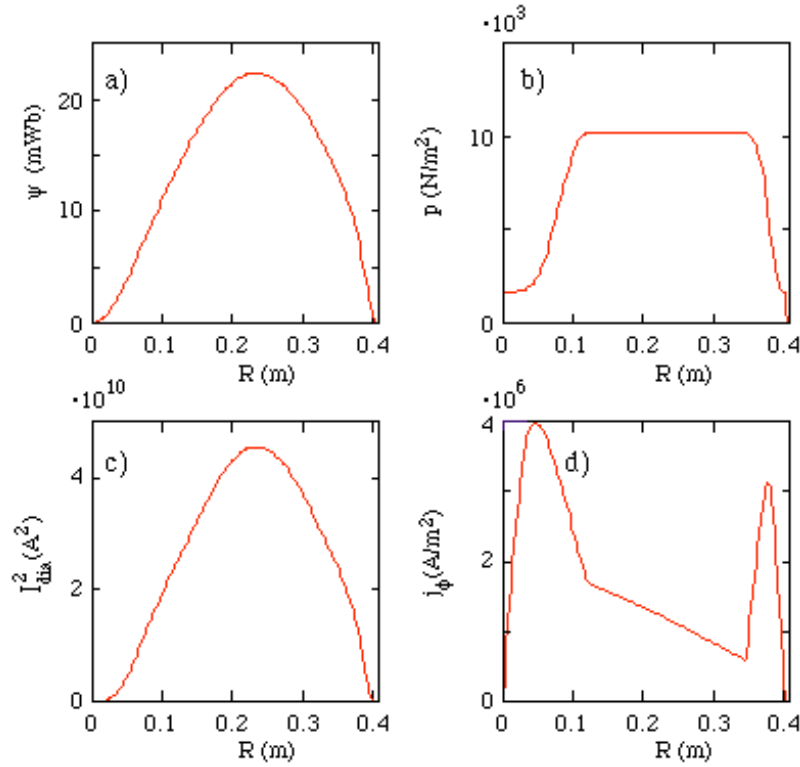


Fig. 4.25. CKF in the modified PROTO-SPHERA: $I_p=327\text{kA}$, $I_e=60\text{kA}$, $\beta_{ST}=50\%$.

Profiles on the equator of: a) Flux function ψ ; b) plasma pressure $p(\psi)$; c) diamagnetic current $I_{dia}^2(\psi)$ and d) toroidal current density j_ϕ .

The main differences with the corresponding quantities for PROTO-SPHERA, shown in Fig. 4.7, are due to the much larger beta value ($\beta_{ST}=2\beta_0<p>_{ST}/<B^2>_{ST}=50\%$ in the modified PROTO-SPHERA, compared to $\beta_{ST}\sim 20\%$ in PROTO-SPHERA).

The main spherical torus has a winding number (inverse of the rotational transform, $q=1/\bar{\nu}$), which is shown in Fig. 4.26. The values are $q_0^{ST}\sim 1.1$ on the magnetic axis and $q_{95}^{ST}\sim 2.0$ at the edge (95% of the poloidal flux of the magnetic separatrix).

This configuration has $\bar{\nu}=\bar{\nu}_0\bar{\mathbf{j}}\cdot\bar{\mathbf{B}}/B^2=\text{constant}$ only at the edge of the plasma ($\bar{\nu}_T=\bar{\nu}_T^{\text{max}}$), as a boundary condition for the MHD equilibrium. The surface averaged value $\langle\bar{\nu}\rangle=\bar{\nu}_0\langle\bar{\mathbf{j}}\cdot\bar{\mathbf{B}}/B^2\rangle$ drops from $\langle\bar{\nu}\rangle=25$, at the edge of the surrounding spheromak, down to $\langle\bar{\nu}\rangle=8$, on the axis of the main spherical torus (see Fig. 4.27). Such $\langle\bar{\nu}\rangle$ profile corresponds to a sustainment obtained by driving current on the flux surfaces of the surrounding spheromak.

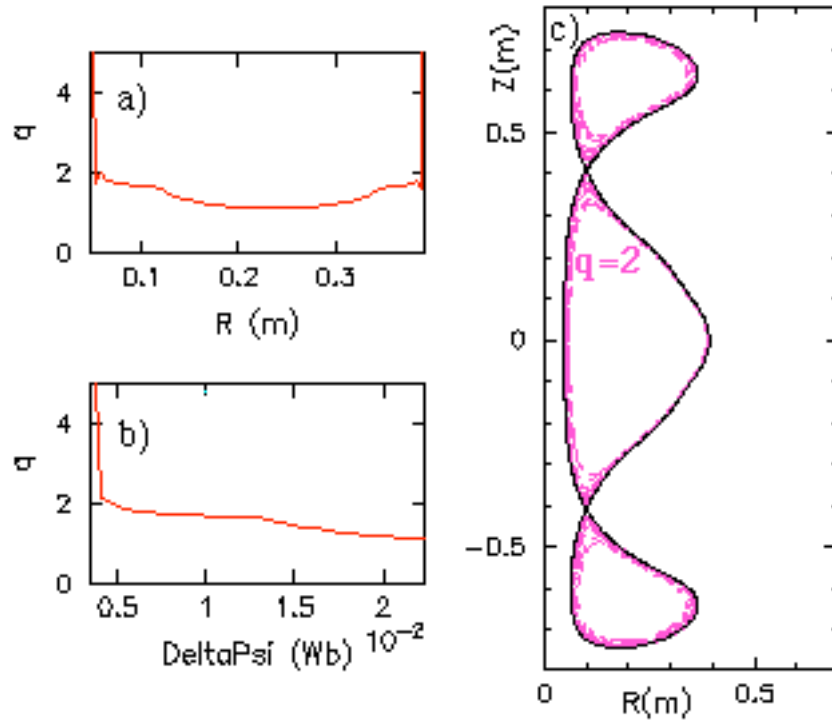


Fig. 4.26. CKF in the modified PROTO-SPHERA: $I_p=327\text{kA}$, $I_e=60\text{kA}$, $\beta_{ST}=50\%$. Winding number q as a function of: a) the major radius on the equator and b) the poloidal flux function inside the main spherical torus. c) Contour plot of q .

The gradient of the pressure profile (see Fig. 4.25.b) is assumed to be concentrated in the same region where the gradient of $\langle\beta\rangle$ has the largest variation.

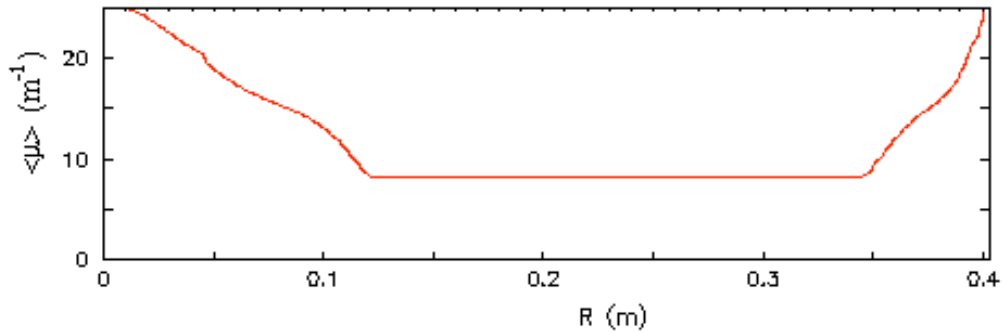


Fig. 4.27. CKF in the modified PROTO-SPHERA: $I_p=327\text{kA}$, $I_e=60\text{kA}$, $\beta_{ST}=50\%$. Profile of the relaxation parameter $\langle\mu\rangle$ (m⁻¹) on the equatorial plane.

5. IDEAL MHD STABILITY

The purpose of this Section is to analyze the ideal MHD stability of the combined screw pinch + spherical torus configuration of PROTO-SPHERA. Although finite amplitude resistive MHD instabilities are required to produce the helicity flow from the screw pinch to the spherical torus, the combined configuration must be stable in the ideal MHD framework. If it were unstable any resistive local finite amplitude displacement would turn into an exponentially growing global ideal MHD mode.

The formation of the PROTO-SPHERA configuration can be described as a tunneling from an ideal MHD stable configuration (a screw pinch at low longitudinal current) to a different ideal MHD stable configuration (a screw pinch at high longitudinal current surrounded by a spherical torus). The tunneling could occur through an ideal MHD unstable region.

The first analysis is that of the stability to rigid vertical shift and tilt displacements. Such an evaluation does not yield a rigorous result, but provides useful indications about two of the most dangerous ideal MHD instabilities, which could be present during the formation of PROTO-SPHERA. The first result is that the metal cases of the poloidal field coils with constant current are sufficiently thick as to stabilize the rigid vertical displacement of the spherical torus during the formation phase. The second result is that the external field, produced by the poloidal field coils, is such as to stabilize the rigid tilt displacement of the spherical torus during the formation phase, even if the effect of the thick metal cases is neglected.

New finite element method ideal MHD stability codes have been developed in order to analyze the combined screw pinch + spherical torus configuration of PROTO-SPHERA. As a matter of fact, no existing code was able to compute the ideal MHD stability of a configuration in which closed and open field lines did coexist. A number of innovative features are contained in these codes.

These codes calculate an ideal MHD upper limit to the value of the ratio between the toroidal ST current I_p and the longitudinal SP current I_e . Such a limit depends upon the volume averaged beta value of the spherical torus, $\beta_{ST} = 2\mu_0 \langle p \rangle_{ST} / \langle B^2 \rangle_{ST}$. With $\beta_{ST} \sim 30\%$ the ideal stability of the configuration is limited to $I_p = I_e$, with $\beta_{ST} \sim 20\%$ I_p can reach a value of $2 \div 3 \cdot I_e$ and with $\beta_{ST} \sim 10\%$ I_p can reach a value of $4 \cdot I_e$ (the design limit of PROTO-SPHERA). The position of the conducting shells near the plasma does not seem to be critical for the ideal MHD stability results, which do not change even removing all the conductors to infinite distances from the plasma.

5.1. Ideal MHD Stability to Rigid Shift and Tilt Displacements of PROTO-SPHERA

The first approach to the problem of the MHD stability of the PROTO-SPHERA configurations is to estimate their stability with respect to rigid vertical shift and to rigid tilt displacements. This approach is not rigorous, as it assumes arbitrarily the plasma perturbed displacement and does not search for the most unstable trial function.

Nevertheless it gives useful indications, as the rigid tilt mode approximates one of the most dangerous instability of a spheromak equilibrium confined by poloidal field coils [80]. The MHD equilibrium requires the magnetic dipole moment of the toroidal plasma to oppose the magnetic dipole moment of the confining poloidal field coils. An instability will try to align the plasma dipole, but the alignment of the dipoles is obviously incompatible with the MHD equilibrium.

Also the rigid vertical instability is well known in the physics of tokamak configurations and makes impossible to exceed, in particular at medium/high aspect ratio, a limit to the elongation of the plasma poloidal cross-section. In the case of PROTO-SPHERA the toroidal plasma formation begins with an aspect ratio of about $A=2$ and with an elongation of about $\kappa=2$ (see Fig. 4.14). Due to the presence of only two groups of serial poloidal field coils it is impossible to reduce the elongation at the beginning of the formation. Therefore the formation phase of the spherical torus could be plagued by a vertical instability.

Operating rigid vertical shift and rigid tilt displacements of all the poloidal field coils and computing the reaction force and torque acting on the unperturbed plasma performs the analysis:

$$\vec{F} = \int_{\text{Plasma}} \vec{j} \cdot \nabla \vec{B} \, dV \quad , \quad \vec{T} = \int_{\text{Plasma}} \vec{j} \times \nabla \vec{B} \, dV \quad ,$$

where $\nabla \vec{B}$ is the $\vec{B}$ change due to the rigid displacement of the coils.

This calculation could overestimate the stability, as the most unstable modes will not be rigid plasma displacements, but it could also underestimate the stability, as the stabilizing effect of the vacuum magnetic energy perturbation is not accounted for.

In the evaluation of the stability to rigid displacements the simplest choice is that of neglecting the effect of the conducting shells around the plasma; in this case the picture of the rigid vertical shift and of the rigid tilt is the one shown in Fig. 5.1.

The effect of the thick metal cases (2 ms time constant) of the poloidal field coils of group "B" (PF2, PF3.1 and PF4) can instead be accounted for, by freezing the shape of the plasma disks near the electrodes. This means that in the two separate electrode chambers the perturbed displacement is zero, as shown in Fig. 5.2. In the case of the vertical displacements, the induced axisymmetric reacting currents in the metal cases produce this effect. Also the tilt instability is counter-reacted by the closed conducting path that can join the electrodes to the PF4 metal cases. This path can allow a freezing effect due to tilted reacting currents.

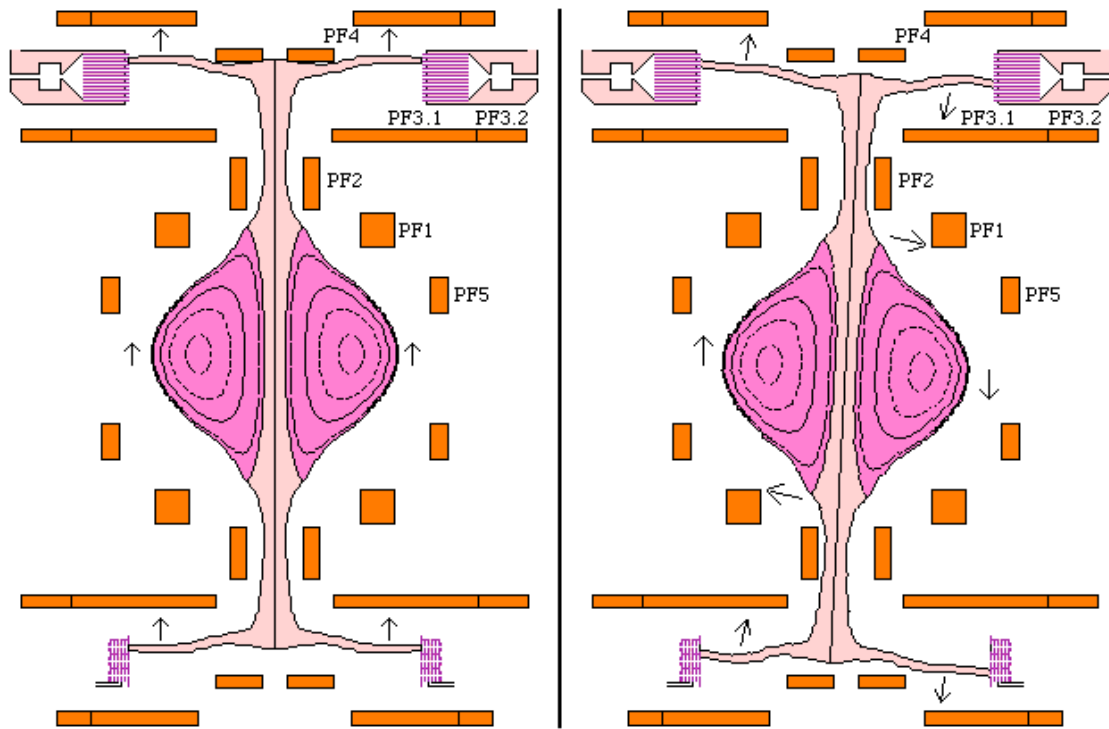


Fig. 5.1. Rigid vertical shift and tilt displacements of PROTO-SPHERA.

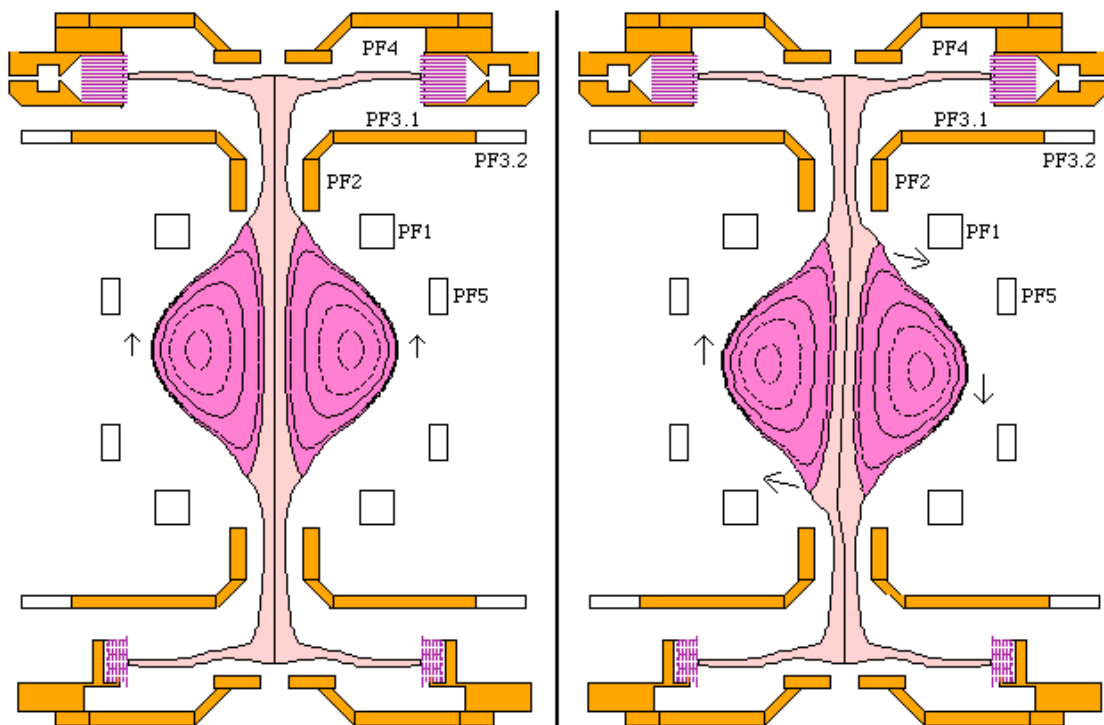


Fig. 5.2. Vertical shift and tilt displacements of PROTO-SPHERA with frozen plasma disks.

In the case of frozen plasma disks, operating a rigid vertical/horizontal shift and a rigid tilt displacement limited to the PF1, PF5 and PF3.2 poloidal field coils and computing the reaction force and torque acting on the unperturbed plasma performs the analysis. The results of both cases are shown in Tab. 5.1.

Shift $\Delta Z=4$ mm,			Tilt $\Delta\theta=0.5^\circ$	
$F_z=+38.9$ N	Unstable	$I_p=30$ kA	$T_y=-2.25$ N·m	Stable
$F_z=-9.92$ N Stabilized ($t<2$ ms) by thick metal cases of PF2,3.1 and 4				
$F_z=+19.7$ N	Unstable	$I_p=60$ kA	$T_y=-9.22$ N·m	Stable
$F_z=-23.5$ N Stabilized ($t<2$ ms) by thick metal cases of PF2,3.1 and 4				
$F_z=-6.9$ N	Stable	$I_p=120$ kA	$T_y=-12.0$ N·m	Stable
Stabilization by thick metal cases of PF2,3.1 and 4 is not needed				
$F_z=-68.1$ N	Stable	$I_p=240$ kA	$T_y=-24.5$ N·m	Stable
Stabilization by thick metal cases of PF2,3.1 and 4 is not needed				

Table 5.1. Results of the evaluation of the stability to rigid shift and tilt displacements.

The shift results mean that the thick metal cases of PF2, PF3.1 and PF4, with a time constant of 2 ms, are sufficient to stabilize the rigid vertical instability, which could operate during the first 500 μ s of the toroidal plasma formation.

The tilt results mean that the magnetic dipole moment of PF2 and PF3 provides the larger part of the disk-shaping field near the electrodes. It can stabilize the rigid tilt instability, as it is aligned with the plasma magnetic dipole moment and dominates over the opposite (destabilizing) dipole moment of PF1, PF5 and PF4.

5.2. Rigorous Ideal MHD Stability

New ideal MHD stability codes, suited for treating magnetic configurations with closed and open field lines, have been built in collaboration with François Rogier (ONERA, Toulouse, France), under an EURATOM mobility scheme. The codes have been validated on the well-known stability results of analytic Solovév tokamak equilibria [93], with fixed boundary conditions at the edge of the plasma, as well as with free boundary conditions in presence of vacuum regions surrounding the plasma. The codes contain a number of new features:

- Boozer coordinates on open field lines are defined and continuously joined to the closed field lines Boozer coordinates at the spherical torus-screw pinch (ST-SP) interface.
- Treatment of the magnetic separatrix at the ST-SP interface.
- Boundary conditions at the ST-SP interface.
- Perturbed vacuum magnetic energy in presence of multiple plasma boundaries.
- 2D finite element method for accounting the perturbed vacuum energy.

5.3. Boozer Coordinates

The Boozer coordinates [87] (φ , θ , ψ) (for closed field lines) simplify the expression of the ideal MHD stability problem [86]. A list of definitions of quantities appearing in the Boozer coordinates follows:

- Radial coordinate $\varphi_T = (\text{toroidal flux})/2\pi$. Rotational transform: $\varphi(\varphi_T) = 1/q$.
- Covariant field: $\vec{B} = \varphi_* \vec{e}_\varphi + I \vec{e}_\theta + f \vec{e}_\psi$. Nonorthogonality term: $\varphi_* = \varphi_*(\varphi_T, \psi)$.
- Normalized toroidal and poloidal currents: $I(\varphi_T) = I_0 I_p / 2\pi$; $f(\varphi_T) = R B_\varphi = I_0 I_{dia} / 2\pi$.
- Contravariant magnetic field: $\vec{B} = \varphi_* \vec{e}_\varphi + I \vec{e}_\theta + f \vec{e}_\psi$.

The Jacobian, $\sqrt{g} = [f(\varphi_\psi) + \varphi(\varphi_\psi) I(\varphi_\psi)] / B^2$, fixes the poloidal angle ψ (see Fig. 5.3) and the toroidal angle φ (which does not coincide with the geometrical azimuth φ_G).

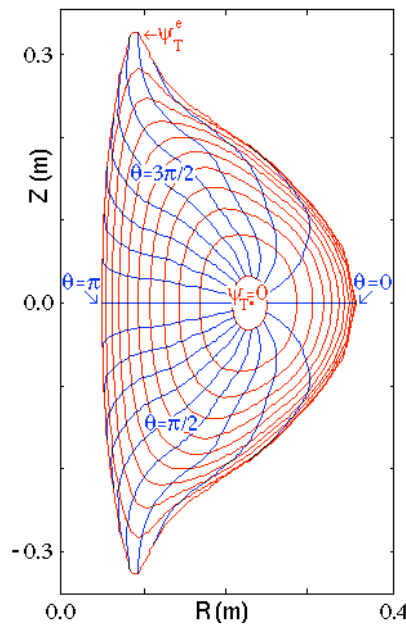


Fig. 5.3. Radial coordinate φ_T and poloidal angle ψ for the ST of PROTO-SPHERA.

5.4. Boozer Coordinates on Open Field Lines

The combined screw pinch + spherical torus configuration is analyzed in term of the poloidal flux function $\psi = 2\pi R A_\phi = 2\pi R A_\psi$ (see Fig. 5.4):

- The open field lines in the screw pinch (SP) span the range $(0 < \psi < \psi_x)$.
- The closed field lines inside the spherical torus (ST) span the range $(\psi_x < \psi < \psi_{\max})$;

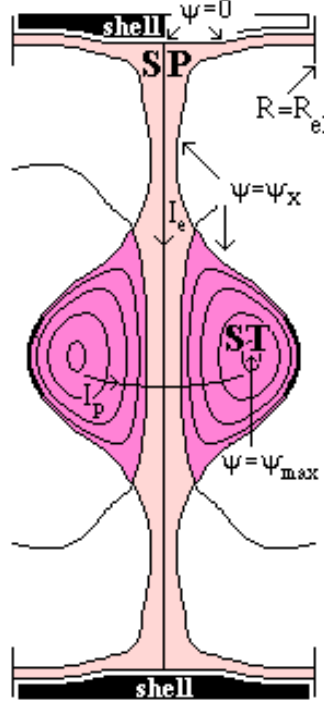


Fig. 5.4. Equilibrium of PROTO-SPHERA: values of the poloidal flux function ψ .

The following conditions are used to extend the Boozer coordinates into the pinch:

- The continuity of $\bar{\psi}$, $\bar{\psi}_T$, $\bar{\psi}$ and $\bar{\psi}$ is imposed at the ST-SP interface ($\bar{\psi} = \bar{\psi}_x$).
- At the ST-SP interface the field lines $\bar{\psi}_0 = \bar{\psi}(\bar{\psi})$ must remain contiguous.
- The continuity of the rotational transform $\bar{\omega}(\bar{\psi}) = \bar{\omega}_x$ is imposed in $\bar{\psi} = \bar{\psi}_x$.
- The continuity of $I(\bar{\psi}) = I_x$ is imposed in $\bar{\psi} = \bar{\psi}_x$, while $f(\bar{\psi})$ is continuous from its equilibrium definition (see Section 4.5).

The Boozer poloidal angle $\bar{\psi}$, toroidal angle $\bar{\phi}$ and radial coordinate $\bar{r}_T$ inside the screw pinch are found with the following procedure:

- 1) The Boozer poloidal angle is fixed to $\bar{\psi}$ on the equator. The length parameter s on the screw pinch side of the magnetic separatrix is calculated, with $s=0$ on the lower electrode and $s=s_{eq}$ on the equatorial plane.
- 2) The length parameter $s=s_0$ at which $\bar{\psi}=0$ on the screw pinch side of the interface ($\bar{\psi} = \bar{\psi}_x - \bar{\psi}_{sp}$) has to be determined:

- Line integral definitions for $I(\psi)$ and $\mathcal{I}(\psi_T)$ are provided inside the SP:

$$I(\psi) = \frac{1}{\psi} \int_{s_0(\psi)}^{s_{eq}(\psi)} \mathbf{B}_p \cdot \hat{\mathbf{e}}_p \cdot d\vec{\mathbf{l}}_p, \quad \mathcal{I}(\psi) = \frac{\psi}{f(\psi) \int_{s_0(\psi)}^{s_{eq}(\psi)} \frac{1}{R^2 B_p} \hat{\mathbf{e}}_p \cdot d\vec{\mathbf{l}}_p};$$

the latter goes to zero (see Fig. 5.5) as $\mathcal{I}(\psi) \propto \sqrt{|\psi - \psi_X|}$ in a very narrow layer near the separatrix $|\psi - \psi_X| \sim 10^{-5} \cdot |\psi_{max} - \psi_X|$.

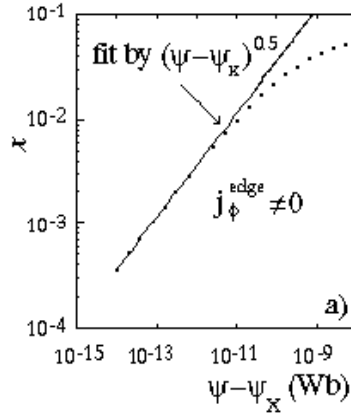


Fig. 5.5. Behavior of $\mathcal{I}(\psi)$ near the separatrix in an ST-SP combined configuration.

- On the ST side ψ_X can be calculated at $\psi = \psi_X + \psi_{ST}$, with $\psi_{ST} \sim 10^{-3} \cdot |\psi_{max} - \psi_X|$.
- On the SP side $\psi = \psi_X$ can also be calculated at $\psi = \psi_X - \psi_{SP}$, but with $\psi_{SP} \neq \psi_{ST}$.
- Given I_X and $\mathcal{I}_X$, ψ is evaluated on the ST-SP interface as:

$$\psi(s) = \psi \left[\frac{\mathcal{I}_X}{f(\psi_X) + \mathcal{I}_X I_X} f^2(\psi_X) \int_s^{s_{eq}(\psi_X - \psi_{SP})} \frac{1}{R^2 B_p} \hat{\mathbf{e}}_p \cdot d\vec{\mathbf{l}}_p + \int_s^{s_{eq}(\psi_X - \psi_{SP})} \mathbf{B}_p \cdot \hat{\mathbf{e}}_p \cdot d\vec{\mathbf{l}}_p \right].$$

This determines the value $s = s_0(\psi_X - \psi_{SP})$, where $\psi = 0$.

- $I(\psi)$ and $\mathcal{I}(\psi)$ can be determined inside the SP if the value of $s_0(\psi)$, for all the flux surfaces in the range $(0 < \psi < \psi_X - \psi_{SP})$, is calculated:

- $s_0(\psi)$ is imposed by the equilibrium relation for a force-free magnetic field: $df/d\psi_T + \mathcal{I}dI/d\psi_T = 0$.
- This provides the first order integro-differential equation for $s_0(\psi)$:

$$\frac{\partial(s_0(\psi))}{\partial\psi} = \frac{\psi_p(s_{eq}(\psi)) \frac{\partial s_{eq}(\psi)}{\partial\psi} + \frac{\psi_0 I_e}{2\psi_X} \int_{s_0(\psi)}^{s_{eq}(\psi)} \frac{B_T}{RB_p} ds + \int_{s_0(\psi)}^{s_{eq}(\psi)} \frac{\partial B_p}{\partial\psi} ds}{\mathcal{I}_p(s_0(\psi))},$$

solved iteratively in the range $[0 \leq \psi \leq \psi_X]$, with boundary condition $s_0(\psi_X - \psi_{SP})$.

This fixes the value of $s_0(\psi)$.

- Knowing $I(\psi)$ and $\mathcal{I}(\psi)$, the behavior of the poloidal angle φ inside the ST is:

$$\varphi(\psi, s) = \varphi \left[\frac{\mathcal{I}(\psi)}{f(\psi) + \mathcal{I}(\psi) I(\psi)} f^2(\psi) \int_s^{s_{eq}(\psi)} \frac{1}{R^2 B_p} \hat{\mathbf{e}}_p \cdot d\vec{\mathbf{l}}_p + \int_s^{s_{eq}(\psi)} \mathbf{B}_p \cdot \hat{\mathbf{e}}_p \cdot d\vec{\mathbf{l}}_p \right]$$

The solution found has the unavoidable feature that the Boozer poloidal angle on the lower electrode is not a fixed value: $\varphi_l(\psi) \neq \varphi_l(\psi_X)$, if $\psi \neq \psi_X$.

Figure 5.6 shows that this procedure makes the Boozer poloidal angle θ quite accurately joined at the ST-SP interface. This interface corresponds, upon the magnetic separatrix, to the range of Boozer poloidal angles $[\theta_x < \theta < 2\pi - \theta_x]$, where θ_x represents the point where the outermost magnetic surface of the lower part of the screw pinch reaches its largest R value near the X-point.

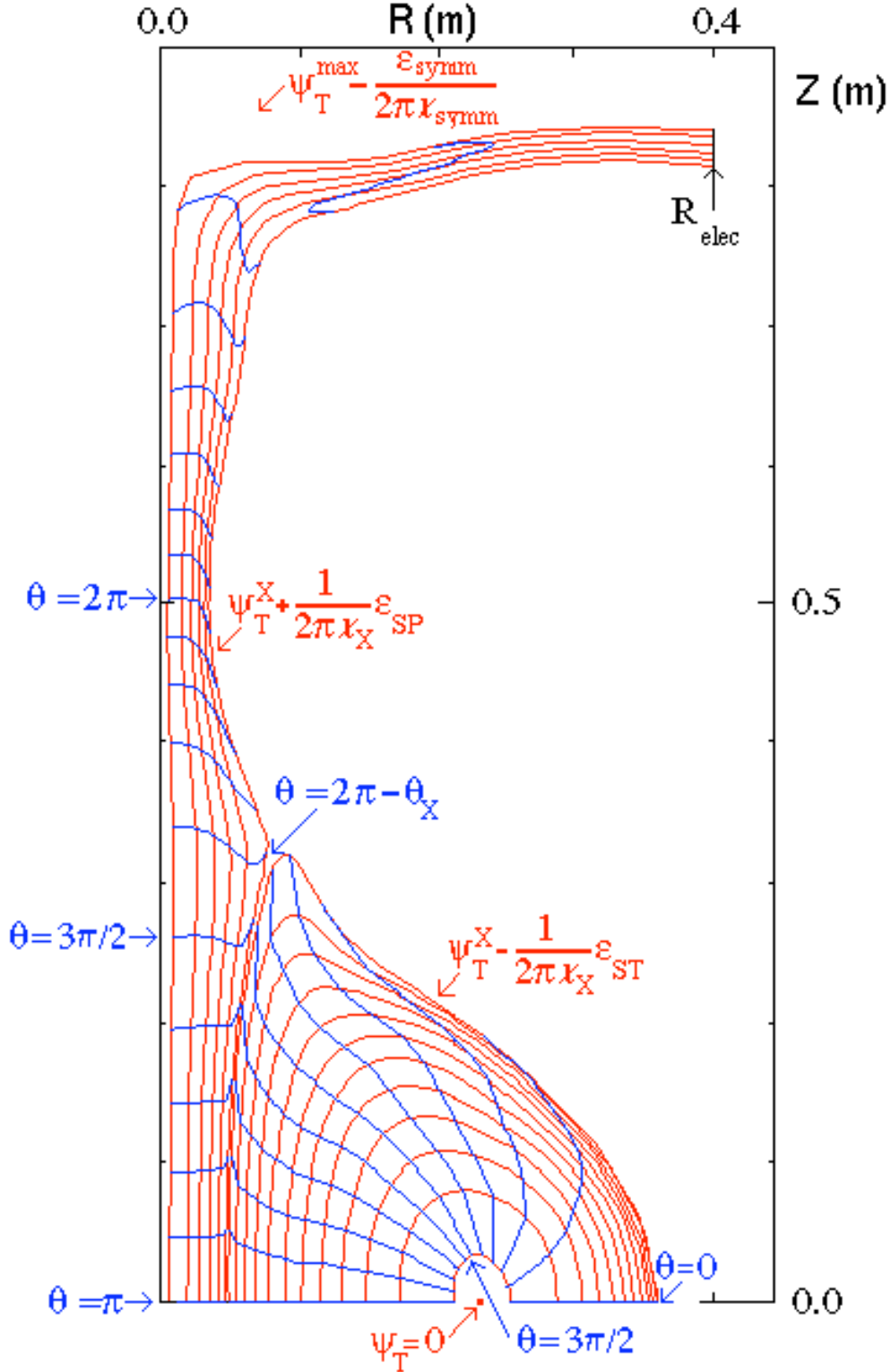


Fig. 5.6. Boozer coordinates for PROTO-SPHERA.

- 5) The Boozer toroidal angle φ is finally:

$$\varphi(\varrho, s) = \varphi_G + \frac{\varphi(\varrho)}{f(\varrho) + \varphi(\varrho)I(\varrho)} \int_{s(\varrho)}^{s_{eq}(\varrho)} \frac{1}{R^2 B_p} \hat{\mathbf{e}}_p \cdot d\vec{\mathbf{l}}_p - \int_{s(\varrho)}^{s_{eq}(\varrho)} \mathbf{B}_p \hat{\mathbf{e}}_p \cdot d\vec{\mathbf{l}}_p$$

- 6) The radial Boozer coordinate ϱ_T inside the screw pinch is defined as:

$$\varrho_T = \varrho_T^x + \frac{1}{2\varrho} \int_{\varrho}^{\varrho^x} \frac{1}{\varphi(\varrho)} d\varrho.$$

This definition is based upon the relationship $\varphi(\varrho) = -d\varphi_p/d\varrho_T$, which holds also inside the ST. However it must be noted that within the SP the radial Boozer coordinate is not anymore the normalized toroidal flux: $2\varrho\varrho_T \neq \int \vec{\mathbf{B}} \cdot d\vec{\mathbf{S}}_T$, evaluated on the surface bounded by the poloidal contour composed by the open flux surface ϱ , the electrodes ($R=R_{el}$) and the symmetry axis $\varrho=0$. The radial Boozer coordinate ϱ_T takes the value 0 on the magnetic axis of the spherical torus. It increases to $\varrho_T = \varrho_T^x$ on the magnetic separatrix and then reaches the maximum value $\varrho_T = \varrho_T^{max}$ on the symmetry axis (see Fig. 5.7).

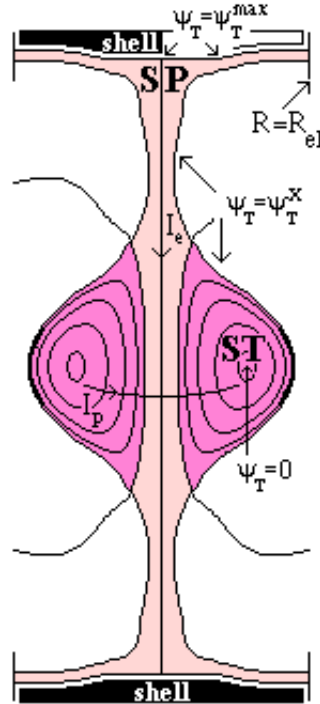


Fig. 5.7. Equilibrium of PROTO-SPHERA: values of the Boozer radial coordinate ϱ_T .

5.5. Energy Principle

The linearized normal-mode equation describing the ideal MHD stability can be expressed in a variational form [85]. Considering the displacement vector $\vec{\xi}$ away from the equilibrium ($\vec{\xi}^*$ its complex conjugate), with time dependence $e^{-i\omega t}$, the perturbed kinetic energy of a plasma with scalar mass density ρ_0 is:

$$\Delta W_k(\vec{\xi}^*, \vec{\xi}) = \frac{1}{2} \int_{V_p} dV \rho_0 (\vec{\xi}^* \cdot \vec{\xi}) .$$

And the perturbed potential magnetic energy of the plasma is:

$$\Delta W_p(\vec{\xi}^*, \vec{\xi}) = \frac{1}{2} \int_{V_p} dV \vec{\xi}^* \cdot \vec{F}(\vec{\xi}) ,$$

with $\vec{F}(\vec{\xi})$ being the self-adjoint force operator. The variational principle states that any function $\vec{\xi}$, which makes stationary the quotient:

$$\frac{\Delta W_p(\vec{\xi}^*, \vec{\xi})}{\Delta W_k(\vec{\xi}^*, \vec{\xi})} = \omega^2(\vec{\xi}^*, \vec{\xi}) = \omega^2 ,$$

is an eigenfunction of the normal-mode equation with eigenvalue ω^2 .

For an arbitrary displacement $\vec{\xi}$ the perturbed magnetic field is $\vec{Q} = \vec{\xi} \times (\nabla \times \vec{B})$ and the energy principle is written as [94]:

$$\Delta W_p = \frac{1}{2} \int_{V_p} dV \left[\vec{C}^2 + \rho_0 (\vec{\xi} \cdot \vec{\xi})^2 + D (\vec{\xi} \cdot \vec{\xi} \times \vec{T})^2 \right] , \quad \text{with } \gamma = 5/3,$$

$$\vec{C} = \vec{\xi} \times (\nabla \times \vec{B}) + \rho_0 \frac{(\vec{j} \times \vec{\xi} \times \vec{T})}{|\vec{\xi} \times \vec{T}|^2} \vec{\xi} \cdot \vec{\xi} \times \vec{T} \quad \text{and} \quad D = 2 \frac{(\vec{j} \times \vec{\xi} \times \vec{T})}{|\vec{\xi} \times \vec{T}|^2} \cdot (\vec{B} \cdot \vec{\xi}) \frac{\vec{\xi} \cdot \vec{\xi} \times \vec{T}}{|\vec{\xi} \times \vec{T}|^2} .$$

It is convenient to decompose the displacement $\vec{\xi}$ in Boozer coordinates, in terms of its normal $\xi^\perp = \vec{\xi} \cdot \vec{\xi} \times \vec{T}$, binormal $\xi^\parallel = \vec{\xi} \cdot (\vec{\xi} \times \vec{T})$ and parallel $\xi^\parallel = \sqrt{g} \vec{\xi} \cdot \vec{\xi}$ components.

The compressible perturbed displacement $(\xi^\perp, \xi^\parallel, \xi^\parallel)$ is expanded in a trigonometric Fourier series; every mode has a single toroidal number n and is a superposition of poloidal numbers m_ℓ , each labeled by an index ℓ :

$$\begin{aligned} \xi^\perp &= \sum_\ell \xi_\ell(\varphi_T) \sin(m_\ell \varphi - n\varphi), & \xi^\parallel &= \sum_\ell \xi_\ell(\varphi_T) \cos(m_\ell \varphi - n\varphi), \\ \xi^\parallel &= \sum_\ell \xi_\ell(\varphi_T) \cos(m_\ell \varphi - n\varphi). \end{aligned}$$

The reduction to a sine component for $\xi^\perp$ and to a cosine component for $\xi^\parallel$ and $\xi^\parallel$ is permitted if up-down symmetric equilibria are assumed, as is the case in the combined ST+SP configurations of PROTO-SPHERA.

By using the Fourier expansions, the perturbed kinetic energy ΔW_k is expressed as a quadratic form of the displacements ξ_r , ξ_ℓ and ξ_ℓ . The perturbed potential magnetic energy ΔW_p is expressed as a quadratic form of the displacements ξ_r , ξ_ℓ and ξ_ℓ and of the radial derivative of the normal displacement $\partial \xi / \partial \varphi_T$.

For the stability calculation the problem is discretized radially covering the $\square_T$ interval inside the ST $[0, \square_T^X]$ by an equidistant mesh with the mesh points $\square_T^i = i \square_T^{ST}$, for $i=0, \dots, N_\square^{ST}$, where $N_\square^{ST} \square_T^{ST} = \square_T^X - \square_{\text{symm}}/2$. The value $\square_T^X$ of the magnetic separatrix is carefully excluded from the mesh in order to avoid the singularity of the Boozer coordinates. Also the degenerate X-points sitting on the symmetry axis $\square_T = \square_T^{\max}$ (see Fig. 5.7) is excluded from the mesh, by putting the last mesh point with $i=N_\square^{ST} + N_\square^{SP}$ at $\square_T = \square_T^{\max} - \square_{\text{symm}}/2$. Inside the SP the $\square_T$ interval $[\square_T^X, \square_T^{\max}]$ is covered by an equidistant mesh with the mesh points $\square_T^i = \square_T^X + \square_{\text{SP}}/2 + (i - N_\square^{ST} - 1) \square_T^{SP}$, for $i = N_\square^{ST} + 1, \dots, N_\square^{ST} + N_\square^{SP}$, where $(N_\square^{SP} - 1) \square_T^{SP} = (\square_T^{\max} - \square_T^X) - \square_{\text{SP}}/2 - \square_{\text{symm}}/2$.

The radial behavior of $\square(\square_T)$, $\square_\ell(\square_T)$, $\square_\ell(\square_T)$ and $\partial \square / \partial \square_T$ is approximated by a one-dimensional finite hybrid element method [88]. For $\square(\square_T)$ the hat functions $e_i(\square_T)$, $i=0, \dots, N_\square^{ST} + N_\square^{SP}$ are used. For $\square_\ell(\square_T)$, $\square_\ell(\square_T)$ and $\partial \square / \partial \square_T$ the piecewise constant functions $c_{i-1/2}(\square_T)$, $i=1, \dots, N_\square^{ST} + N_\square^{SP}$ are used. The finite hybrid element representation is then: $\square^i \equiv \square(\square_T^i)$, $\square_\ell^i \equiv \square_\ell(\square_T^{i-1/2})$, $\square_\ell^i \equiv \square_\ell(\square_T^{i-1/2})$ and $\partial \square / \partial \square_T = (\square^i - \square^{i-1}) / (\square_T^i - \square_T^{i-1})$.

5.6. Boundary Conditions at the Magnetic Separatrix and at the Symmetry Axis

The discontinuity of the normal component ($\square^\perp$) of the perturbed displacement $\vec{\square}$ at the ST-SP interface is forbidden in ideal MHD [88], as it would give rise to flux generation and would provide an unavoidable divergence of the perturbed plasma potential energy. On the other hand, discontinuities of the tangential components ($\square_\ell, \square$) of perturbed displacement $\vec{\square}$ are allowed for at the ST-SP interface.

The code STABLE solves the eigenvalue problem $\vec{\mathbf{W}} \cdot |\vec{\mathbf{x}}\rangle = \square^2 \vec{\mathbf{K}} \cdot |\vec{\mathbf{x}}\rangle$, where the total potential magnetic energy matrix $\vec{\mathbf{W}}$ and the (positive definite) kinetic energy matrix $\vec{\mathbf{K}}$ are symmetric and block-diagonal, $\vec{\mathbf{x}} \equiv (\square_\ell^i, \square_\ell^i, \square^i)$ is the eigenvector and $\square^2$ the eigenvalue. The system is solved by an inverse iteration method, which finds all the lowest discrete eigenvalues and the corresponding eigenvectors.

A further difficulty appears in PROTO-SPHERA, as the plasma extends until the symmetry axis $R=0$. As a matter of fact on the symmetry axis $R=0$ ($\square_T = \square_T^{\max}$), $\vec{\square} \square_T$ goes to zero like $|\vec{\square} \square_T| \propto R$, therefore the regularity condition at the symmetry axis $\square^\perp(\square_T^{\max})=0$ should be imposed in the stability solution. However, along the plasma disks, on top and bottom of the configuration, after the degenerate X-point ($B=0$), the flux surface $\square_T = \square_T^{\max}$ branches from the symmetry axis (see Fig. 5.7) and has $\vec{\square} \square_T \neq 0$. Therefore no regularity condition on $\square^\perp(\square_T^{\max})$ should be imposed on the plasma disks, unless upper and lower stabilizing conducting plates (see Fig. 4.1) are close fitting the screw pinch plasma, forcing $\square^\perp(\square_T^{\max})=0$. One can so make either of the two choices,

imposing $\bar{\psi}(\psi_T^{\max})=0$ or leaving free the behavior of $\bar{\psi}(\psi_T^{\max})$, and compare the results obtained by the code STABLE.

The only proper way of treating the flux surface $\psi_T=\psi_T^{\max}$ is to change the radial displacement variable to $\tilde{\psi}=\bar{\psi}/|\bar{\psi}_T|$; with this new definition no regularity condition has to be imposed at the symmetry axis.

Also, at the degenerate X-points ($B=0$) on the symmetry axis, the prescription that $\bar{\psi}(\psi_T \rightarrow \psi_T^{\max}) \sim O(r^{1/2+\epsilon})$ must at least hold. So a further regularity condition $\bar{\psi}(\psi_T^{\max})=0$ should be imposed in the stability solution, but this cannot be done, as the radial derivative of the binormal displacement $\partial\bar{\psi}/\partial\psi_T$ does not appear in the energy principle. The proper way out of treating the degenerate X-points is to change the binormal displacement variable to $\tilde{\psi}=\bar{\psi}/B$; with this new definition no regularity condition has to be imposed on the degenerate X-points.

In terms of the new variables $(\tilde{\psi}, \tilde{\theta}, \tilde{\eta})$ the perturbed displacement becomes:

$$\tilde{\psi} = \tilde{\psi} \frac{\bar{\psi}_T}{|\bar{\psi}_T|} + \frac{\tilde{\theta}}{B} - \frac{\tilde{\eta}}{B} \frac{|\bar{\psi}_T|}{B} \tilde{\theta} + \frac{I}{B} \tilde{\psi} - \frac{\tilde{\eta}}{B} \tilde{\theta}, \text{ instead of}$$

$$\tilde{\psi} = \tilde{\psi} \frac{\bar{\psi}_T}{|\bar{\psi}_T|^2} + (\tilde{\theta} - \tilde{\eta} \tilde{\theta}) \frac{\tilde{\theta}}{B^2} + \frac{\tilde{\eta}}{B^2} \tilde{\theta} + \frac{I}{B^2} \tilde{\psi} - \frac{\tilde{\eta}}{B} \tilde{\theta},$$

with $\tilde{\theta} \frac{\bar{\psi}_T}{|\bar{\psi}_T|} = \tilde{\psi}_T \cdot \frac{\bar{\psi}_T}{|\bar{\psi}_T|}$.

The modified code STABLEC solves the eigenvalue problem, $\tilde{\mathbf{W}} \cdot |\tilde{\mathbf{x}}\rangle = \lambda^2 \tilde{\mathbf{K}} \cdot |\tilde{\mathbf{x}}\rangle$, where the total potential magnetic energy matrix $\tilde{\mathbf{W}}$ and the (positive definite) kinetic energy matrix $\tilde{\mathbf{K}}$ are symmetric and block-diagonal, $\tilde{\mathbf{x}} \equiv (\tilde{\psi}_\ell^i, \tilde{\theta}_\ell^i, \tilde{\eta}_\ell^i)$ is the eigenvector and λ^2 the eigenvalue.

The compressible perturbed displacement $(\tilde{\psi}, \tilde{\theta}, \tilde{\eta})$ is expanded in a trigonometric Fourier series; every mode has a single toroidal number n and is a superposition of poloidal numbers m_ℓ , each labeled by an index ℓ :

$$\tilde{\psi} = \sum_\ell \tilde{\psi}_\ell(\psi_T) \sin(m_\ell \psi - n\eta), \quad \tilde{\theta} = \sum_\ell \tilde{\theta}_\ell(\psi_T) \cos(m_\ell \psi - n\eta), \quad \tilde{\eta} = \sum_\ell \tilde{\eta}_\ell(\psi_T) \cos(m_\ell \psi - n\eta).$$

The modified code STABLEC has however two disadvantages with respect to the code STABLE:

- The expression of the perturbed potential energy is much more complicated.
- The convergence of λ^2 by extending the range of the poloidal numbers m_ℓ is quite slower.

5.7. Vacuum Magnetic Energy with Multiple Plasma Boundaries

The treatment of the vacuum magnetic energy contribution in PROTO-SPHERA is complicated by the presence of three plasma-vacuum surfaces (see Fig. 5.8):

$$\begin{aligned}\psi_T^{v1} &= \psi_T^X - \psi_{ST}/2\pi\chi_X \quad (i=N_\psi^{ST}), & \text{with rotational transform } \vartheta^1 = \chi_X; \\ \psi_T^{v2} &= \psi_T^X + \psi_{SP}/2\pi\chi_X \quad (i=N_\psi^{ST}+1), & \text{with rotational transform } \vartheta^2 = \chi_X; \\ \psi_T^{v3} &= \psi_T^{\max} - \psi_{\text{symm}}/2\pi\chi_{\text{symm}} \quad (i=N_\psi^{ST} + N_\psi^{SP}), & \text{with rotational transform } \vartheta^3 = \chi_{\text{symm}}.\end{aligned}$$

In general, couplings in the matrix elements among all the three surfaces can exist.

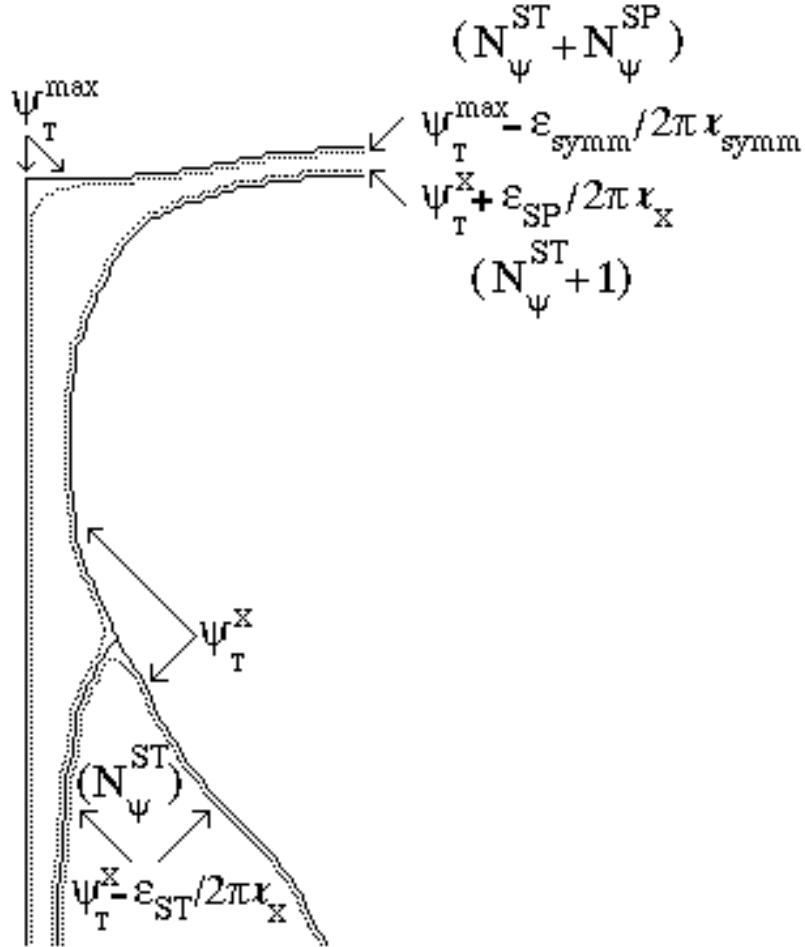


Fig. 5.8. The three plasma-vacuum surfaces of PROTO-SPHERA.

If $\tilde{\psi} = \tilde{\psi}^{nc} \cos(n\chi_G) + \tilde{\psi}^{ns} \sin(n\chi_G)$ is the 3D scalar magnetic potential in the vacuum region, the 2D quantities $\tilde{\psi}^{nc}$ and $\tilde{\psi}^{ns}$, obey in cylindrical coordinates (R, χ_G, Z) the equation:

$$\frac{1}{R} \frac{\partial}{\partial R} \left(R \frac{\partial \tilde{\psi}^{nc}}{\partial R} \right) + \frac{\partial^2 \tilde{\psi}^{nc}}{\partial Z^2} - \frac{n^2}{R^2} \tilde{\psi}^{nc} = 0,$$

with boundary conditions:

$$\left. \frac{\partial \tilde{\eta}^{nc}}{\partial n} \right|_{s_c} = 0, \quad \text{on the perfectly conducting shells, and}$$

$$\left. \frac{\partial \tilde{\eta}^{nc}}{\partial n} \right|_{s_{\phi}^{(i)}} = \sum_{\ell} \frac{\eta_{\ell}(\phi_T^{(i)}) \tilde{\eta}_{\ell}^{(i)} m_{\ell} - n \tilde{\eta}}{\phi_0 \sqrt{g^{(i)}} |\tilde{\eta}_{\phi_T}^{(i)}|} \cos[m_{\ell} \phi^{(i)} - n \phi(\phi^{(i)})], \quad \text{or}$$

$$\left. \frac{\partial \tilde{\eta}^{ns}}{\partial n} \right|_{s_{\phi}^{(i)}} = \sum_{\ell} \frac{\eta_{\ell}(\phi_T^{(i)}) \tilde{\eta}_{\ell}^{(i)} m_{\ell} \phi - n \tilde{\eta}}{\phi_0 \sqrt{g^{(i)}} |\tilde{\eta}_{\phi_T}^{(i)}|} \sin[m_{\ell} \phi^{(i)} - n \phi(\phi^{(i)})],$$

on each $\phi_T^{v(i)}$ surfaces, where $\phi(\phi) = \phi(\phi) - \phi_G$ is the difference between ϕ and the geometrical azimuth ϕ_G .

The vacuum energy of the magnetic surface ϕ_T^{v3} is an additional term on the $\langle \tilde{\eta} | \tilde{W} | \tilde{\eta} \rangle$ components of the $\tilde{W}$ potential energy matrix. It is however decoupled from the surfaces ϕ_T^{v1} and ϕ_T^{v2} by the closed conducting path of the screw pinch current I_c , which flows into the electrodes, then into the return legs and finally into the coaxial feeder at the top (/bottom) of the machine (see Fig. 5.9). This conducting path is assumed to be axisymmetric in the MHD stability calculation.

The other two surfaces ϕ_T^{v1} and ϕ_T^{v2} are very near, have been chosen in such a way as to have the same rotational transform $\tilde{q}_k$ and furthermore the continuity condition on the normal perturbed displacement imposes $\phi(\phi_T^{v1}) = \phi(\phi_T^{v2})$. These choices eliminate any coupling between the vacuum energy of the magnetic surfaces ϕ_T^{v1} and ϕ_T^{v2} .

The equation for $\tilde{\eta}^{nc}$ and $\tilde{\eta}^{ns}$ are therefore solved in the larger vacuum region shown in Fig. 5.9, on a unique plasma surface. Such a surface is composed by the screw pinch surface ($\phi_T^U = \phi_T^{v2}$), in the two disconnected ranges of poloidal Boozer angles $[\phi_{EL} \leq \phi^U < \phi_X]$ and $[2\pi - \phi_X < \phi^U \leq 2\pi - \phi_{EL}]$, and by the ST surface ($\phi_T^U = \phi_T^{v1}$), in the intermediate range $[\phi_X \leq \phi^U \leq 2\pi - \phi_X]$ at the ST-SP interface (see Fig. 5.6).

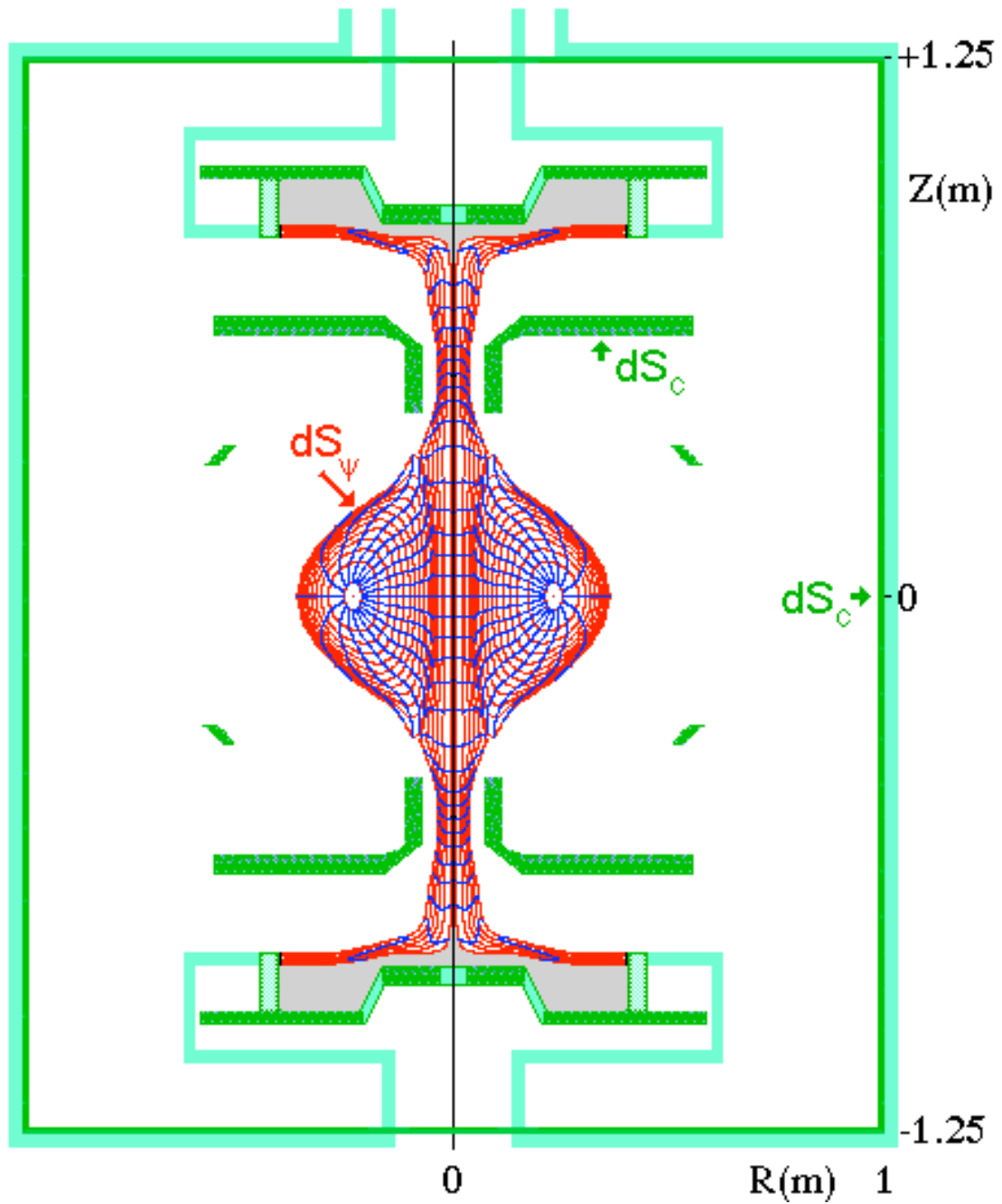


Fig. 5.9. The closed conducting path of the screw pinch current I_c decouples the two small vacuum regions (shaded) on top and bottom of PROTO-SPHERA.

The vacuum problem is solved both in the smaller as well as in the larger vacuum region by a 2D finite element method that can fit any shape of the plasma and of the surrounding conductors (see Fig. 5.10).

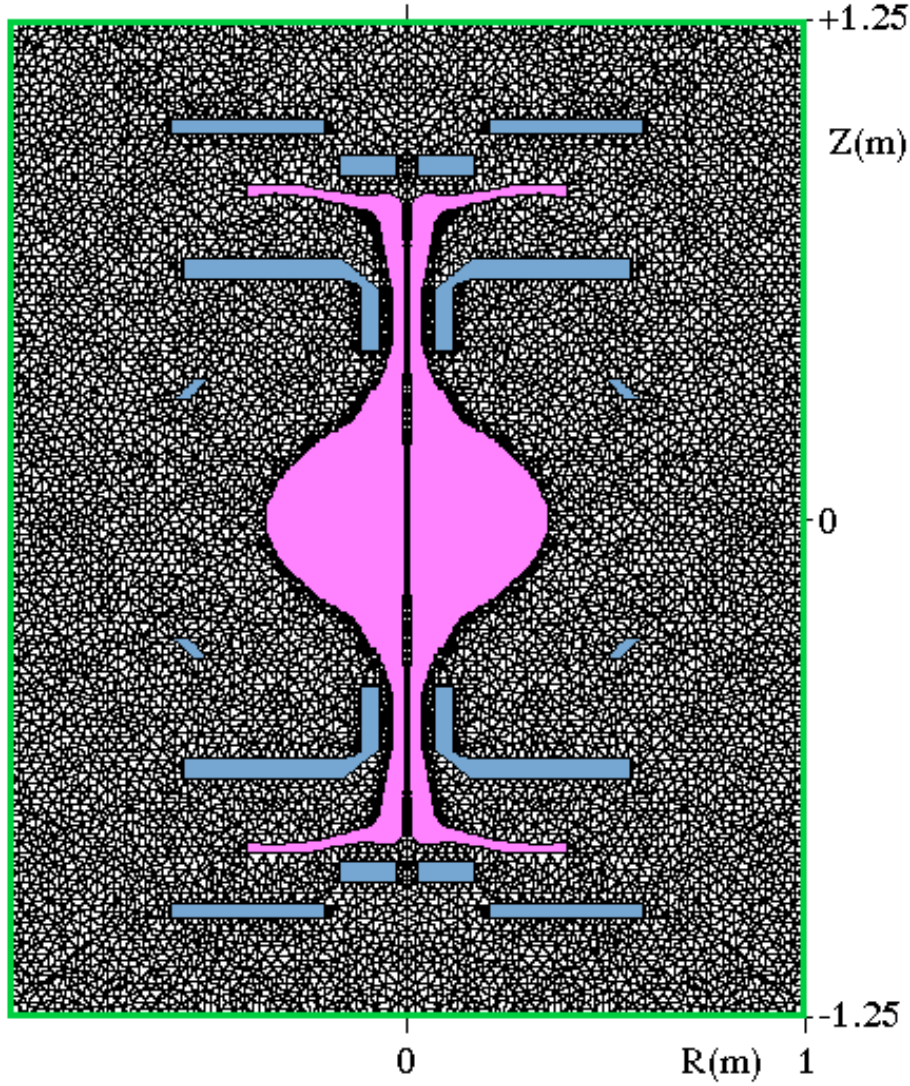


Fig. 5.10. Example of 2D finite element mesh used to solve the vacuum problem.

The perturbed vacuum magnetic energy is then integrated into the energy principle in its final form:

$$\frac{\Delta W_p(\vec{\alpha}^*, \vec{\alpha}) + \sum_{i=1}^3 \Delta W_{v(i)}(\alpha_{v(i)}^*, \alpha_{v(i)})}{\Delta W_k(\vec{\alpha}^*, \vec{\alpha})} = \alpha^2(\vec{\alpha}^*, \vec{\alpha}) = \alpha^2.$$

The vacuum energy contribution of the magnetic surface α_T^{v3} are additional terms on the $\langle \alpha | \vec{W} | \alpha \rangle$ components of the $\vec{W}$ potential energy matrix:

$$\Delta W_{v3} = \frac{1}{2\alpha_0} \sum_{\ell, \ell'} \left\{ \left(\alpha_\ell(\alpha_T^{v3}) \right) \left[\left(\alpha_{\text{ymm}} m_\ell \cdot n \right) \left(\alpha_{\text{ymm}} m_{\ell'} \cdot n \right) R_{\ell\ell'}^{v3} \right] \left(\alpha_{\ell'}(\alpha_T^{v3}) \right) \right\}$$

The vacuum energy contributions of the two surfaces α_T^{v1} and α_T^{v2} are additional terms on the $\langle \alpha | \vec{W} | \alpha \rangle$ components of the $\vec{W}$ potential energy matrix:

$$\Delta W_{v1} = \frac{1}{2\alpha_0} \sum_{\ell, \ell'} \left\{ \left(\alpha_\ell(\alpha_T^{v1}) \right) \left[\left(\alpha_k m_\ell \cdot n \right) \left(\alpha_k m_{\ell'} \cdot n \right) R_{\ell\ell'}^{11} \right] \left(\alpha_{\ell'}(\alpha_T^{v1}) \right) \right\},$$

$$\Delta W_{v2} = \frac{1}{2\alpha_0} \sum_{\ell, \ell'} \left\{ \left(\alpha_\ell(\alpha_T^{v2}) \right) \left[\left(\alpha_k m_\ell \cdot n \right) \left(\alpha_k m_{\ell'} \cdot n \right) R_{\ell\ell'}^{22} \right] \left(\alpha_{\ell'}(\alpha_T^{v2}) \right) \right\},$$

where the coupling coefficients $R_{\ell\ell'}^{v3}$, $R_{\ell\ell'}^{11}$ and $R_{\ell\ell'}^{22}$ are calculated by the 2D finite

element method. In the finite hybrid element method, which solves the eigenvalue problem $\vec{W} \cdot |\vec{x}\rangle = \lambda^2 \vec{K} \cdot |\vec{x}\rangle$, the vacuum additional terms influence only the $\vec{W}$ potential energy matrix elements which multiply the $i=N_{\square}^{ST}$ components $\square_{\square}^{N_{\square}^{ST}}$, the $i=N_{\square}^{ST}+1$ components $\square_{\square}^{N_{\square}^{ST}+1}$ and the last ($i=N_{\square}^{ST}+N_{\square}^{SP}$) components $\square_{\square}^{N_{\square}^{ST}+N_{\square}^{SP}}$ of the eigenvector $\vec{x} = (\square_{\square}^i, \square_{\square}^i, \square_{\square}^i)$.

5.8. Ideal MHD Stability of TS-3

In order to obtain a comparison of the stability code with an actual experiment, the results of the TS-3 flux-core-spheromak experiment have been simulated (Fig. 5.11).

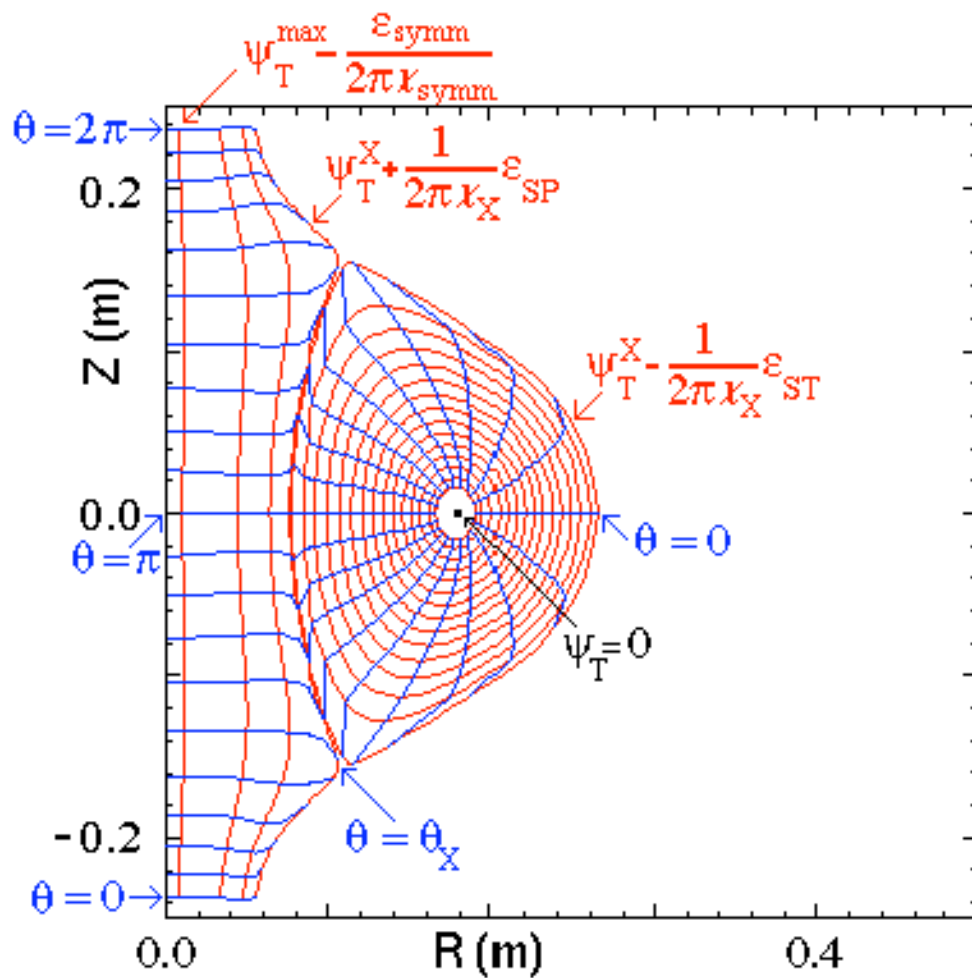


Fig. 5.11. Mesh of Boozer coordinates ($\square_T$ radial, $\square$ poloidal) for TS-3 with $I_e=40$ kA, $I_p=50$ kA and $\square \sim 12\%$.

The calculation of the ideal MHD stability of TS-3 has also other reasons of interest. TS-3 produced an oblate flux-core-spheromak (whereas the ST of PROTO-SPHERA will be prolate). Furthermore TS-3 did not have plasma disks inside separate electrode chambers, but a simple screw pinch fed by cylindrical electrodes. Finally TS-3 had a q profile completely different from the q profiles expected from PROTO-SPHERA (see Fig. 5.12).

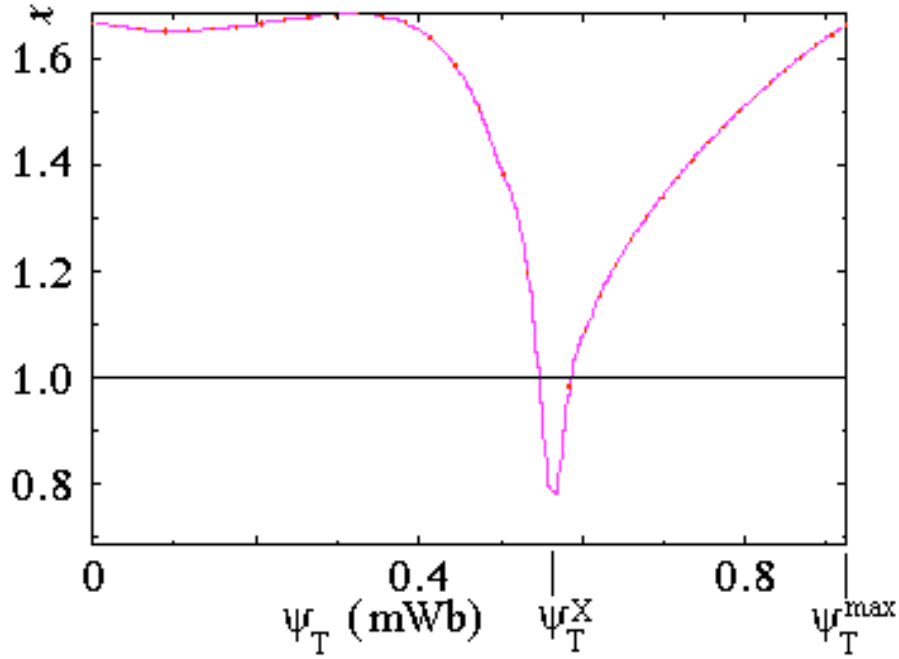


Fig. 5.12. Profile of χ versus ψ_T ($\psi_T=0$ on the ST magnetic axis, $\psi_T=\psi_T^{\max}$ on the symmetry axis) for TS-3 with $I_e=40$ kA, $I_p=50$ kA and $\beta \sim 12\%$.

The first equilibrium of TS-3 considered for the ideal MHD stability calculation has a longitudinal pinch current $I_e=40$ kA, a toroidal ST current $I_p=50$ kA and a volume averaged beta inside the ST $\beta_{ST}=12.0\%$ (defined as $\beta_{ST}=2\beta_0\langle p \rangle_{ST}/\langle B^2 \rangle_{ST}$). The conducting shell is limited to the vacuum vessel, as shown in Fig. 5.13.

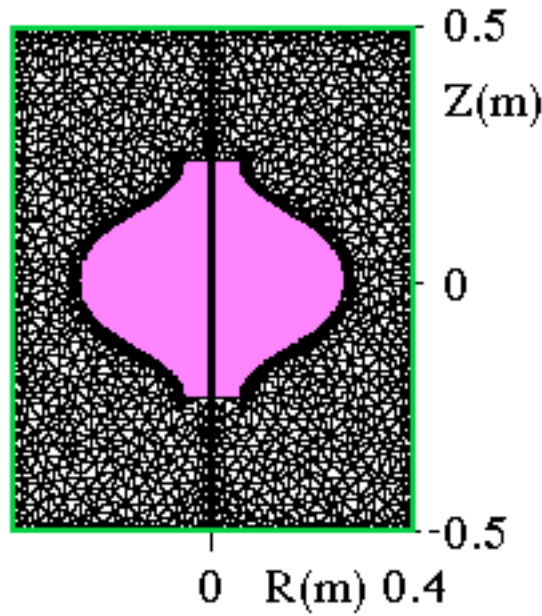


Fig. 5.13. Vacuum 2D finite element mesh used for TS-3 at $I_p=50$ kA, with conducting shells marked in green

In this Section the results for the growth rate are expressed as the ratio between the square of the growth rate of the instability γ^2 and the Alfvén rate: $\gamma_A^2 = B_0^2 / \mu_0 \rho R_0^2$, where B_0 is the magnetic field on the ST magnetic axis R_0 .

Running the code STABLE with the range of poloidal mode numbers $m_r = [-5, 10]$, the TS-3 plasma is calculated to be stable with a positive $\beta^2/\beta_A^2 = +1.33 \cdot 10^{-7}$, when $\beta^2(\beta_T^{\max}) = 0$ is imposed at the symmetry axis. Conversely an unstable plasma is calculated with a negative $\beta^2/\beta_A^2 = -1.03$, when no condition at the symmetry axis applies.

Figure 5.14 shows the perturbed displacement plots in 3D, as poloidal plane cross-sections inside the ST and as horizontal plane cross-sections inside the screw pinch. It confirms that the two conditions at the symmetry axis give quite different eigenfunctions.

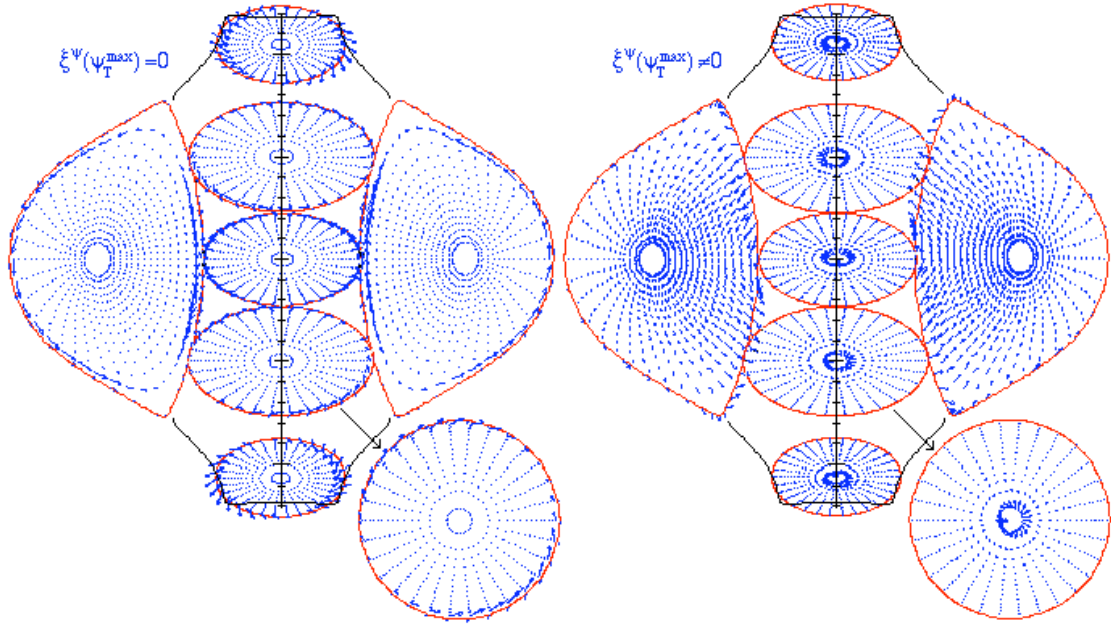


Fig. 5.14. Displacement arrow plots for the smallest eigenvalue mode with $n=1$ for TS-3 with $I_p=50$ kA, $A=1.7$ and $\beta_{ST} \sim 12\%$. The left side is for the stable mode calculated by STABLE with the condition $\beta^2(\beta_T^{\max}) = 0$; the right side for the unstable mode calculated by STABLE without conditions at the symmetry axis $\beta_T = \beta_T^{\max}$. A different normalization of the arrow lengths is used inside the ST and the SP.

The stable eigenfunction represents a stable oscillating motion around the $q=1$ resonance (see Fig. 5.12), inside the ST as well as inside the SP. The unstable eigenfunction represents a globally unstable mode, which takes the character of a kink mode inside the SP and of a tilt mode inside the ST. Figure 5.15 shows that, in the case of TS-3, these features are also clear when the perturbed displacement plots are limited to the poloidal cross-section, both inside the ST as well as inside the SP.

The modified code STABLEC calculates a stable plasma with $\beta^2 = +1.35 \cdot 10^{-7}$, which is quite similar in eigenvalue to the one obtained by the code STABLE, when $\beta^2(\beta_T^{\max}) = 0$ is imposed at the symmetry axis.

Figure 5.16 shows the perturbed displacement plots in 3D, as poloidal plane cross-sections inside the ST and as horizontal plane cross-sections inside the screw pinch. It confirms that code STABLE, with the condition $\beta^2(\beta_T^{\max}) = 0$ at the symmetry axis, and the modified code STABLEC calculate roughly the same eigenfunction,

which is a stable oscillating motion around the $q=1$ resonance, inside the ST as well as inside the SP.

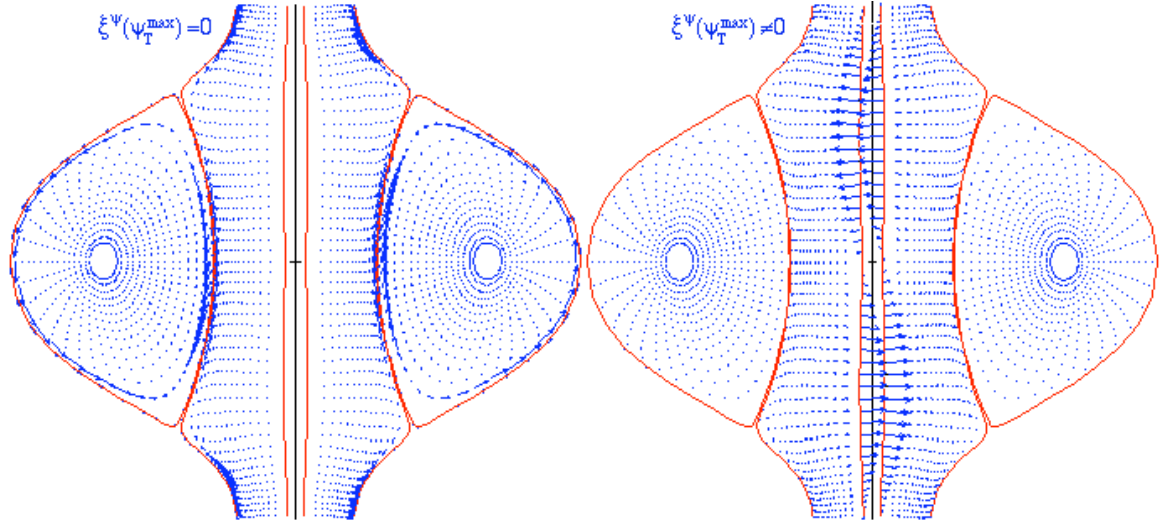


Fig. 5.15. Displacement arrow plots for the smallest eigenvalue mode with $n=1$ for TS-3 with $I_p=50$ kA, $A=1.7$ and $\beta_{ST}\sim 12\%$, all in the poloidal cross-section. The left side is for the stable mode calculated by STABLE with the condition $\bar{\eta}(\bar{\eta}_T^{\max})=0$; the right side for the unstable mode calculated by STABLE without conditions at the symmetry axis $\bar{\eta}_T=\bar{\eta}_T^{\max}$. The same normalization of the arrow lengths is used all over the plasma.

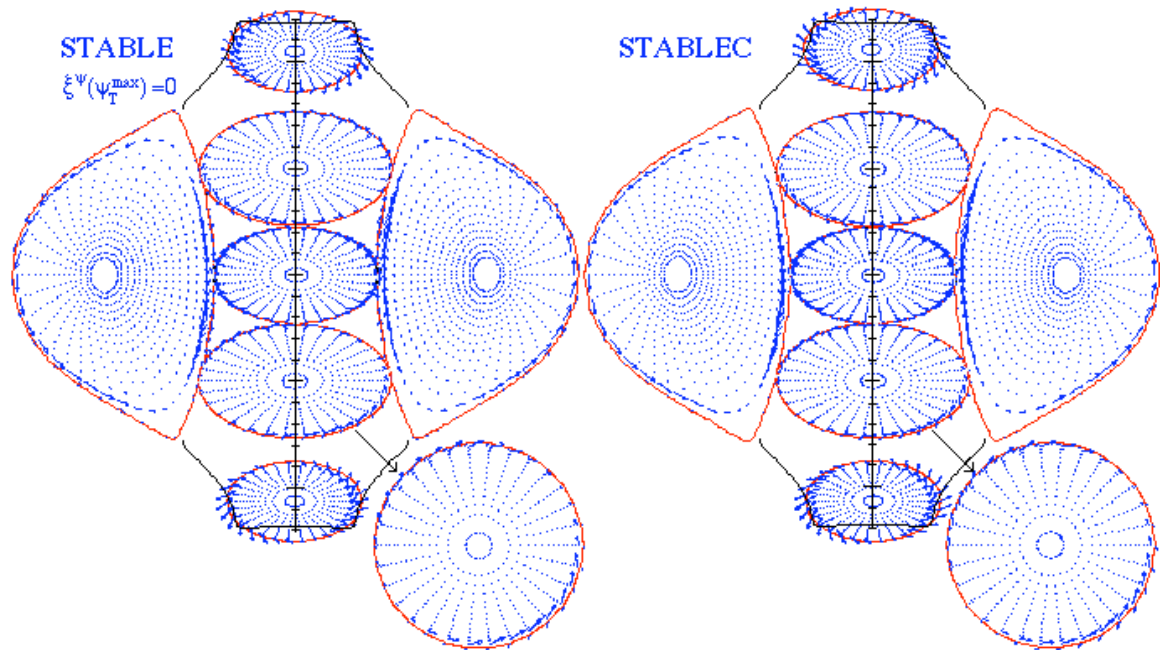


Fig. 5.16. Displacement arrow plots for the smallest eigenvalue stable mode with $n=1$ for TS-3 with $I_p=50$ kA, $A=1.7$ and $\beta_{ST}\sim 12\%$. The left side plots are calculated by STABLE with the condition $\bar{\eta}(\bar{\eta}_T^{\max})=0$ at the symmetry axis; the right side plots are calculated by STABLEC.

Figure 5.17 shows that, in the case of TS-3, the similarity of the two eigenfunctions is also clear when the perturbed displacement plots are limited to the poloidal cross-section both inside the ST as well as inside the SP.

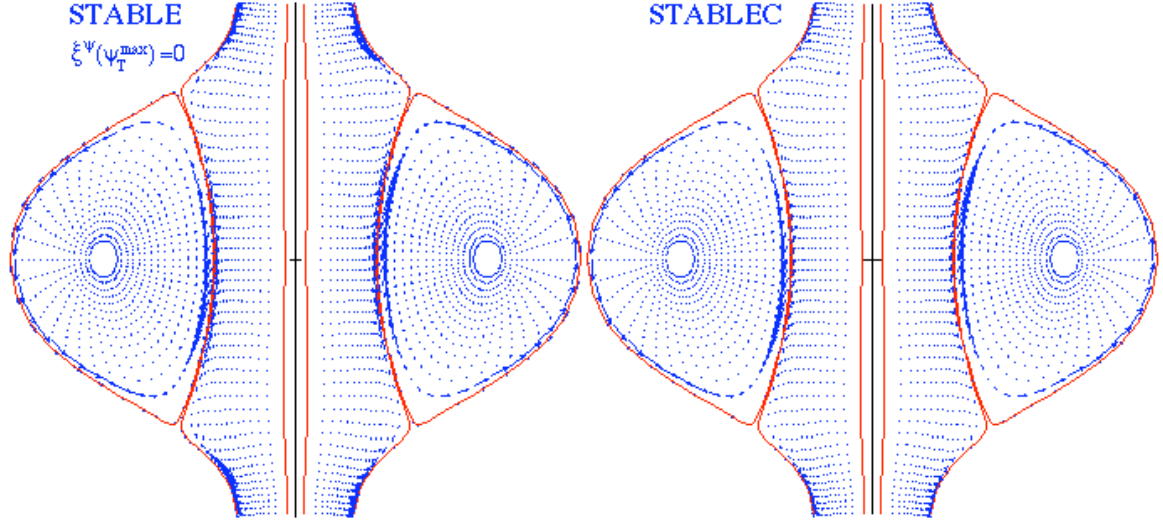


Fig. 5.17. Displacement arrow plots for the smallest eigenvalue stable mode with $n=1$ for TS-3 with $I_p=50$ kA, $A=1.7$ and $\beta \sim 12\%$, all in the poloidal cross-section. The left side plots are calculated by STABLE with the condition $\xi^{\psi}(\psi_T^{\max})=0$ at the symmetry axis; the right side plots are calculated by STABLEC.

The second equilibrium of TS-3 considered for the ideal MHD stability calculation has a longitudinal pinch current $I_e=40$ kA, a toroidal ST current $I_p=100$ kA and a volume averaged beta inside the ST $\beta_{ST}=13.8\%$. The conducting shell is limited to the vacuum vessel, as shown in Fig. 5.13.

In this second case the condition imposed at the symmetry axis turns out to be irrelevant. Running the code STABLE with the range of poloidal mode numbers $m_r=[-5,10]$, an unstable plasma is calculated, with a negative β^2 , which decreases, from $\beta^2/\beta_A^2=-1.1$, when $\xi^{\psi}(\psi_T^{\max})=0$ is imposed, down to $\beta^2/\beta_A^2=-3.92$, when no condition at the symmetry axis applies. Each of the two unstable eigenfunctions represents a globally unstable mode, which takes the character of a kink mode inside the SP and of a tilt mode inside the ST. Also the modified code STABLEC calculates an unstable plasma with $\beta^2/\beta_A^2=-2.73$.

Figure 5.18 shows the perturbed displacement plots in 3D, as poloidal plane cross-sections inside the ST and as horizontal plane cross-sections inside the screw pinch. It confirms that the code STABLE, with the condition $\xi^{\psi}(\psi_T^{\max})=0$ at the symmetry axis, and the modified code STABLEC give roughly the same eigenfunction.

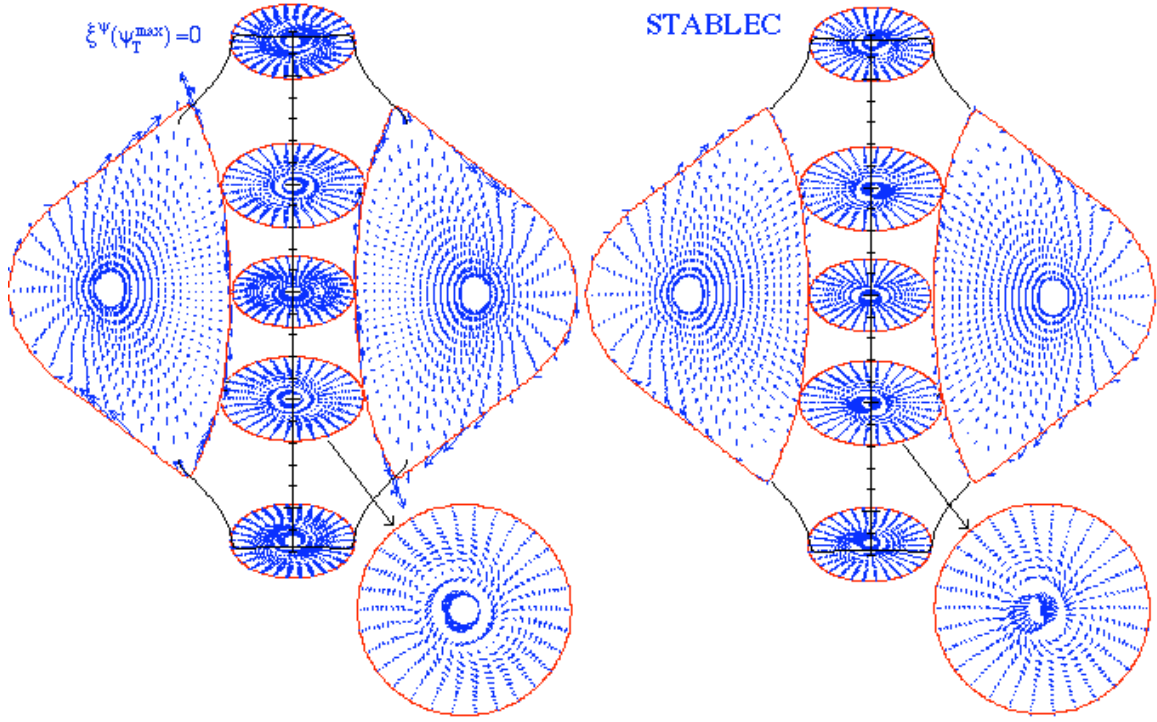


Fig. 5.18. Displacement arrow plots for the largest eigenvalue unstable mode with $n=1$ for TS-3 with $I_p=100$ kA, $A=1.5$ and $\beta_{ST}\sim 14\%$. The left side plots are calculated by STABLE with the condition $\xi^\psi(\psi_T^{\max})=0$ at the symmetry axis; the right side plots are calculated by STABLEC.

Figure 5.19 shows that, in the case of TS-3, the similarity of the two eigenfunctions is also clear when the perturbed displacement plots are limited to the poloidal cross-section inside both the ST and the SP.

The ideal MHD stability calculations agree with the experimental results, confirming that the TS-3 flux-core-spheromak could not achieve a toroidal ST current much higher than $I_p=50$ kA and that its ST was limited to an aspect ratio not lower than $A=1.6$.

For all the equilibria of TS-3, the code STABLE with the condition $\xi^\psi(\psi_T^{\max})=0$ at the symmetry axis and the modified code STABLEC calculate very similar ideal MHD stability results. As a matter of fact, as in TS-3 there is not degenerate X-point, the flux surface $\psi_T=\psi_T^{\max}$ does always coincide with the symmetry axis $R=0$, where ξ^ψ must go to zero like $|\xi^\psi| \propto R$. It is therefore quite reasonable that the regularity condition $\xi^\psi(\psi_T^{\max})=0$ should be imposed in the stability calculations of the TS-3 equilibria.

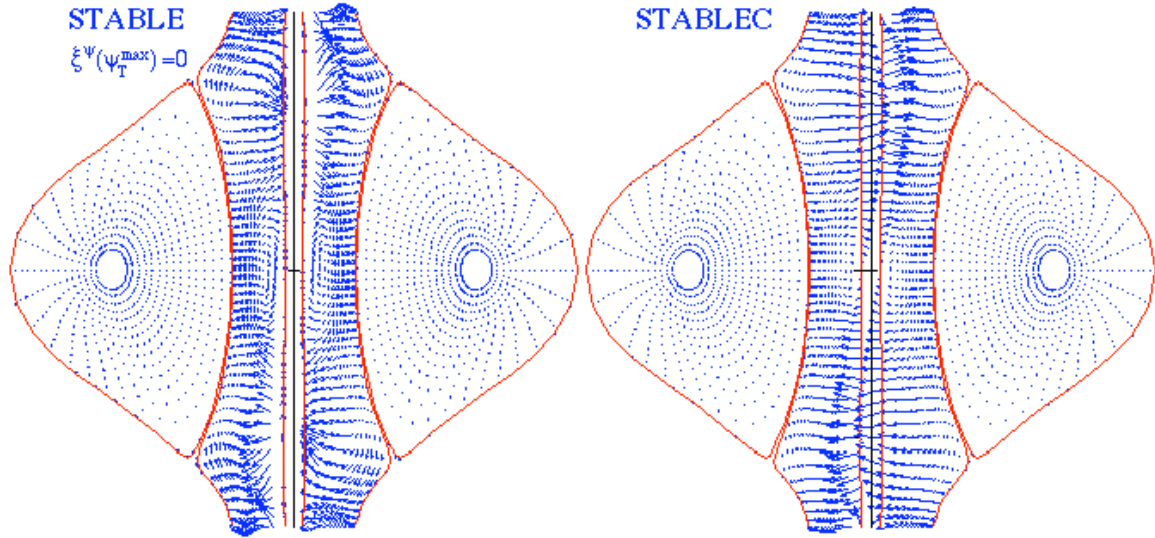


Fig. 5.19. Displacement arrow plots for the smallest eigenvalue unstable mode with $n=1$ for TS-3 with $I_p=100$ kA, $A=1.5$ and $\beta_{ST} \sim 14\%$, all in the poloidal cross-section. The left side plots are calculated by STABLE with the condition $\xi^{\psi}(\psi_T^{\max})=0$ at the symmetry axis; the right side plots are calculated by STABLEC.

5.9. Ideal MHD Current Limits in PROTO-SPHERA

Three cases of PROTO-SPHERA have been selected with growing ratio between the toroidal ST and the longitudinal SP current, respectively $I_p/I_e = 2, 3$ and $7/2$. All of these cases have a volume averaged beta of the spherical torus $\beta_{ST} \sim 20\%$ (defined as $\beta_{ST} = 2\beta_0 \langle p \rangle_{ST} / \langle B^2 \rangle_{ST}$). It has to be remarked that this β_{ST} value, when reduced to the definition commonly used in spherical tokamaks, i.e. the vacuum toroidal beta $\beta_{T0} = 2\beta_0 \langle p \rangle_{ST} / B_0^2$, would correspond to $\beta_{T0} \sim 80\%$. Furthermore all the three cases have spherical tori which are ideal MHD stable, when analyzed on their own (i.e. limiting the radial range to $0 \leq r \leq r_T^x$), assuming fixed boundary conditions $\xi^{\psi}(r_T^x) = 0$ at their edge.

In the case of PROTO-SPHERA at the time slice T5 (with $I_e=60$ kA, $I_p=120$ kA and $\beta_{ST}=23.1\%$), with the conducting shell limited to the outermost vacuum vessel shown in Fig. 5.9, the code STABLE calculates, with both conditions at the symmetry axis, an ideal MHD stable plasma.

Running the code STABLE with the range of poloidal mode numbers $m_r = [-5, 10]$, the positive β^2 decreases, from $\beta^2/\beta_A^2 = +3.15 \cdot 10^{-6}$, when $\xi^{\psi}(\psi_T^{\max}) = 0$ is imposed, to $\beta^2/\beta_A^2 = +2.31 \cdot 10^{-6}$, when no condition at the symmetry axis applies. Also the modified code STABLEC calculates a stable plasma. Figure 5.20 shows the perturbed displacement plots in 3D, as poloidal plane cross-sections inside the ST and as horizontal plane cross-sections inside the screw pinch. It confirms that the code STABLE, with no condition on $\xi^{\psi}(\psi_T^{\max})$ at the symmetry axis, and the modified code STABLEC give roughly the same eigenfunction. It is a stable oscillating motion around the $q=1$ and $q=2$ resonances inside the ST and around the $q=3$ resonance inside the SP.

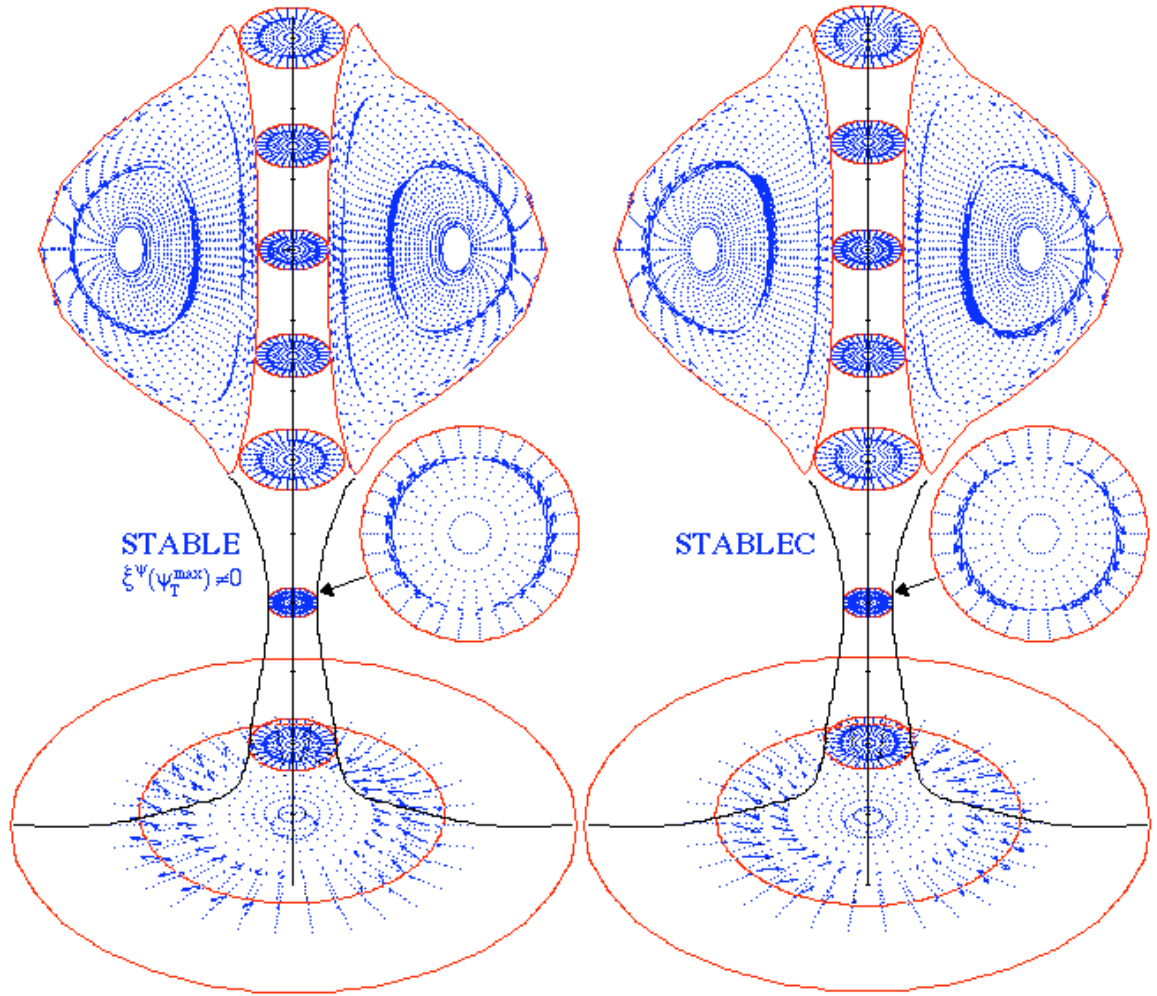


Fig. 5.20. Displacement arrow plots for the smallest eigenvalue stable mode with $n=1$ for PROTO-SPHERA with $I_p=120$ kA, $A=1.3$ and $\beta_{ST}\sim 20\%$. The left side plots are calculated by STABLE without conditions on $\bar{\eta}(\bar{\eta}_T^{\max})$ at the symmetry axis; the right side plots are calculated by STABLEC.

Instead in the case of PROTO-SPHERA at the time slice T6 (with $I_c=60$ kA, $I_p=180$ kA and $\beta_{ST}=22.4\%$), with the conducting shell limited to the outermost vacuum vessel shown in Fig. 5.9, the two conditions on $\bar{\eta}(\bar{\eta}_T^{\max})$ give respectively a stable and an unstable plasma. Running the code STABLE with the range of poloidal mode numbers $m_r=[-5,10]$, a stable plasma with a positive $\bar{\eta}^2$ is $\bar{\eta}^2/\bar{\eta}_A^2=+7.77\cdot 10^{-7}$ is calculated, when $\bar{\eta}(\bar{\eta}_T^{\max})=0$ is imposed, but an unstable plasma with a negative $\bar{\eta}^2/\bar{\eta}_A^2=-4.89$ is computed, when no condition at the symmetry axis applies.

Figure 5.21 shows the perturbed displacement plots in 3D, as poloidal plane cross-sections inside the ST and as horizontal plane cross-sections inside the screw pinch. It confirms that the two conditions at the symmetry axis give completely different eigenfunctions. The stable eigenfunction represents a stable oscillating motion around the $q=1$ resonance inside the ST and around the $q=4$ resonance inside the SP. The unstable eigenfunction represents a globally unstable mode, which takes the character of a kink mode inside the SP and of a tilt mode inside the ST.

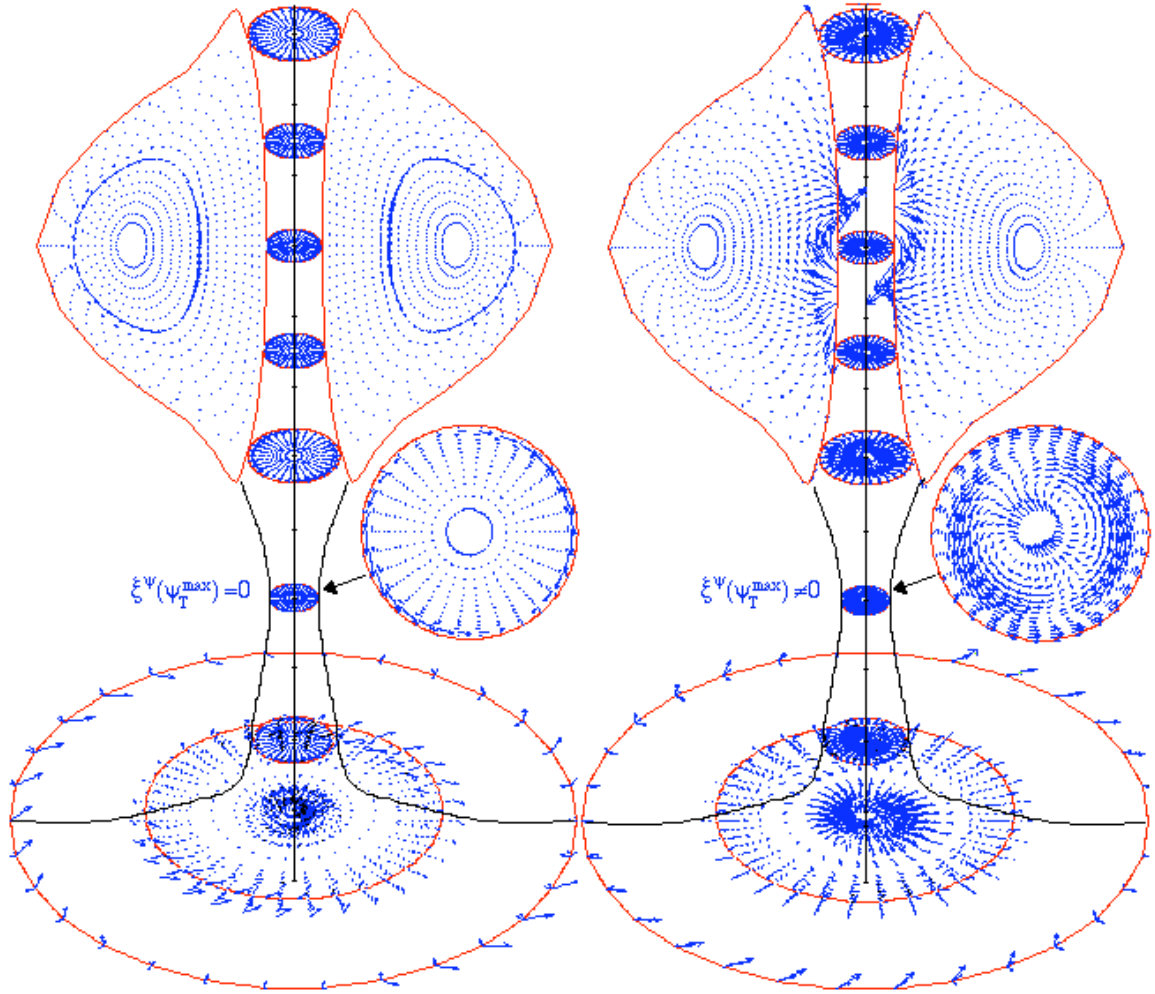


Fig. 5.21. Displacement arrow plots for the smallest eigenvalue mode with $n=1$ for PROTO-SPHERA with $I_p=180$ kA, $A=1.25$ and $\beta_{ST} \sim 20\%$. The left side is for the stable mode calculated by STABLE with the condition $\xi^{\psi}(\psi_T^{\max})=0$; the right side for the unstable mode calculated by STABLE without conditions at the symmetry axis. A different normalization of the arrow lengths is used inside the ST and inside the SP.

The modified code STABLEC calculates an unstable plasma, which is similar in eigenvalue to the one obtained by the code STABLE when no condition was imposed at the symmetry axis. Figure 5.22 shows the perturbed displacement plots in 3D, as poloidal plane cross-sections inside the ST and as horizontal plane cross-sections inside the screw pinch. It confirms that the code STABLE, with no condition on $\xi^{\psi}(\psi_T^{\max})$ at the symmetry axis, and the modified code STABLEC give roughly the same eigenfunction. Each of the two unstable eigenfunctions represents a globally unstable mode, which takes the character of a kink mode inside the SP and of a tilt mode inside the ST.

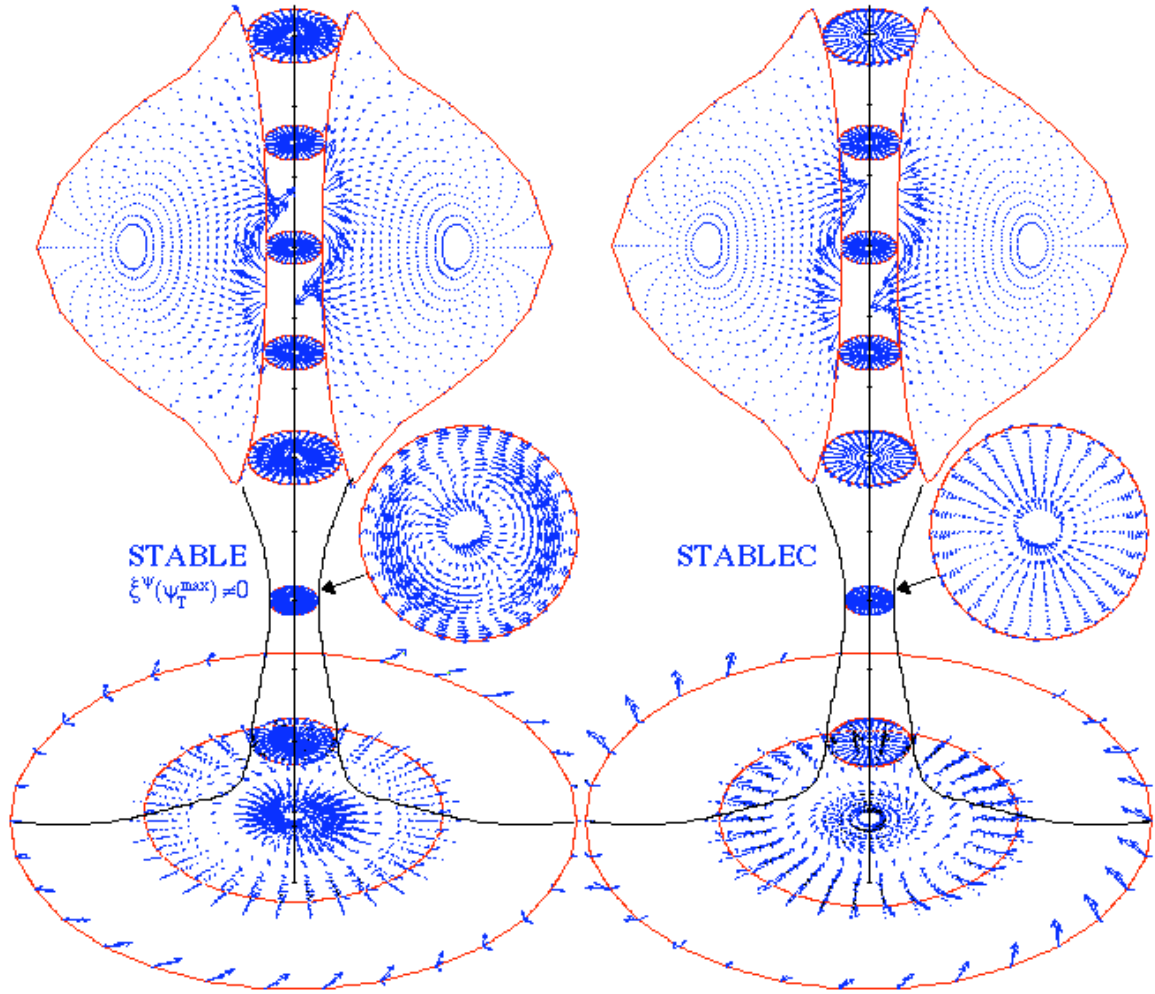


Fig. 5.22. Displacement arrow plots for the smallest eigenvalue unstable mode with $n=1$ for PROTO-SPHERA with $I_p=180$ kA, $A=1.25$ and $\beta_{ST}\sim 20\%$. The left side plots are calculated by STABLE without conditions on $\bar{\psi}(\bar{\psi}_T^{\max})$ at the symmetry axis; the right side plots are calculated by STABLEC.

In the case of PROTO-SPHERA at the time slice T7 (with $I_e=60$ kA, $I_p=210$ kA and $\beta_{ST}=20.6\%$), with the conducting shell limited to the outermost vacuum vessel shown in Fig. 5.9, both conditions on $\bar{\psi}(\bar{\psi}_T^{\max})$ give an unstable plasma.

Running the code STABLE with the range of poloidal mode numbers $m=[-5,10]$, the negative $\bar{\psi}^2$ decreases from $\bar{\psi}^2/\bar{\psi}_A^2=-3.22\cdot 10^{-3}$, when $\bar{\psi}(\bar{\psi}_T^{\max})=0$ is imposed, to $\bar{\psi}^2/\bar{\psi}_A^2=-8.59$, when no condition at the symmetry axis applies. Also the modified code STABLEC calculates an unstable plasma.

Figure 5.23 shows the perturbed displacement plots in 3D, as poloidal plane cross-sections inside the ST and as horizontal plane cross-sections inside the screw pinch. It confirms that the code STABLE, with no condition on $\bar{\psi}(\bar{\psi}_T^{\max})$ at the symmetry axis, and the modified code STABLEC give roughly the same eigenfunction. Each of the two unstable eigenfunctions represents a globally unstable mode, which takes the character of a kink mode inside the SP and of a tilt mode inside the ST.

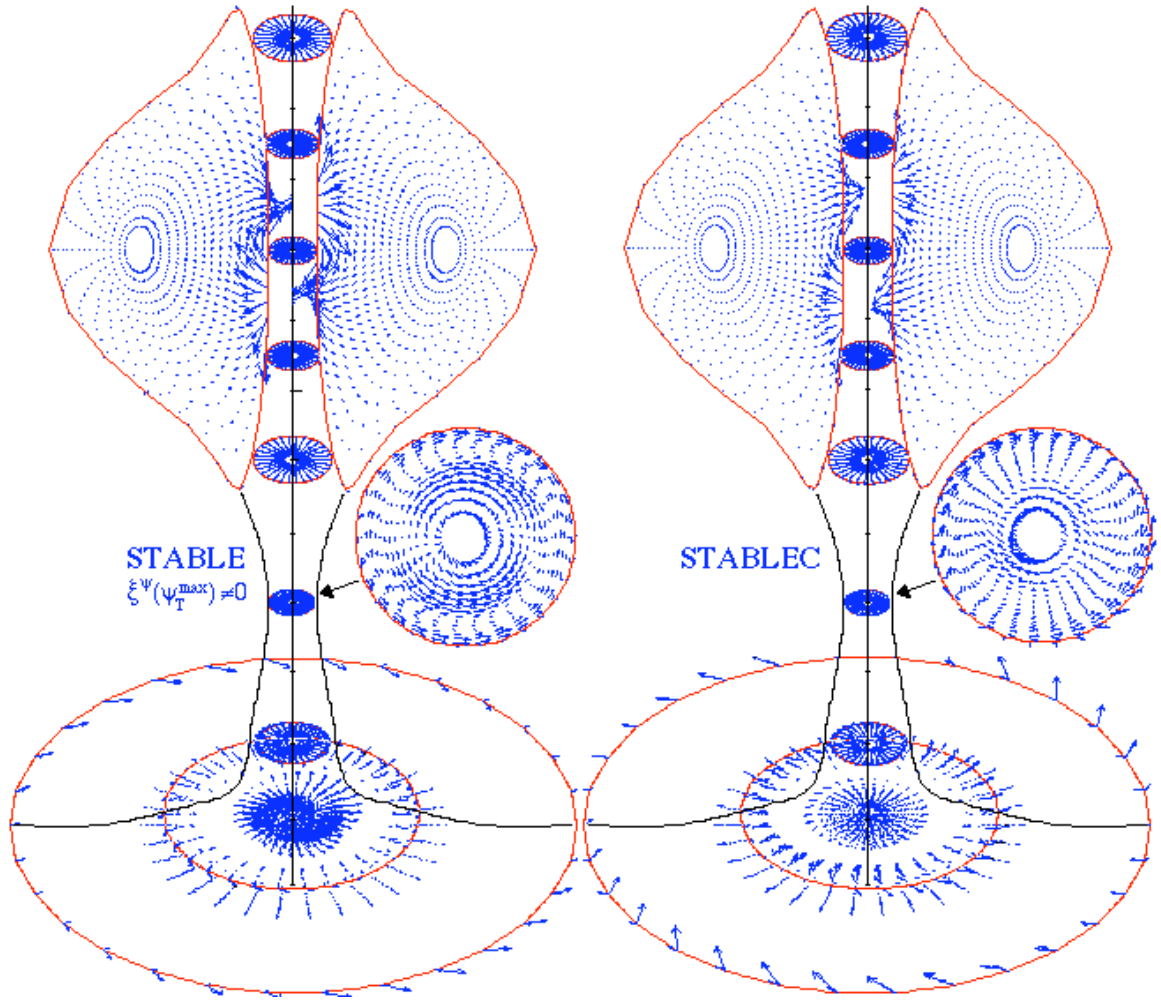


Fig. 5.23. Displacement arrow plots for the smallest eigenvalue unstable mode with $n=1$ for PROTO-SPHERA with $I_p=210$ kA, $A=1.25$ and $\beta_{ST} \sim 20\%$. The left side plots are calculated by STABLE without conditions on $\bar{\psi}(\psi_T^{\max})$ at the symmetry axis; the right side plots are calculated by STABLEC.

For all the equilibria of PROTO-SPHERA, the code STABLE with no condition on $\bar{\psi}(\psi_T^{\max})$ at the symmetry axis and the modified code STABLEC calculate very similar ideal MHD stability results. As a matter of fact, in PROTO-SPHERA there are degenerate X-points ($B=0$), where the flux surface $\psi_T = \psi_T^{\max}$ branches from the symmetry axis $R=0$ and has $\bar{\psi} \neq 0$. Therefore no regularity condition on $\bar{\psi}(\psi_T^{\max})$ should be imposed on the plasma disks, unless upper and lower stabilizing conducting plates (see Fig. 5.7) are close fitting the screw pinch plasma, forcing $\bar{\psi}(\psi_T^{\max})=0$.

5.10. Ideal MHD β Limits in PROTO-SPHERA

After the detailed study of the ideal MHD stability of PROTO-SPHERA at fixed $\beta_{ST} \sim 20\%$, the effect of different β_{ST} values has been explored and is here summarized:

- At $\beta_{ST} \sim 5\%$ PROTO-SPHERA is stable up to:
 $I_p/I_e=8$ ($I_p=240$ kA, $I_e=30$ kA), $A=1.16$ if $\bar{\psi}(\psi_T^{\max})=0$ is imposed in the code STABLE

- At $\beta_{ST} \sim 10\%$ PROTO-SPHERA is stable up to:
 $I_p/I_e=4$ ($I_p=240$ kA, $I_e=60$ kA), $A=1.2$ if $\psi(\psi_T^{\max})=0$ is imposed in the code STABLE
 $I_p/I_e=2$ ($I_p=120$ kA, $I_e=60$ kA), $A=1.3$ if $\psi(\psi_T^{\max}) \neq 0$ (with STABLE and STABLEC)
- At $\beta_{ST} \sim 20\%$ PROTO-SPHERA is stable up to:
 $I_p/I_e=3$ ($I_p=180$ kA, $I_e=60$ kA), $A=1.25$ if $\psi(\psi_T^{\max})=0$ is imposed in the code STABLE
 $I_p/I_e=2$ ($I_p=120$ kA, $I_e=60$ kA), $A=1.3$ if $\psi(\psi_T^{\max}) \neq 0$ (with STABLE and STABLEC)
- At $\beta_{ST} \sim 30\%$ PROTO-SPHERA is stable only up to:
 $I_p/I_e=1$ ($I_p=60$ kA, $I_e=60$ kA), $A=1.5$, irrespective of the conditions $\psi(\psi_T^{\max})=0$ or $\psi(\psi_T^{\max}) \neq 0$. However, at high β_{ST} and high I_p , the spherical tori of PROTO-SPHERA become ideal MHD unstable, when analyzed on their own (i.e. limiting the radial range to $0 \leq R \leq R_T^X$), even assuming fixed boundary conditions $\psi(R_T^X)=0$ at their edge.

These results confirm that the formation sequence of PROTO-SPHERA is ideal MHD stable, provided that it occurs at $\beta_{ST} \leq 20\%$.

Furthermore, if the condition $\psi(\psi_T^{\max})=0$ is not imposed in PROTO-SPHERA, the limit to the ST toroidal plasma current should be $I_p=120$ kA. Other stabilizing effects, not included in the ideal MHD description, such as plasma sheared velocity, could be operating in PROTO-SPHERA. However if these stabilizing effects will not show up, it could be important to insert stabilizing plates on top and bottom of the screw pinch, in order to force the condition $\psi(\psi_T^{\max})=0$. These plates will have to be so near to the plasma that they should be in contact with it, acting as limiters for the screw pinch (see Fig. 5.24, Fig. 4.1 and Section 7.5.2).

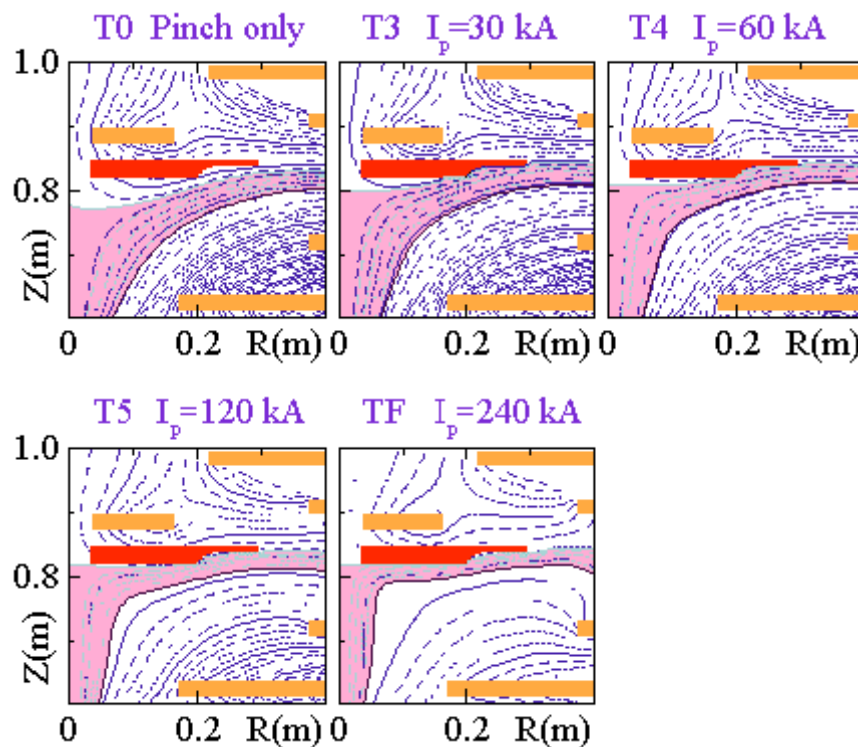


Fig. 5.24. Upper part of the screw pinch discharge of PROTO-SPHERA, showing in red the stabilizing limiter plates in contact with the plasma.

5.11. Convergence Studies

The study of the convergence of the least stable (smallest) eigenvalue calculated by the ideal MHD code STABLE is presented in Fig. 5.25. The same three cases of PROTO-SPHERA, at fixed $\beta_{ST} \sim 20\%$, with $I_p = 120$ kA, $I_p = 180$ kA and $I_p = 210$ kA respectively, already analyzed in Section 5.9, are considered. Increasing the range of the poloidal mode numbers m_r included in the stability calculation leads in all (stable and unstable) cases to a destabilization (decrease) of the smallest eigenvalue. The use of all the poloidal mode numbers in the range $m_r = [-5, 10]$ yields an acceptable convergence. The studies of the convergence of the code STABLE upon the well-known stability results of the analytic Solovév tokamak equilibria [93] confirm that the choice of the range $m_r = [-5, 10]$ is adequate.

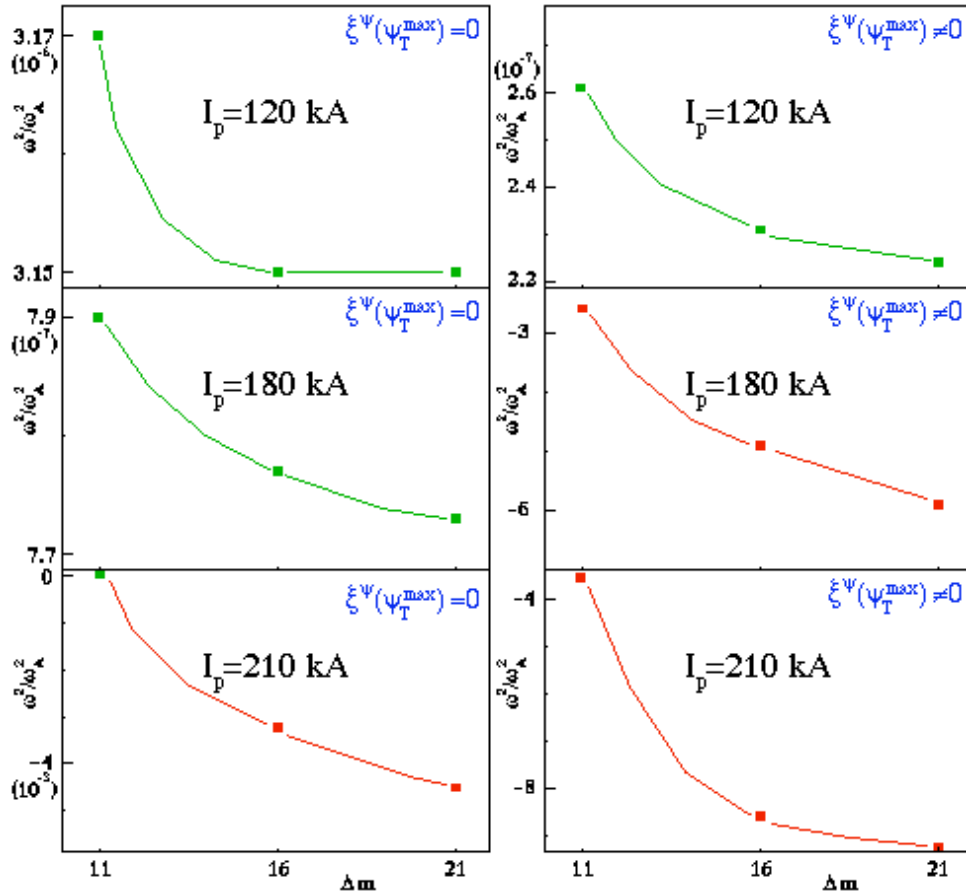


Fig. 5.25. Behavior of the smallest eigenvalue of the PROTO-SPHERA stability calculation as a function of the spectrum of the poloidal mode numbers. $\Delta m = 11$ means the range $m = [-3, +7]$, $\Delta m = 16$ means $m = [-5, +10]$ and $\Delta m = 21$ means $m = [-5, +15]$. The left side plots are with the condition $\xi^{\Psi}(\psi_T^{\max}) = 0$; the right side plots are without conditions at the symmetry axis.

The value ψ_T^X of the magnetic separatrix is carefully excluded from the mesh in order to avoid the singularity of the Boozer coordinates. However it is necessary to investigate the effect that the distance of the two nearest mesh points ($\psi_T^{N_{ST}} = \psi_T^X - \beta_{ST}/2\pi R_X$ and $\psi_T^{N_{ST}+1} = \psi_T^X + \beta_{ST}/2\pi R_X$) from the magnetic separatrix has upon the results of the ideal MHD stability calculation. Two equilibria of

PROTO-SPHERA at $I_p=120$ kA and $I_p=240$ kA, both with $I_e=60$ kA, but slightly different from the cases discussed in Section 5.9, have been examined. The code STABLE has been run using all the poloidal mode numbers in the range $m_r=[-5,10]$, with the boundary condition $\psi(\psi_T^{\max})=0$ at the symmetry axis.

The distance Δ_{ST} of the nearest mesh points from the magnetic separatrix, on the ST side, has been varied; $\Delta_X=\Delta(\Delta_X+\Delta_{ST})$ is calculated at $\Delta=\Delta_X+\Delta_{ST}$. Correspondingly the condition $\Delta(\Delta_X-\Delta_{SP})=\Delta_X$ determines the distance Δ_{SP} of the nearest mesh points from the magnetic separatrix, on the screw pinch side (see Section 5.4). The distance Δ_{ST} has been decreased, in the case of $I_p=240$ kA, from $\Delta_{ST}=2\cdot 10^{-3}$ (which corresponds to a safety factor $q_X=1/\Delta_X=4.4$) down to $\Delta_{ST}=3\cdot 10^{-4}$ (which corresponds to a safety factor $q_X=1/\Delta_X=5.5$). Figure 5.26 shows the effect upon the ratio ω^2/ω_A^2 . Although ω^2 are always found to be negative, an acceptable convergence of the most unstable (smallest) eigenvalue is obtained only when $\Delta_{ST}\leq 10^{-3}$.

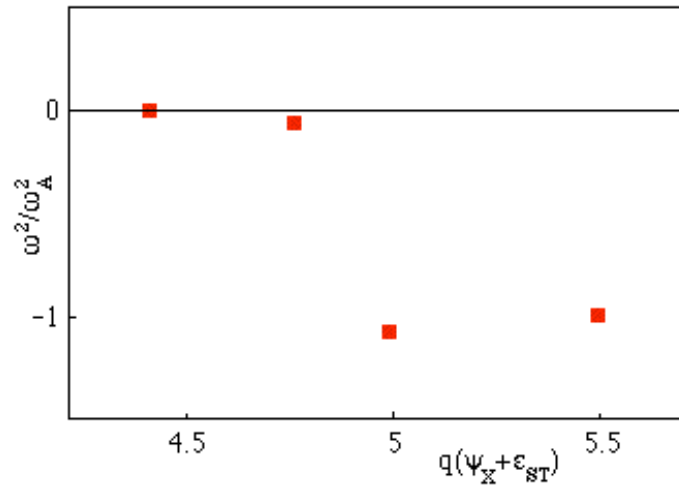


Fig. 5.26. Behavior of the smallest eigenvalue ω^2/ω_A^2 of a PROTO-SPHERA unstable equilibrium at $I_p=240$ kA, as a function of the distance of the nearest mesh points from the magnetic separatrix, expressed by the safety factor $q_X=q(\Delta_X+\Delta_{ST})$. Red squares are calculated with all the poloidal mode numbers in the range $m_r=[-5,+10]$.

The distance Δ_{ST} has been decreased, in the case of $I_p=120$ kA, from $\Delta_{ST}=4\cdot 10^{-3}$ (which corresponds to a safety factor $q_X=1/\Delta_X=4.3$) down to $\Delta_{ST}=1\cdot 10^{-3}$ (which corresponds to a safety factor $q_X=1/\Delta_X=4.8$). Figure 5.27 shows the effect upon the ratio ω^2/ω_A^2 . Although ω^2 is always found to be positive, an acceptable convergence of the least stable (smallest) eigenvalue is obtained only when $\Delta_{ST}\leq 2\cdot 10^{-3}$.

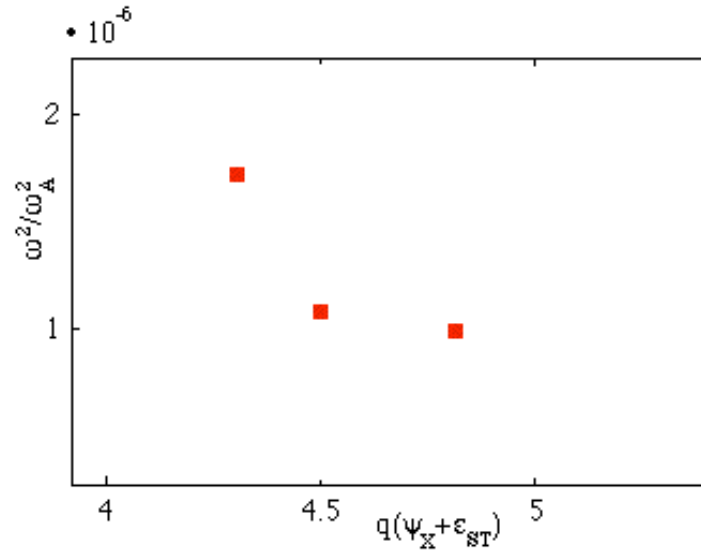


Fig. 5.27. Behavior of the smallest eigenvalue ω^2/ω_A^2 of a PROTO-SPHERA stable equilibrium at $I_p=120$ kA, as a function of the distance of the nearest mesh points from the magnetic separatrix, expressed by the safety factor $q_X=q(\psi_X+\epsilon_{ST})$. Red squares are calculated with all the poloidal mode numbers in the range $m_r=[-5,+10]$.

The effect of accounting all the walls shown in Fig. 5.9 as conducting shells, or of considering as a conducting shell only the outermost vacuum vessel, does affect only marginally the results of the stability calculations. Conversely the effect of a complete (but incorrect) removal of all the vacuum magnetic energy contribution can be much larger. Taking as an example the case of PROTO-SPHERA at the time slice T5 (with $I_e=60$ kA, $I_p=120$ kA and $\epsilon_{ST}=23.1\%$), when $\psi(\psi_T^{\max})=0$ is imposed at the symmetry axis, the (incorrect) removal of the vacuum energy term does not affect (to the third digit) the result $\omega^2/\omega_A^2=+3.15 \cdot 10^{-6}$. However, when no condition at the symmetry axis applies, the (incorrect) removal of the vacuum energy term changes the positive (stable) $\omega^2/\omega_A^2=+2.31 \cdot 10^{-6}$ into a negative (unstable) $\omega^2/\omega_A^2=-1.12$.

Also in the case of the modified code STABLEC the effect of accounting all the walls shown in Fig. 5.9 as conducting shells, or of considering as a conducting shell only the outermost vacuum vessel, does affect only marginally the results of the stability calculations. Conversely the effect on the results of STABLEC of the complete (but incorrect) removal of the vacuum magnetic energy contribution can be much larger, decreasing by about 50% the result for ω^2/ω_A^2 , both in the stable as well as in the unstable cases.

6. ELECTRODE EXPERIMENT

The purpose of this Section is to illustrate how the electrodes for PROTO-SPHERA have been developed. They were the most unconventional items and among the major concerns, when the design of PROTO-SPHERA was started. It was not clear that a feasible solution did exist, allowing for almost steady-state (1 second) emission from a cathode at a plasma current density level of 1 MA/m^2 at the plasma-cathode interface. Neither was it clear that a working solution did exist for an anode able to withstand $20\div 30 \text{ MW/m}^2$ for the same discharge duration. Other concerns were about the endurance to many hundred discharges and about the contamination of the plasma. A final concern was the breakdown voltage, which could cause insulation problems in the PROTO-SPHERA load assembly.

In order to investigate these points the PROTO-PINCH benchmark of one anode and one cathode module has been built. It is similar to PROTO-SPHERA in physical dimensions and in the strength of the magnetic fields near the electrodes. PROTO-PINCH has produced, within a Pyrex vacuum vessel, Hydrogen and Helium arcs in the form of screw pinch discharges, stabilized by two poloidal field coils located outside the vacuum. Following a trial and error procedure about 3 anode prototypes and 10 cathode prototypes have been tested on PROTO-PINCH from October 1998 to April 2001. The technical solutions are a W-Cu(5%) hollow anode and an AC directly heated cathode, composed by pure Tungsten helical filaments. The W-Th alloy, normally used in cathode where high current emission is required, has been substituted by pure W. As a matter of fact, the temperature of the filaments rises to about 2700°C , as soon as the arc starts, and exceeds by far the Thorium melting point (1730°C). The anode and cathode prototypes can work as they increase the effective plasma-electrode interaction area by a factor of about 16, reducing the effective loads to the level of 6A/cm^2 for the current density and of $1.4\div 2 \text{ MW/m}^2$ for the power density.

The final result is that a cathode and an anode module, able to withstand the required current and power densities, have been built and have survived to many hundreds plasma shots. A further remarkable result is that the produced Hydrogen plasma has turned out to be almost free of impurities. The final relevant result is that the breakdown is produced at a low voltage ($\sim 100 \text{ V}$) in the same filling pressure range of a standard tokamak discharge ($\sim 10^{-3}\div 10^{-2} \text{ mbar}$).

6.1. PROTO-PINCH

Before building PROTO-SPHERA, the electrodes' benchmark PROTO-PINCH (see scheme in Fig. 6.1) has been built and operated, with the goal of testing modular units of the cathode and of the anode of PROTO-SPHERA.

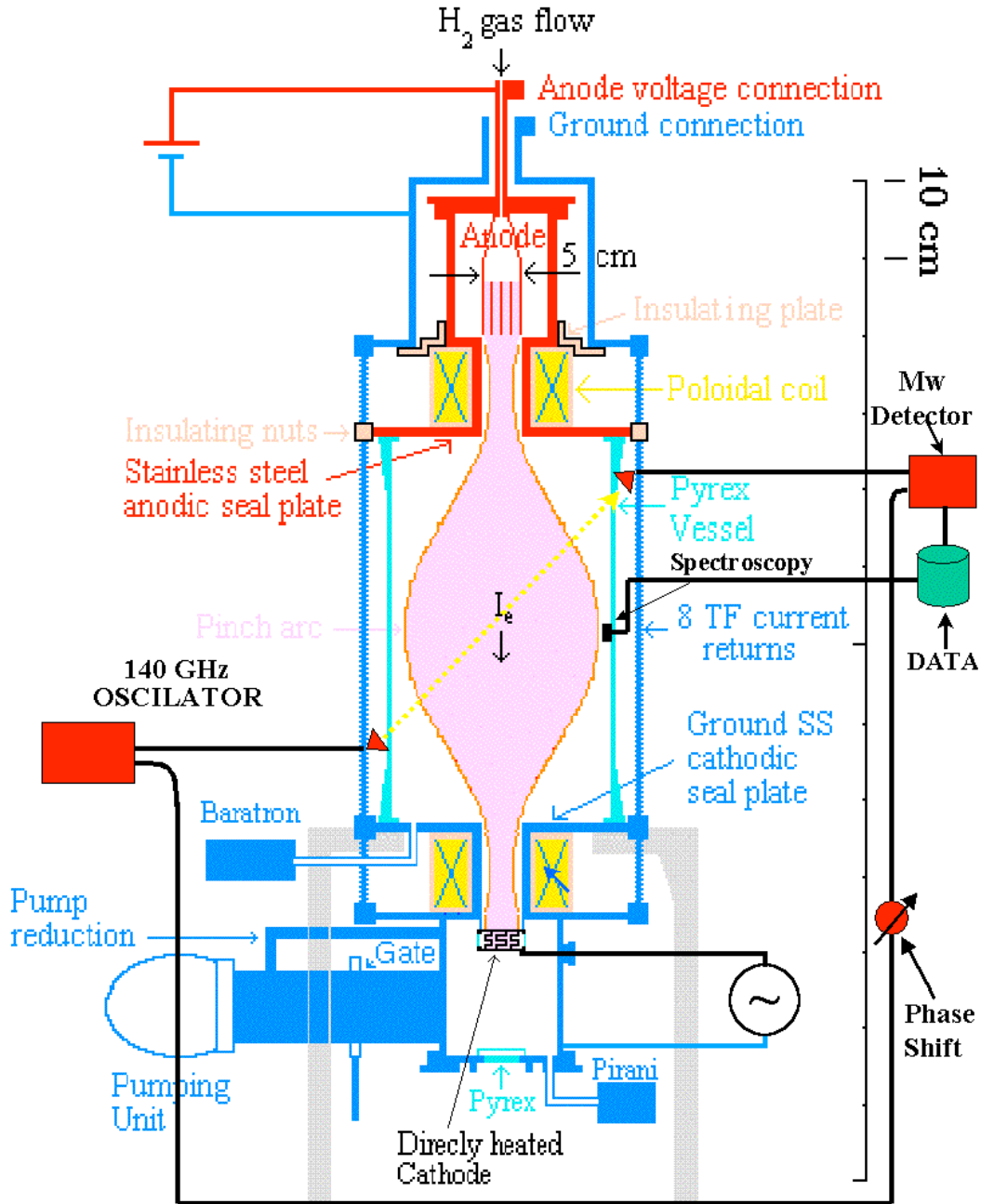


Fig. 6.1. Scheme of the electrode testbench PROTO-PINCH, showing the main diagnostics: 140 GHz interferometer and spectroscopy.

PROTO-PINCH, with an anode-cathode distance of 0.75 m and a stabilizing magnetic field up to $B=1.0$ kG, has a current capability of $I_c=0.67$ kA, (with a safety factor $q_{\text{Pinch}} \geq 2$). PROTO-PINCH has a Pyrex vacuum vessel, is stabilized by two poloidal field coils located outside the vacuum, has a coaxial I_c feeding structure and 8 copper conductors for the I_c current return (Fig. 6.2).

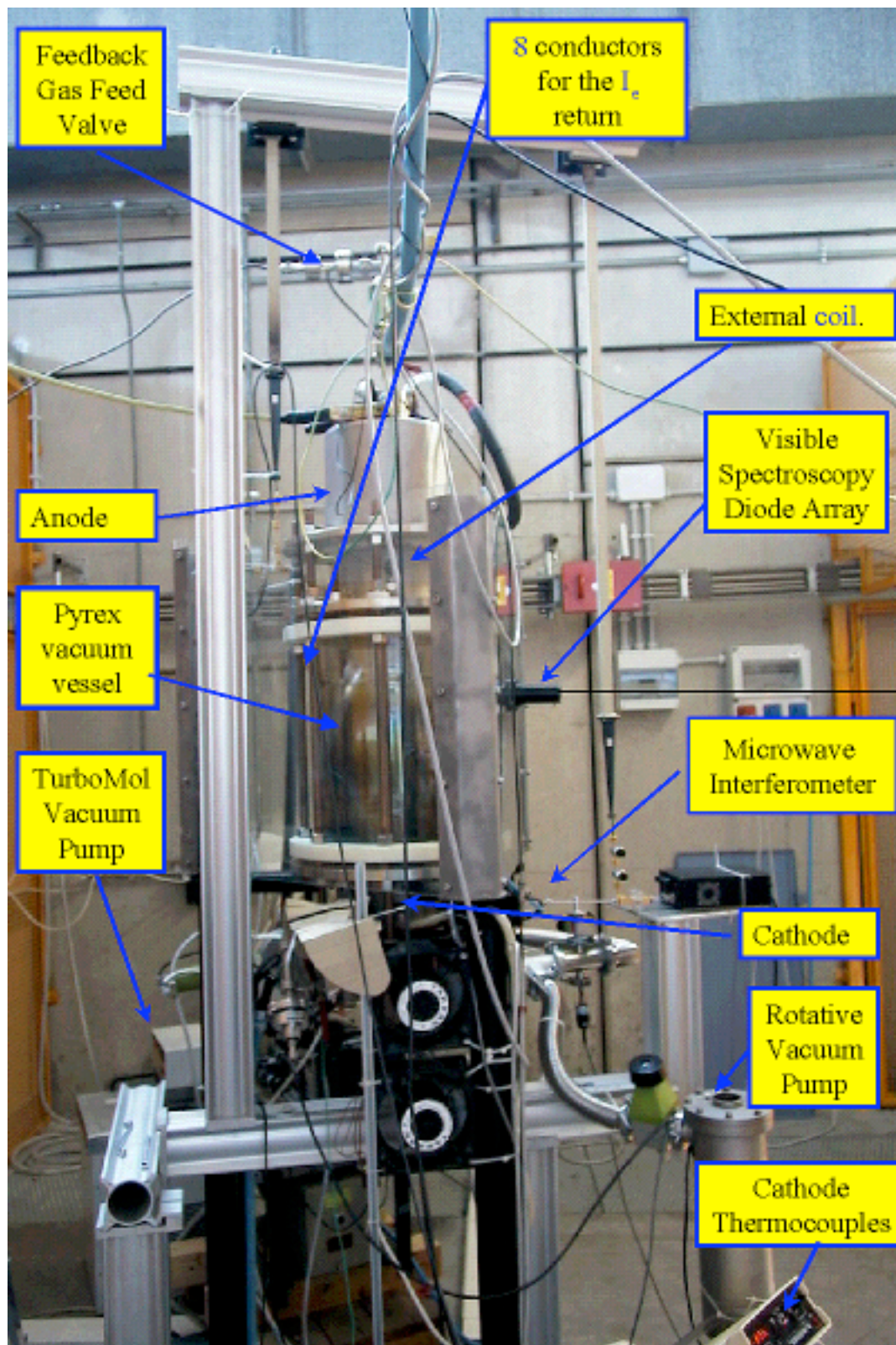


Fig. 6.2. Photograph of the electrode testbench PROTO-PINCH, showing the main diagnostics: 140 GHz interferometer and spectroscopy.

PROTO-PINCH has produced Hydrogen (see Fig. 6.3) and Helium arcs in the form of screw pinch discharges.

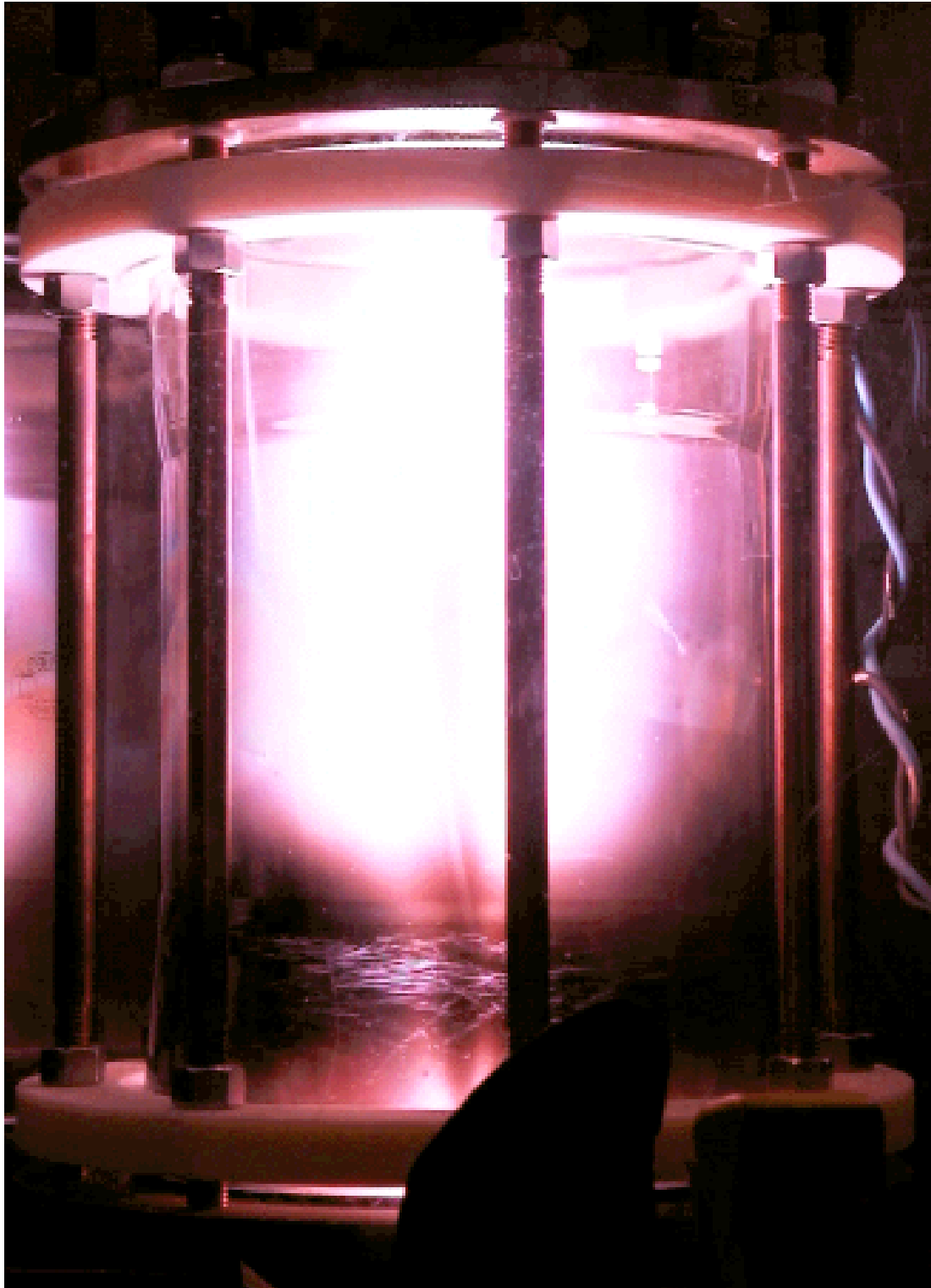


Fig. 6.3. Image of PROTO-PINCH Hydrogen plasma with $I_c=600$ A, $B=1$ kG.

Following a trial and error procedure 3 anode prototypes and 10 cathode prototypes have been tested on PROTO-PINCH from October 1998 to April 2001 (see Fig. 6.4).

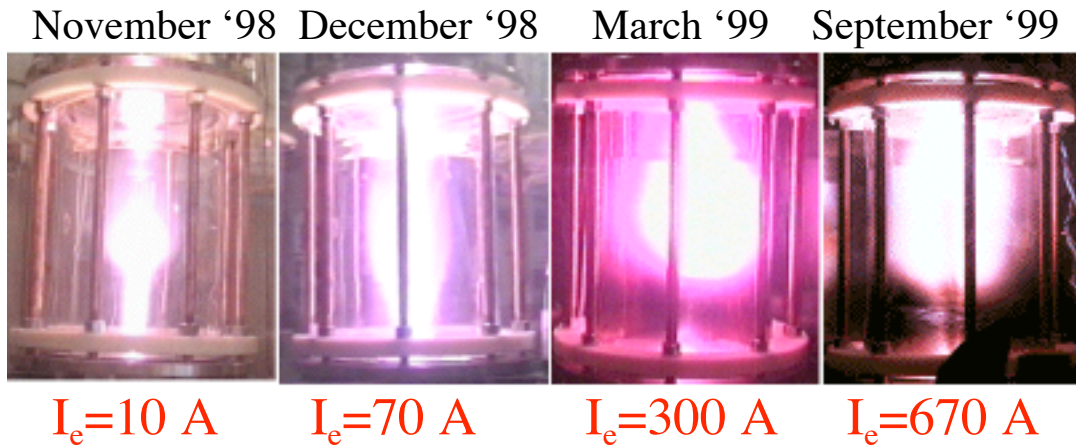


Fig. 6.4. Progress of the PROTO-PINCH experiment.

The main results of the PROTO-PINCH testbench are:

- The technical solution for the 5-cm diameter electrodes are: a directly heated (AC) Tungsten cathode and a Cu-W hollow anode, with H_2 (or He) puffed through it.
- The Hydrogen pinch breakdown occurs in the filling pressure range $p_H=10^{-3} \div 10^{-2}$ mbar, which is the same of a standard tokamak discharge.
- The pinch breakdown voltage is $V_e \leq 100$ V, which means that the insulation problems in PROTO-SPHERA should be quite easy to deal with.
-
- The typical duration of a plasma pulse at $I_e=600$ A is $2 \div 5$ s, limited by thermo-mechanical properties of the W cathode, heating of Pyrex, rubber seals, etc...
-
- The arc plasma is very clean: a few barely measurable impurity lines appear in Hydrogen and in Helium discharges only at the lowest filling pressures ($p_H=1 \div 2 \cdot 10^{-3}$ mbar).
- The final anode and cathode prototypes have withstood 400 discharges at the current and the power densities required for PROTO-SPHERA.

6.2. Cathode

The directly heated (AC) cathode is composed (see Fig. 6.5) by two Molybdenum plates connected by Tantalum columns embedded in insulating Alumina.

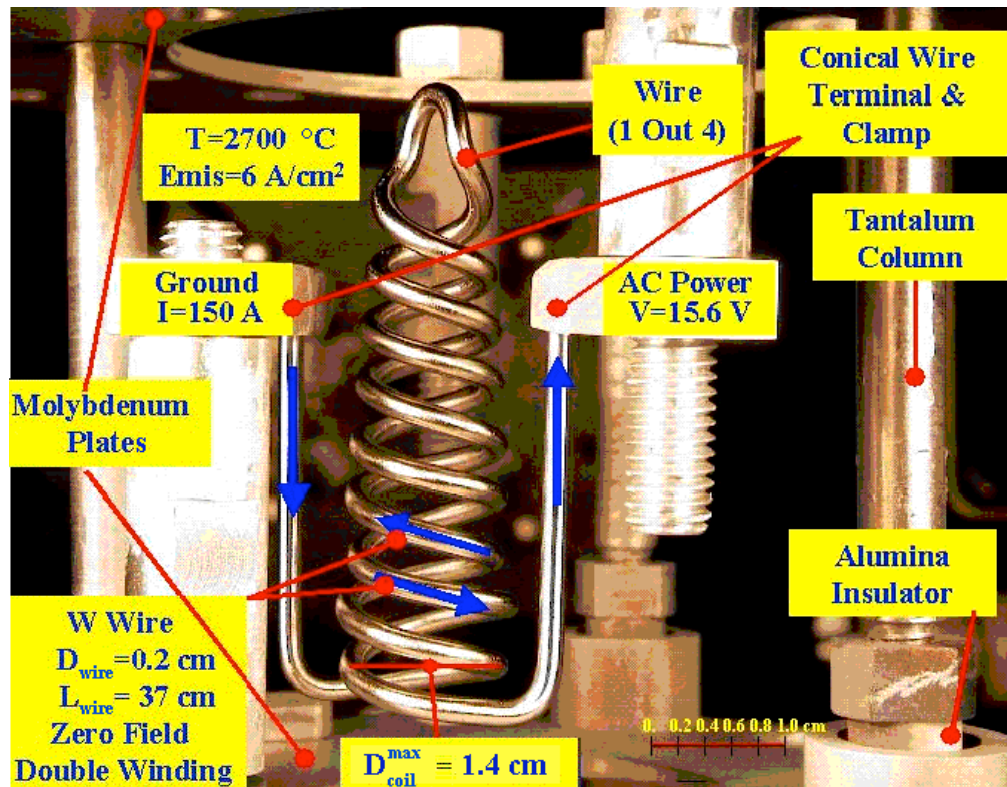


Fig. 6.5. Scheme of the directly heated (AC) cathode of PROTO-PINCH, with the final version of the Tungsten filament.

Four helical filaments of pure Tungsten, fixed by Tantalum nuts and bolts, connect in parallel the two plates (see Fig. 6.6).

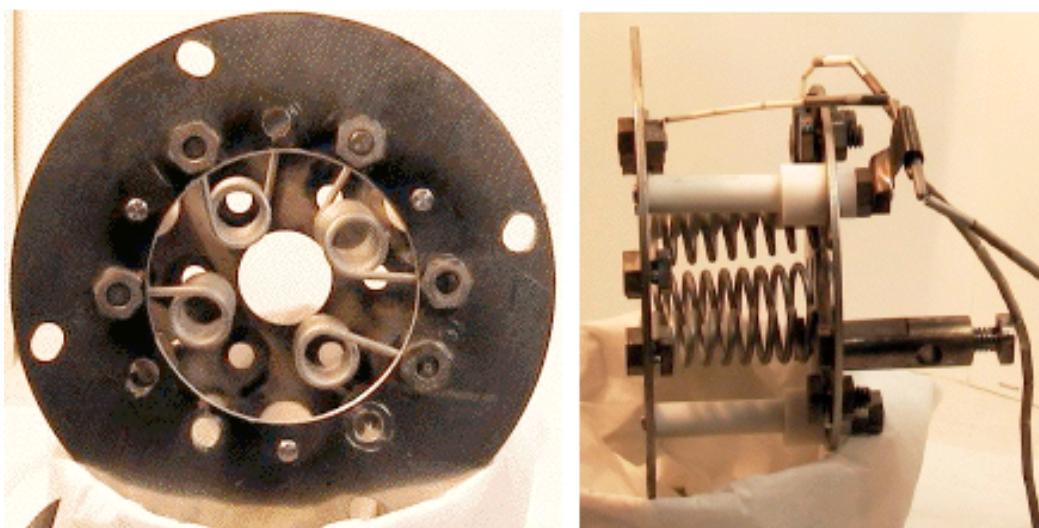


Fig. 6.6. Pictures (plasma facing view and side view) of the directly heated (AC) cathode of PROTO-PINCH, with an earlier version of the filament.

The wire diameter is $D_{\text{wire}} = 2.0$ mm, the total length of each filament is $L_{\text{wire}} = 370$ mm and the maximum diameter of each filament is $D_{\text{coil}}^{\text{max}} = 14$ mm. These parameters have been chosen in order to guarantee the emission of 150 A of plasma current from each filament, while assuring enough mechanical resilience at the working temperature of 2700 °C.

Each of the four filaments has an effective emitting surface of 20 cm². They are heated up to 2600 °C, by a total AC current $I_{\text{cath}} = 590$ A (rms). During the arc discharge the temperature rises up to 2750 °C. Pinch discharges have been obtained with $B = 0.8$ kG, $I_e = 600 \div 700$ A and $V_e = 70 \div 120$ V. So the filament surface emissivity is about 8 A/cm². The cross-section emissivity of each spiral filament along its axis, i.e. in the direction of the impinging plasma, is about 100 A/cm².

A number of cathode treats and recipes are required to achieve this result:

- An AC current must be used for the direct heating of the cathode, in order to spread the ion plasma current over the filaments. DC heated cathodes did work but were prone to systematic damages at their most negative voltage point.
- The time required for heating up the cathode before the plasma shot is about 15 s.
- The AC current is switched off as soon as the Hydrogen arc breaks down.
- The AC heating current required for emitting $I_e = 600 \div 700$ A of plasma current is $I_{\text{cath}} = 550 \div 590$ A (rms.) at a voltage $V_{\text{cath}} = 14.5$ V (rms). So the reference ratio between the emitted plasma current and the AC cathode heating current is, for each filament, $I_e/I_{\text{cath}} \sim 1$.
- A cathode heating power $P_{\text{cathode}} \sim 8.5$ kW allows to inject into the screw pinch plasma a total power $P_e \sim 50 \div 85$ kW.

The PROTO-SPHERA cathode will be equivalent to 100 modules similar to the PROTO-PINCH cathode, connected in parallel in six groups (six-phased power supply): $I_e = 60$ kA = 100 modules • 600 A. Then the extrapolation to the PROTO-SPHERA cathode is:

1. The overall AC current required to heat the cathode will be $I_{\text{cathode}} = 60$ kA (rms.) at $V_{\text{cathode}} < 20$ V (rms.); it could be composed by a six-phased power supply able to deliver 10 kA per phase.
2. The overall cathode heating power will be $P_{\text{cathode}} \sim 850$ kW. This peak power will be required only for about 1 s before the arc breakdown. As a matter of fact, during the 15 seconds, required for bringing the temperature of the filament to 2600 °C, the heating power will be growing almost linearly toward 850 kW.
3. The total power required by the PROTO-SPHERA screw pinch can be estimated in two different ways. The rougher estimate is just to scale, by a factor 100, the power injected into PROTO-PINCH: $P_e = 100$ modules • $50 \div 85$ kW $\sim 5.0 \div 8.5$ MW. A more refined, but non less uncertain, estimate will be detailed in Section 6.5 and shows that the power injected into the PROTO-SPHERA electrode sheaths will be $P_{\text{Pinch}}^{\text{el}} \leq 4.6$ MW. The ohmic input $P_{\text{Pinch}}^{\square} \leq 5.4$ MW (see Section 4.9) and the helicity injection power, required for sustaining the spherical torus, $P_{\text{HI}} \sim 0.6$ MW (see Section 4.8) will have to be added, summing up to a total power $P_e \leq 10.6$ MW. A screw pinch power supply able to deliver 60 kA at 300 V will be adequate with both estimates (see Section 8.4).

Fig. 6.7 shows the waveforms of I_e and V_e obtained in PROTO-PINCH; the rise time of the pinch current is limited to 600 A/0.4 s by the characteristics of the power supply, which was originally the feeder of a klystron filament.

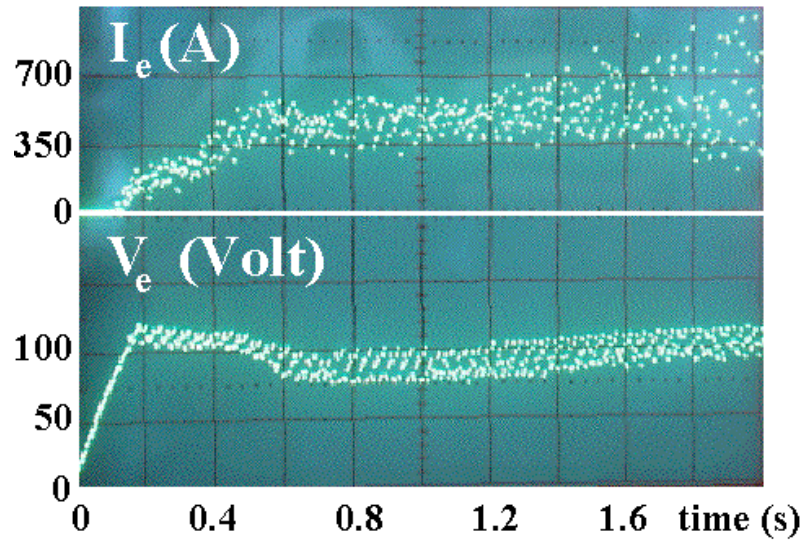


Fig. 6.7. Waveform of I_e and V_e in PROTO-PINCH. The ripples are modulations due to the AC cathode heating.

6.3. Anode

The PROTO-PINCH anode is a cylinder of Copper with 7 passing holes, of diameter 9 mm, drilled into it. Seven inserts of W95% -Cu5% protect the tips of the holes, on the plasma facing side. Scheme and pictures of the anode are shown in Fig. 6.8.

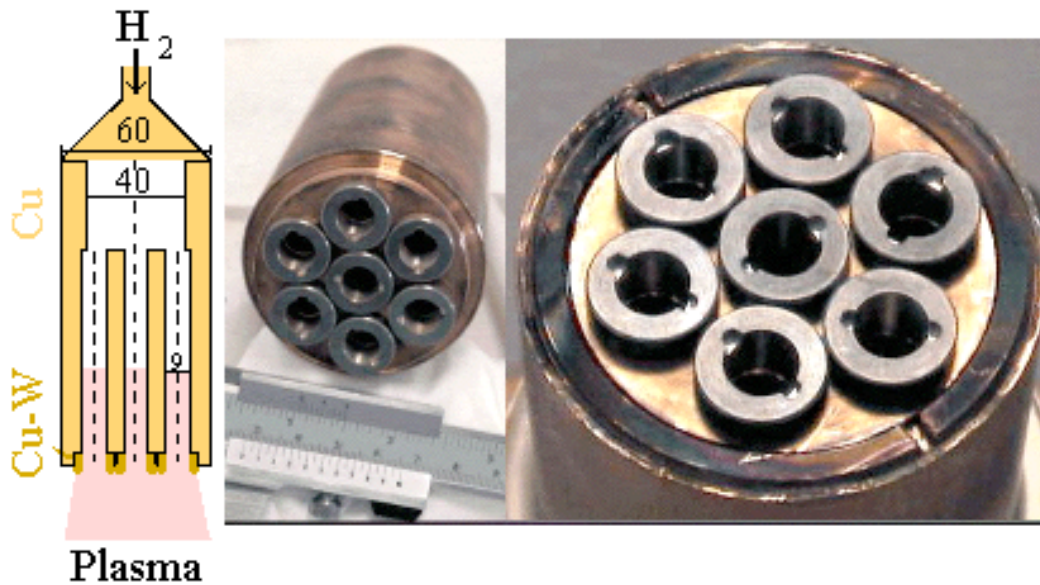


Fig. 6.8. Scheme and pictures of the PROTO-PINCH anode.

The PROTO-PINCH anode results can be so summarized:

- A feedback controlled system puffs gas at the rear of the holes and the emerging gas acts as a virtual anode for the plasma.
- The gas feedback system keeps p_H =constant inside the vacuum vessel.
- The PROTO-PINCH anode has not suffered any damage after more than 1000 plasma discharges.

While the cathode was DC heated there was sometimes evidence of anode arc anchoring (Fig. 6.9), where only one hole tip was emitting plasma. This dangerous phenomenon has disappeared after switching to AC cathode heating.

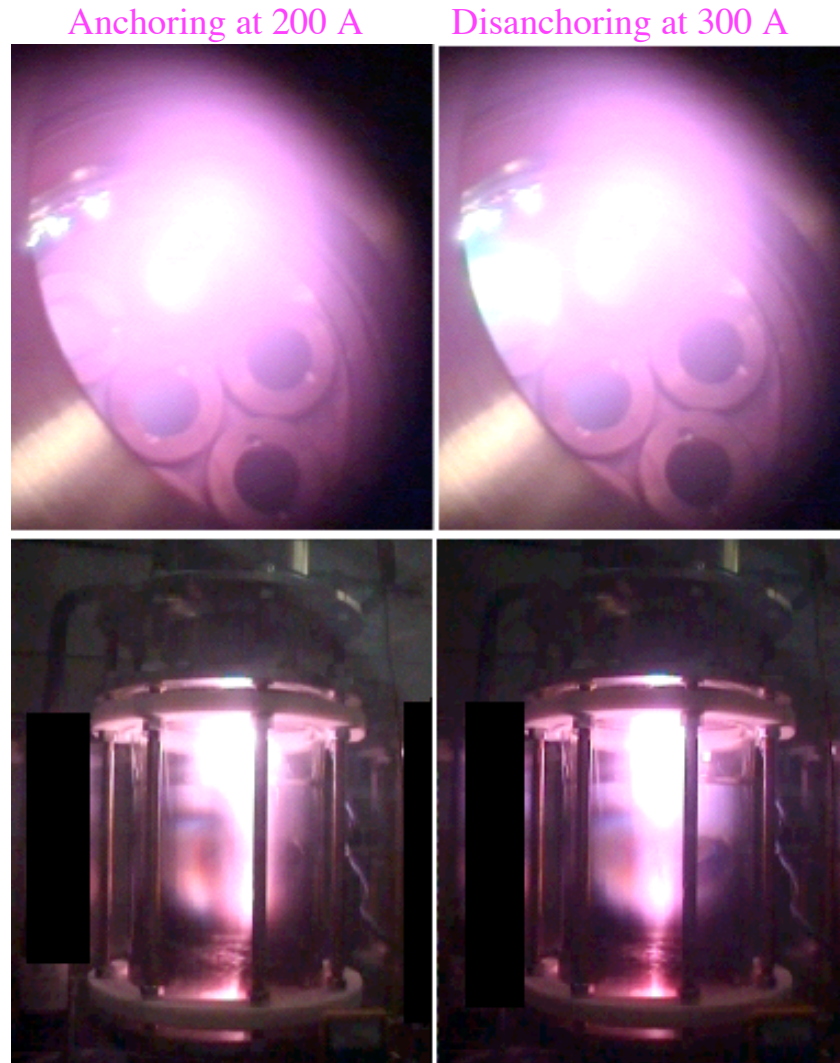


Fig. 6.9. Anode arc anchoring in PROTO-PINCH, while the cathode was DC heated.

Saddle coils near the PROTO-SPHERA electrodes could however be inserted, in order to provide a rotating magnetic field able to contrast arc anchoring, should this phenomenon reappear in PROTO-SPHERA. Four saddle coils, able to inject a torsional Alfvén wave with toroidal number $n=2$, could be placed below and above the anode and the cathode, respectively. The rotation frequency (of about 500 kHz) should be comparable to the Alfvén transit time in the plasma disk near the electrodes.

6.4. Diagnostics

The visible light emitted by the plasma of PROTO-PINCH has been collected on the equatorial plane by a telescope and focused onto 1-mm diameter optical fiber (see Fig. 6.1 and 6.2). An intensified diode array with 1024 pixels and spectral resolution of $3 \text{ \AA}/\text{pixel}$ has been used as detector. Visible spectroscopy of the Hydrogen plasma shows a single (unidentified) line near the H_{η} ($\approx 4303 \text{ \AA}$) at a count level of about 10^{-2} of the largest H_{η} line counts. (Fig. 6.10 and 6.11).

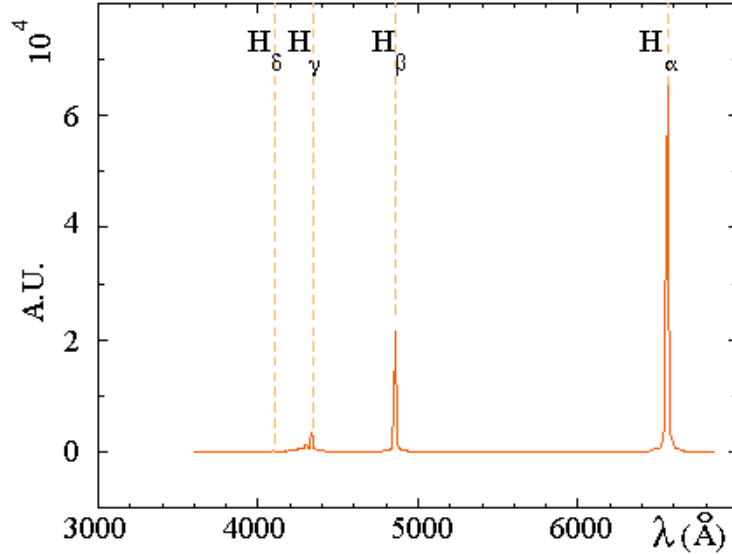


Fig. 6.10. Hydrogen plasma visible spectrum in PROTO-PINCH.

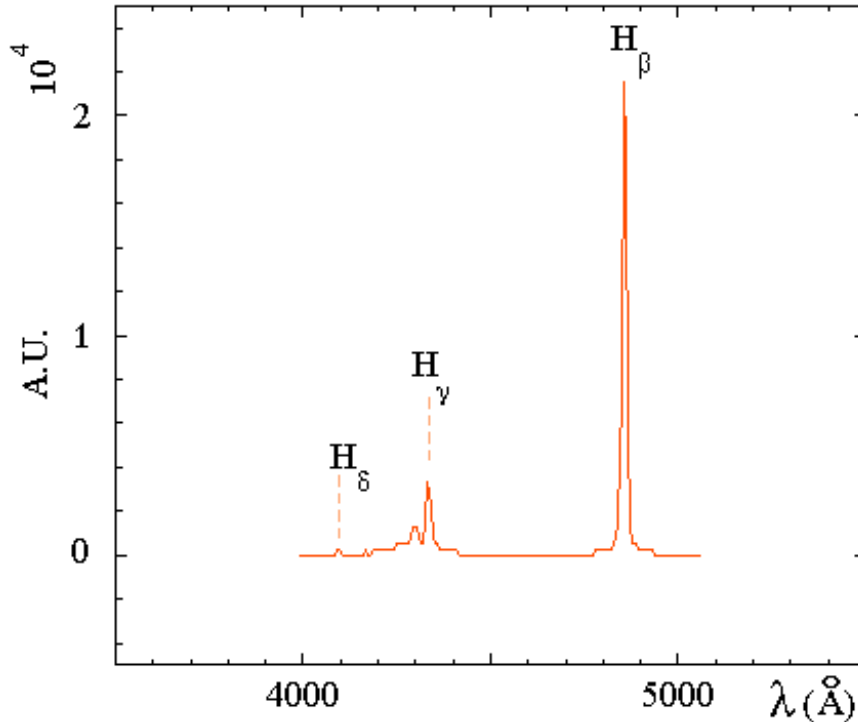


Fig. 6.11. Enlarged Hydrogen plasma visible spectrum in PROTO-PINCH. An unidentified line near the H_{η} ($\approx 4303 \text{ \AA}$) appears.

Even when Helium plasma discharges were run, the spectroscopic measurements did show barely perceptible impurity lines at a count level of about 10^{-2} of the largest Helium line counts (see Fig. 6.12)

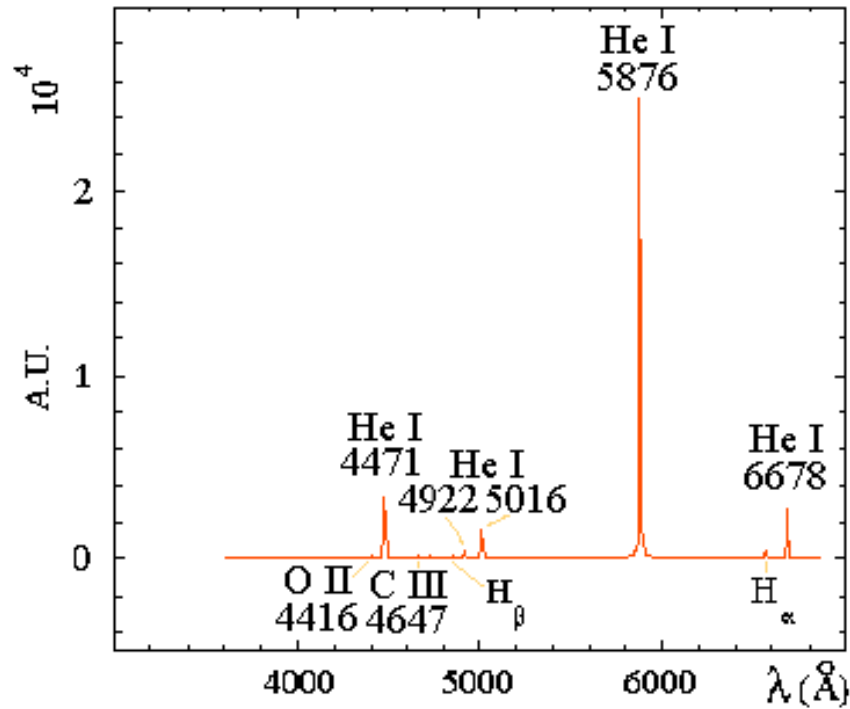


Fig. 6.12. Helium plasma spectrum in PROTO-PINCH at low filling density, showing the presence of impurity lines O II ($\lambda \approx 4416$ Å) and C III ($\lambda \approx 4647$ Å). Hydrogen lines are still present.

Density measurements has been done by means of a 2 mm microwave interferometer, whose scheme is shown in Fig. 6.1 and 6.2, using a 140 GHz oscillator. The density measurements have been successful only in Helium discharges at low arc currents ($I_e < 200$ A). They indicate that the line-averaged electron density of Helium discharges increases linearly with the current I_e , see Fig. 6.13. With a filling pressure $p_{\text{He}} = 4 \cdot 10^{-3}$ mbar, at $I_e = 200$ A the Helium ionization degree is about 16%.

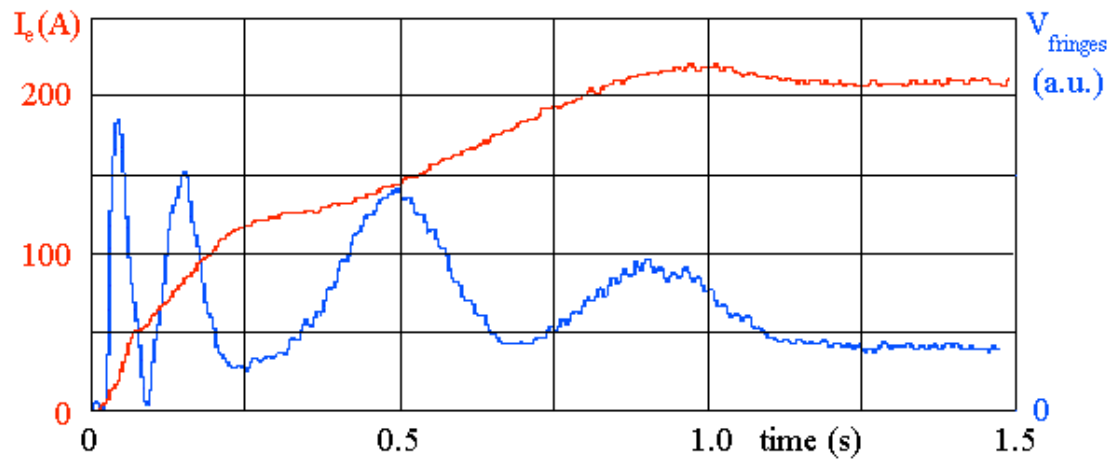


Fig. 6.13. Waveform of I_e and of the analogue signal of 2-mm interferometer fringes in a Helium discharge of PROTO-PINCH.

6.5. Screw Pinch Modeling and Extrapolation to PROTO-SPHERA

The PROTO-PINCH discharge at $I_e=600$ A is modeled as a straight cylinder of radius $\langle r_{\text{Pinch}} \rangle = 0.10$ m. Since the magnetic field lines are helices and the pinch safety factor at $B=1$ kG is $q_{\text{Pinch}} \sim 2$, the connection length (i.e. the length of a field line from one electrode to the other) is taken as $1.33 \cdot L_{\text{Pinch}}$ (~ 1.0 m). Under the assumption of constant electrical conductivity in the screw pinch, $\eta = \text{constant}$, one obtains for the ohmic power input:

$$P_{\text{Pinch}}^{\eta} = \eta \langle j_{\text{Pinch}} \rangle^2 \langle r_{\text{Pinch}} \rangle^2 1.33 \cdot L_{\text{Pinch}} / \eta,$$

where $\langle j_{\text{Pinch}} \rangle = 1.9 \cdot 10^4$ A/m² and $\eta = 2 \cdot 10^2 \ln \eta T_e^{3/2} / Z_{\text{eff}}$.

In the main body of the discharge, neglecting the radiation losses, the convected flux due to the electron flow toward the anode is the main loss:

$$P_{\text{Pinch}}^{\text{an}} = (5/2) \langle j_{\text{Pinch}} \rangle T_e \eta \langle r_{\text{Pinch}} \rangle^2.$$

Balancing the two terms:

$$(5/2) \langle j_{\text{Pinch}} \rangle T_e \eta \langle r_{\text{Pinch}} \rangle^2 = \eta \langle j_{\text{Pinch}} \rangle^2 \langle r_{\text{Pinch}} \rangle^2 1.33 \cdot L_{\text{Pinch}} Z_{\text{eff}} / (2 \cdot 10^2 \ln \eta T_e^{3/2}).$$

Substituting the values $L_{\text{Pinch}}=0.75$ m, $\langle j_{\text{Pinch}} \rangle = 1.9 \cdot 10^4$ A/m², $\ln \eta = 8$, $Z_{\text{eff}}=2$, the temperature of the main body of the PROTO-PINCH discharge is obtained: $T_e = 2.45$ eV. This temperature is roughly in agreement with the spectroscopic estimate $1 \leq T_e \leq 3$ eV.

Assuming that at $I_e=600$ A the plasma is 50% ionized, a filling pressure $p_H = 8 \cdot 10^{-3}$ mbar of H_2 corresponds to an electron density $n_e = 2.15 \cdot 10^{20}$ m⁻³ in the main body of the plasma.

However the total ohmic power injected into the main body of the discharge turns out to be only $P_{\text{Pinch}}^{\eta} = \eta \langle j_{\text{Pinch}} \rangle^2 \langle r_{\text{Pinch}} \rangle^2 1.33 \cdot L_{\text{Pinch}} Z_{\text{eff}} / (2 \cdot 10^2 \ln \eta T_e^{3/2}) \sim 4$ kW, whereas the total power injected through the electrodes is much larger, $P_{\text{Pinch}} \sim 50$ kW. This means that most of the power in PROTO-PINCH is injected into the electrode plasma sheaths, $P_{\text{Pinch}}^{\text{el}} \sim 46$ kW.

The power injected into the electrode plasma sheaths can be written as:

$P_{\text{Pinch}}^{\text{el}} = 1.6 \cdot 10^{-19} \cdot S_{\text{el}} \cdot \eta k_s \cdot n_{\text{el}} \cdot T_{\text{el}}^{3/2}$ Watt, where η is the energy transmission factor through the sheath ($\eta \sim 8$) and k_s the numerical coefficient for the sound velocity ($k_s = 9.78 \cdot 10^3 / A^{1/2}$ m/s, A being the mass number of the incident ion). The electron temperature and density T_{el} , n_{el} near the electrodes can be guessed from this formula, by assuming furthermore that the electron pressure is constant and the same at the electrodes as well as in the body of the pinch. Accounting for the electron flow at a distance from the sheaths: $2n_{\text{el}} \cdot T_{\text{el}} = n_e \cdot T_e = 5.3 \cdot 10^{20}$ eV/m³.

A total effective electrode area $S_{\text{el}} = 0.02$ m² is used, composed by 100 cm² of cathode filaments (see Section 6.2) and by 100 cm² of anode plasma wetted surface (see Section 6.3). The prescription that $4.6 \cdot 10^4 = 1.6 \cdot 10^{-19} \cdot 0.02 \cdot \eta k_s \cdot n_{\text{el}} \cdot T_{\text{el}}^{3/2}$ Watt, means $n_{\text{el}} \cdot T_{\text{el}}^{3/2} = 1.85 \cdot 10^{20}$, which together with $2n_{\text{el}} \cdot T_{\text{el}} = 5.3 \cdot 10^{20}$, gives: $T_{\text{el}} = 0.49$ eV, of the same order as the temperature of the cathode filaments, and $n_{\text{el}} = 5.4 \cdot 10^{20}$ m⁻³.

Intersecting the cross-section of both electrodes can perform an independent evaluation of the power injected into the electrode plasma sheaths. This gives a much

smaller surface $S_{el}=3.9 \cdot 10^{-3} \text{ m}^2$, but using the parameters of the main body pinch discharge: $n_e=2.15 \cdot 10^{20} \text{ m}^{-3}$, $T_e=2.45 \text{ eV}$, the estimate becomes $P_{Pinch}^{el}=40 \text{ kW}$. This is in reasonable agreement with $P_{Pinch}^{el}=46 \text{ kW}$, found using the electrode plasma parameters.

For the extrapolation to PROTO-SPHERA, one can assume that the plasma parameters near the electrodes will be the same as in PROTO-PINCH. This seems reasonable as the electrode modules will be almost the same and as the current and power densities at the electrode-plasma interface will also be the same. The plasma of PROTO-SPHERA shown in Fig. 4.8 ($I_p=180 \text{ kA}$, $I_e=60 \text{ kA}$, $\square_{Pinch}(0)=0.04 \text{ m}$), with $T_{el}=0.49 \text{ eV}$ and $n_{el}=5.4 \cdot 10^{20} \text{ m}^{-3}$, will have a temperature in the main body of the pinch discharge $T_e=36 \text{ eV}$ (see Section 4.9). Using $2n_{el} \cdot T_{el}=n_e \cdot T_e=5.3 \cdot 10^{20} \text{ eV/m}^3$, the result is that $n_e=1.5 \cdot 10^{19} \text{ m}^{-3}$, i.e. the main body of the PROTO-SPHERA screw pinch discharge will have an electron density much lower than PROTO-PINCH.

In PROTO-SPHERA the effective electrode surface will be 100 times the one of PROTO-PINCH: $S_{el}=2 \text{ m}^2$. Therefore the power injected into the electrode plasma sheaths will be: $P_{Pinch}^{el}=1.6 \cdot 10^{-19} \cdot 2 \cdot [k_s \cdot n_{el} \cdot T_{el}]^{3/2}=4.6 \text{ MW}$, which is obviously 100 times the one of PROTO-PINCH.

As a matter of fact, the radiative losses have been neglected in the analysis of the data of PROTO-PINCH; their proper accounting could reduce the estimate of the power injected into the PROTO-PINCH electrode plasma sheaths even by a factor of two. That would reduce the power required for PROTO-SPHERA, whose central screw pinch is predicted to work at a plasma density much lower with respect to PROTO-PINCH, with negligible radiation losses (0.1 MW). Therefore the estimate of the total power required for the screw pinch of PROTO-SPHERA, consisting of $P_{Pinch}^{\square}=5.4 \text{ MW}$, $P_{Pinch}^{el}=4.6 \text{ MW}$, $P_{Pinch}^{HI}=0.6 \text{ MW}$ and summing up to $P_e=10.6 \text{ MW}$, must be considered as a conservative upper bound.

The power injected into the electrode plasma sheaths can be considered to be constant $P_{Pinch}^{el}=4.6 \text{ MW}$ during all the toroidal plasma formation at $I_e=60 \text{ kA}$. It will be instead much lower during the stable pinch formation at $I_e=8.5 \text{ kA}$ (see Section 4.4), where $P_{Pinch}^{el}=0.65 \text{ MW}$, along with the ohmic power input $P_{Pinch}^{\square} \sim 0.1 \text{ MW}$.

The power density impinging on the electrodes of PROTO-PINCH, considering the sum of the cross-section of both electrodes, is:
 $P_e/S_{el}=50 \div 85 \text{ kW} / 3.9 \cdot 10^{-3} \text{ m}^2=13 \div 22 \text{ MW/ m}^2$.

The same figure for PROTO-SPHERA, that has a sum of the cross-section of both electrodes $S_{el}=3.5 \cdot 10^{-1} \text{ m}^2$, is:
 $P_e/S_{el}=5 \div 10 \text{ MW} / 3.5 \cdot 10^{-1} \text{ m}^2=14 \div 28 \text{ MW/ m}^2$.

7. MECHANICAL ENGINEERING

The PROTO-SPHERA machine consists of the following components: the vacuum vessel (VV), poloidal field (PF) coil system, the internal support, the anode and cathode and the machine support (MS), see Fig. 7.1. In addition a number of screening plates protect the coils from the hot anode and cathode. Stabilizing limiters, divertor protection plates and reflection rings are also required. The main parameters of the machine are given in Tab. 7.1.

Spherical Torus (ST) diameter	0.7 m
Longitudinal Screw Pinch current	60 kA
Toroidal ST current	120÷240 kA
Plasma pulse duration	1 s
Minimum time between two pulses	5 min.
Maximum heat loads on first wall components in divertor region	$\sim 2 \text{ MW/m}^2$
Maximum heat loads on rest of first wall	3 MW/m^2 , for 0.5 ms
Maximum current density on the plasma-electrode interface	0.8 MA/m^2

Table 7.1. Machine Parameters.

□

The basic principle of the mechanical engineering of PROTO-SPHERA is for a substantial VV, which provides both the ultra-high vacuum enclosure and contains the PF coils, the anode, the cathode and the other components. The PF coils are located very close to the plasma and therefore must be positioned inside the VV. In order to achieve the required ultra high vacuum conditions ($\sim 10^{-8}$ mbar), each coil will be enclosed in a vacuum tight metal case.

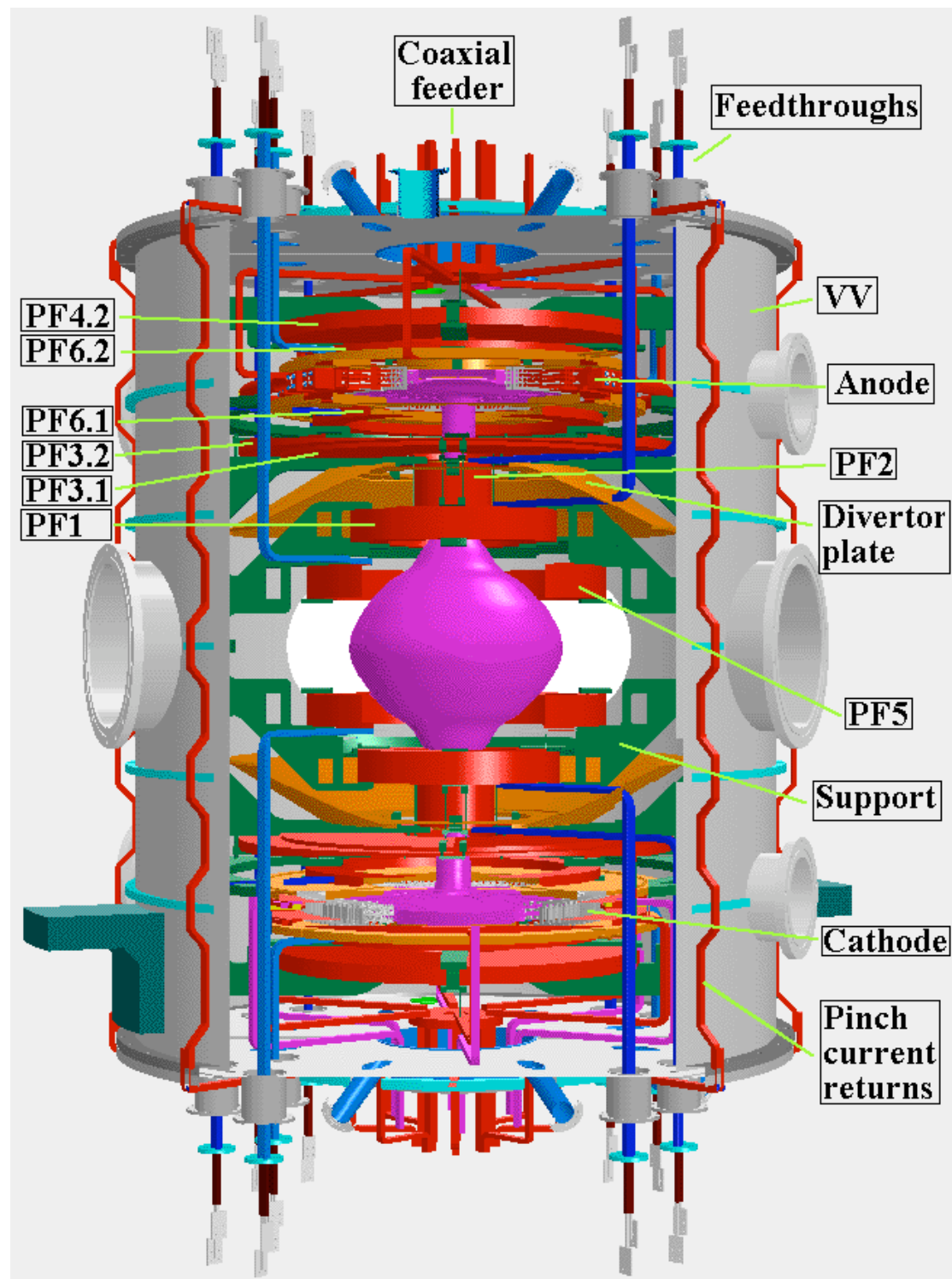
The plasma arc inside the machine is produced by two electrodes, anode and cathode, which are (particularly the cathode) the most unconventional and technologically demanding components.

The primary aim is to produce a design as simple as possible, easily assembled, with good access, particularly to anode and cathode, which are critical components and may require frequent maintenance/repair. Considering the experience in PROTO-PINCH, no major problems are also expected with the electrodes, the only unconventional components.

In order to enhance the reliability and maintainability, all connections for the PF coils are external to the VV. All the feeds come from the top and bottom flanges, leaving space for diagnostic ports in the main body of the VV. Each coil has a separate feed connected to the access flange by a flexible bellows arrangement, in order to adjust its position. Provisions will be made in the design to minimize the stray magnetic field, particularly in region near the spherical torus.

Particular care has been exercised to maintain the appropriate potential in each component, to avoid hot spots (circa 90°C) in the coils and to accommodate the electromagnetic stresses during plasma disruptions. Insulation plates have been used where appropriate, while no coil can see directly the cathode.

The PF coil system, the anode and the cathode will be pre-assembled outside the VV to check and adjust their relative positions. They will then be installed inside the VV, which will be closed by the top and bottom flanges.



□ Fig. 7.1. 3D outline of PROTO-SPHERA, showing the main machine components.
 □
 □

7.1. Vacuum Vessel



Figure 7.2 gives the basic machine geometry. The VV is a non-magnetic Stainless Steel (AISI 304L) vessel, 2 m in diameter and ~2.5 m in height. The thickness of the VV will be ~18 mm while the flat top and bottom flanges will be ~30 mm, in order to resist effectively the vacuum forces (~300 kN per flange). Flat flanges (albeit with increased thickness with respect to the VV cylinder) have been chosen to generate space for the coil feedthroughs, and facilitate interfaces.

The VV has a large number of ports for diagnostic purposes and vacuum pumping. Eight 500 mm and eight 250 mm ports are foreseen in total. In order to accommodate the vacuum forces and avoid distortion (ovality) of the ports, stiffening ribs will be incorporated as required. Note that in the top and bottom flanges, viewing ports will be employed to check the condition and operation of anode and cathode, see Fig. 7.2. During normal conditions the VV will be at room temperature (20 °C) with a vacuum of $\sim 10^{-8}$ mbar. However provision will be made to bake the machine up to 80-90 °C. Such a baking temperatures are effective in removing water vapor. They allow for the use of Viton O-rings and simplify the flange design. Finally they avoid any control of the PF coil insulation, which should be maintained always lower than 100 °C, in order not to run any risk of damage due to excessive temperatures. In addition the choice of a relatively low baking temperature of 80-90 °C will also result in a lower total cost for the machine. The predicted total outgassing rate by the O rings and the VV Stainless Steel is $\sim 3 \cdot 10^{-5}$ mbar·l/s. Such an outgassing rate, together with that of anode and cathode, can be easily accommodated by turbomolecular pumps, considering the port areas present in the VV.

The baking temperature will be reached by electrical heating tapes located on the external surface of the VV and of the top and bottoms flanges. The size of the machine requires a supply of ~25 kW for baking, which will take about 3÷4 hours to heat the assembly. Note that in order to speed up the baking cycle, avoid hot spots (dangerous for the coils) inside the machine, minimize thermal gradients and avoid relying only on conduction and radiation to heat the internal components, contact dry Nitrogen gas at ~1 mbar will be used during the temperature ramp-up phase. At this filling pressure convection starts to become effective. Thermal insulating material outside the VV would reduce losses to the environment and speed up also the baking cycle.

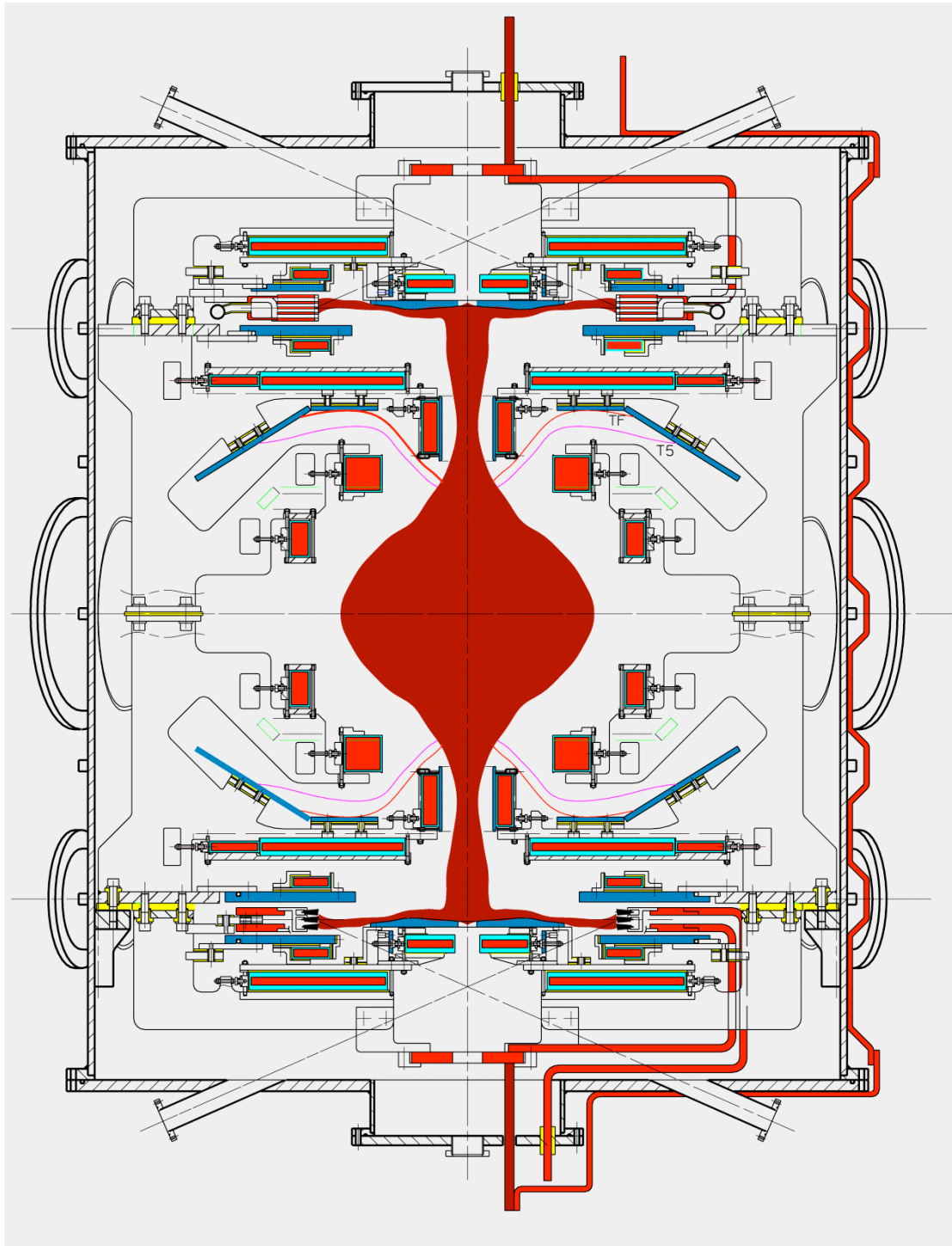
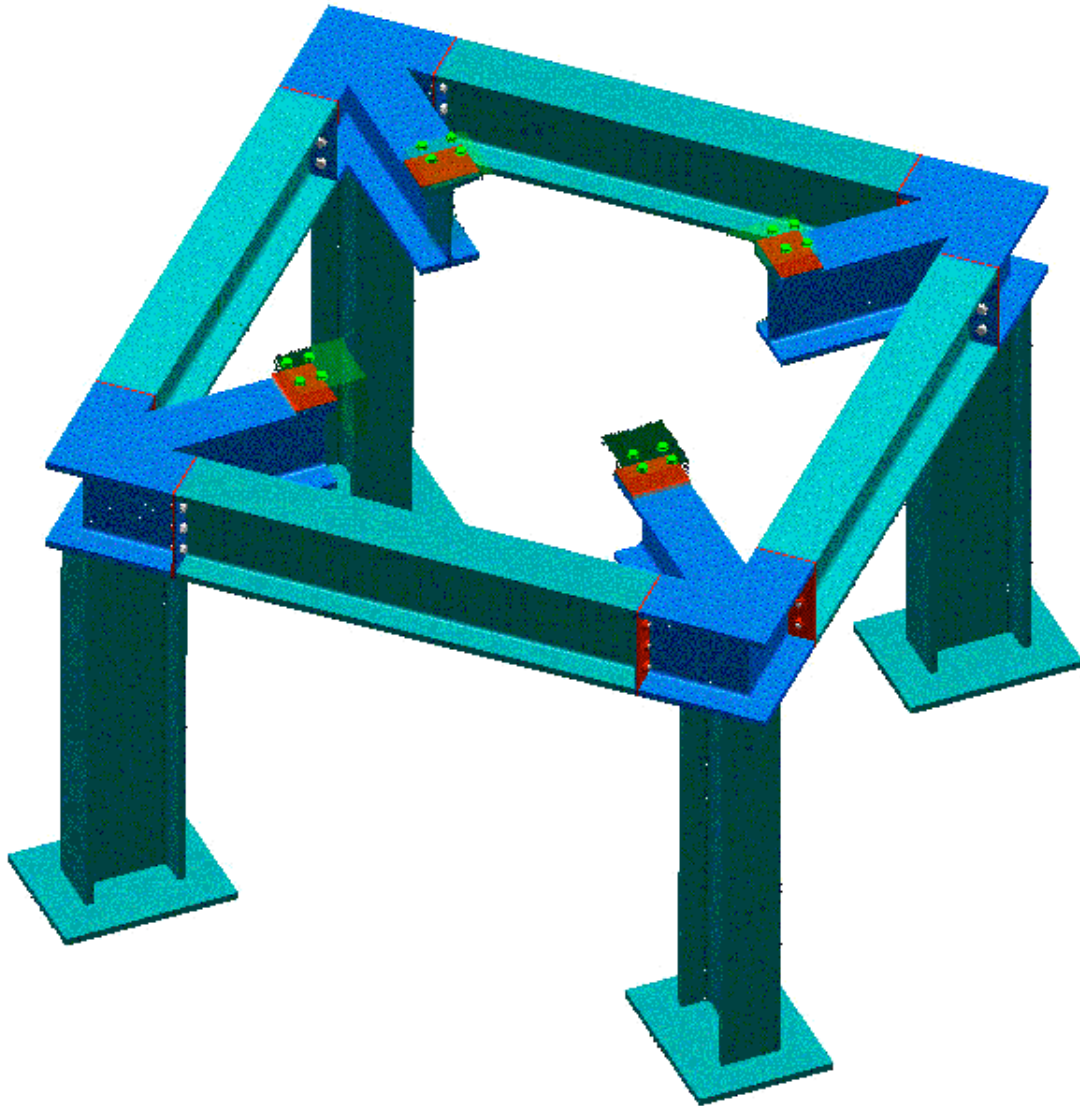


Fig. 7.2. Basic machine geometry as a cross-section outline.

In Fig. 7.2 the internal coil support structure is also shown. This supports mainly the PF coils, anode and cathode and consists of a rigid mechanical framework in which toroidal eddy currents are limited. The structure is divided vertically into 4 parts, which can be connected at different levels of potential. The 3 upper parts are electrically insulated from each other and from the VV. Alumina or other suitable material will be adopted for the insulation. The coil support structure will have to withstand the electromagnetic forces generated during normal operation and plasma

formation and will incorporate suitable mechanical system to adjust to the required alignments. Figure 7.3 shows the support of the VV, which has to accommodate thermal expansions during baking, in addition to the ~110 kN weight of the machine. It will be made from non-magnetic Stainless Steel (AISI 304L) to limit the stray field in the plasma region. This support arrangement provides also space for access to remove the top and bottom flanges, as required for the anode/cathode maintenance.



□

Fig. 7.3. Support structure of the vacuum vessel.

The VV will be designed in detail and manufactured according to pressure vessel requirements (ASME), with limited weld radiography where possible. Where not possible, welder qualifications will suffice. Good ultra high vacuum practice (no blind holes, clean conditions, etc) will be naturally employed, while all components will be vacuum baked to at least 150 °C prior to final installation.

□

7.2. Poloidal Field Coils

□

There will be two sets of poloidal field coils in PROTO-SPHERA, see Fig. 7.4: type 'B', the set of coils which shape the screw pinch and whose currents do not vary during the plasma evolution; type 'A', the set of coils which compress the ST and whose currents vary during the plasma evolution. As the formation time of the configuration will be 1 ms, the coils whose variable currents compress the ST will be shielded inside thin metal cases (time constant $\sim 200 \mu\text{s}$). On the other hand the coils with constant current will have to be enclosed inside thick conductors (time constant $> 2 \text{ ms}$) in order to stabilize the formation phase. As a consequence the type 'A', PF coils will be enclosed in an Inconel case of 1.5 mm thickness, while the type 'B' coils in a Stainless Steel (AISI 304L) case of $\sim 10 \text{ mm}$ thickness. Note that the two PF2 coils, Fig. 7.4, require an additional cylindrical shield facing the plasma to reach the required time constant of 2 ms. This will be made from Copper-Tungsten alloy and, due to space restrictions, can act also as a first wall protection for the coil.

□

All coils at present are designed considering normal operating conditions, i.e. using PF current waveforms consistent with the proposed plasma current and shape (see Section 8). This is due to the fact that the time variations of PF current waveforms during the plasma formation result in flux variations similar to those occurring in tokamak disruptions. Fault conditions will however be considered in detail in future design stages.

□

The coils are arranged coaxially and sustained by the support structure, Fig. 7.4. The coils and their supports are designed to withstand electromagnetic forces during normal and fault conditions. They can also accommodate thermal expansion during baking and normal operation.

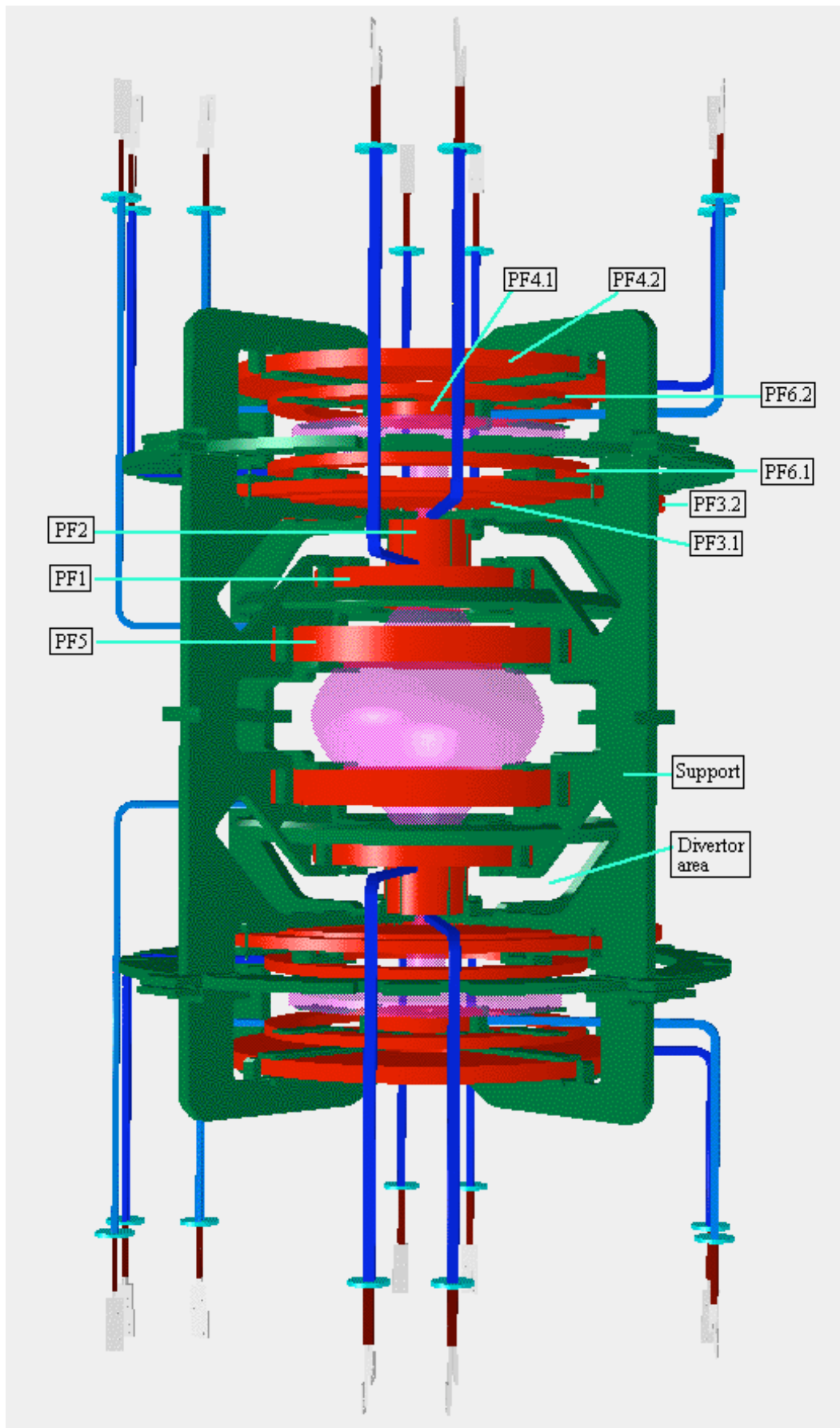


Fig. 7.4. Poloidal field coil arrangement and support structure.

The metal cases of all PF coils will be kept individually floating. □



7-8

Table 7.2 gives the electrical, geometrical and thermal coil characteristics. The coils need to be cooled between pulses within ~5 min. A maximum ΔT after a pulse of ~35 °C has been predicted in the PF2 coil with pessimistic assumptions; this ΔT is generated from the coil current (Joule effect), from the plasma, which is at close proximity, and from the anode or cathode. Water has been chosen as coolant in order to limit the pressure drop ΔP , which was too high in case of gas cooling (He or N₂). With water and a 6-mm hollow conductor, the ΔP will be limited to a few bar (~4 bar).

Coil □	N° of turns	Maximum Current per turn [A]	Mean Radius [mm]	Coil z* location [mm]	Coil size □r•□z [mm ²]	Approx. Coil Weight [N]	Current density [A/mm ²]	Total Coil □T [°C]
PF 1	64	1156	280	375	98.92	530	11.56	3
PF 3.2	10	1156	625	625	129.26	440	4.59	25
PF 5	32	1156	450	200	51.94	450	11.56	3
PF 2	48	1875	100	500	43.138	295	25.51	35
PF 3.1	24	1875	362	625	383.26	1480	5.21	25
PF 4.1	18	1875	100	885	138.26	200	11.16	25
PF4.2	18	1875	400	985	365.26	1565	3.72	25
PF6.1	8	1156	420	720 (710)	93.21	332	6.8	2
PF6.2	8	1156	420	910	93.21	332	6.8	2

* vertical distance from machine center line

□

Table 7.2. PF coil characteristics.

□

Table 7.3 gives a preliminary estimate of the coil vertical electromagnetic forces generated during normal operation. These forces would be accommodated by the support systems. A preliminary assessment of the hoop stresses generated in the Cu conductors gives a value of only a few MPa.

□

COILS	without Plasma	with Plasma
PF1+PF5	-8.2	-1.6
PF2+PF3.1+PF3.2	6.9	0
PF4.1+PF4.2	-2.7	0

□

Table 7.3. Coil electromagnetic forces [kN] during operation; see Fig. 7.4.

□

Figures 7.5 and 7.6 show typical coil details. Each coil turn is wrapped with half-lapped glass fiber and Kapton tape up to 0.6 mm, where necessary to meet the voltage requirements. The inter-layer insulation will be made from the same material, but 1.8 mm thick.

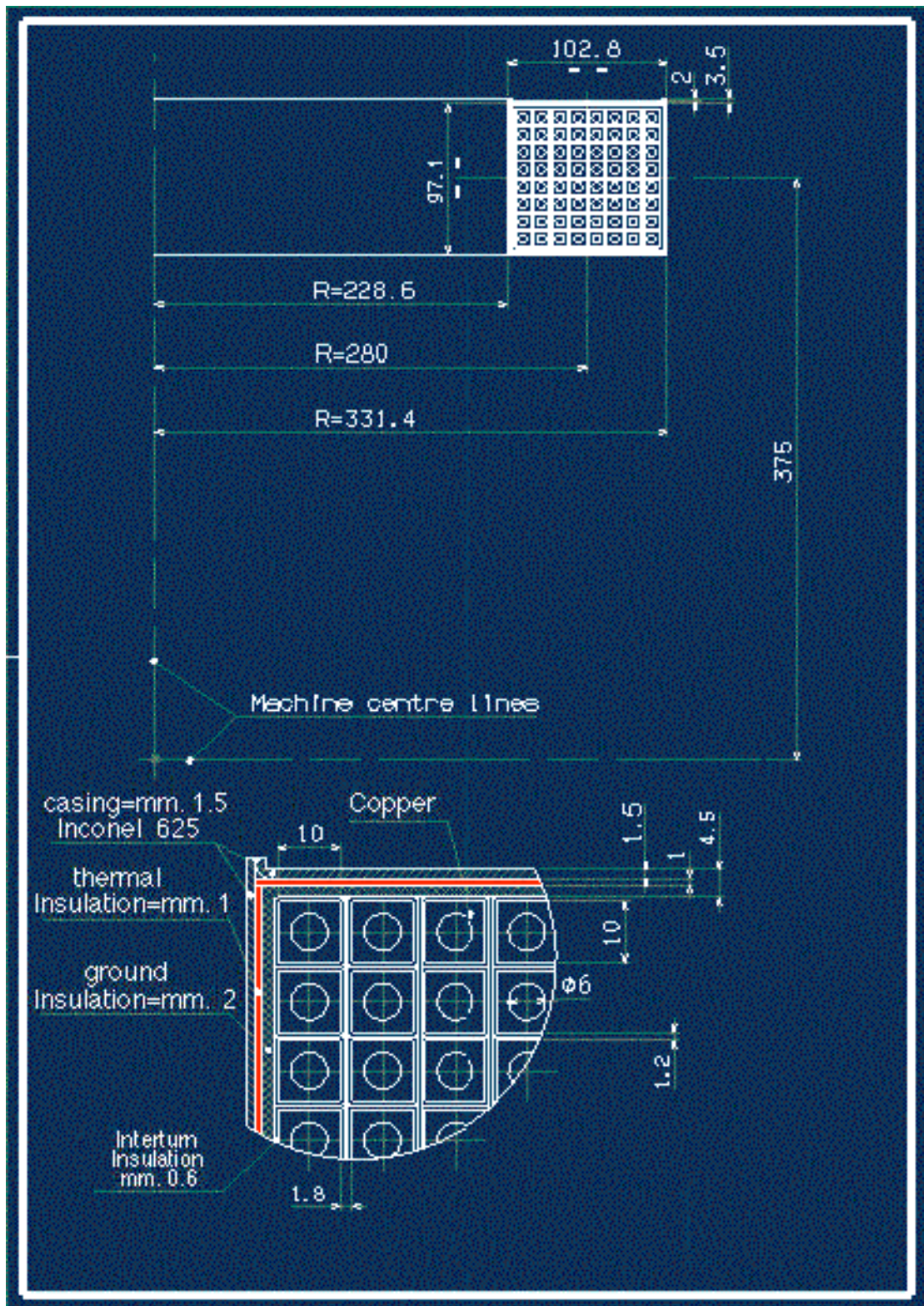
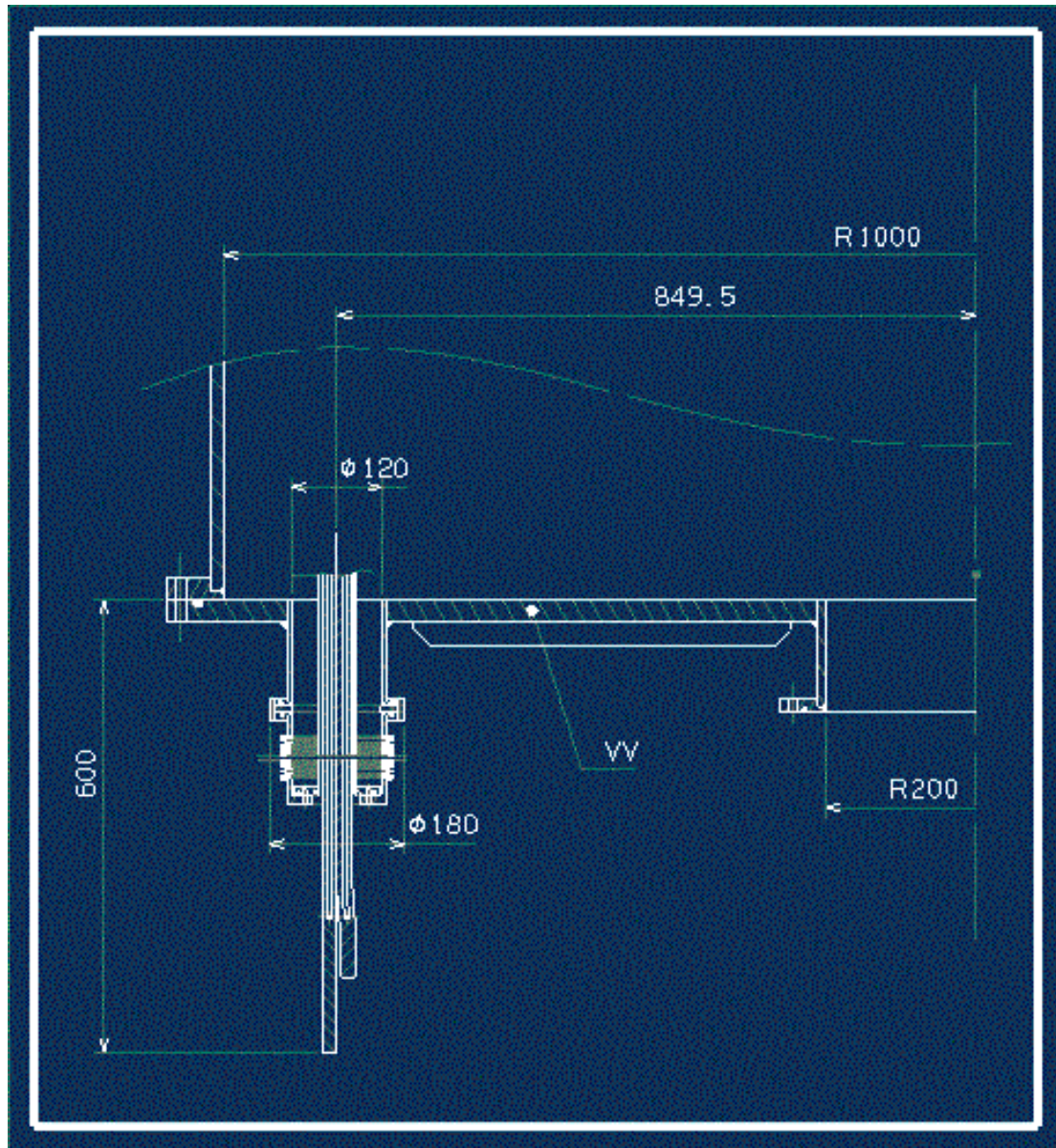


Fig. 7.6. PF1 as an example of a group 'A' poloidal field coil.

The ground insulation will be up to 2 mm thick. Figure 7.7 gives details of the coil feedthroughs and the associated bellows arrangement to accommodate coil alignment requirements and thermal movements. An electrical break, vacuum-sealed, assures the electrical insulation between the VV and the coil metal cases. The coils after the manufacture of interturn and interlayer insulation will be vacuum impregnated with

epoxy resin prior to casing. Then the ground insulation will be made and the coils will be positioned inside their cases, and a thick layer of high temperature thermal insulation will be placed between the case and the coil. The final welding is done in a lap joint of the metal case to avoid damage in the insulation. The whole assembly is then evacuated and vacuum impregnated with epoxy resin.

□



□

Fig. 7.7. Detail of the feedthrough of a poloidal field coil.

□

Note that stray fields can be generated in the plasma region. Stray fields can be due to induced currents in the VV, support structure and coil metal cases, to misaligned position of the coils, to the detailed geometry of the turns, to the electrical feeders and to the presence of ferromagnetic materials. The significance of such error fields is being assessed and suitable provisions are being adopted: a precise alignment procedure has been studied. The effect of induced currents will be computed and an ad hoc insulation will be introduced, if required. The joggles in the PF coil turns will be localized if necessary in order to compensate, as much as possible, the vertical component of the current in the helical winding type coils. The two electrical feeders

of each coil will be maintained very close to each other and will be connected to the coil as far away as possible from the plasma region. Finally non-magnetic materials will be used.

The coil metal cases and the support structure need to be protected from the plasma heat loads. A max power density of $\sim 2 \text{ MW/m}^2$ for $\sim 1 \text{ s}$ is expected in the divertor region (Figs. 4.16 and 4.17), and significantly lower heat loads elsewhere in the machine. Such a power can be accommodated with conventional AISI 304L tiles. For a very short time ($\sim 0.5 \text{ ms}$) during the plasma start-up phase (Figs. 4.5, 4.14 and 4.15), a thermal load of 3 MW/m^2 has been estimated on the cylindrical shields of the PF2 coils.

□

□

7.3. Electrodes

□

The anode and cathode, the two electrodes for producing the screw pinch plasma that characterizes the machine, are perhaps the most technologically demanding components. Fig. 7.8 and Tab. 7.4 show the main characteristics and a preliminary design of the anode. This cylindrical component is formed by six 60° sectors, each with 5 modules. Each module is made from OFHC Cu, with its surface, exposed to the plasma arc, protected by an alloy of W-Cu(5%) to resist excessive transient temperatures ($\sim 1000^\circ \text{C}$). Gas puff in each individual module, summing up to $30 \text{ mbar}\cdot\text{l/s}$, is performed through 20, 10-mm diameter holes, see Fig. 7.8, to spread the arc energy and avoid melting. The modular design of the anode permits replacement of each module individually.

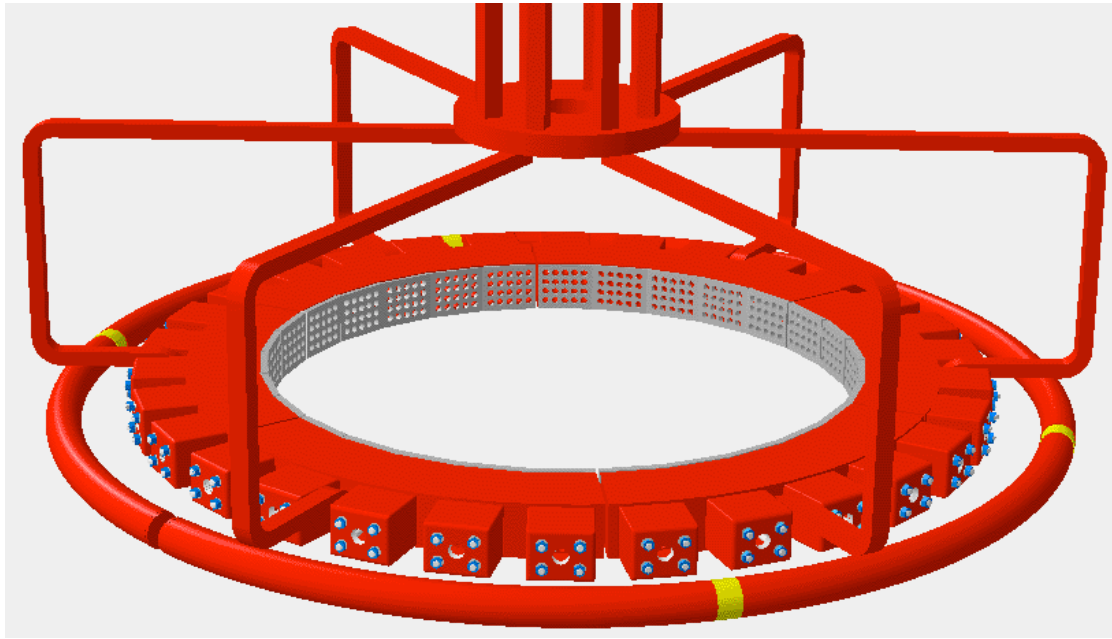
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Main Sectors:	6	Total Module Number:	30
Module Material:	Cu	Total Anode Holes:	600
Nuts & Bolts:	Inconel or Ta	Energy for each hole 1 sec:	6.7 kJ
Modules per Sector:	5	Total Arc Current:	60 kA
Protection Tile Material:	W-Cu (5%)	Arc Voltage:	100 V
Tile Max. Temperature:	$\sim 1000^\circ \text{C}$	Arc duration:	1 s
Module Hole Number:	20	Energy Distribution: Anode	2/3
Hole Diameter:	10 mm	Energy Deposition:	4 MJ
Module-Plasma Surface:	H 85mm • L 70mm	□	□

□

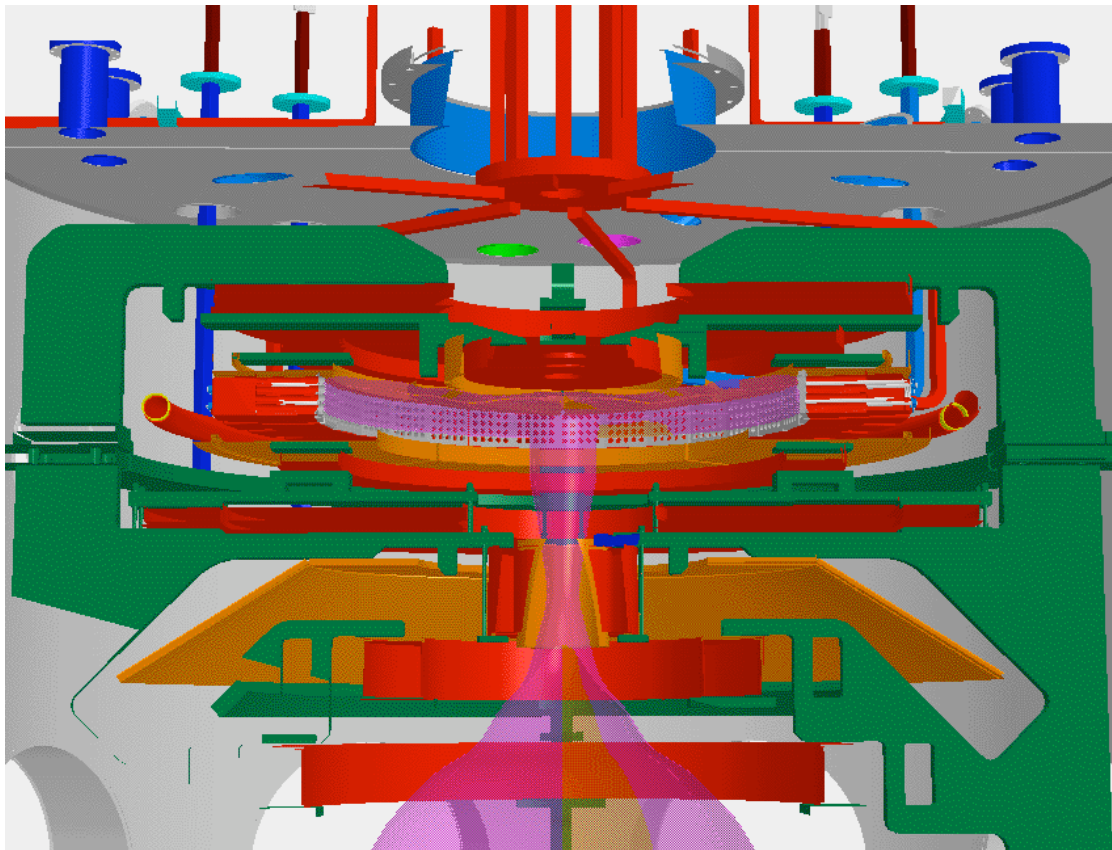
Table 7.4. Anode main features.

□



□
Fig. 7.8. Outline of the anode. Tubes (non-indicated) will connect each anode module to the gas distribution torus.
 □

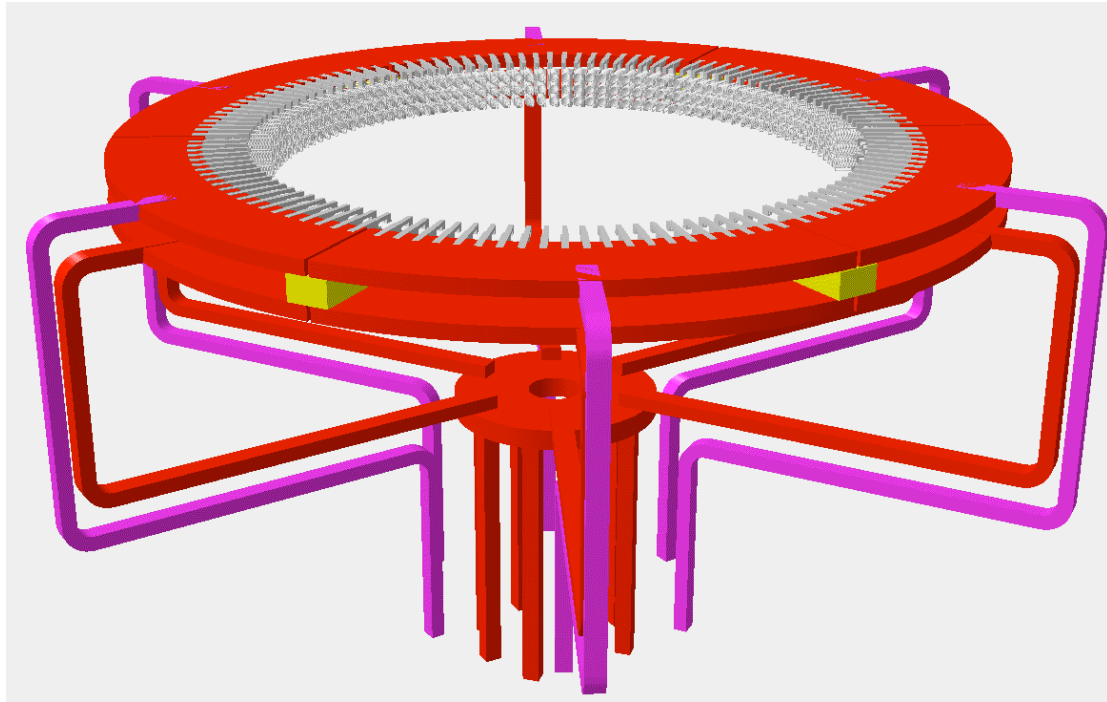
Figure 7.9 gives a view of the anode and the top part of the machine load-assembly.



□
Fig. 7.9. 3D view of the anode inside the PROTO-SPHERA anode chamber.
 □

Figure 7.10 and Tab. 7.5 show the main features and a preliminary design of the cathode.

□



□

Fig. 7.10. Outline of the cathode.

□

□

Main Sectors:	6	Wire Length:	40 cm
Dispenser Material:	Mo	Coil Surface:	25 cm ²
Nuts & Bolts:	Tantalum	Total Coil Number:	378
Insulators:	Alumina	Electron Emission Density:	6 Amp/cm ²
Dispenser per Sector:	24	Emission for each coil:	150 A
Coils per Dispenser:	3	Max electron Emission:	64.8 kA
Coil Material:	W	Voltage Power Supply:	15 V
Coil Work. Temper.	2750 °C	Total Cathode Current:	60 kA
Turns Number:	8	Heating Time:	15 sec
Wire Diameter:	2 mm	Est. Heating Energy:	8 MJ
Coil Diameter:	14 mm	Est. Arc Energy Deposition:	2 MJ
Coil Length:	50 mm	□	□

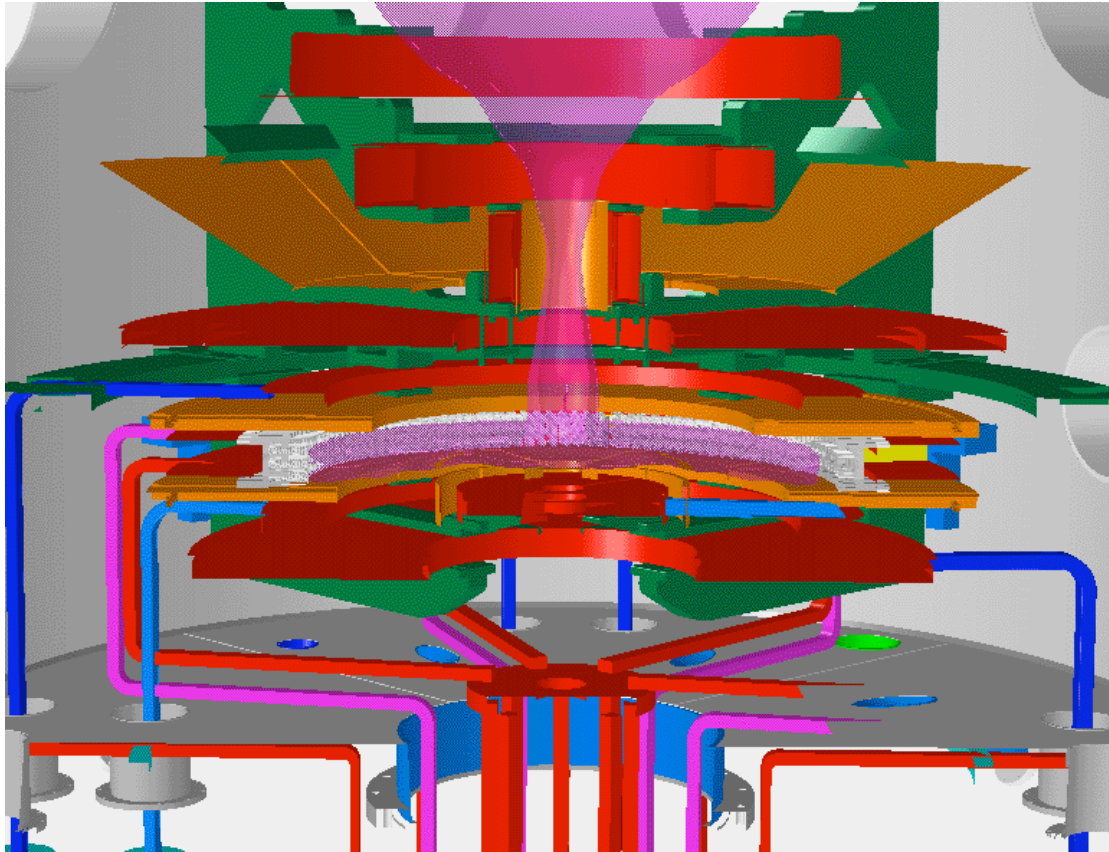
□

Table 7.5. Cathode main features.

□

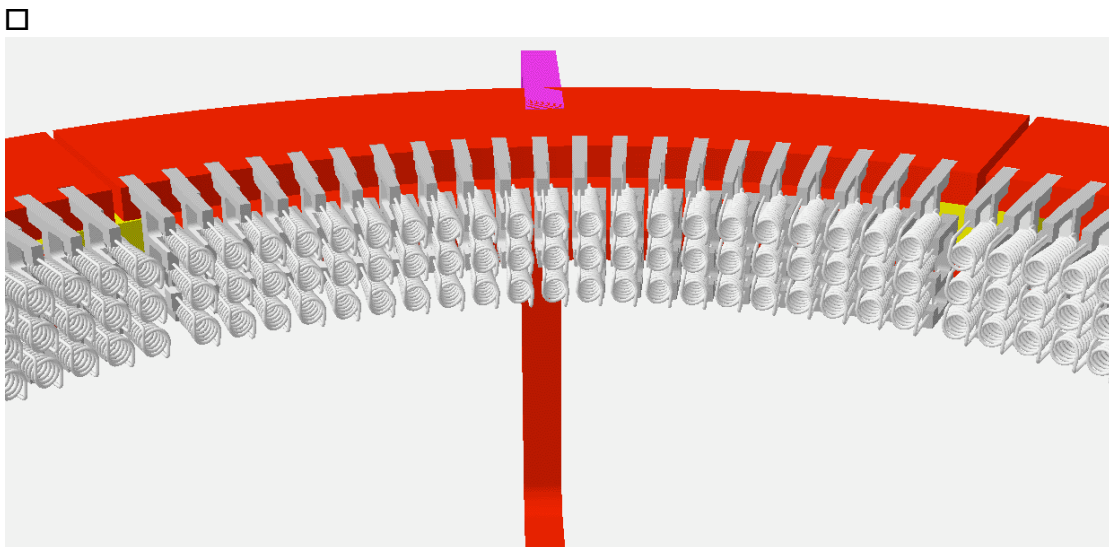
The cylindrical component is made from 378 coils supported by a dispenser assembly, see Figs. 7.11 and 7.12, which also feeds the current to the W coils. The dispensers are made from Mo to resist to high temperatures, which in the coils can reach up to 2750 °C. The cathode is composed from 6 sectors, each powered by a six-phased AC power supply. 24 dispensers form each sector, each carrying 3 coils of null field type, see Figs. 7.10, 7.11 and 7.12.

□



□ Fig. 7.11. 3D view of the cathode inside the PROTO-SPHERA cathode chamber.

□ The design is such that each dispenser can be individually replaceable. The six-phased AC power supply gives 8 MJ to the cathode. The heating time to the working temperature (2600 °C) of the coil wires is 15 s. As soon as the screw pinch plasma breaks down, the coil temperature increases to a maximum of 2750 °C.



□ Fig. 7.12. 3D front view of the cathode.

Concerning the reliability of the W filaments that operate at high temperatures, preliminary analytical and experimental work indicates no significant problems. The

predicted stresses are much less than the ultimate W strength at 2750 °C (circa 40 MPa) and result also in no creep at these temperatures for several thousand pulses. The machine duty cycle will be determined by the cooling time of the electrodes. The interpulse cooling of the anode and cathode will be mainly done by radiation. In order to achieve a machine duty cycle of 5 min, the global temperature (after a few successive pulses) must be 380 °C for the anode and 450 °C for the cathode. To facilitate the radiation cooling, the electrodes will be plasma-sprayed (where possible) by $\text{Al}_2\text{O}_3+\text{TiO}_2$, in order to enhance their radiation emissivity. Also the conduction cooling, via the electrodes supports and the copper conductors, although not yet quantified, would significantly facilitates the cooling.

Note that optical diagnostics are used to view directly most of the anode and cathode from the top and bottom flanges, in order to monitor any degradation. The design of the electrodes is modular so that local replacements can be done at minimum cost.

7.4. Divertor

□

Among the coils of the poloidal system of PROTO-SPHERA, some are very near and in direct view of the plasma and thus can be subjected to thermal loads. In addition the double X-point configuration requires target plates, where the thermal power diverted from the spherical torus can be dumped. Only the "normal" operation is considered here, while the problems that could arise in pathological events are only indicated.

The thermal flux impinging upon the divertor plates in the steady-state phase of the discharge is first evaluated, assuming that the spherical torus can be sustained for 1 s. Based on the calculated equilibrium configurations, the position of the divertor protection plates have been chosen, as indicated in Fig. 7.13. The rationale of this choice is to provide a large enough separation from the plasma to the divertor plates and to allow for the positioning of the target at a sufficiently small angle with the projection of the separatrix on the poloidal cross-section.

□

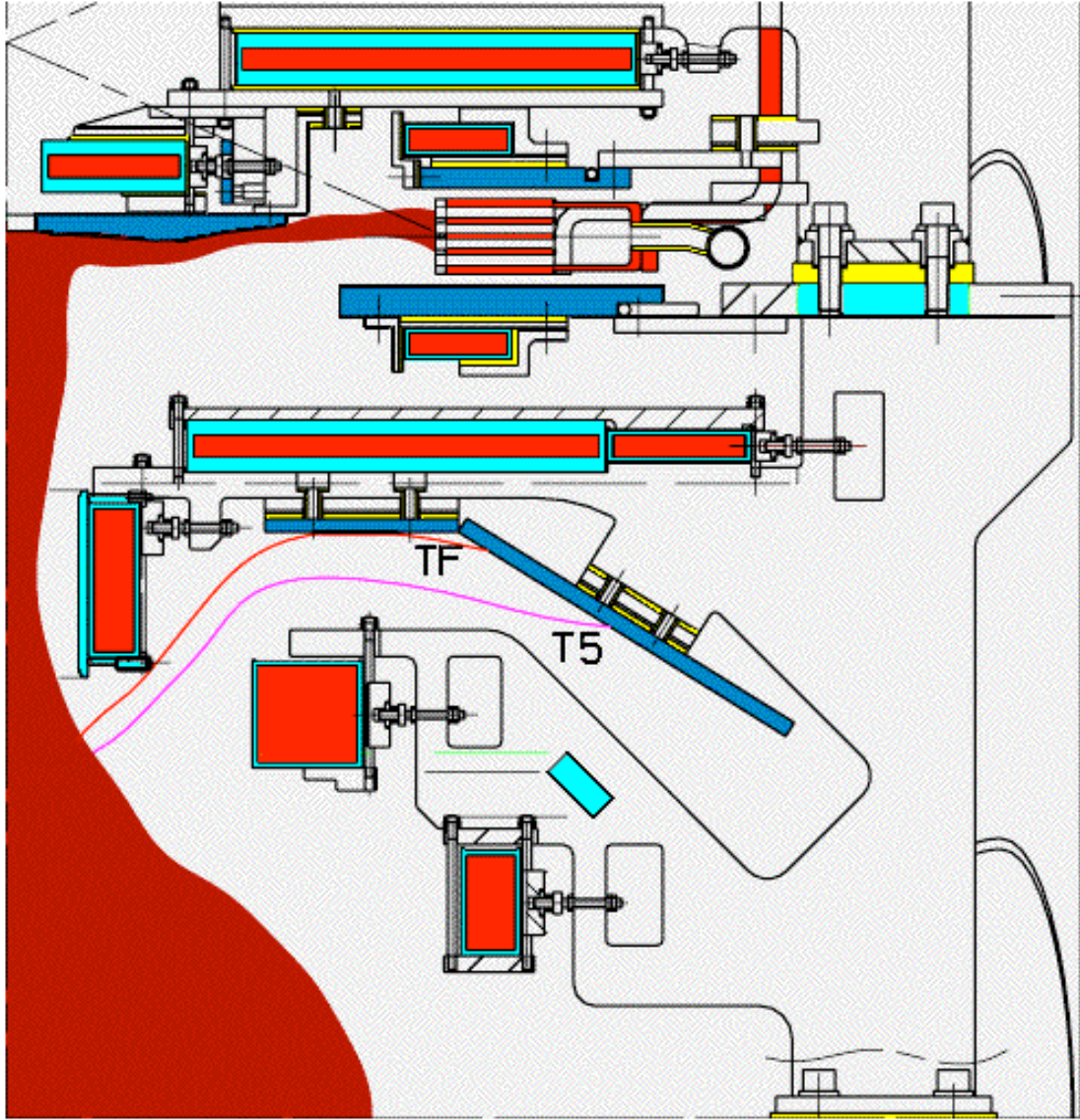


Fig. 7.13. Position of the divertor protection plates.

□

The total power required by the helicity injection through the X-points amounts to $P_{OH} \sim 0.58$ MW. A fraction of 50% of this power is assumed to be lost by radiation, due to the impurity content of the plasma. The remaining power is conducted/convected to the two (top and bottom) target plates through the scrape-off-layer (SOL). A further assumption is that all this power goes to the outer leg of the separatrix.

The total target surface wetted by the SOL plasma is given by

$$S_t = 2 \cdot 2\pi R \cdot \lambda_E \cdot e_{flux} \cdot 1/\sin\theta,$$

where R is the distance from the axis of the separatrix strike point on the target, λ_E is the energy decay length at the ST midplane, e_{flux} is the magnetic flux expansion at this distance, and θ the poloidal angle between the separatrix and the target surface. At the stated position, the following numbers are plugged into the equation:

$R = 0.45$ m, $e_{flux} = 2.5$ and $\theta = 20^\circ$, λ_E is taken as 1 cm, which is typical for the energy decay length at the midplane of a conventional tokamak. Thus the average thermal flux on the divertor plates is $P_{OH}/S_t = 0.7$ MW/m². One could expect that the corresponding peak thermal flux would be larger than this value by a factor of 3.

□

This thermal flux is easily manageable by any material we can think for the divertor plates, so that the choice of this material can be based on other issues. The most convenient one would be Stainless Steel. It is to be noted that the divertor plates configuration just described is rather unconventional with respect to tokamak experiments, and it could offer some advantages:

- The wetted surface is quite far away from the plasma, so that the impurity flux to the plasma, due to generation at the plates, could be lower than in more conventional configurations.
- Also the recycling should be quite different: neutrals emitted from the target can reenter the plasma only after recirculation through the vacuum chamber volume. This could result into a very diffuse refueling and into an effective recycling coefficient substantially smaller than 1.
- The target plates are accessible for optical, bolometric and thermographic diagnostics.

□

The target tiles have to be properly aligned in order to avoid formation of hot spots due to exposed edges. A possible problem to be examined is the abnormal behavior in presence of runaway electrons and disruptions.

□

The PF2 coils are situated in the private region of the ST divertor and are quite near to the screw pinch discharge. The contribution of the thermal flux due to the screw pinch can be evaluated by considering that the power loss due to transport across the magnetic field of the pinch can be approximated by: $Q_{per} = n_e \kappa_{eff} (T_e + T_i) / a_{pinch}$,

where n_e is the pinch electron density, κ_{eff} the conductivity coefficient, T_e and T_i the peak electron and ion temperature in the screw pinch plasma, and a_{pinch} the pinch radius at the PF2 position. Assuming radial transport of the Bohm type, $T_i = T_e \sim 30$ eV and a magnetic field of 0.25 T, we have $\kappa_{eff} \sim 35$ m²/s, and for a pinch density $n_e = 1.5 \cdot 10^{19}$ m⁻³:

$Q_{per} \sim 0.1$ MW/m², which can be considered negligible.

□

The thermal load impinging upon the surfaces of the coils, due to heat transport in the private region of the divertor, is quite difficult to evaluate. Anyway, on the basis of the data from conventional tokamaks, taking into account the distance from the separatrix, we can estimate that only a few percent of the power flowing in the SOL will impinge on the PF2 surface. Thus this heat flux would be of the same order of magnitude as that delivered by the screw pinch discharge.

Finally the PF1 coil intercepts the separatrix during the first millisecond of the ST formation (see Fig. 7.14 and 4.15). Even if in this case a power flux of ~ 3 MW/m² can hit the surface of the PF1 metal case, the total energy deposited there will be insufficient to increase its temperature by more than a few °C.

□

7.5. Protection Components

7.5.1. Coil Screening Plates

The coil screening plates, Fig. 7.14, are made out of copper. Their task is to contain the temperature increase of the coils that can see directly the cathode or anode to much less than 100 °C. The cathode can radiate up to 3 MW/m² when it reaches its maximum temperature. With this power density the metal cases of the coils, if directly exposed to the radiation, could reach locally temperatures well in excess of 100 °C in less than 1 sec. Such temperatures are dangerous for the insulation, which should not rise to more than 90 °C.

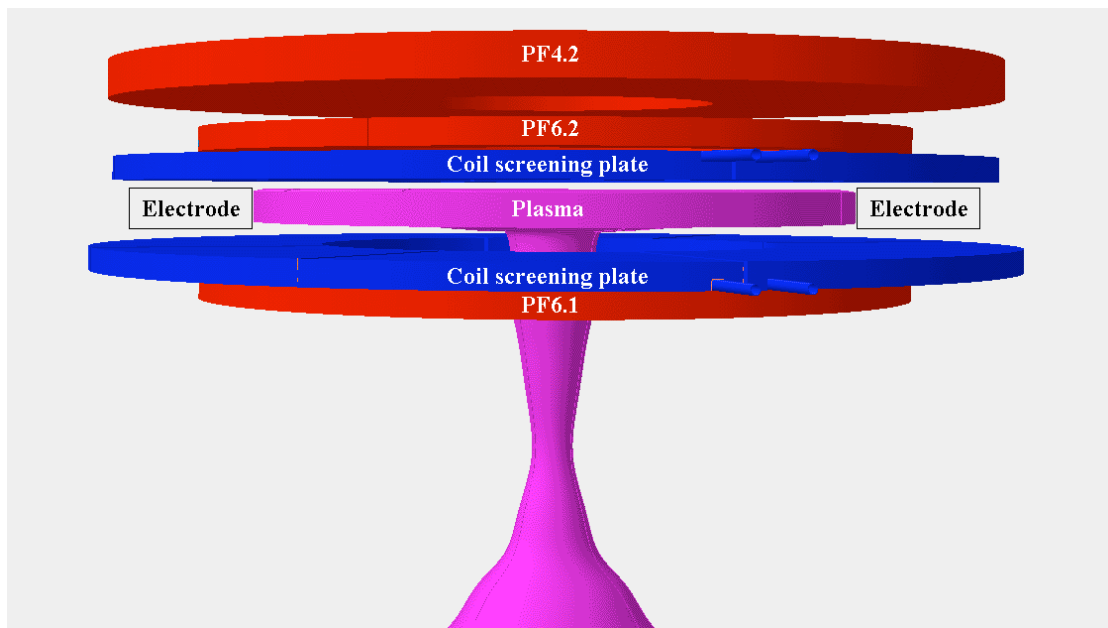


Fig. 7.14. Outline of the upper PF6 coils screening plates.

The coil screening plates are designed so that the electrodes viewing factors towards the coils is eliminated and their thermal mass is such that the maximum operating temperature is always less than 100 °C. Furthermore the plates are water-cooled in between pulses. In the 10-mm cooling pipes, a pressure drop Δp of less than 2 bar is expected for a flow velocity of about 4 m/sec. The conduction and convection resistances of the plates permit a cooling capacity of at least 20 kW, which allows for cooling within 5 min.

The surfaces of the screening plates will be plasma sprayed wherever possible. This will facilitate the total energy collection (about 16 MJ) at known resilient components and the radiation cooling, which will reach 1 kW/m², adding up at least 2 kW to the 20 kW cooling of the water circuit.

In order to avoid axisymmetric eddy currents, all the coil screening plates are subdivided in four 90° sectors. A pessimistic estimate of $B = 500$ G and of $dB/dt = 50$ T/s, provides maximum stresses not exceeding 100 MPa, even neglecting skin effects. Due to the high copper electrical conductivity and small formation time (~ 1 ms), skin effects will however be significant and would reduce the generated moments and stresses significantly.

Finally the PF2 coil is also screened by 10-mm thick collar made of W-Cu, Fig. 7.15. This component must also operate at less than 100 °C. Thus the maximum power density and energy it can accept are about 2 MW/m² and 120 kJ, respectively. It should be thermo-coupled, to avoid energy accumulation after a few successive pulses, since its maximum cooling capacity is limited to about 40 W.

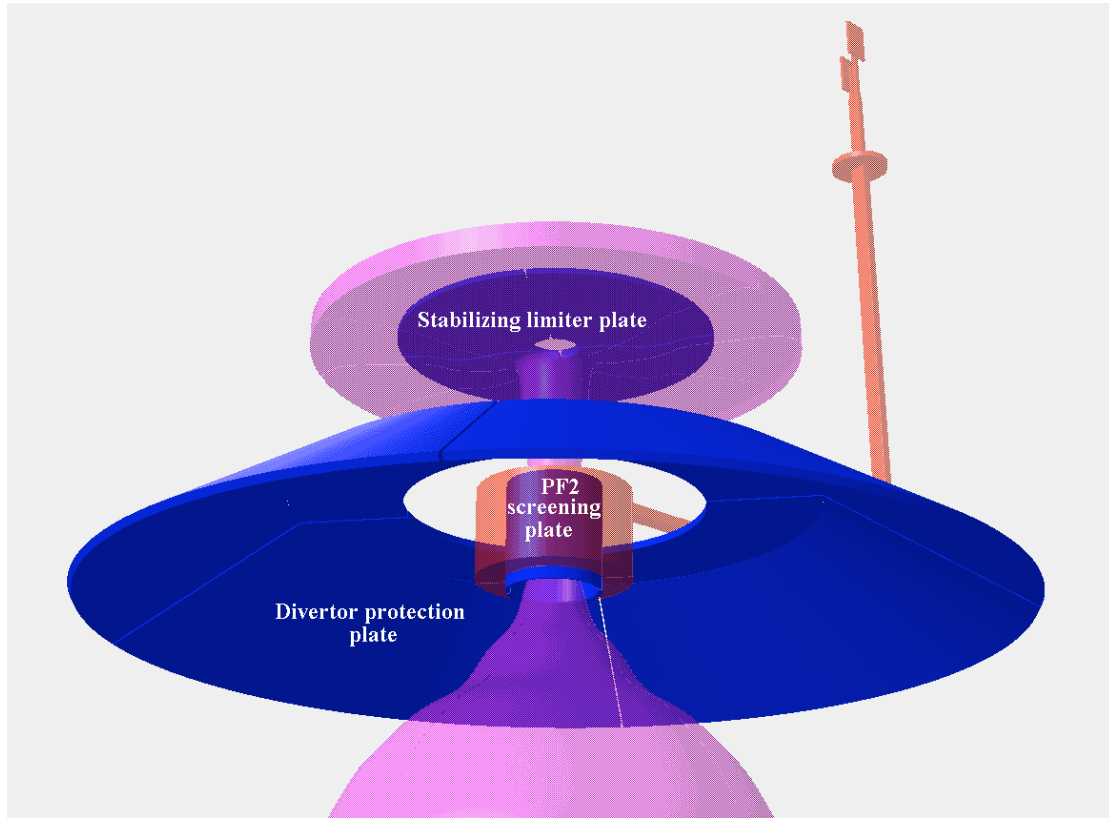


Fig. 7.15. Outline of the upper divertor protection plate, upper PF2 screening plate and upper stabilizing limiter plate.

7.5.2. Stabilizing Limiter Plates

The stabilizing limiter plates, shown in Fig. 7.15, are made from Cu with W plasma sprayed black and their plasma surface is profiled to follow the evolving plasma shape. They can absorb up to 1 MJ of heat, without endangering the coil PF4.1, radiating from about 100 °C. Thus their maximum radiation cooling capacity will be 600 W/m² (120 W). Also the stabilizing limiter plates should be thermo-coupled in order to avoid energy accumulation after a few successive pulses. Finally these components will also be subdivided in four 90° sectors, in order to avoid axisymmetric eddy currents.

7.5.3. Divertor Protection Plates

The divertor protection plates are made from 10 mm AISI 304L and have a mass of about 30 kg and 100 kg, see Fig. 7.15. As the maximum power density impinging upon them will be 2 MW/m^2 for 1 sec, a maximum temperature increase of 120°C is expected locally in the front surface. The maximum bulk temperature raise is only 60°C . Under this condition, the maximum radiation capacity is about 300 W/m^2 . To avoid energy accumulation, blackening of the rear surfaces of these plates ($\text{Al}_2\text{O}_3+\text{TiO}_2$) may be used to raise the radiation capacity to 700 W/m^2 . In addition thermo-couples may be also employed to monitor the temperature.

7.5.4. Reflection Rings

Two reflecting rings will be incorporated around the coils PF4.1, to protect them from the hot anode and cathode. They will be silver-plated in the outer surface, to reflect the radiated energy from the anode and cathode. Their dimensions are such that their operating temperature, considering solid view-angle from the electrodes, is less than 100°C .

7.6 Allowable and Permitted Stresses

The maximum eddy current stresses during disruptions are designed to be in all component less than 100 MPa. In addition in the divertor protection plates, a power density of 2 MW/m^2 , 1 s could give significant local thermal stresses of up to 320 MPa.

For AISI 304L plate, according to ASMEIII-NB3221, the allowable stress is 200 MPa for 100°C (min. of $1/3$ of ultimate strength or of $2/3$ of yield strength). According to this code, the eddy current stresses (100 MPa) must be less than $200 \cdot 1.5 = 300 \text{ MPa}$. The sum of all (including thermal) stresses ($100+320=420 \text{ MPa}$) must be less than $200 \cdot 3 = 600 \text{ MPa}$. It is clear therefore that significant safety factors are incorporated in the design.

Concerning the low temperature Cu components, an allowable of more than 70 MPa at 100°C is required to accommodate the eddy current stresses of less than 100 MPa. Therefore Cu slightly hardened with a yield strength more than 115 MPa and ultimate strength more than 230 MPa at 20°C is required.

Finally care requires to be exercised for the electrode Cu, where high strength at relatively high temperature may be required.

7.7. Machine Services

Figures 7.16 and 7.17 show the top and bottom flanges of the machine with the ports dedicated for the services. The water requirements are $2\div3 \text{ kg/s}$ per flange with a pressure drop of $4\div5 \text{ bar}$. The relatively high pressure drop is determined by the hydraulic resistance of the coils which, even with this pressure drop, have a flow velocity of about 1 m/s ($\text{Re}=6000$), the minimum acceptable.

Top View

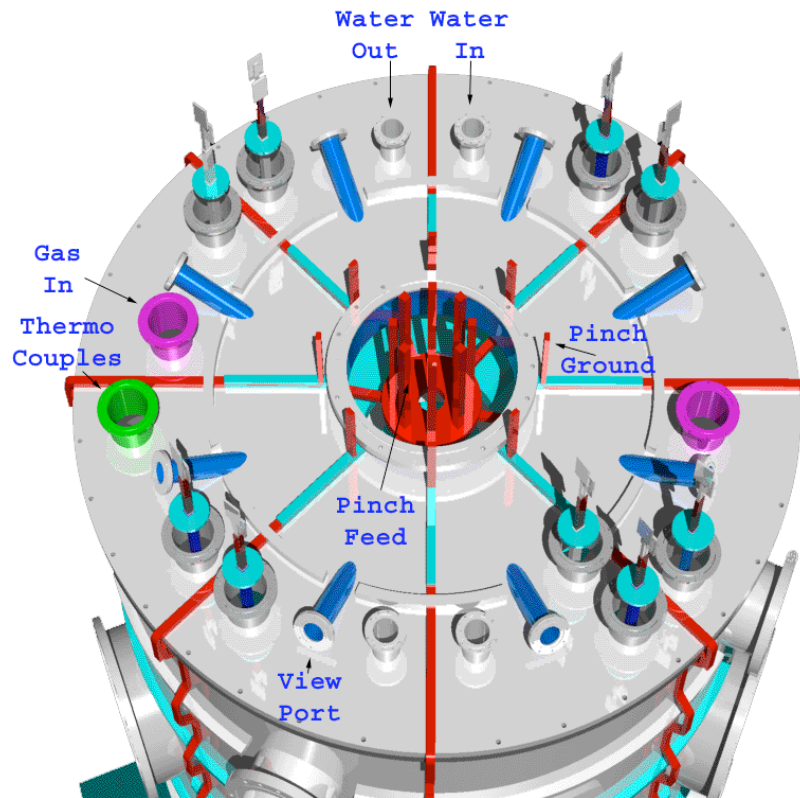


Fig. 7.16. Top flange of the PROTO-SPHERA machine.

Bottom View

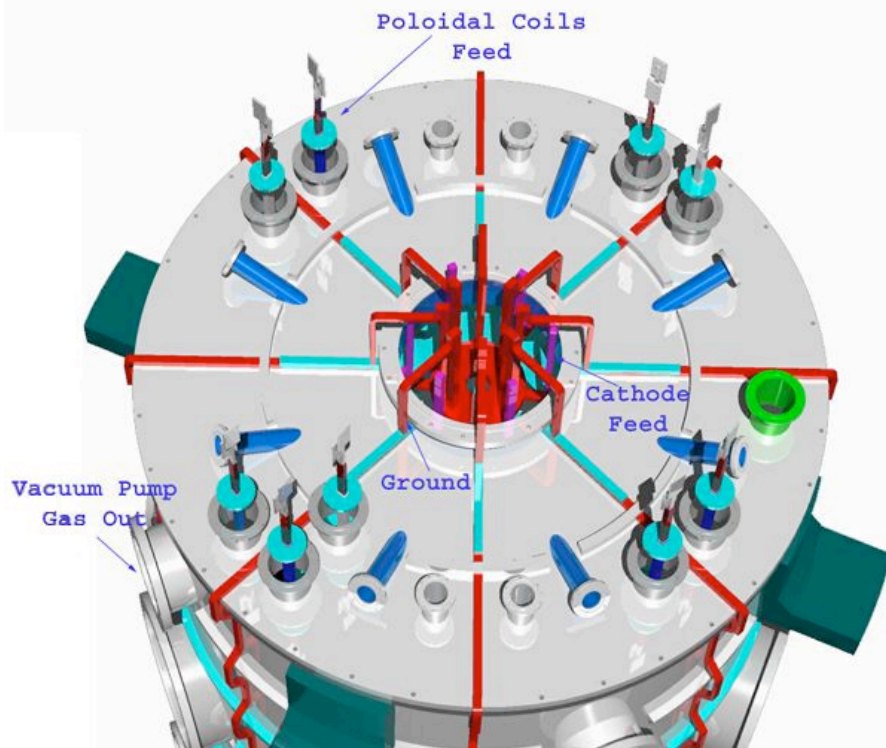


Fig. 7.17. Bottom flange of the PROTO-SPHERA machine.

Gas flow needs are 30 mbar·l/sec through the two flanges shown in Fig. 7.16. Electric services are composed by eighteen feedthroughs for poloidal coils (nine for each main flange), eight insulated flanges and eight return ground connectors in the top main flange and six insulated connector in the bottom main flange (for more details about anode and cathode insulated connectors through the top and bottom flanges, see Fig. 7.2). For the voltage and current requirements for electric services, see Section 8.

7.8. Assembly and Maintenance

□

To facilitate the assembly and maintenance, the machine services are routed through ports at the bottom and top flat flanges. Thus no internal, to the vacuum, connections to the services are needed. Furthermore the design of the coil feedthrough and of the other services is such as to avoid any cutting and re-welding when the machine is partially dismantled for access to the electrodes, see Fig. 7.7.

The PF coils, anode, cathode and their support structure will be pre-assembled on a customized jig outside the VV. The relative position of the coils will be adjusted to guarantee the accuracy of the magnetic field. The magnetic field will be measured with a magnetic probe system, which would record the value and direction of the field. In addition the position of the probe(s) in relation to datum points together with these of anode, cathode and PF Coil system will also be carefully measured. Then the PF coils, anode, cathode and their supports will be installed inside the VV, which will be closed by the top and bottom flanges. These flanges can be removed in situ for repair of the anode/cathode as required.

8. CURRENT WAVEFORMS

The purpose of this Section is to specify the waveforms of the currents that have to be fed into the cathode, into the poloidal field coils and into the screw pinch plasma in order to form and to sustain, through magnetic reconnections, the ST of PROTO-SPHERA.

The value of the cathode current is based on the results of the PROTO-PINCH electrode testbench (see Section 6). The values of the other currents are based on the predictive equilibrium calculations, detailed in Section 4, and on the mechanical design of the poloidal field coils, presented in Section 7.

The time-scale connecting the sequence of equilibrium calculations is derived scaling up the time-scale obtained in the TS-3 flux-core spheromak experiment by the square root of the magnetic Lundquist number, $S^{1/2}$. This scaling is the one predicted by the Sweet-Parker reconnection theory.

8.1. Formation Time-Scale

In order to model the formation of the ST in PROTO-SPHERA, the experimental results of TS-3 [67, 68] have been used as a reference basis. The case with $I_e=40$ kA, $I_p=50$ kA, which has a large beta value ($\beta_{ST}=29\%$, $\beta_p^{ST}=0.6$), has been chosen. It is the same case whose MHD equilibrium is shown in Section 4.5 (see Figs. 4.11 and 4.12) and is calculated with the same profile assumptions used for PROTO-SPHERA.

Its parameters are:

Major radius $R=0.178$ m, minor radius $a=0.103$ m, elongation $\kappa=1.62$.

Radius of the magnetic axis $R_{axis}=0.19$ m, poloidal cross-section area $S_{pol}=0.045$ m².

Total ST plasma volume $V_p=0.046$ m³, radius of pinch at the X-point $\kappa_{Pinch}=0.10$ m.

Toroidal field: on axis $B_{axis}\sim 871$ G (paramagnetism=2.08), at X-point $B_{\kappa_{Pinch}}\sim 777$ G.

Toroidal ST plasma current $I_p=0.05$ MA, $\langle j_{\kappa} \rangle = I_p/S_{pol}=1.1$ MA/m², $q_{95}=1.0$, $M=1$ (H₂).

The density limit of the ST of TS-3 is evaluated by using the Greenwald density limit [90], which fits well the START data [47]: $\langle n_G \rangle = \kappa \langle j_{\kappa} \rangle$,

where $\langle n_G \rangle$ is in units of 10^{20} m⁻³, $\langle j_{\kappa} \rangle$ is in MA/m², the averages are on the plasma poloidal cross-section area and κ is the elongation.

Therefore the density limit for TS-3 is: $\langle n_G \rangle = 1.8 \cdot 10^{20}$ m⁻³.

The total energy confinement time is evaluated from the semi-empirical Lackner-Gottardi L-mode plateau-scaling [91]:

$$\tau_E^{LG} = 120 I_p^{0.8} R^{1.8} a^{0.4} \langle n_e \rangle^{0.4} q_{95}^{0.4} M^{0.5} P^{-0.6} \kappa^{0.8} / (1 + \kappa) \quad [\text{ms; MA, m, } 10^{20} \text{ m}^{-3}, \text{ a.m.u., MW}].$$

The total power required for the helicity injection through the X-points of the configuration is taken as $P_{HI}=4 \cdot P_{ST}^{oh}$ [78], where P_{ST}^{oh} the equivalent ohmic power required for sustaining the spherical torus. It is assumed that half of this power is dissipated inside the spherical torus, therefore $P=P_{ST}^{HI}=2 \cdot P_{ST}^{oh}$ is used in the confinement scaling. The estimation of the ohmic power $P_{ST}^{oh}=I_p V_{loop}$ comes from the Spitzer conductivity [92]:

$$1/\kappa = 2 \cdot 10^2 \ln \kappa T_e^{3/2} / Z_{eff} [\text{Siemens; Coulomb logarithm, eV, ion-effective/proton charge}].$$

The energy confinement is calculated by an iterative procedure, starting from a guessed volume average electron temperature $\langle T_e \rangle$, evaluating a provisional ohmic input power and a provisional energy confinement, which give a revised

$$\langle T_e \rangle = \tau_E^{LG} P_{\kappa\kappa} / (3 \cdot 1.6 \cdot 10^{-19} \langle n_e \rangle V_p) \quad [\text{eV; s, W, m}^{-3}, \text{ m}^3],$$

and so on, until the convergence of $\langle T_e \rangle$ is obtained.

Choosing for the spherical torus of TS-3: $\langle n_e \rangle = 0.55 \cdot 10^{20}$ m⁻³, $Z_{eff}=2$ and $\ln \kappa=12$,

the procedure converges to $\langle T \rangle = 55$ eV, which means

$$V_{loop}=2.5 \text{ V}, P_{OH}=125 \text{ kW}, P_{HI}=0.5 \text{ MW}, \tau_E^{LG} \sim 0.27 \text{ ms}.$$

The profiles of electron density, temperature and total kinetic pressure of the equilibrium of Fig. 4.12 are shown in Fig. 8.1, as a function of the normalized equivalent cylindrical radius κ/a .

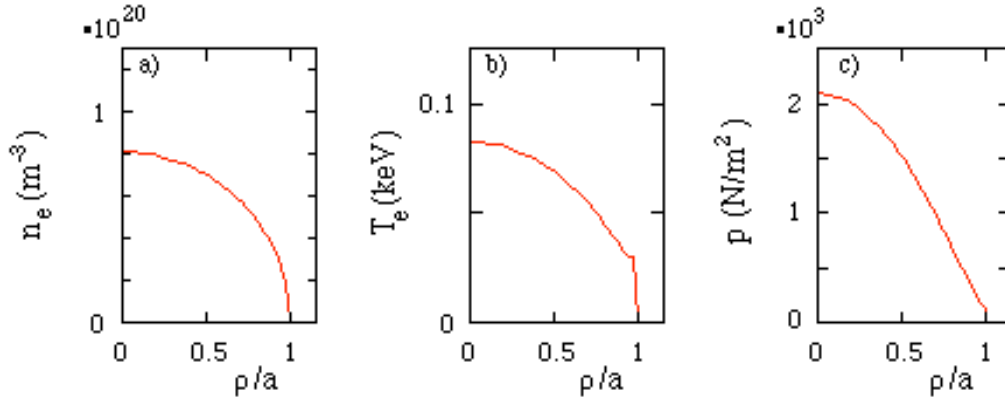


Fig. 8.1. Equilibrium calculation for TS-3: $I_p=50$ kA, $I_e=40$ kA, $\beta_{ST}=0.29$.

Profiles inside the ST of: a) electron density n_e ; b) electron temperature T_e ; c) kinetic pressure p , versus the normalized equivalent cylindrical radius ρ/a .

The Alfvén time is calculated as $\tau_{Aaxis}=R_{axis}/v_A$ on axis, $\tau_{AX}=q_{95}\tau_{pinch}/v_A$ at X-points:

$$\tau_{Aaxis}=19.1[\text{cm}] \cdot (m_i/m_p)^{1/2} n[\text{cm}^{-3}]^{1/2} / (2.18 \cdot 10^{11} \text{B[G]}) \sim 0.75 \mu\text{s},$$

$$\tau_{AX}=1.03 \cdot 10[\text{cm}] \cdot (m_i/m_p)^{1/2} n[\text{cm}^{-3}]^{1/2} / (2.18 \cdot 10^{11} \text{B[G]}) \sim 0.45 \mu\text{s}.$$

The resistive time is calculated as $\tau_k = \tau_0 a^2 / \eta \sim 6.5$ ms.

The representative Lundquist number of TS-3 is $S = \tau_k / \tau_A \sim 9 \cdot 10^3$.

For PROTO-SPHERA the waveforms of the screw pinch current I_e , of the ST toroidal plasma current I_p and of the compression coil current I_A , are calculated using the same inputs that produced the sequence of formation equilibria discussed in Section 4, shown in Figs. 4.5, 4.14, 4.15, 4.16 and 4.17. What is added in this Section is the time-scale that connects these equilibria.

Magnetic reconnections are required to form the ST from the screw pinch, either if the formation is mainly due to the inductive flux delivered by the PF compression coils to the pre-existing screw pinch or if the formation is dominated by the helicity injection from the unstable screw pinch to the spherical torus. Therefore the time required for the formation of PROTO-SPHERA must be extrapolated from the experimental results of the TS-3 flux-core spheromak by using the reconnection time-scale. TS-3 needed 80 μs to reach a ratio $I_p/I_e=50$ kA/40 kA. The Sweet-Parker reconnection theory [95, 96] predicts that the reconnection rate scales like $S^{1/2}$, the square root of magnetic Lundquist number. If the formation time is assumed to scale as $S^{1/2}$, as the Lundquist number of TS-3 was $S = \tau_k / \tau_{Aaxis} \sim 9 \cdot 10^3$, the prescription for the formation time $\tau_{form} = 1.12 \cdot S^{1/2} \tau_A$, applied to TS-3 ($\tau_{Aaxis}=0.75 \mu\text{s}$), gives the measured time $\tau_{form}=80 \mu\text{s}$.

The time-scale for the formation of PROTO-SPHERA ($S=1.2 \cdot 10^5$, $\tau_{Aaxis}=0.55 \mu\text{s}$) can be calculated applying the prescription $\tau_{form} = 1.12 \cdot S^{1/2} \tau_A = 210 \mu\text{s}$ in order to reach the same ratio $I_p/I_e=5/4$, which means $I_p=75$ kA. As a consequence a minimum resistive MHD time-scale of 350 μs is required in order to reach $I_p=120$ kA. However, as it will be detailed in the following, the effect of the eddy currents over all the passive components inside the vacuum vessel introduces a further 650 μs delay, therefore an overall time-scale of 1 ms is required for achieving $I_p=120$ kA.

8.2. Cathode Heating Current Waveform

The first current waveform to be applied is obviously the cathode heating current waveform I_K . The cathode heating current waveform I_K is shown in Fig. 8.2. From the results of PROTO-PINCH a total cathode AC heating current $I_K=60$ kA (rms.) at $V_K \leq 20$ V (rms.) is required. The time for heating the cathode to 2600 °C and for reaching cathode filaments temperature equilibration is estimated to be ≥ 15 s, but a longer duration up to 30 s must be possible, in order to compensate additional unpredicted power losses from the cathode.

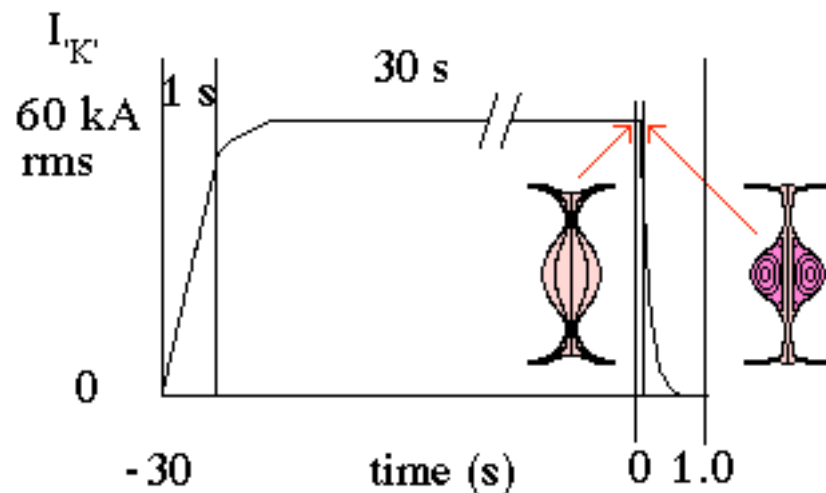


Fig. 8.2. Waveform of the cathode heating current I_K . The two sketches mark with arrows the formation times of the stable screw pinch and the full current of the toroidal plasma.

The cathode current rise time will be 1 s, whereas the cathode voltage rise time will coincide with the filament heating time and will be ≥ 15 s. The cathode heating current I_K will be switched off as soon as possible (presumably when the pinch current will have achieved its flat top value $I_c=60$ kA), with a decay time of about 100 ms.

8.3. Pinch Shaping Current Waveform

The second current waveform to be applied is the constant current PF coil waveform I_B (group 'B'), which can be called the pinch shaping current. It starts at $t=-0.1$ s, rises up to its plateau value $I_B=1875$ A in 0.1 s (see Fig. 8.3). It can be held at the plateau value for (at most) 1 s, with a voltage $V_B=350$ V and with a ripple $\Delta I_B/I_B$ of less than 5%.

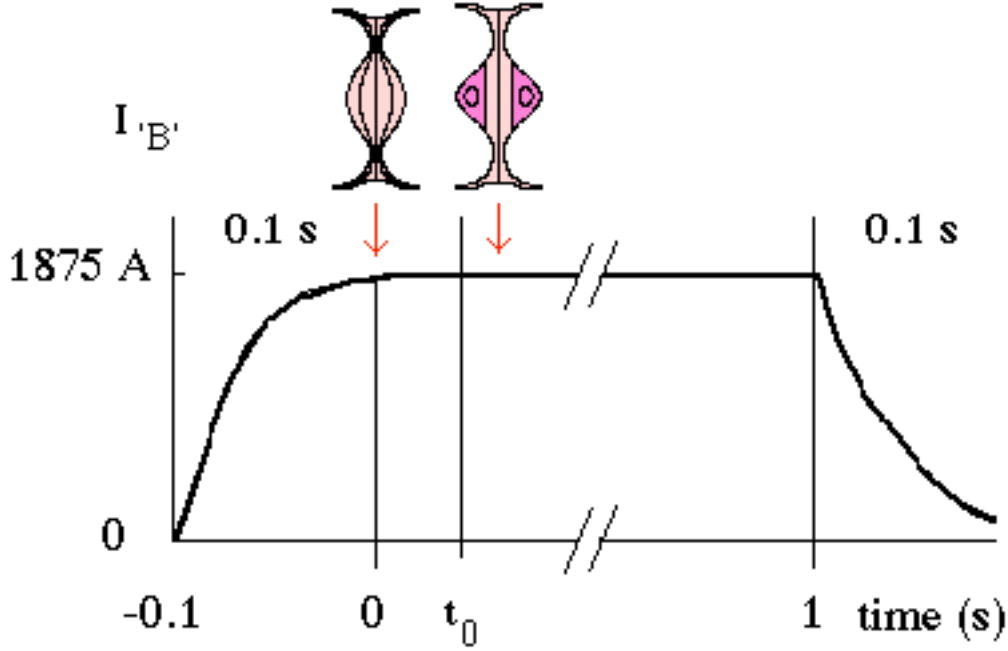


Fig. 8.3. Waveform of the constant current PF coils I_B . The two sketches mark with arrows the formation times of the stable screw pinch and of the toroidal plasma.

8.4. Screw Pinch Current Waveform

The third current waveform to be applied is the pinch current waveform I_e . It starts at $t=0.1$, when the pinch shaping current is fully established, rises up to its stable pinch value $I_B=8.5$ kA, where it is maintained, with a voltage $V_e \leq 90$ V (0.8 MW/8.5 kA) for at least 0.1 s (see Fig. 8.4).

The current I_e is thereafter increased, starting at $t \sim t_0 - 100$ μ s, with a ramp-up time of about 500 μ s. As the inductance of the arc discharge is about $L_e = 0.8$ μ H, the ramp-up of I_e requires an additional voltage of about $\Delta V_e = 100$ V, before the formation of the toroidal plasma. Therefore in this phase the total voltage on the pinch is $V_e \leq 220$ V (2.7 MW/22 kA + 100 V). At $t=t_0$, when the current I_e has achieved about 22 kA ($q_{\text{pinch}} \sim 1$), the compression current I_A is switched on.

At $t \sim t_0 + 120$ μ s, when the current I_e has achieved about 35 kA, the formation of the toroidal plasma begins. As the inductance of the toroidal plasma is about $L_p = 80$ nH and the rate of increase of the toroidal current is 120 kA in 1 ms, a loop voltage of at most $V_{\text{loop}} = L_p (dI_p/dt) = 9.6$ V is required. This voltage is almost entirely provided by

the flux swing associated with the increase of the compression current, which creates over the region of the spherical torus a compressional loop voltage of about $V_{\text{comp}}=9.1 \text{ mWb}/1 \text{ ms}=9.1 \text{ V}$. The missing loop voltage (if any) ($V_{\text{loop}}-V_{\text{comp}}\sim 0.5 \text{ V}$): should be provided by helicity injection according to the formula $\Delta_2 V_e=(V_{\text{loop}}-V_{\text{comp}})\cdot(I_p/I_e)/0.25\sim 8\cdot(V_{\text{loop}}-V_{\text{comp}})\sim 4 \text{ V}$. As the current I_e increases, during the formation of the toroidal plasma, with a longer ramp-up time of about 1 ms, an additional voltage of about $\Delta_1 V_e=50 \text{ V}$, is required. Therefore in this phase the total voltage on the pinch is $V_e\leq 190 \text{ V}$ ($8.0 \text{ MW}/60 \text{ kA}+50 \text{ V}+4 \text{ V}$).

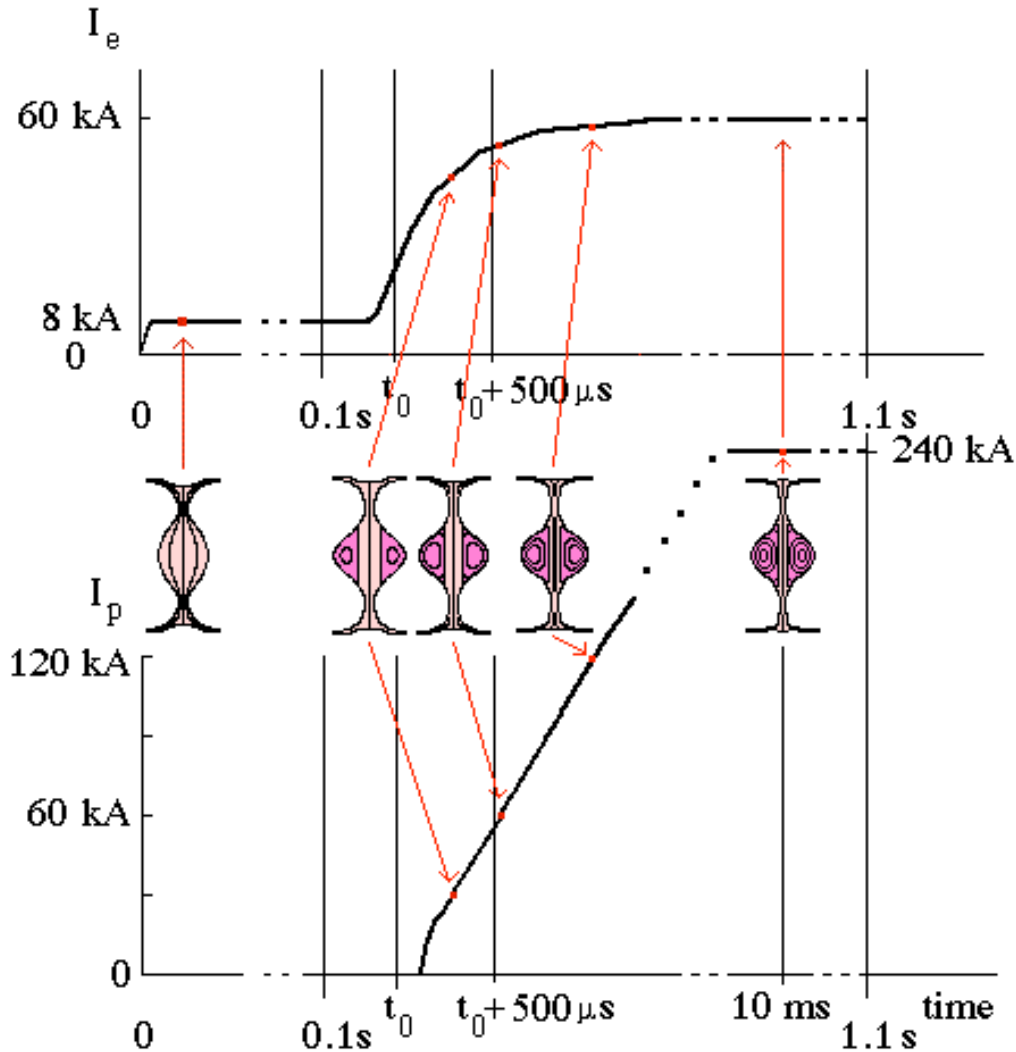


Fig. 8.4. Waveform of the pinch current I_e , shown along with the toroidal plasma current I_p . The sketches mark with arrow the formation times of the stable screw pinch and of the compression of the toroidal plasma.

The plateau value $I_e=60 \text{ kA}$ is reached at $t\sim t_0+1.0 \text{ ms}$, where a toroidal current $I_p=120 \text{ kA}$ is achieved. The estimated power for helicity injection $P_{\text{HI}}\sim 0.6 \text{ MW}$ implies that an additional voltage of about $\Delta V_e^{\text{HI}}=P_{\text{HI}}/I_e\sim 10 \text{ V}$ is required for sustaining the toroidal plasma current. The plateau value $I_e=60 \text{ kA}$ is maintained by a voltage $V_e\leq 180 \text{ V}$ ($10.0 \text{ MW}/60 \text{ kA} + 10 \text{ V}$) for at most 1 s, with a response time to perturbations of about 5 ms.

8.5. Compression Current Waveform

The last current waveform to be applied is the compression current waveform $I_{A'}$, flowing in the poloidal field coils of group 'A'. It is switched on at $t=t_0$ (see Fig. 8.5).

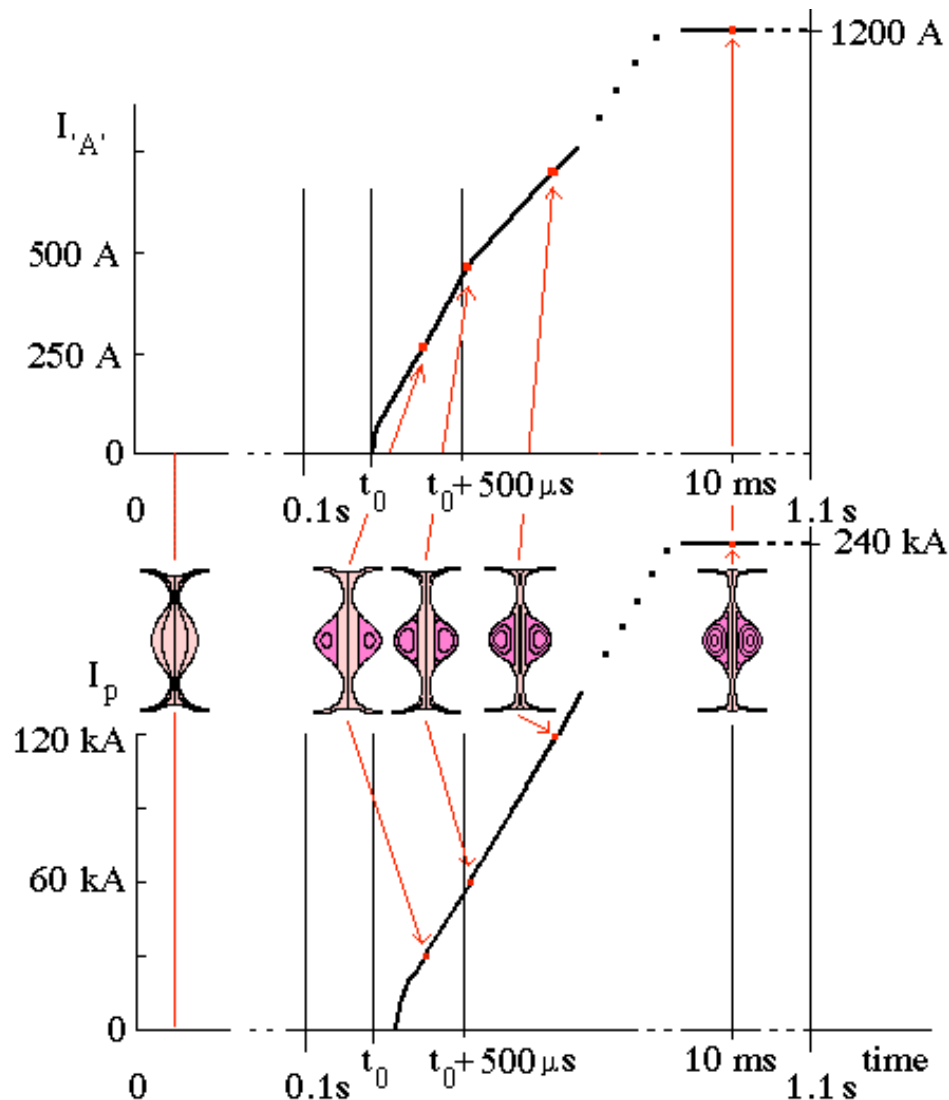


Fig. 8.5. Waveform of the compression current $I_{A'}$, shown along with the toroidal plasma current I_p . The sketches mark with arrows the formation times of the stable screw pinch and of the compression of the toroidal plasma.

It increases up to $I_{A'}=500$ A in about 0.5 ms. As the total inductance of the PF coils of group 'A' is $L_{A'}=14.2$ mH, a maximum voltage up to $V_{A'}\leq 15$ kV is required during the first 500 μs of the compression (see Fig. 8.6). Then the toroidal current in the spherical torus is increased by helicity injection to $I_p=240$ kA in about 10 ms; correspondingly the compression current has to increase to $I_{A'}=1200$ A, requiring a maximum voltage of about $V_{A'}\leq 1$ kV. Finally the equilibrium is sustained at $I_{A'}\sim 1200$ A with $V_{A'}\sim 100$ V.

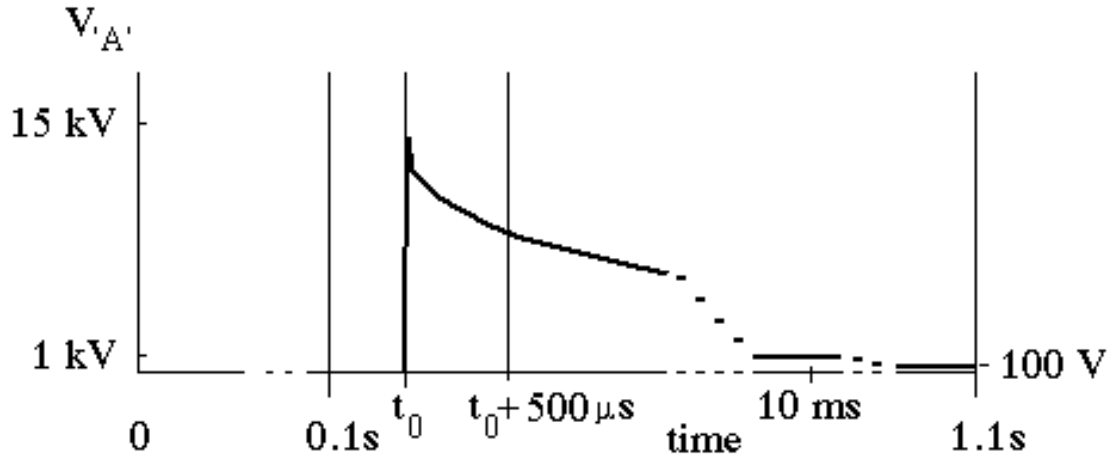


Fig. 8.6. Waveform of the compression voltage $V_{A'}$.

8.6. Eddy Currents Effect on the Formation Scenario

The effect of the eddy currents on the PROTO-SPHERA formation scenario has been evaluated by the finite element code ANSYS. This code takes into account the time evolution of the eddy current distribution over all the passive components inside the machine vacuum vessel. The active/passive elements included in the model are shown in Fig. 8.7, all of them are axisymmetric and continuous.

In more detail, the passive elements are the following:

- Thin casings of the "Group A" poloidal field coils (PF1, PF5, PF3.2, PF6.1 and PF6.2), made in Inconel 625 with a resistive time constant of $\sim 200 \mu\text{s}$;
- Thick casings of the "Group B" poloidal field coils (PF2, PF3.1, PF4.1 and PF4.2) made in AISI 304 Stainless Steel with a resistive time constant of $\sim 2 \text{ ms}$;
- Two protection rings of coil PF2 (Ring5 in AISI 304 Stainless Steel and Ring2 in Tungsten-Copper);
- Two rings that sustain the overall poloidal field coil structure (Ring1 and Ring3, AISI 304 Stainless Steel);
- The machine vacuum vessel in AISI 304 Stainless Steel.

The spherical torus and the screw pinch plasma shapes have been fixed for all the formation sequence (see Fig. 8.7). All the plasma current values at the various time slices have been provided by the equilibrium code, but the second-order inductive effects due to the variations of the eddy currents inside the spherical torus and inside the screw pinch have not been included in the computation of the time evolution of the discharge.

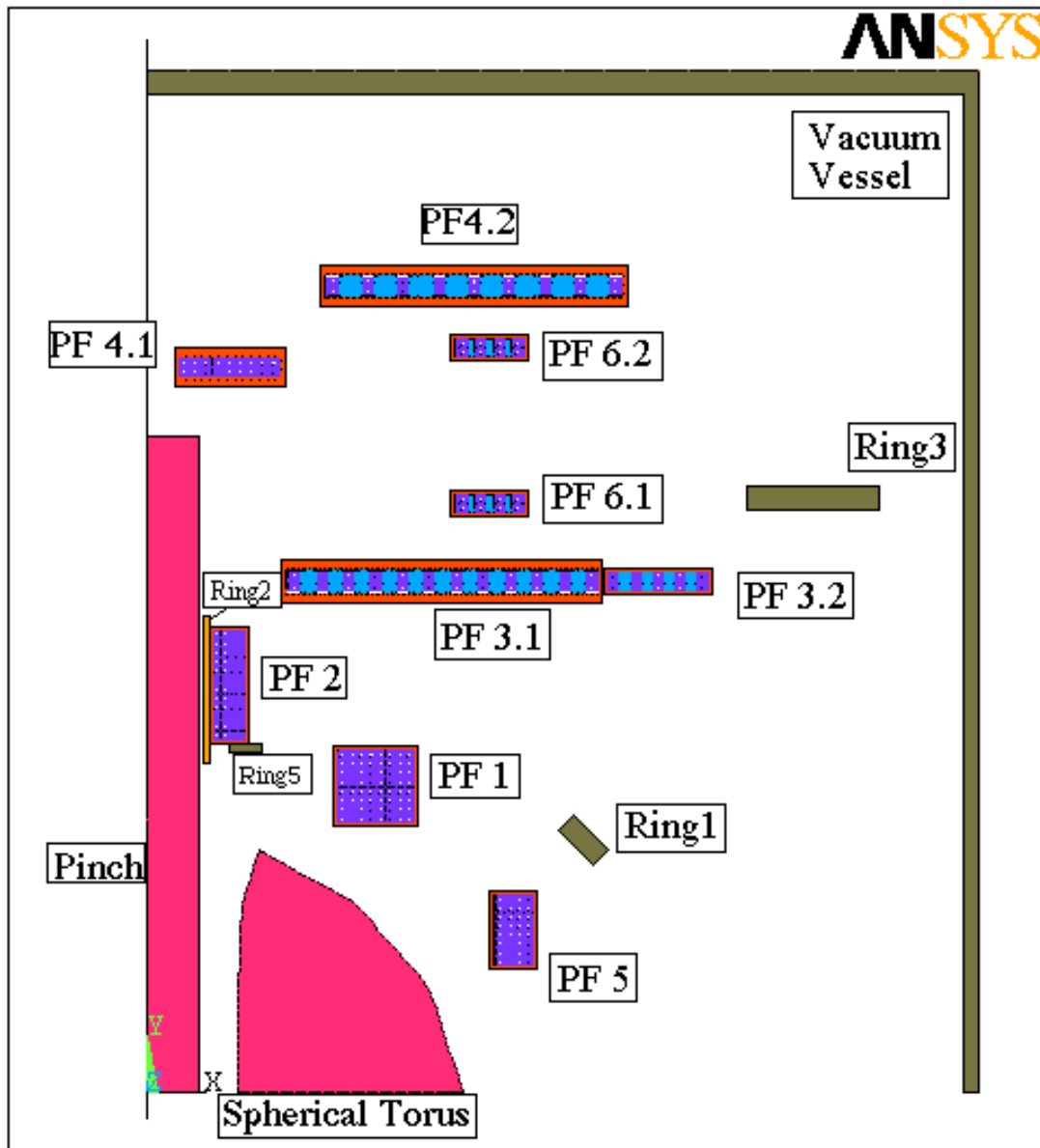


Fig. 8.7. Active (poloidal field coils) and passive elements (rings and coil metal cases) included in the ANSYS model for the eddy current evolution. Also a scheme of the spherical torus and of the screw pinch plasma is shown.

The waveforms of the currents in the poloidal field coils have been imposed, since the power supplies are considered as ideal.

The behavior of the induced currents (see Tab. 8.1) has been determined iteratively, keeping constant the current waveforms in the poloidal field coils. As a first guess, as input to the ANSYS code, the time sequence described by the equilibria of Section 4 has been used: $I_p=30$ kA at $t=t_0+250$ μ s; $I_p=60$ kA at $t=t_0+500$ μ s; $I_p=120$ kA at $t=t_0+1$ ms; $I_p=240$ kA at $t=t_0+10$ ms (see Fig. 8.4).

Then the resulting eddy currents have been fed back into the free boundary equilibrium code, by changing the ULART toroidal current I_p in such a way as to make it compatible with the external field. The process has been iterated until all the plasma currents in the spherical torus and in the screw pinch as well as the eddy

currents in the passive elements inside the vacuum vessel have reached convergence (few %). The method converges typically in three or four iteration.

Coil/mat.	Total current [A] = $\sum j \, dS$ vs. time [ms]				
	0.00	0.25	0.50	1.00	10.00
PF5/Copper	0.0	-9254.2	-14472.2	-22978.	-37000.
Case/Inconel	-1.2	3687.8	676.0	-884.1	-167.0
PF1/Copper	0.0	-18507.	-28941.9	-45954.	-73999.
Case/Inconel	-3.1	7708.7	3790.6	1679.	74.6
PF2/Copper	89996.	89994.	90000.	90001.	89997.
Case/S.S.	-15.1	418.9	410.8	110.7	-120.3
PF3.1/Copper	44999.	45001.	45002.	45001.	45000.
Case/S.S.	-132.3	1285.0	2941.7	4038.7	-214.3
PF3.2/Copper	0.0	2868.9	4498.8	7163.	11560.8
Case/Inconel	-2.0	-890.5	-665.3	-616.0	-97.3
PF4.1/Copper	-33750.	-33748.	-33748.4	-33749.	-33748.
Case/S.S.	27.4	-500.2	-886.3	-805.9	-138.4
PF4.2/Copper	-33750.	-33750.	-33750.	-33750.	-33750.
Case/S.S.	77.4	-1066.5	-1962.6	-3008.4	-1138.6
PF6.1/Copper	0.0	-2299.3	-3605.3	-5735.1	-9249.1
Case/Inconel	-1.9	612.3	376.3	326.1	12.1
PF6.2/Copper	0.0	2299.4	3605.6	5735.3	9249.0
Case/Inconel	1.2	-585.3	-392.8	-353.0	-57.2
Vac.Vessel/S.S.	46.7	120.1	18.3	-776.6	-3453.3
Ring1/S.S.	-8.2	1679.1	2614.4	1425.5	-455.9
Ring2/W-Cu	-732.2	-7973.4	-14571.7	-18526.2	-10204.2
Ring3/S.S.	-8.8	-312.2	-612.6	-1054.6	-732.8
Ring5/S.S.	-8.7	855.5	868.0	187.6	-98.6
SphericalTorus	0.0	15001.	50004.6	120011.	240022.
Screw Pinch	0.0	103469.	212570.	299506.	421959.

Table 8.1. Time evolution of the PROTO-SPHERA eddy currents.

The result of this procedure is that the formation scenario reported in Tab. 8.1 is compatible with the eddy current of the PROTO-SPHERA device, i.e.:

$I_p=15$ kA at $t=t_0+250 \, \mu\text{s}$; $I_p=50$ kA at $t=t_0+500 \, \mu\text{s}$; $I_p=120$ kA at $t=t_0+1$ ms; $I_p=240$ kA at $t=t_0+10$ ms.

Fig. 8.8 shows the comparison between the "ideal" equilibrium of Section 4 at $t=t_0+500 \, \mu\text{s}$ ($I_p=60$ kA) and the "effective" equilibrium obtained at the same time slice, including the eddy current effects ($I_p=50$ kA), with the same currents in the poloidal field coils. Also the difference in the equilibrium vacuum field is shown.

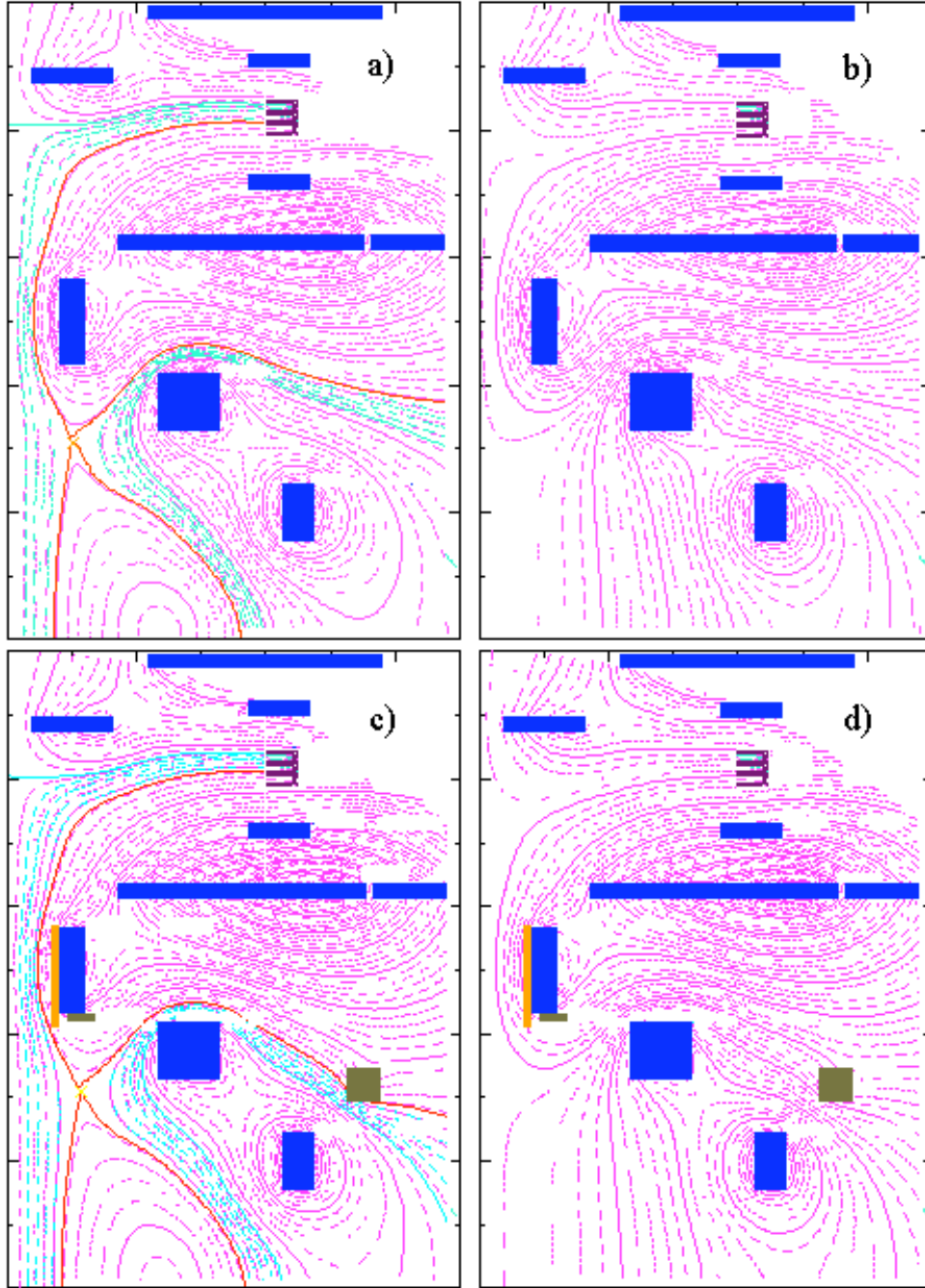


Fig. 8.8. Comparison between "ideal" and "effective" PROTO-SPHERA equilibria at $t=t_0+500 \mu\text{s}$. Poloidal flux maps of:
a) Equilibrium without eddy currents ($I_p=60 \text{ kA}$).
b) Vacuum field without eddy currents.
c) Equilibrium with eddy currents ($I_p=50 \text{ kA}$).
d) Vacuum field with eddy currents.

A similar comparison for "ideal" and "effective" equilibria at $t=t_0+1 \text{ ms}$ ($I_p=120 \text{ kA}$) shows that at this time slice the eddy current effects are weak enough to allow for obtaining the same plasma current as in the "ideal" case of Section 4.

In conclusion, the eddy current analysis provides the following results.

- The rise of the toroidal current in the spherical torus is slightly delayed. In fact I_p is ~ 15 kA 250 μ s after t_0 (it would be 30 kA, if the eddy currents were neglected) and $I_p \sim 50$ kA 500 μ s after t_0 (it would be 60 kA, if the eddy currents were neglected). This effect could be probably minimized with an earlier discharge of the capacitor bank feeding the PF coils of "Group A".
- It is not possible to rise I_p up to the maximum value of 240 kA in about 1 ms. This is essentially due to the eddy currents induced by coils PF6.1 and PF6.2. These two coils are required in order to tailor the shape of the screw pinch near the electrodes (cathode and anode). If a faster rise of their currents were applied, the screw pinch would not fit the electrodes, due to the eddy currents that the PF6.1 and PF6.2 would induce in the thick metal cases of PF3.1 and PF4.2. This effect implies that, after the first ms, the ST current must raise up to $I_p = 240$ kA quite slowly (~ 10 ms), relying upon an effective helicity injection.
- The eddy current density is quite high in the passive element Ring2 (the Tungsten-Copper protection of coil PF2): it locally rises up to the value of 34.1 A/mm² at $t = t_0 + 500$ μ s. Anyhow, the short duration of the formation scheme prevents all thermal load problems to this protection ring. Even the electromagnetic forces due to the eddy currents are a small perturbation with respect to the forces acting among the PF coils.

9. POWER SUPPLIES AND LAYOUT

The PROTO-SPHERA power supplies system includes dedicated units to feed the cathode, the central screw pinch, the poloidal field coils 'A' (which compress the spherical torus) and the poloidal field coils 'B' (which shape the screw pinch). It has been designed to perform the scenario depicted in Section 8. The related electrical general schematic is shown in Fig. 9.1.

The PROTO-SPHERA power supply system is composed by the existing substation 150/20 kV, by the new HV/LV board and the new thyristor amplifiers. The required additional transformers are dry-type ones and are located in the HV/LV board together with additional AC filter.

As shown in Fig. 9.1, the units described in the following compose the PROTO-SPHERA poloidal field amplifier system.

9.1. Coils 'A' Amplifier

The 'A' coil current ramp-up (with two initial slopes 1000 kA/s and 70 kA/s as depicted in Section 8) is performed by two capacitor banks (charging voltage 16 kV and 2 kV respectively) switched on by thyristors.

A 6 pulse-2 quadrant thyristor bridge charges the lower voltage capacitor bank and controls the flat top and the terminal part of the current rise. The thyristor amplifier voltage and current rates are 2 kV-1200 A, respectively, with a required load current ripple lower than 10%.

The amplifier fault protection system is based on fuse apparatus on the MV side, one fuse per each thyristor, on the electronic protection (amplifier into "ondulator" mode) and on crowbar on DC side. The plasma shape feedback control will be operated by the coils 'A' amplifier; a detailed analysis of the plasma control is in progress and it could substantially change the power supply rating, configuration and cost.

9.2. Coils 'B' Amplifier

As indicated in Section 8, the DC nominal voltage and current rates are 350 V-1.9 kA and the required load current ripple is less than 10%.

As a consequence the coil 'B' amplifier is composed by a 6 pulse-2 quadrant thyristor bridge protected by fuses and electronic protection. A LR circuit is included into the 'B' circuit to reduce the current induced by fast current variation in coils 'A'.

9.3. Pinch Amplifier

The performances of the screw pinch amplifier are very peculiar due to the high arc current and low arc voltage, to the short arc-current rise time (10 μ 60 kA in about 0.5 ms, Fig. 8.4) and to the low current ripple (lower than 3%). The study of this power supply has been performed using a dedicated code in collaboration with a firm with high expertise in the metallurgic and electrochemical fields, where the above mentioned features are currently required. The simulations made on the model verified the agreement with the system specifications.

The electric scheme of Fig. 9.2 shows the result of this study. The amplifier consists of three units, the first unit (Rect1) is used to rise the arc current up to 8÷10 KA, the second unit is a condenser bank which provides the short (0.5 ms) current rise from 10 to 60 kA, while the third unit is used to feed and regulate the current plateau at 60 KA. This last unit is composed by two sub-units (RL5 & RL6), each one able to provide 350 V-25 kA, in order to minimize the arc current ripple. The current regulation is obtained by an inductor (about 15 μ H) and also by the non-linear characteristic of the arc discharge, which saturates at ~60 kA (see Fig. 9.3), due to the limit to the electron emission, provided that the cathode is properly regulated in temperature. A typical result of the arc current simulation is shown in Fig. 9.4.



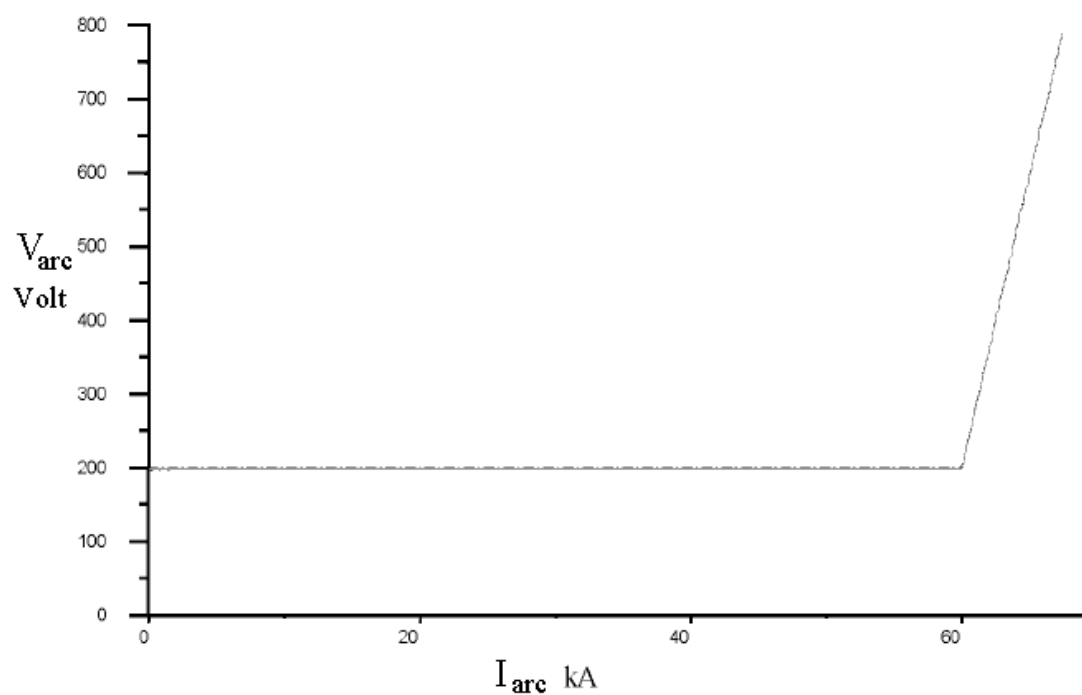


Fig. 9.3. Voltage-current characteristic of the pinch discharge.

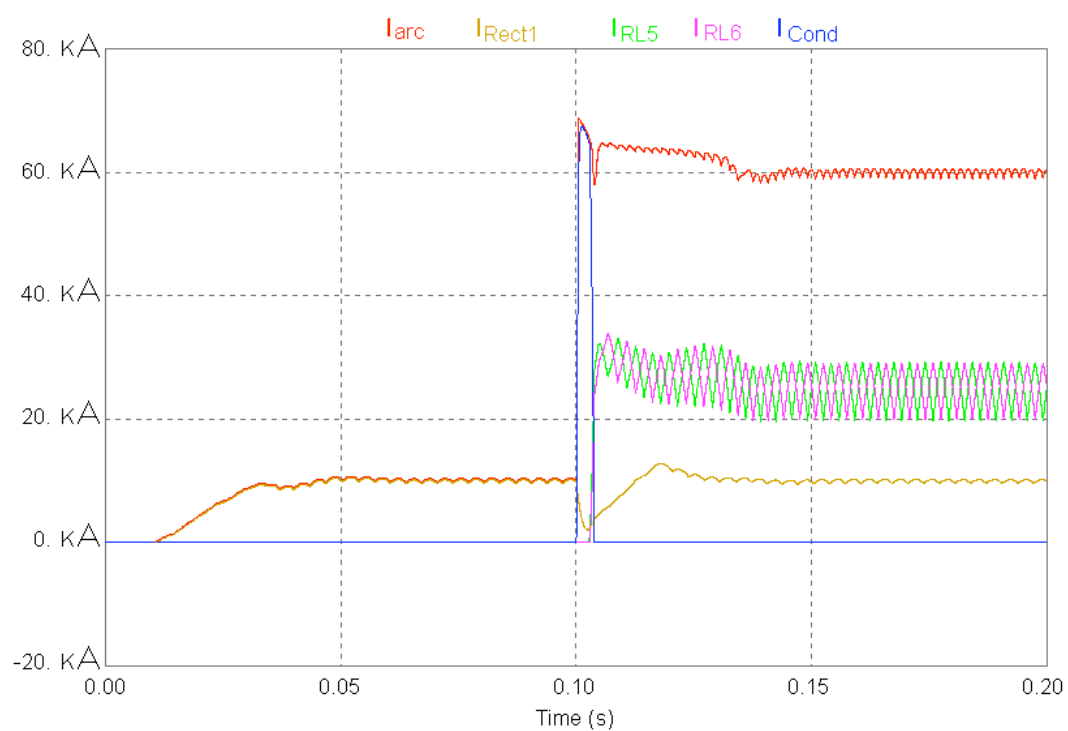


Fig. 9.4. Time behavior of the full arc current (I_{arc}), first pinch unit current (I_{Rect1}), third pinch sub-units currents (I_{RL5} & I_{RL6}), condenser bank current (I_{Cond}).

9.4. Cathode Amplifier

The cathode (more than 400 parallel filaments) requires voltage and current rates of about 60 kA and 25 V AC.

The cathode power supply is composed, as reported in the general schematic Fig. 9.1, by a transformer with a six-phase secondary winding, in order to split the load resistance. A thyristor voltage regulator connected on the MV side controls the average cathode temperature. The regulation of this amplifier will be in voltage. In the voltage mode, it will be possible to have a precise average cathode temperature regulation, exploiting the increase of the filament resistance with the temperature. The use of the AC output permits to spread the current load due to the arc current on the whole filament length. Moreover the filament power will be switched off during the arc current ramp-up. This technique has been demonstrated on PROTO-PINCH, yielding a very long life of the cathode filaments.

9.5. Machine Layout

The size and the feature of PROTO-SPHERA experiment has required an investigation on Frascati Laboratory site to select a possible proper location which provides the appropriate facilities.

The former FT machine experimental hall appears to be a suitable location for the new experiment. The PROTO-SPHERA experimental machine, as proposed in the layout drawings (Fig. 9.5 and Fig. 9.6), can be installed in the same position of the tokamak FT, which will be removed.

The height and the plan arrangement of the platform will allow:

- a good compatibility with the existing RF test bed;
- a good access underneath the machine for the various services;
- a full extraction of the interior part of machine from its tank by the existing top crane;
- a free loading area from the main hall entrance;
- an accessible and adequate area around the new machine for diagnostics, service routes and maintenance;
- a short distance from the available area of the basement where is possible to install most of the power supply equipment;
- a good contiguous room, at the experiment level, available for the installation of engineering and diagnostic controls apparatus.

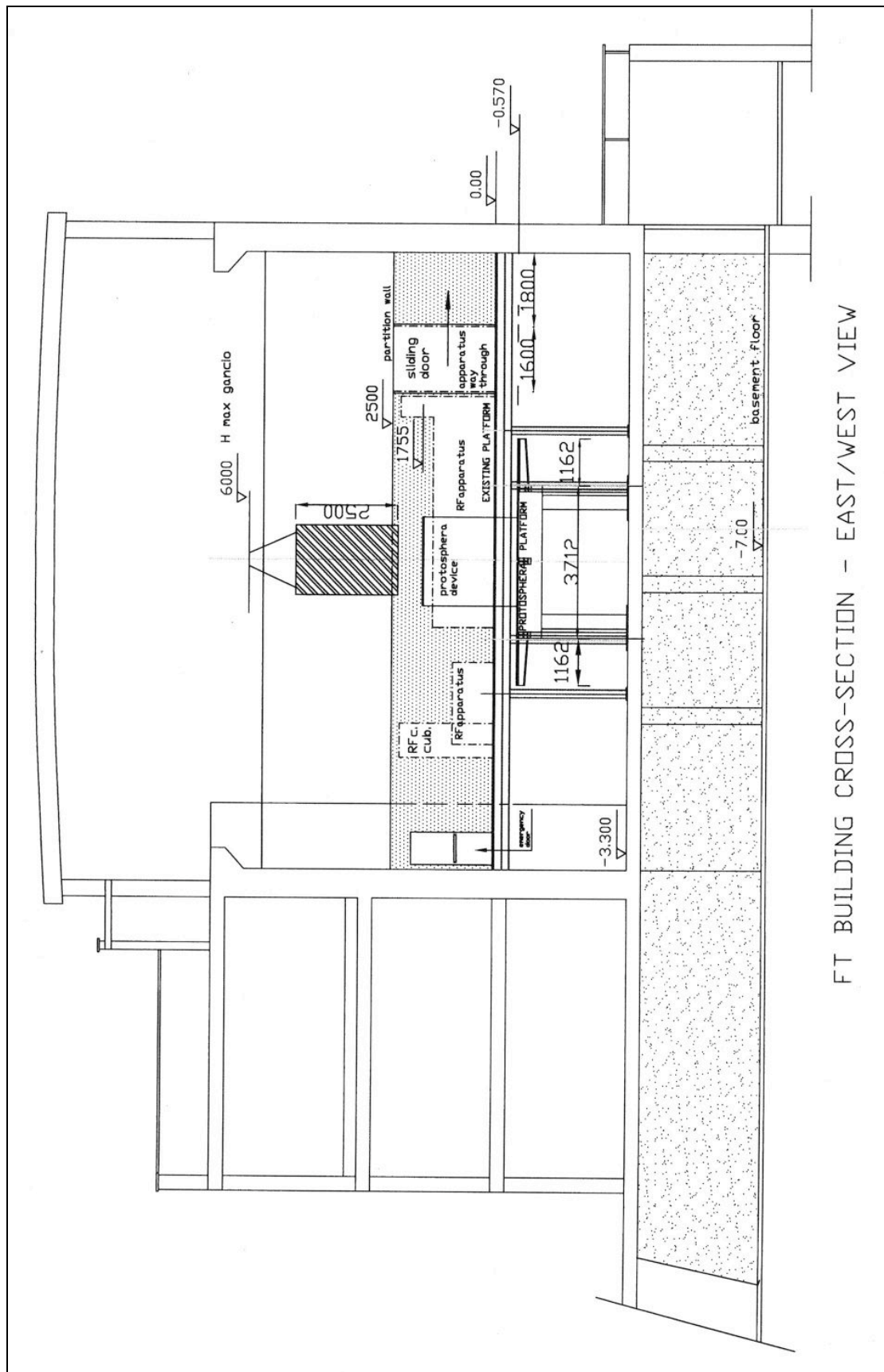


Fig. 9.5. EAST-WEST view of FT building cross-section.

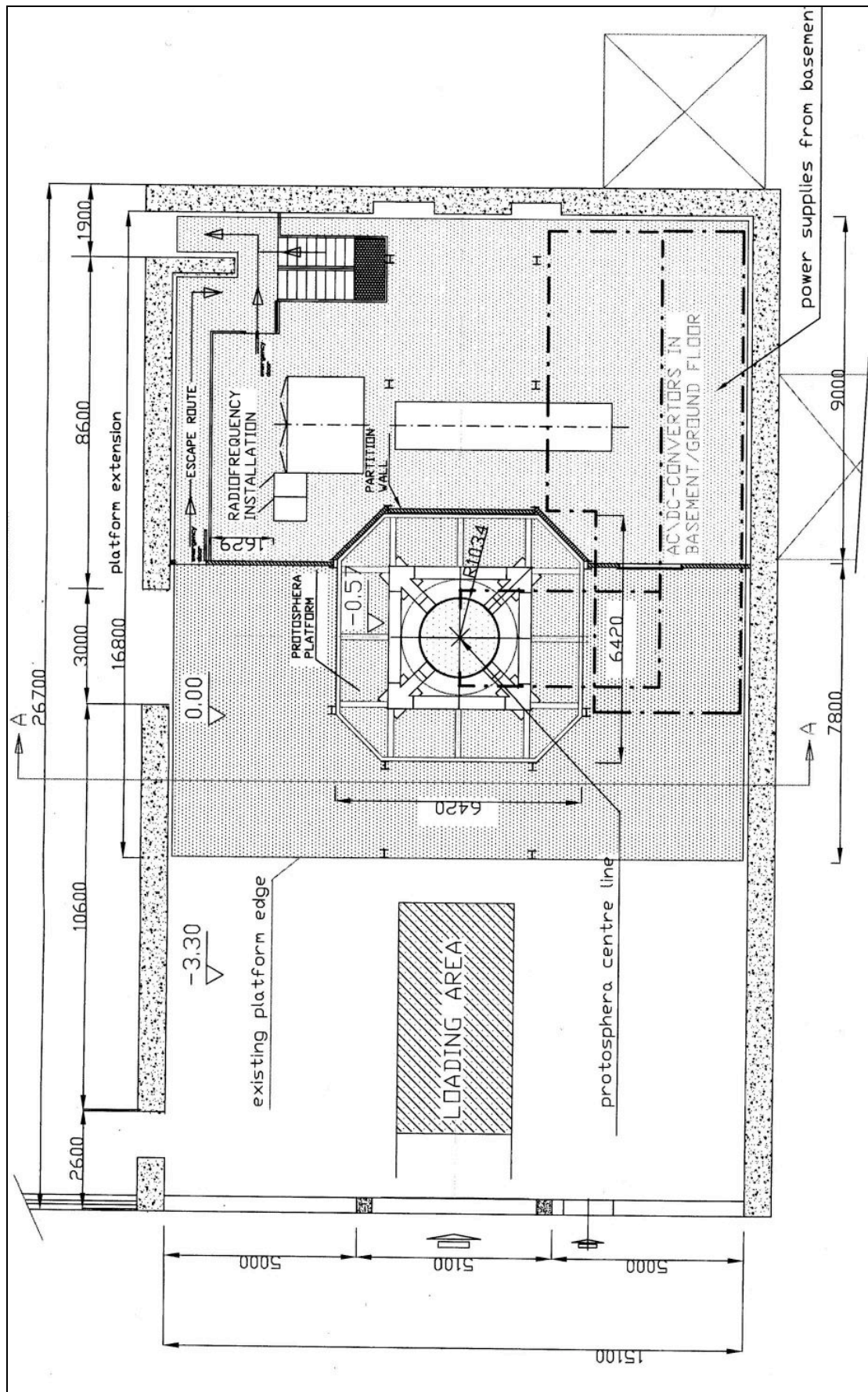


Fig. 9.6. Top view of FT building cross-section.

10. DIAGNOSTICS

The diagnostics of PROTO-SPHERA will include the essential diagnostics used in small tokamak experiments. In the equatorial plane of the machine will be present a **CO₂ interferometer** able to follow the electron density n_e during the ultra-fast breakdown (time-scale $\sim 10^{-8}$ s) of the central screw pinch and during the fast formation (time-scale ~ 1 ms) of the spherical torus. Also in the equatorial plane a multi-point Thomson **scattering** system will measure the electron temperature T_e . **Spectroscopic and visible light measurements** will look at the screw pinch, at the spherical torus and at the X-point region from the equatorial ports and at the screw pinch along the symmetry axis of the machine. **Soft-X ray tomographic arrays** should monitor the MHD activity connected with the helicity injection. **Thermography** will measure the temperature of the cathode, of the anode and of the protection plates.

At present only the **magnetic measurements** have been detailed, as an obvious question comes to mind: which information can be derived from the magnetic measurements, in absence of magnetic probes in the plasma-filled spherical torus hole of PROTO-SPHERA? The answer is that, in presence of a non-magnetic measurement (spectroscopic or interferometric) of the radius of the interface between the spherical torus and the screw pinch on the equatorial plane, a reasonable magnetic reconstruction of the PROTO-SPHERA plasma is feasible.

10.1. Magnetic Reconstruction

In a flux-core-spheromak configuration like PROTO-SPHERA, the magnetic probes cannot be present in the hole of the ST, so loop voltages and poloidal pick-up coils must be located only around the plasma sphere (see Fig. 10.1). With this geometry of the magnetic sensors it has to be clarified whether a magnetic reconstruction is still possible: in particular will the toroidal plasma current I_p be measurable, in absence of a Rogowsky coil around the ST cross-section?

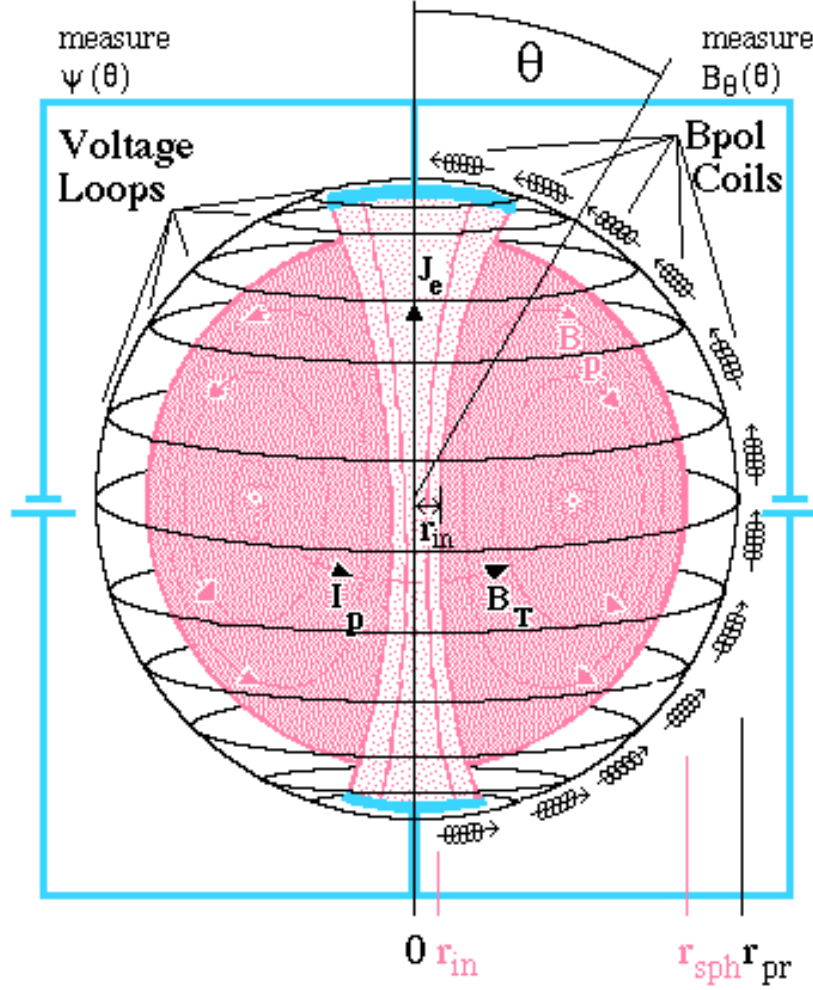


Fig. 10.1. Schematic of the magnetic sensors for a flux-core spheromak configuration.

The magnetic signals have been calculated as an output of the free boundary predictive equilibrium code (see Sect. 4.5). The reconstruction algorithm is based upon the expansion of the flux function ψ in spherical coordinates (r, θ, ϕ) :

$$\psi = \sum_{n=1}^{N_{\max}} \left[M_n^i(r) r^{-n} + M_n^e(r) r^{n+1} \right] \sin \theta P_n^1(\cos \theta) \quad ;$$

where $M_n^i(r)$ and $M_n^e(r)$ are the internal and external spherical multipolar moments. The magnetic signals (typically the flux function and the B_{pol} measurements) are best-fitted through an iterative equilibrium solution, by using a functional parameterization of the sources of the Grad-Shafranov equation: the plasma pressure $p(\psi)$ and the diamagnetic current $I_{\text{dia}}(\psi)$.

The same coefficients are used both for the ST as well as for the Pinch:

$$\begin{aligned}
p(\square) &= A_1 \frac{\square_1}{\square_x} && \text{Inside the ST,} \\
p(\square) &= A_1 && \text{Inside the force-free pinch,} \\
I_{\text{dia}}^2(\square) &= \sum_{i=1}^{N_F} B_i \frac{\square_i}{\square_x} && \text{Inside the ST,} \\
I_{\text{dia}}^2(\square) &= \sum_{i=1}^{N_F} B_i \frac{\square_i^2}{\square_x} && \text{Inside the force-free pinch,} \\
\text{with the obvious constraint } &\sum_{i=1}^{N_F} B_i = I_e^2.
\end{aligned}$$

Moreover it has to be remarked that:

- The best-fit of the data can determine the multipolar moments up to $N_{\text{max}}=7$ [89].
- The number of $p(\square)$ and $I_{\text{dia}}^2(\square)$ functional parameters must be kept as low as possible in order to avoid numerical instabilities during the iterations ($N_F=3$ has been chosen).
- As the plasma pressure $p(\square)$ can be measured on an experiment, the exponent of the pressure has been fixed to $\square_1=1.1$, just the same value used in the predictive equilibrium code. On the other hand the squared diamagnetic current $I_{\text{dia}}^2(\square)$ cannot be directly measured, so the arbitrary choice $\square_1=0.5$, $\square_2=1.0$ and $\square_3=1.5$ has been made.

The most accurate magnetic reconstruction is obtained by putting the magnetic sensors on a constant $r=r_{\text{pr}}$ surface. With this choice it is easy to separate the external and internal current density contributions to the flux function expansion: the values of $M_n^i(r_{\text{pr}})$ and $M_n^e(r_{\text{pr}})$ at $r=r_{\text{pr}}$ are computed from the best-fit of the magnetic measurements once for all, before starting the iterative solution. It comes out that the magnetic probes around the spherical plasma are not sufficient for obtaining an equilibrium reconstruction, so an additional constraint is needed [89]. In particular the addition of the radius of the ST-SP interface r_{in} on the equatorial plane is required. This datum should be derived from non-magnetic measurements (e.g. spectroscopy or interferometry).

Therefore there are 4 unknowns $[A_1, B_1, B_2, B_3]$ and 6 data: $[\sum_{i=1}^{N_F} B_i = I_e^2, r_{\text{in}}, M_1^i(r_{\text{pr}}), M_3^i(r_{\text{pr}}), M_5^i(r_{\text{pr}}), M_7^i(r_{\text{pr}})]$. The Grad-Shafranov equation is iteratively solved by adopting the following scheme. A tentative Multipolar Moments $\tilde{M}_n^{\{e\}}(r)$ expansion is calculated from the tentative $\square, \tilde{A}_1, \tilde{B}_1, \tilde{B}_2, \tilde{B}_3$; then a linear overdetermined system (6 equations, 4 unknowns) finds the correcting factors $[\tilde{A}_1', \tilde{B}_1', \tilde{B}_2', \tilde{B}_3']$ by matching: $\tilde{M}_n^i(r_{\text{pr}}) = M_n^i(r_{\text{pr}})$ for $n=1,3,5,7$; $\square(r_{\text{in}}, \square) = \square_x$ and $\sum_{i=1}^{N_F} B_i = I_e^2$ (exactly). At this point a new set of $\square, [\tilde{A}_1, \tilde{B}_1, \tilde{B}_2, \tilde{B}_3]$ is calculated and the process is iterated up to convergence.

10.2. Magnetic Sensors

The magnetic sensors of PROTO-SPHERA (Fig. 10.2) will be subdivided in two groups:

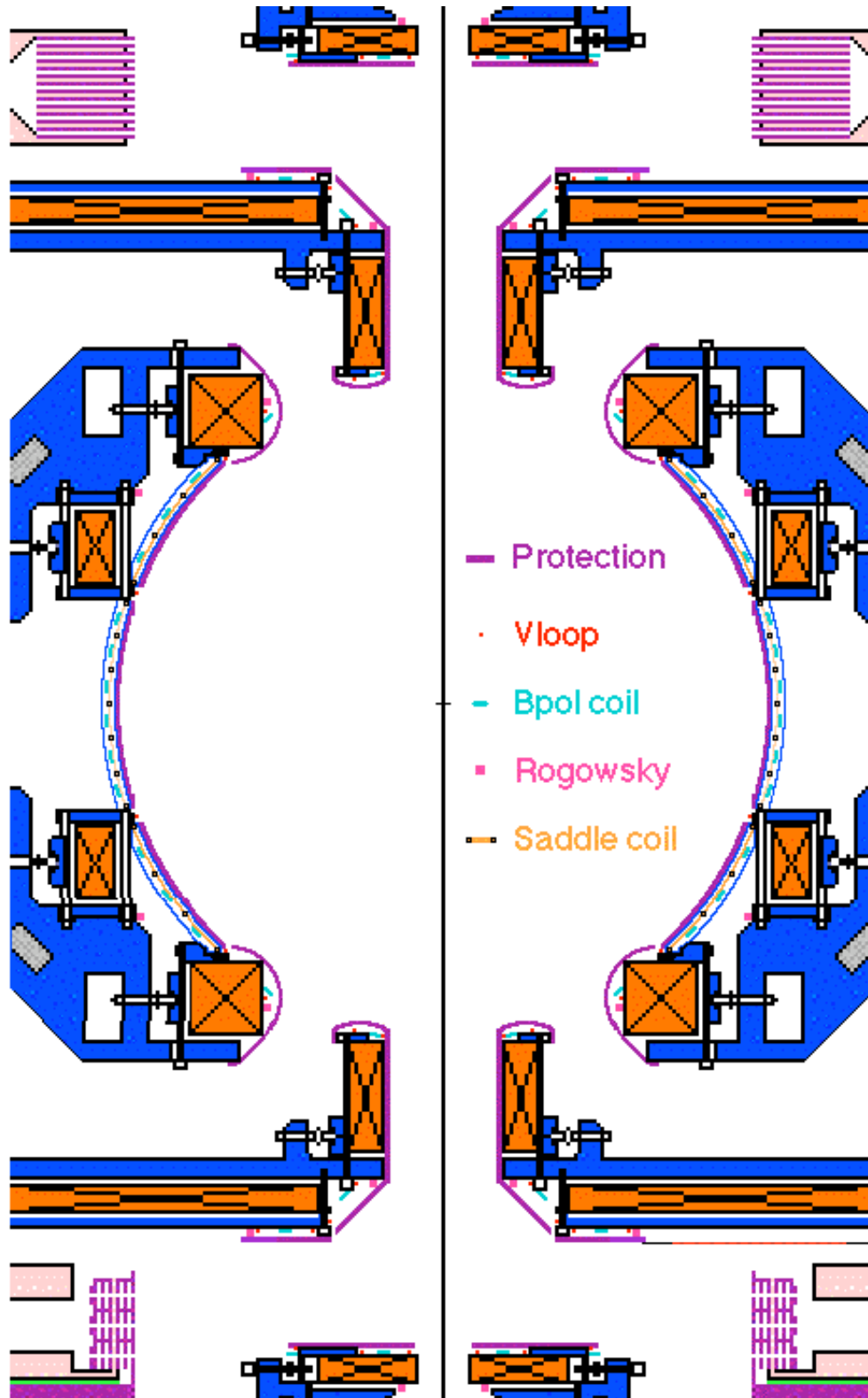


Fig. 10.2. Magnetic probes for PROTO-SPHERA.

- i) Sensors lying on a sphere ($r_{pr}=42$ cm):
 - 10 V-loops
 - 12 Saddle coils (4 inserts)
 - 16 Pick-up coils (4 inserts).
- ii) Sensors for the upper/lower pinch reconstruction:
 - 14 V-loops
 - 8 Pick-up coils (4 inserts)
 - 10 Rogowsky coils for the pinch current I_e .

Introducing a gaussian error on the magnetic measurements and a fixed error on the r_{in} position determination has checked the resilience of the reconstruction. The poloidal flux and the magnetic field due to the poloidal field coils has been subtracted from the measured signals, in order to avoid the use of very high order spherical harmonics.

10.3. Results of the Magnetic Reconstruction

The results of the reconstruction of $(I_p, \bar{\psi}_p, I_{Pinch}^{\square})$ and (q_{95}, q_0) are shown in Tab. 10.1 and Tab. 10.2.

Time-slice	I_p [kA]	$\bar{\psi}_p$	$I_{Pinch}^{\square}$ [kA]	q_{95}	q_0
T0 ($I_e = 8.25$ kA)	0.0	---	2.87	---	---
T3 ($I_e = 60$ kA)	30.0	1.15	169	3.39	1.18
T4 ($I_e = 60$ kA)	60.0	0.50	225	2.87	1.08
T5 ($I_e = 60$ kA)	120.0	0.30	283	2.68	0.97
TF ($I_e = 60$ kA)	240.0	0.15	382	2.83	1.03

Table 10.1. Values of I_p , $\bar{\psi}_p$, $I_{Pinch}^{\square}$, q_{95} and q_0 , from the predictive equilibrium calculation.

Time-slice	I_p [kA]	$\bar{\kappa}_p$	$I_{Pinch}^\square$ [kA]	q_{95}	q_0
T0 ($I_e = 8.25$ kA)	0.0 ---	--- ---	2.97 $\pm 5\%$	--- ---	--- ---
T3 ($I_e = 60$ kA)	29.9 $\pm 8\%$	0.0/2.05 >100%	165 $\pm 1\%$	3.00 $\pm 17\%$	1.30 $\pm 29\%$
T4 ($I_e = 60$ kA)	58.0 $\pm 4.9\%$	0.54 $\pm 47\%$	202 $\pm 1.7\%$	2.71 $\pm 7.5\%$	0.86 $\pm 3.5\%$
T5 ($I_e = 60$ kA)	116.5 $\pm 1.3\%$	0.30 $\pm 66\%$	260 $\pm 4.8\%$	2.56 $\pm 17\%$	0.79 $\pm 11\%$
TF ($I_e = 60$ kA)	249.0 $\pm 3.6\%$	0.19 $\pm 53\%$	361 $\pm 8.5\%$	2.79 $\pm 8.3\%$	0.83 $\pm 12\%$

Table 10.2. Values of I_p , $\bar{\kappa}_p$, $I_{Pinch}^\square$, q_{95} and q_0 from the equilibrium reconstruction.

An error of $\pm 1\%$ has been assumed with the exception of the time-slice T3, in which the error has been increased up to $\pm 2\%$; an indetermination of $r_{in} = \pm 2.5$ mm has been used with the exception of T3, in which it has been increased up to $\pm 5/-10$ mm. The results are that:

- The ST toroidal current I_p can be measured with an error of about $\pm 3.6\%$ to $\pm 8\%$.
- The toroidal component of the screw pinch current $I_{Pinch}^\square$ can be measured with an error of about $\pm 1\%$ to $\pm 8.5\%$.
- An accurate $\bar{\kappa}_p$ estimate is impossible with magnetic measurements alone (the error is always greater than $\pm 47\%$).

The results about the estimate of the toroidal current density $j_\parallel$ profile are shown in Fig. 10.3 and the results about the estimate of the safety factor q profile are shown in Fig. 10.4.

The strong toroidicity effects allow for a good reconstruction of the q profile, but the reconstruction of the $j_\parallel$ profile is much less accurate.

Figures 10.5 and 10.6 show the quality of the plasma boundary reconstruction at the beginning of the spherical torus formation and at the flat-top. There is some inaccuracy in the reconstruction of the shape of the flat-top plasma (which has a very compressed pinch), probably due to the need of very high order spherical harmonics.

In order to have a better magnetic reconstruction of the plasma disks near the electrodes it would be probably necessary to switch to cylindrical coordinates.

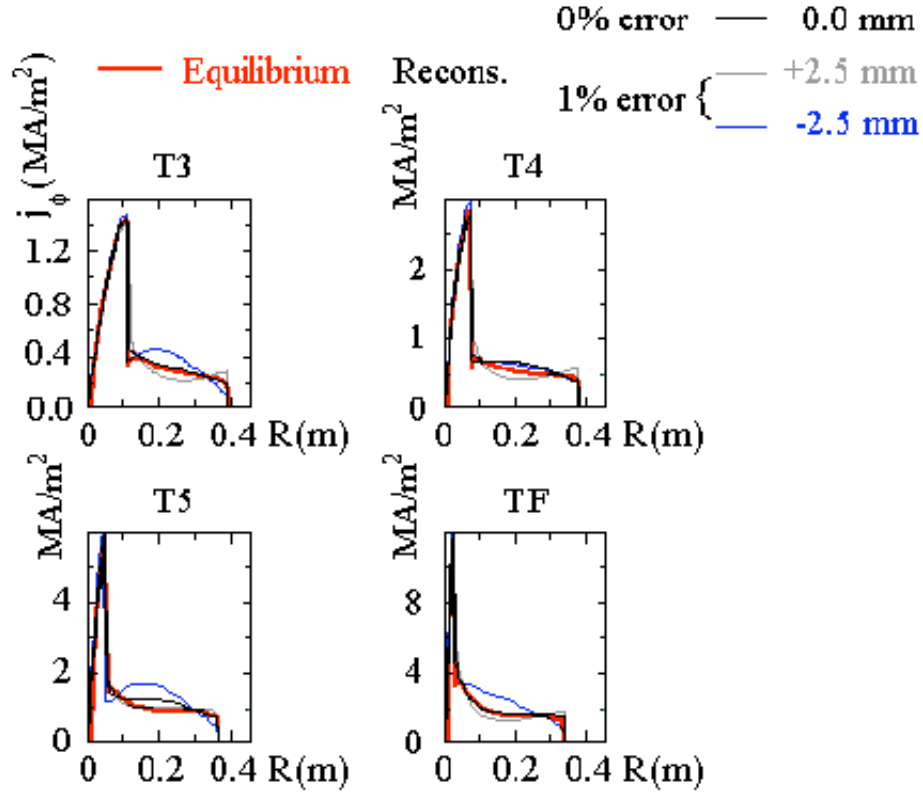


Fig. 10.3. Comparison among the j_0 profiles calculated by the predictive equilibrium code and the ones obtained from the magnetic reconstruction for the time-slices T3, T4, T5 and TF.

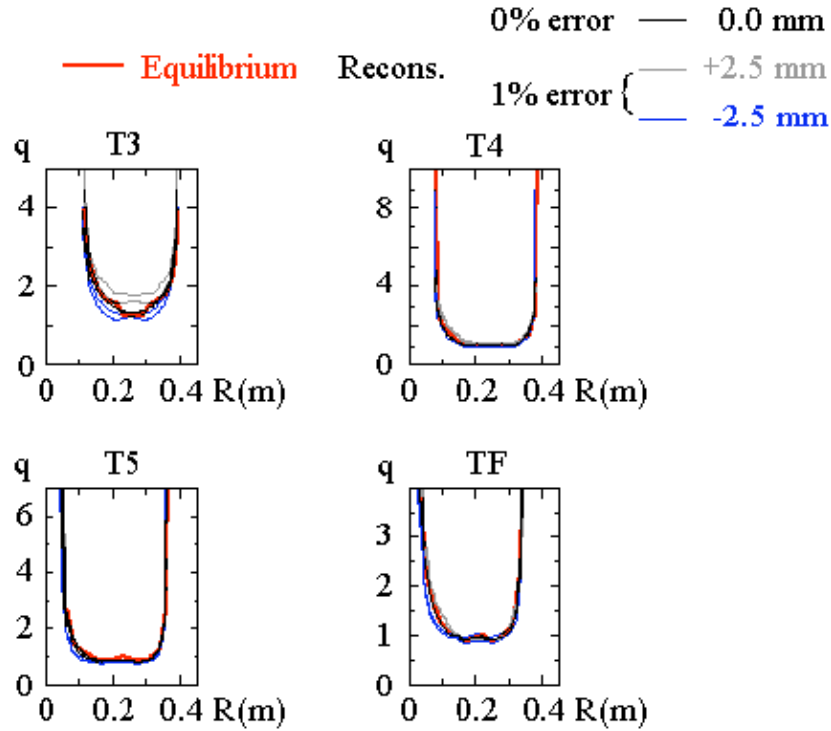


Fig. 10.4. Comparison among the q profiles calculated by the predictive equilibrium code and the ones obtained from the magnetic reconstruction for the time-slices T3, T4, T5 and TF.

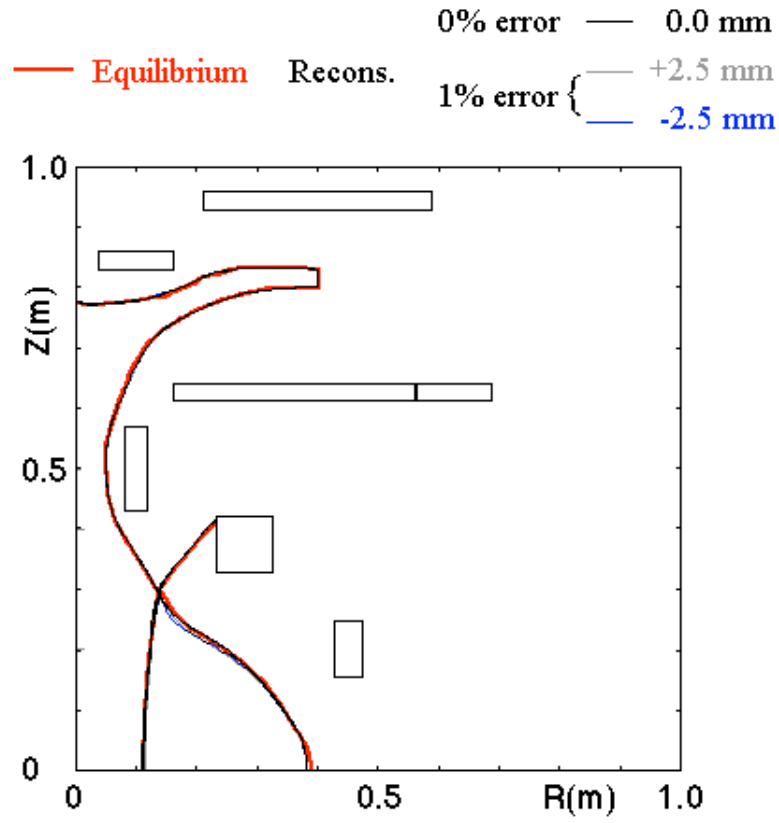


Fig. 10.5. Reconstruction of the plasma boundary at time-slice T3 ($I_p=30$ kA, $I_e=60$ kA).

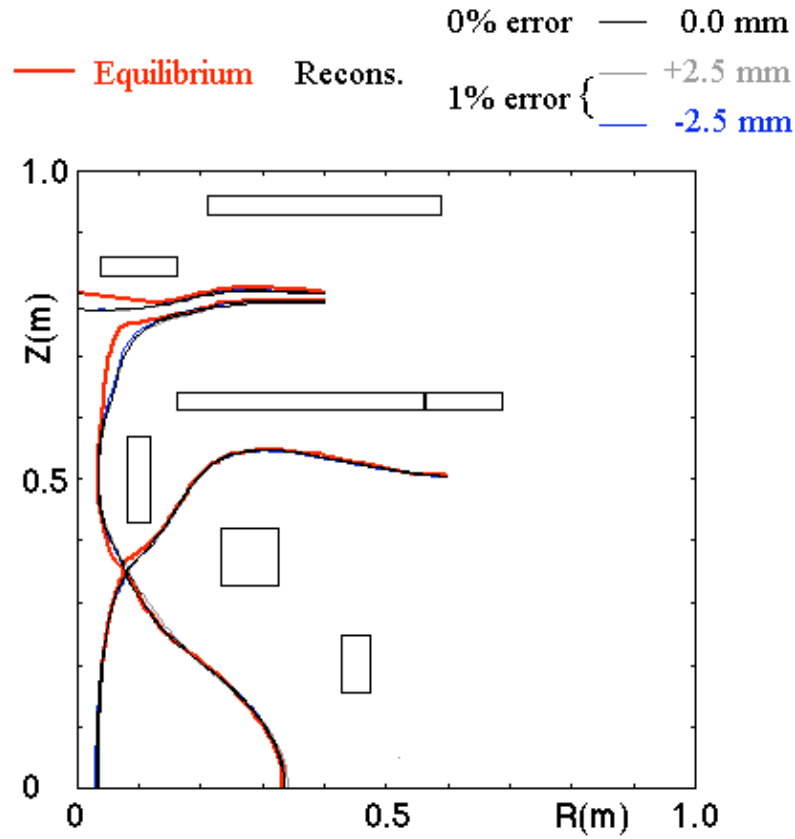


Fig. 10.6. Reconstruction of the plasma boundary at time-slice TF ($I_p=240$ kA, $I_e=60$ kA).

So, although the magnetic sensors cannot (obviously) surround the toroidal plasma (as in a standard tokamak), it is possible to reconstruct the overall configuration of PROTO-SPHERA by using standard magnetic measurement, if:

- The sensors are located on a sphere.
- The information about the inboard plasma boundary r_{in} (from non-magnetic measurements) is added.
- Care is taken to subtract the equilibrium coils contributions from the measured signals.

The ST toroidal current I_p can be detected with an error ranging from $\pm 3.6\%$ to $\pm 8\%$. The toroidal component of the screw pinch current I_{Pinch}^{ϕ} can be measured with an error of about $\pm 1\%$ to $\pm 8.5\%$.

An accurate ϕ_p measurement is impossible with magnetic measurements alone (the error is always greater than $\pm 47\%$).

The strong toroidicity effects allow for a good reconstruction of the q profile, but the reconstruction of the j_{ϕ} profile is much less accurate.

11. EJECTION OF PLASMA TOROIDS FROM TWISTED FLUX TUBES IN ASTROPHYSICS

The purpose of this section is to show that in astrophysical gravity-confined systems, unstable twisted magnetic flux tubes are able to produce, through magnetic reconnection, helically twisted toroidal plasmoids. The fate of these toroids is to expand and to be expelled from the generating gravity-confined parent systems. In this process the system is able to eject helicity and to shed a relevant magnetic flux, with a negligible loss of mass. These phenomena bear a strong resemblance to the formation of the plasma in PROTO-SPHERA, but occur at magnetic Lundquist numbers which are much larger ($S \approx 10^8 - 10^{13}$) than the magnetic Lundquist number of PROTO-SPHERA ($S \approx 10^5$). Also the range of β at which these phenomena occur span a much larger range of values: $\beta \ll 1$ in the solar corona, $\beta \leq 1$ in collapsing magnetized clumps inside giant molecular clouds and $\beta \gg 1$ in protostar magnetized accretion disks. Nevertheless an accurate study of a laboratory plasma like the one of PROTO-SPHERA could provide useful information on some of these phenomena.

Another common feature to all these astrophysical systems is the presence of plasma motions in the form of torsional Alfvén waves (TAW). These waves act in many astrophysical systems (either being injected from the outside or being produced inside) as current drivers. As obviously there are no externally applied electromotive forces in the cosmos, the drivers are convective forces pushing the fluid, whose motion $\vec{u}$ deforms $\vec{B}$, the deformed $\vec{B}$ creates $\vec{\nabla} \times \vec{B}$ and induces $\vec{j}$.

11.1. Solar Flares

After the launch of the solar observatory "Yohkoh" in 1991 there is increasing evidence that X-ray toroidal plasmoid ejections occur in both long duration events (LDE, lasting more than 1 h) as well as in short duration impulsive solar flares. These plasmoids are helically twisted flux ropes in 3D space.

Flares seem to be triggered by the emergence of twisted flux tubes from the photosphere of the Sun $R=R_{\odot}$ into the solar corona (see Fig. 11.1).

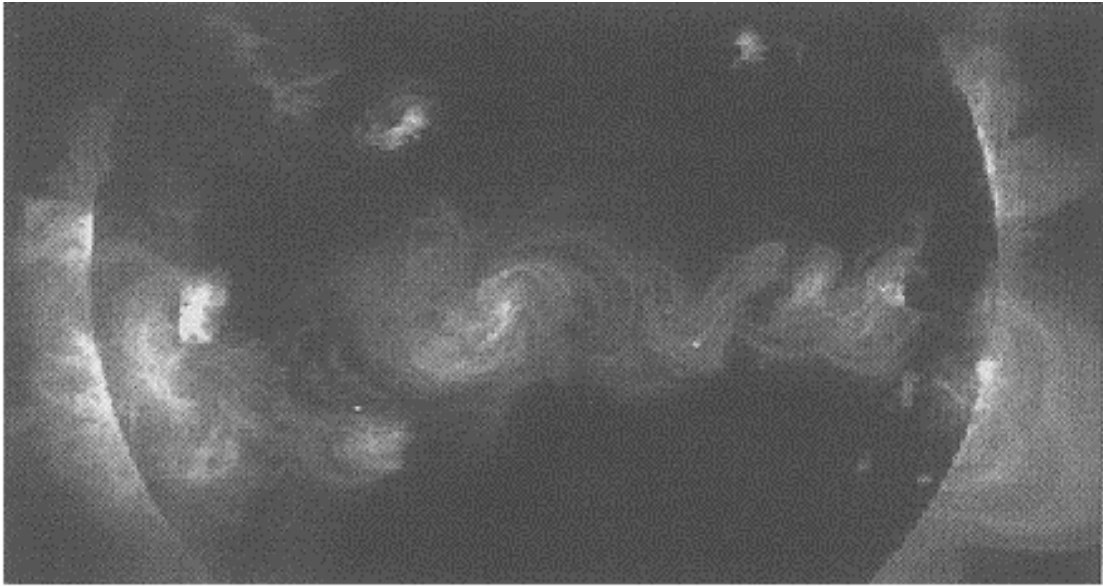


Fig. 11.1. Yohkoh SXT X-ray image of the solar corona. S-shaped sigmoidal structure are contained within magnetic flux systems [97].

The twisted flux tubes are of subphotospheric origin (see Fig. 11.2) and are produced by the solar dynamo acting at core-convection zone interface ($R \approx 0.7 \cdot R_{\odot}$) [98].

The magnetic helicity produced by the dynamo has opposite signs in the northern and southern solar hemispheres (and does not change sign from one 11 year period to the next). The twisted flux tubes rise through the convection zone plasma ($\tau \gg 1$), where their twist (high rotational transform) is what opposes their fragmentation [99].

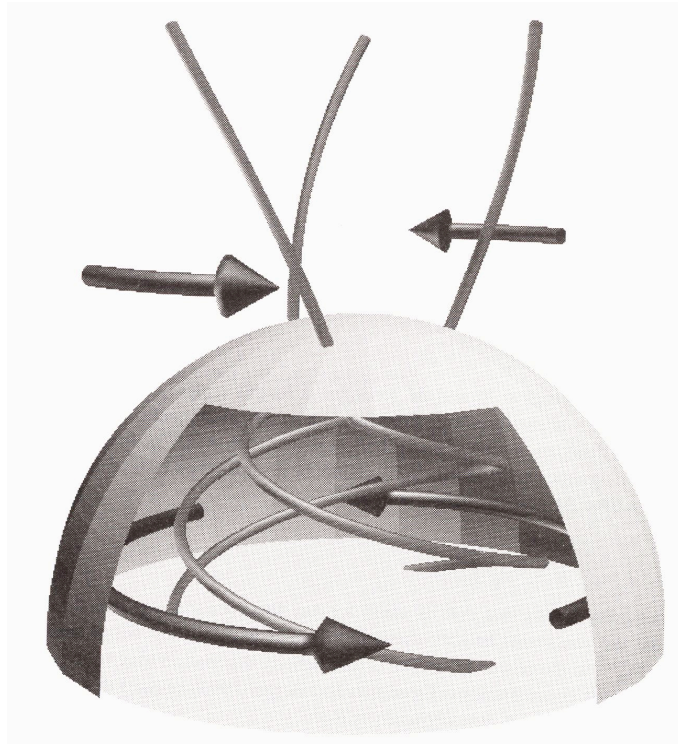


Fig. 11.2. In the solar convection zone the equator rotates faster than the pole. The differential rotation injects helicity into the solar corona.

The electric current systems thread through the photosphere (Fig. 11.3) and pass into the corona ($\beta \ll 1$), where their twist (high rotational transform) is what destabilizes kink modes [100].

The coronal field has not an infinite capacity for the helicity, so the injected helicity must be ejected into the interplanetary space. The corona plays the role of a helicity channel, connecting the sun and the interplanetary space.

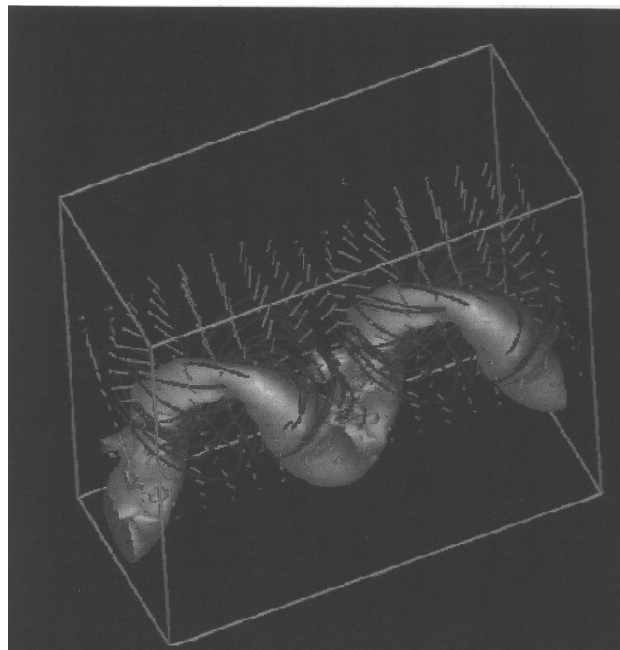


Fig. 11.3. Simulation of emergence of twisted flux tube from the solar photosphere [101].

Intermittent plasmoid ejections (see Fig. 11.4), associated with magnetic reconnections of twisted flux tubes, produce recurrent behavior of solar LDE flares [102] (magnetic Lundquist number $S=\eta_r/\eta_A \approx 10^8$ in the solar corona).

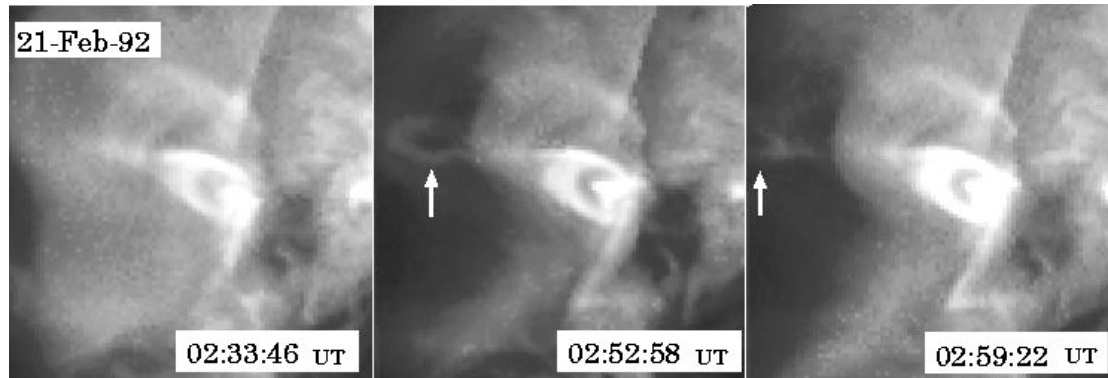


Fig. 11.4. X-ray toroidal plasmoid (arrows) ejection during an LDE solar flare, observed from YOHKOH Soft-X telescope [103].

The plasmoid induced reconnection model proposed by K. Shibata [104] postulates that an unstable emerging twisted flux loop produces (through helicity ejection by magnetic reconnection) an X-ray toroidal plasmoid, which triggers on its turn an increased reconnection rate: a downward jet collides with the top of the SXR loop, producing an MHD fast shock observed in the HXR images (Fig. 11.5). Thereafter the plasma toroid is ejected into the interplanetary space.

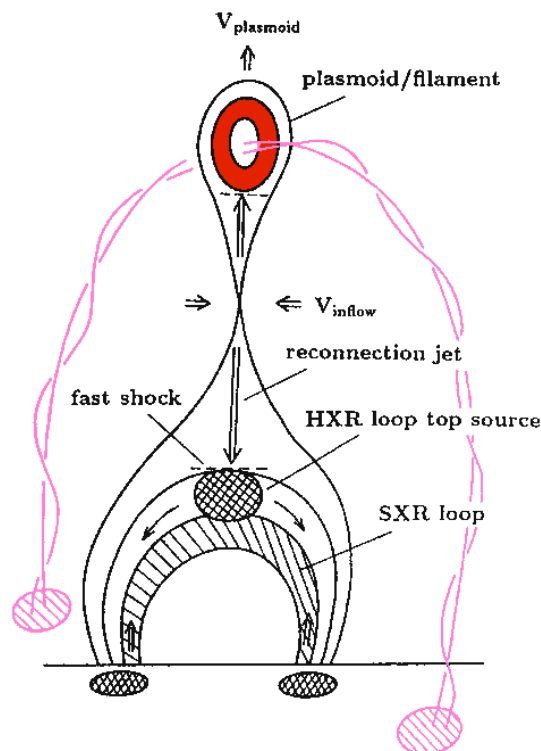


Fig. 11.5. Scheme of the 'plasmoid-induced-reconnection' solar flare model [104].

In small scale flares [105] the plasmoid collides and reconnects with the ambient field, generating a jet of torsional Alfvén waves (TAW), leading to X-ray jets and spinning $H_{\square}$ surges (Fig. 11.6).

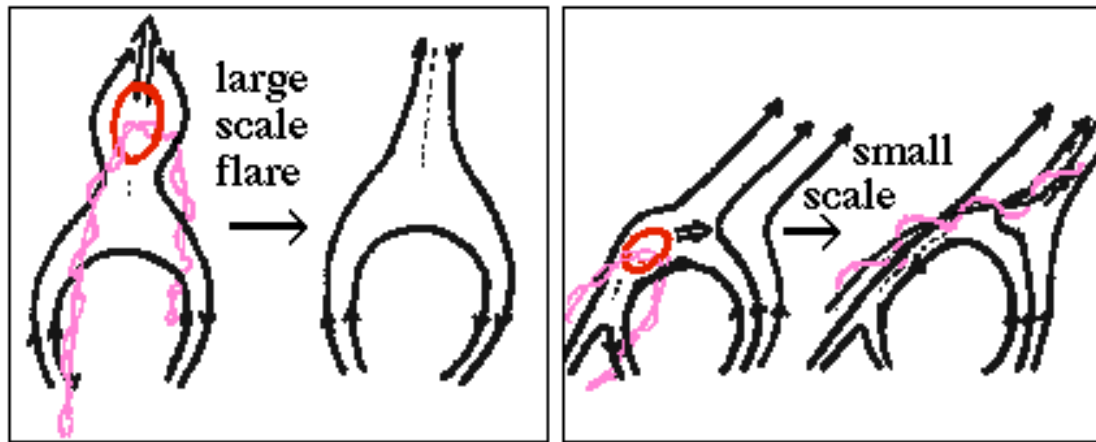


FIG. 11.6. Comparison between a large scale flare, where a cusp structure remains after the plasma loop is ejected, with a small scale flare, where the loop reconnects with the ambient field emitting torsional Alfvén waves [105].

11.2. Protostellar Flares

There is also growing evidence that the ejection of plasma toroids from twisted flux tubes could play a role in the star formation process, by allowing a fast shedding of the magnetic flux from the star condensation region. Giant molecular clouds (GMCs) contain weakly ionized (10^{-4} - 10^{-6}) mass condensations of scale length ≤ 0.1 pc, called clumps; they are sites of massive star formation. The gravitational collapse of clumps is opposed by strong magnetic fields ($\beta < 1$) and by Alfvén waves turbulent energy (Alfvén Mach number $m_A \approx 1$). Magnetized clumps can condense via ambipolar diffusion of the magnetic field, which decouples the ionized component of the cloud from the self gravitating neutrals; but the ambipolar diffusive time-scale for a clump is $\geq 2 \cdot 10^7$ yr., longer than the lifetime of a GMC. However the field often appears to be in filaments, with Lundquist number $S \approx 10^{11}$: magnetic helicity injected by torsional Alfvén waves (TAW) can drive longitudinal current instabilities [106].



Fig. 11.7. Hint of a twisted magnetic field jet on 0.1 pc scale length, from Hubble Space Telescope (HST).

The folding of the filaments by MHD instabilities and their break-off in fast reconnection processes ($20 \cdot \beta_k$), with time-scales $\sim 1-3 \cdot 10^6$ yr., can be a faster trigger of massive star formation.

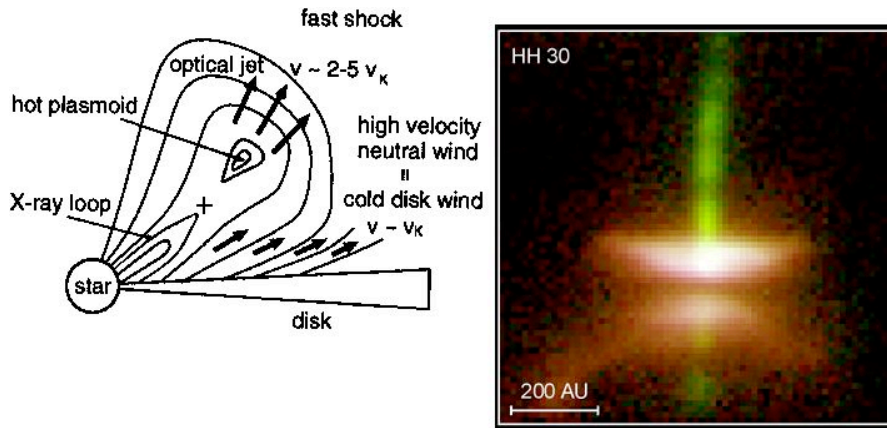


Fig. 11.8. Schematic picture of the interaction between a magnetic field and a Keplerian disk [107]. Obscuring torus and high velocity bipolar jet from a protostar (HST).

In collapsing protostars X-ray flares are observed, along with an obscuring torus ($\beta \gg 1$, scale length $\leq 10^{-3}$ pc) and high velocity neutral winds. High velocity collimated ionized bipolar jets ($\beta \approx 1$) emanates from the central region, see Fig. 11.8.

One of the current explanations of what is observed around protostars is the model of the magnetically driven jet bipolar jet. This model assumes that when an accretion disc is threaded by large scale poloidal magnetic field, centrifugal force and magnetic pressure drive outflows, as shown in Fig. 11.9.

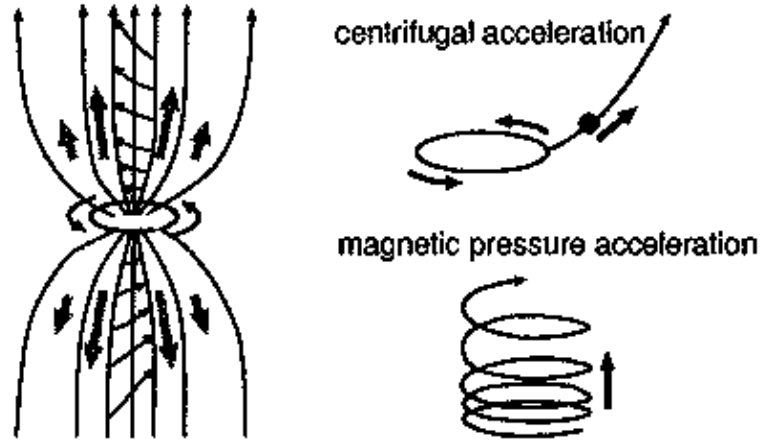


Fig. 11.9. Schematic picture of a magnetically driven bipolar jet.

Numerical simulations [107] of a differentially rotating cylinder with vertical magnetic field, shows the appearance of non axisymmetric instabilities (Fig. 11.10). The generation and relaxation of magnetic twist is driven by the rotation of the disk, the outflows are collimated along the rotation axis, due to the magnetic pinch effect and the twist relaxes by emitting torsional Alfvén waves (TAW). Magnetic reconnection takes place intermittently ($S \approx 10^{11}$). A rotating spheromak ($\beta \leq 1$) carries away magnetic flux, magnetic helicity and angular momentum.

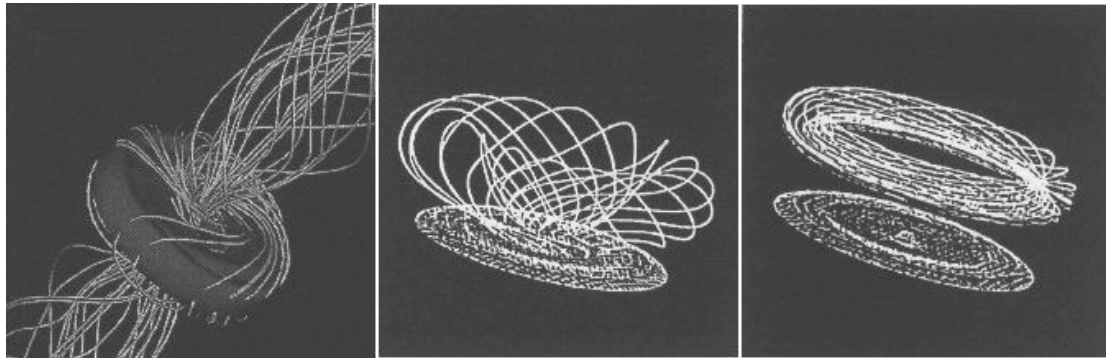


Fig. 11.10. Structure of $m=1$ instability of a magnetized differentially rotating cylinder, showing the effect of the magnetic reconnection [107].

12. REACTOR EXTRAPOLATION

PROTO-SPHERA is an experiment containing a component of scientific risk, but its success could lead to a larger size and more fusion oriented experiment. The three major and down-to-earth points that have to be demonstrated on PROTO-SPHERA are that the formation scheme is effective and reliable, that the combined configuration can be sustained in 'steady-state' by DC helicity injection and that the energy confinement is not worse than the one measured on spherical tori. If all these three points are met, the race toward small and compact fusion reactor could be enriched by a new competitor.

It is possible to show that scaling by a factor of about 5.5 in linear dimensions the PROTO-SPHERA experiment and increasing the current density both in the plasma as well as in the coils by a factor of 2, one could approach a D-T burning plasma. However the resistivity of a screw pinch plasma centerpost remains always larger than the resistivity of a corresponding copper centerpost (which is usually assumed to be technically limited to $I_p/I_{tr} \sim 1$), unless a plasma temperature $T_{Pinch} \sim 700$ eV is achieved. Nevertheless, if a screw pinch plasma centerpost configuration yields by helicity injection $I_p/I_e \sim 4$, the increase in linear dimensions and in current density makes the plasma centerpost competitive for the power dissipation with the copper centerpost, even if only the lower plasma temperature $T_{Pinch} \sim 300$ eV is achievable. The increase of the temperature of the screw pinch to a many-hundreds-eV temperature regimes remains indeed the most critical point in a direct reactor extrapolation of PROTO-SPHERA, unless a real breakthrough could be achieved in understanding and designing the electrode-pinch plasma interaction.

On the base of the present knowledge one has better to consider as reactor-oriented a configuration in which the plasma is initiated by electrodes, like in PROTO-SPHERA, but is then sustained in absence of electrodes. The configuration considered belongs to the family of the unrelaxed Chandrasekhar-Kendall-Furth equilibria, illustrated in Section 3. Any of them can be viewed as a spherical torus (with winding numbers $q_0^{ST} \sim 1.0$ on axis and $q_{95}^{ST} \sim 2.0$ at the magnetic separatrix) surrounded by a spheromak endowed with high elongation and therefore with high winding number ($q_0^P \sim 3$ on the symmetry axis and $q_{95}^P \sim 5$ at the separatrix). If the spheromak discharge can be sustained by driving current on its closed flux surfaces, magnetic helicity, flowing down the $\langle \nabla \cdot \mathbf{B} \rangle$ gradient, will be injected into the main spherical torus, through magnetic reconnections at the X-points. Unrelaxed CKF configurations can be stable to all low- n ideal MHD modes, up to unity beta values $\beta = 2\langle p \rangle_{vol} / \langle B^2 \rangle_{vol} = 1$. Unrelaxed CKF fusion reactors with the right helicity injection, β limit and energy confinement, will allow for an unimpeded outflow of the high energy charged fusion products, easing direct energy conversion and the use of the burner as a space thruster.

12.1. Burner Parameters

The fusion performances of an unrelaxed CKF configuration are studied as a function of the geometric size, of the plasma temperature $T_e=T_i$ and of the total current I_p , both for a D-T burner as well as for a D- ^3He burner. The minimum geometrical size considered is shown in Fig. 12.1. It will be referred to as " $\square L=1$ " in the following. Its (ideal MHD stable) equilibrium has been calculated with $\square_{ST}=78\%$ in the spherical torus. The nuclear burning is limited to the spherical torus (ST), which has the following parameters:

major radius	R_{ST}	=	1.0 m	$\square L$
minor radius	a_{ST}	=	0.80 m	$\square L$
elongation	$\square_{ST}$	=	2.37	
total ST poloidal cross-section	S_{ST}	=	3.51 m ²	$\square L^2$
total ST plasma volume	V_{ST}	=	17.92 m ³	$\square L^3$
safety factor at 95% of edge flux	q_{ST}	=	2.0	
toroidal ST plasma current	I_p	=	MA	free parameter
averaged total field	$\sqrt{\langle B^2 \rangle}$	=	0.133 $\square I_p$ Tesla	$\square 1/L$
ST electron averaged temperature	$\langle T_e \rangle$	=	keV	free parameter
ST electron averaged density	$\langle n_e \rangle$	=	10^{20} m^{-3}	derived parameter.

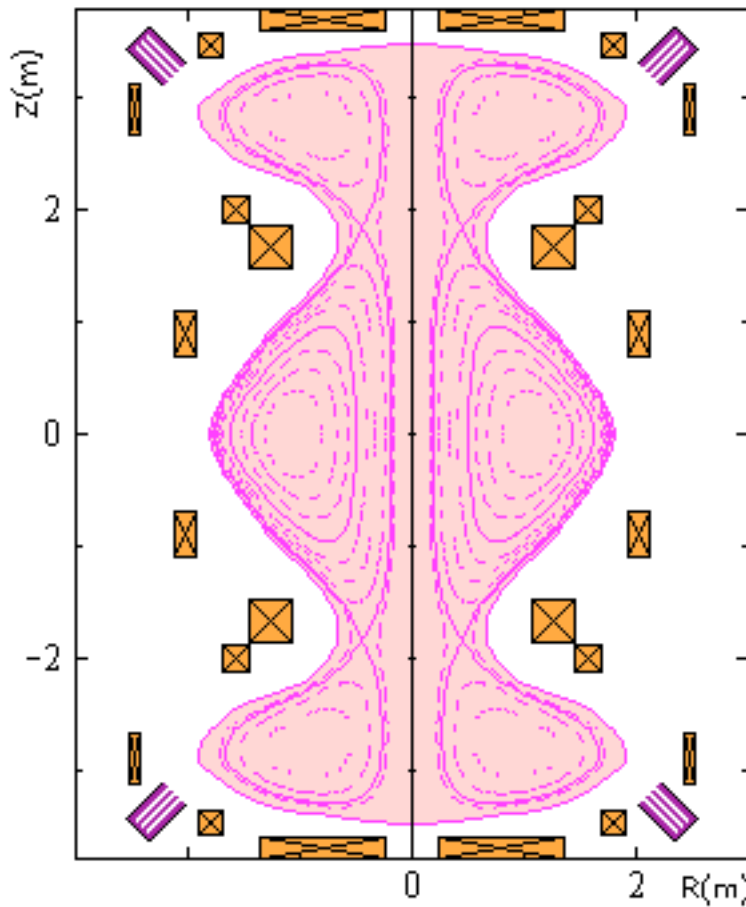


Fig. 12.1. Geometry for the $\square L=1$ standard size CKF burner.

The minimum geometrical size ($\square L=1$) gives for the surrounding spheromak discharge:

Equatorial radius of the spheromak	$\square_p$	=	0.20 m	$\square L$	.
Total spheromak plasma volume	V_p	=	9.84 m ³	$\square L^3$	.
Spheromak length	L_p	=	7 m	$\square L$	.
Maximum total field at the edge	B_m^e	=	0.149 $\square I_p$ Tesla	$\square 1/L$	.
Maximum field on symmetry axis	B_m^i	=	0.371 $\square I_p$ Tesla	$\square 1/L$	.
Averaged poloidal field	B_p	=	0.182 $\square I_p$ Tesla	$\square 1/L$	.
Poloidal spheromak plasma current	I_e	=	$I_p/6.6$	.	.

The same $\square L=1$ minimum geometrical size reactor corresponds, for the closed field line surrounding spheromak discharge, to the simplified geometry shown in Fig. 12.2.

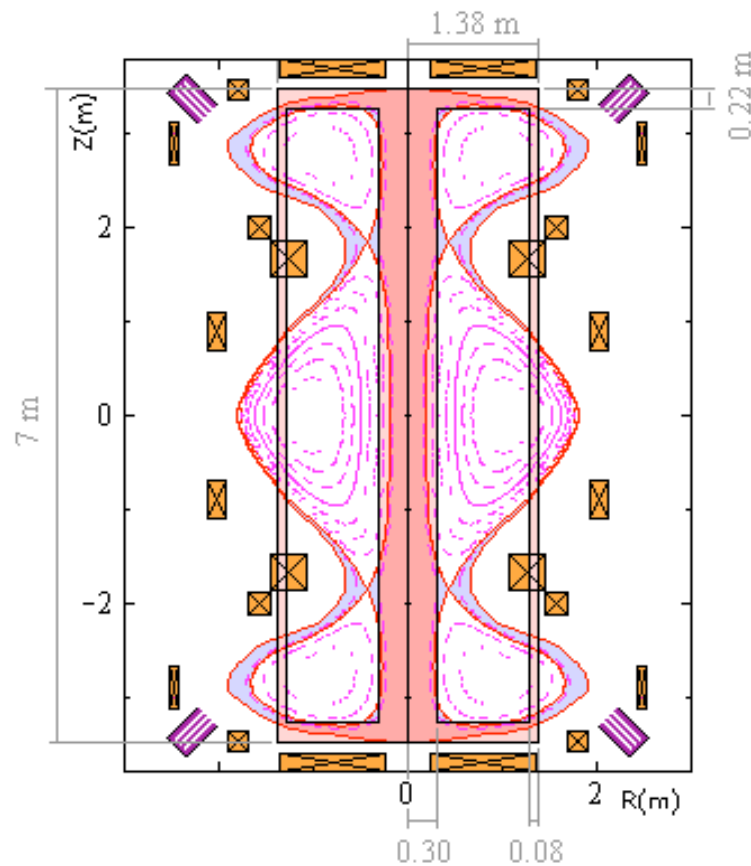


Fig. 12.2. Simplified geometry of the $\square L=1$ standard size surrounding spheromak discharge.

Operating the average of $(1/\text{surface})^2$ (which is the relevant quantity for the resistive dissipation power) on the simplified geometry and converting the spheromak volume and the resulting cross-section into an equivalent toroidal circular discharge, one gets:

Spheromak averaged surface	S_p	=	0.35 m ²	$\square L^2$	.
Spheromak major radius	R_p	=	4.46 m	$\square L$	.
Spheromak minor radius	a_p	=	0.334 m	$\square L$	.
Spheromak elongation	$\square_p$	=	1.0	.	.
Spheromak safety factor	q_p	=	0.15	.	.

The ion mass is $M=2.5$ a.m.u., for $n_D=n_T$ and for $n_D=n_{3He}$.

The total energy confinement time for the spherical torus is evaluated from the semi-empirical Lackner-Gottardi L-mode plateau-scaling:

$$\tau_E = 0.12 \cdot H_{ST} \cdot I_p^{0.8} R_{ST}^{1.8} a_{ST}^{0.4} \langle n_e \rangle^{0.4} q_{ST}^{0.4} M^{0.5} P_{input}^{-0.6} \tau_{ST} / (1 + \tau_{ST})^{0.8}$$

[s; MA, m, 10^{20} m^{-3} , a.m.u., MW], where a confinement improvement factor H_{ST} has been inserted. The surrounding spheromak is assumed to have a worse energy confinement than the ST: $H_p = H_{ST}/2$:

$$\tau_E = 0.12 \cdot H_p \cdot I_e^{0.8} R_p^{1.8} a_p^{0.4} \langle n_e \rangle^{0.4} q_p^{0.4} M^{0.5} P_{input}^{-0.6} \tau_p / (1 + \tau_p)^{0.8} \text{ [s; MA, m, } 10^{20} \text{ m}^{-3} \text{, a.m.u., MW].}$$

The charged-particle fusion power produced inside the spherical torus ($T_e = T_i$) is:

$$P_{\square} = 1.7 \cdot 10^{-3} \langle n_e \rangle^2 \langle T_e \rangle^2 V_{ST} \quad [\text{MW; } 10^{20} \text{ m}^{-3}, \text{ keV, m}^3] \text{ for D-T reactions}$$

in the range $7 < T_e < 25$ keV, where $P_{fus} = 5 P_{\square}$, and

$$P_{fus} = P_{charged} = 7 \cdot 10^{-5} \langle n_e \rangle^2 \langle T_e \rangle^2 V_{ST} \quad [\text{MW; } 10^{20} \text{ m}^{-3}, \text{ keV, m}^3] \text{ for D-}^3\text{He reactions}$$

in the range $50 < T_e < 80$ keV.

The estimations of the resistive power are derived from the Spitzer conductivity:

$$\square = 3 \cdot 10^3 \langle T_e \rangle^{3/2} / Z_{eff} \text{ [Siemens; Coulomb logarithm } \ln \square = 15, \text{ eV, ion effective/proton charge, with } Z_{eff} = 1 \text{ in D-T and } Z_{eff} = 5/3 \text{ in D-}^3\text{He].}$$

The total power required for the helicity injection through the X-points of the configuration is taken as $P^{HI} = 4 \cdot P_{ST}^{oh}$ [78], where P_{ST}^{oh} the equivalent resistive power required for sustaining the spherical torus. It is assumed that half of this power is dissipated inside the spherical torus, therefore $P_{ST}^{HI} = 2 \cdot P_{ST}^{oh}$. The other half is dissipated inside the surrounding spheromak, therefore $P_p = P_p^{oh} + 2 \cdot P_{ST}^{oh}$, where P_p^{oh} is the equivalent resistive power required for sustaining the spheromak. Therefore the overall power required for sustaining the magnetic configuration is $P = P_p^{oh} + 4 \cdot P_{ST}^{oh}$.

The energy confinement is estimated by an iterative procedure, starting from a guessed volume average electron temperature $\langle T_e \rangle$, evaluating either a resistive or a fusion charged-particle input power and a provisional energy confinement, obtaining a revised:

$$\langle T_e \rangle = \square_E^{LG} P_{input} / (3 \cdot 16 \langle n_e \rangle V) \quad [\text{eV; s, W, } 10^{20} \text{ m}^{-3}, \text{ m}^3] \text{ in D-T or}$$

$$\langle T_e \rangle = \square_E^{LG} P_{input} / (2.5 \cdot 16 \langle n_e \rangle V) \quad [\text{eV; s, W, } 10^{20} \text{ m}^{-3}, \text{ m}^3] \text{ in D-}^3\text{He,}$$

and so on until the convergence of $\langle T_e \rangle$ is obtained.

The beta value of the spherical torus is estimated as:

$$\beta = 8.0 \cdot 10^{-2} \langle n_e \rangle \langle T_e \rangle / B^2 \text{ [} 10^{20} \text{ m}^{-3}, \text{ keV, T]} \text{ for D-T plasmas and}$$

$$\beta = 6.7 \cdot 10^{-2} \langle n_e \rangle \langle T_e \rangle / B^2 \text{ [} 10^{20} \text{ m}^{-3}, \text{ keV, T]} \text{ for D-}^3\text{He plasmas.}$$

A number of limits have been considered in performing the exercise:

- The density limit of the ST is evaluated by using the Greenwald density limit, which fits well the START data: $n_e \leq 0.8 n_G$, with $n_G = \square \langle j \rangle_{ST}$ [10^{20} m^{-3} , MA/m²].
- The technological limit $B \leq 13$ T for superconducting coils.
- The technological limit $P_{neut}/S_{wall} \leq 5$ MW/m² for the continuous D-T neutron wall loading, where the surface of the wall is assumed for the $\square L = 1$ size $S_{wall} = 112 \text{ m}^2$.
- The largest considered size has been a $3 \square L$ unit size (means a cylinder 21-m high).

12.2. D-³He Burners

As far as the D-³He scenario is concerned, if no confinement improvement is introduced ($H_{ST}=1$), Table 12.1 is obtained. In order to achieve burning in the temperature range $64 < T_e < 76$ keV, the required toroidal current inside the ST exceeds 180 MA. Neither the density limit ($n_e \leq 0.2 n_G$), nor the β limit ($\beta \leq 0.21$) do pose any problems, whereas the 13 T limit constrains the size to the minimum $\beta \sim 2$ option. A neutron power emission (from D-D reactions) $P_{neut} \sim 60$ MW is still present.

R_{ST} [m]	I_p [MA]	I_e [MA]	$\langle T_e \rangle$ [keV]	$\langle n_e \rangle$ 10^{20} [m ⁻³]	B_m^e [T]	β	τ_E [s]	P_{fus} [MW]	P_{fus}/S_{wall} [MW/m ²]	$\langle j \rangle_{ST}$ [MA/m ²]
2.06	180	27.3	64	4.9	13	0.15	1.8	1078	2.3	12.1
2.06	180	27.3	70	5.2	13	0.18	1.6	1453	3.1	12.1
2.06	180	27.3	76	5.7	13	0.21	1.3	2058	4.3	12.1
2.18	190	28.8	64	3.6	13	0.11	2.5	671	1.3	11.4
2.18	190	28.8	70	3.9	13	0.13	2.1	954	1.8	11.4
2.18	190	28.8	76	4.2	13	0.16	1.8	1324	2.5	11.4

Table 12.1. D-³He burner scenarios without confinement improvement $H_{ST}=1$.

Always for the D-³He scenario, if a confinement improvement $H_{ST}=2$ is allowed for, Table 12.2 is obtained. In order to achieve burning in the temperature range $64 < T_e < 76$ keV, the required toroidal current inside the ST exceeds 95 MA.

R_{ST} [m]	I_p [MA]	I_e [MA]	$\langle T_e \rangle$ [keV]	$\langle n_e \rangle$ 10^{20} [m ⁻³]	B_m^e [T]	β	τ_E [s]	P_{fus} [MW]	P_{fus}/S_{wall} [MW/m ²]	$\langle j \rangle_{ST}$ [MA/m ²]
1.09	95	14.4	64	6.8	13	0.21	1.3	306	2.3	23.2
1.09	95	14.4	70	7.4	13	0.26	1.1	435	3.3	23.2
1.09	95	14.4	76	8.1	13	0.31	0.9	604	4.6	23.2
1.54	95	14.4	64	3.6	9.2	0.23	2.5	236	0.9	11.4
1.54	95	14.4	70	3.9	9.2	0.27	2.1	336	1.3	11.4
1.54	95	14.4	76	4.2	9.2	0.31	1.8	467	1.8	11.4

Table 12.2. D-³He burner scenarios with confinement improvement $H_{ST}=2$.

Neither the density limit ($n_e \leq 0.2 n_G$), nor the β limit ($\beta \leq 0.31$) do pose any problems, whereas the 13 T limit constrains the size to the minimum $\beta \sim 1.1$ option. However if the averaged current density $\langle j \rangle_{ST}$ is maintained to the level of the previous $H_{ST}=1$ case, a less demanding scenario is obtained with the $\beta \sim 1.5$ size option, losing only 30% of the fusion power. A neutron power emission (from D-D reactions) $P_{neut} \sim 20$ MW is still present.

Finally for the D-³He scenario, if the very large confinement improvement $H_{ST}=4$ is allowed for, Table 12.3 is obtained. In order to achieve burning in the temperature range $64 < T_e < 76$ keV, the required toroidal current inside the ST exceeds 47 MA. The density limit does not pose any problems ($n_e \leq 0.2 n_G$), the Q is quite high ($Q \leq 0.68$), whereas the 13 T limit would allow the size to the minimum $Q \sim 0.6$ option. However if the averaged current density $\langle j \rangle_{ST}$ is maintained to the level of the previous $H_{ST}=1$ case, a much less demanding scenario is obtained with the $Q \sim 1$ size option, while loosing only 15% of the fusion power at the highest temperature. A neutron power emission (from D-D reactions) $P_{neut} \sim 8$ MW is still present.

R_{ST} [m]	I_p [MA]	I_e [MA]	$\langle T_e \rangle$ [keV]	$\langle n_e \rangle$ 10^{20} [m ⁻³]	B_m^e [T]	Q	Q_E [s]	P_{fus} [MW]	P_{fus}/S_{wall} [MW/m ²]	$\langle j \rangle_{ST}$ [MA/m ²]
0.56	47.5	7.2	64	15	13	0.48	0.6	194	5.7	43.2
0.56	47.5	7.2	70	16	13	0.57	0.5	271	8	43.2
0.56	47.5	7.2	76	18	13	0.68	0.4	382	11.2	43.2
1.09	47.5	7.2	64	3.6	9.2	0.43	2.7	75	0.6	11.4
1.09	47.5	7.2	70	3.9	9.2	0.52	2.2	108	0.8	11.4
1.09	47.5	7.2	76	4.2	9.2	0.61	1.9	149	1.1	11.4

Table 12.3. D-³He burner scenarios with confinement improvement $H_{ST}=4$.

From Tables 12.1, 12.2 and 12.3 it is clear that huge plasma currents have to be driven inside a D-³He burner, and that the current density is equal or larger than the one achieved in tokamak plasmas. An improved confinement would obviously help in reducing the size of the burner, but at the price of increasing the Q value.

Three representative cases have been studied as examples of D-T start-up for the D-³He ignition. D-T preignition requires that 25-30% of Tritium is added the $n_D=n_{3He}$ mixture, see Table 12.4.

R_{ST} [m]	I_{ST} [MA]	n_e^{form} 10^{20} [m ⁻³]	H_{ST}	T_p [keV]	P_p [MW]	T_{ST}^{HI} [keV]	P_{ST}^{HI} [MW]	Q_{form} [s]	Q^{D-3He} [s]
2.18	190	4.1	1	12.6	31.8	14	19.9	860	6.0
1.54	95	4.7	2	11.9	12.3	13.2	7.6	410	4.3
1.09	47.5	4.9	4	11.8	4.4	13.2	2.8	210	4.1

Table 12.4. D-³He burner scenarios; start-up in D-³He adding up 25÷30% of T.

The times Q_{form} and Q^{D-3He} represent, respectively, the time needed to form the configuration at full current and the time needed to reach ignition in D-³He, through D-T preliminary ignition. Many hundreds of seconds are required to build up the huge currents required for a D-³He burner (unless fast reconnections are so effective as to shorten Q_{form} appreciably). The helicity injection input power to the spherical torus $P_{ST}^{HI}=2 \cdot P_{ST}^{oh}$ and the total input power to the surrounding spheromak $P_p = P_p^{oh} + 2 \cdot P_{ST}^{oh}$

are required for the start-up of the configuration. Due to the huge currents, they are quite high and produce such high temperatures T_p and T_{ST}^{HI} in the helicity injected spherical torus and in the surrounding spheromak, that no further additional power is required for the D-T ignition. The transition from D-T ignition to D- ^3He ignition takes a time of about 4÷6 s; during that time a pulse of neutron power up to about 0.5÷1.5 GW will be released to the wall, with a transient neutron wall loading of 3÷4 MW/m².

12.3. D-T Burners

As far as the D-T scenario is concerned, if a confinement deterioration $H_{ST}=0.5$ is assumed, Table 12.5 is obtained. In order to achieve burning in the temperature range $7 < T_e < 12$ keV, the Greenwald density limit ($n_e \leq 0.8 n_G$) requires the toroidal current inside the ST to exceed 66 MA. The technological limit $P_{neut}/S_{wall} \leq 5$ MW/m², for the continuous D-T neutron wall loading, is the effective constraint for the minimum size option (which varies from $\square L=1.2$ to $\square L=3$ with the burning temperature). Neither the $\square$ limit ($\square \leq 0.36$), nor the 13 T limit ($B_e^m < 8.3$ T), introduce any further constraint.

R_{ST} [m]	I_p [MA]	I_e [MA]	$\langle T_e \rangle$ [keV]	$\langle n_e \rangle$ 10^{20} [m ⁻³]	B_e^m [T]	$\square$	$\square_E$ [s]	P_{fus} [MW]	P_{neut}/S_{wall} [MW/m ²]	$\langle j \rangle_{ST}$ [MA/m ²]
1.45	66	10	7	8.1	6.8	0.12	0.5	1475	5	8.9
2.35	66	10	10	4.3	4.2	0.25	0.7	3705	5	3.4
3.00	66	10	12	3.2	3.3	0.36	0.7	5915	5	2.1
1.25	70	10.6	7	8.4	8.3	0.09	0.5	1030	5	12.7
2.00	70	10.6	10	4.8	5.3	0.17	0.6	2750	5	5
2.60	70	10.6	12	3.3	4	0.25	0.7	4300	5	2.9

Table 12.5. D-T burner scenarios with confinement deterioration $H_{ST}=0.5$.

Always for the D-T scenario, if no confinement improvement is introduced ($H_{ST}=1$), Table 12.6 is obtained. In order to achieve burning in the temperature range $7 < T_e < 12$ keV, the Greenwald density limit $n_e \leq 0.8 n_G$ requires the toroidal current inside the ST to be more than 32 MA. The technological limit $P_{neut}/S_{wall} \leq 5$ MW/m², for the continuous D-T neutron wall loading, is the effective constraint for the minimum size option (which varies from $\square L=0.8$ to $\square L=1.8$ with the burning temperature). Neither the high $\square$ value ($\square \leq 0.7$), nor the 13 T limit ($B_e^m < 6.5$ T), introduce any further constraint.

R_{ST} [m]	I_p [MA]	I_e [MA]	$\langle T_e \rangle$ [keV]	$\langle n_e \rangle$ 10^{20} [m ⁻³]	B_e^m [T]	β	β_E [s]	P_{fus} [MW]	P_{neut}/S_{wall} [MW/m ²]	$\langle j \rangle_{ST}$ [MA/m ²]
0.95	32	4.9	7	10	5.1	0.28	0.4	640	5	10
1.60	32	4.9	10	5.2	3	0.57	0.5	1685	5	3.6
1.80	32	4.9	11	4.6	2.7	0.7	0.6	2275	5	2.8
0.80	35	5.3	7	11	6.5	0.17	0.4	420	5	15.2
1.25	35	5.3	10	6.1	4.2	0.36	0.5	1105	5	6.4
1.41	35	5.3	11	5.2	3.7	0.42	0.5	1410	5	5

Table 12.6. D-T burner scenarios without confinement deterioration $H_{ST}=1$.

Finally for the D-T scenario, if a confinement improvement $H_{ST}=2$ is allowed for, Table 12.7 is obtained. In order to achieve burning in the temperature range $7 < T_e < 12$ keV, the Greenwald density limit $n_e \leq 0.8 n_G$ requires the toroidal current inside the ST to be more than 16 MA. The β limit ($\beta \sim 1$) requires furthermore that the toroidal current inside the ST exceeds 17.5 MA. The technological limit $P_{neut}/S_{wall} \leq 5$ MW/m², for the continuous D-T neutron wall loading, is the effective constraint for the minimum size option (which varies from $\beta L=0.5$ to $\beta L=1.0$ with the burning temperature). The β value is near the limit ($\beta \sim 1$), but the 13 T limit ($B_e^m < 5.5$ T) does not introduce any further constraint.

R_{ST} [m]	I_p [MA]	I_e [MA]	$\langle T_e \rangle$ [keV]	$\langle n_e \rangle$ 10^{20} [m ⁻³]	B_e^m [T]	β	β_E [s]	P_{fus} [MW]	P_{neut}/S_{wall} [MW/m ²]	$\langle j \rangle_{ST}$ [MA/m ²]
0.60	16	2.4	7	13	3.9	0.58	0.3	255	5	12.3
1.00	16	2.4	10	6.7	2.4	1.22	0.4	685	5	4.6
1.30	16	2.4	12	4.8	1.8	1.8	0.5	1110	5	2.7
0.47	17.5	2.7	7	14	5.5	0.34	0.3	160	5	21.9
0.80	17.5	2.7	10	7.4	3.3	0.70	0.4	425	5	7.6
1.00	17.5	2.7	12	5.6	2.6	1.02	0.4	690	5	5

Table 12.7. D-T burner scenarios with confinement improvement $H_{ST}=2$.

Comparing Tables 12.1, 12.2 and 12.3 with Tables 12.5, 12.6 and 12.7, it is evident that the total plasma currents needed to drive D-³He burners are by a factor of 2.7 greater than the total plasma currents in D-T reactors, if a confinement improvement $H_{D-3He}=2 \cdot H_{D-T}$ is assumed (the ratio of the needed current would double to 5.4 if $H_{D-3He}=H_{D-T}$). The required β values for a D-T reactor is larger than in a D-³He burner and, obviously, the neutron wall loading is much worse with respect to that of a D-³He burner. The improved confinement would obviously help in reducing the size of the burner, but at the price of increasing the β value, which could be pushed beyond the stability limit.

Three representative cases have been studied for the start-up of a D-T burner, see Table 12.8. The start-up of all configurations is assumed to occur at an electron density that is 70% of the final electron density.

R_{ST} [m]	I_p [MA]	n_e^{form} 10^{20} [m ⁻³]	H_{ST}	T_p [keV]	P_p [MW]	T_{ST}^{HI} [keV]	P_{ST}^{HI} [MW]	P_{add} [MW]	τ_{form} [s]	τ^{D-T} [s]
2.18	70	2.8	0.5	2.8	24.9	3.1	15.6	80	134	4.6
1.54	35	3.1	1	2.7	9.4	3.0	5.8	42	64	3.8
1.09	17.5	3.2	2	2.7	3.4	3.0	2.1	19	32	3.3

Table 12.8. D-T burner scenarios: start-up.

The times τ_{form} and τ^{D-T} are, respectively, the time needed to form the configuration at full current and the time needed to reach D-T ignition, through additional power P_{add} injected into the spherical torus. Many tens of seconds are required to build up the large currents required for a D-T burner (unless fast reconnections are so effective as to shorten τ_{form} appreciably). The helicity injection input power to the spherical torus $P_{ST}^{HI}=2 \cdot P_{ST}^{oh}$ and the total input power to the surrounding spheromak $P_p= P_p^{oh}+2 \cdot P_{ST}^{oh}$ are required for the start-up of the configuration. Due to the lower currents with respect to the D-³He cases, the start-up temperatures are too low to achieve the D-T ignition without additional power P_{add} . The additional power required to reach the ignition is in the range of many tens of MW and must be applied for 3÷5 s.

12.4. Comparison

A comparison between D-T and D-³He burners is shown in Table 12.9, for three geometrical sizes ($\square L=1.09$, $\square L=1.54$ and $\square L=2.18$), while the averaged current densities inside the ST $\langle j \rangle_{ST}$ and the surrounding spheromak discharge $\langle j \rangle_P$ have been kept constant for the same isotope mix (all current densities increase by a factor 2.7 moving from the D-T to the D-³He burners).

Larger values of confinement improvement ($H_{ST}=1, 2$ and 4) have considered in the case of the D-³He burners with respect to D-T reactors ($H_{ST}=0.5, 1$ and 2): the reason is just that in case of a degraded confinement ($H_{ST}=0.5$) a D-³He burner could presumably become too big or unfeasible.

isotope mix	R_{ST} [m]	H_{ST}	I_p, I_e [MA]	$\langle T_e \rangle, T_P$ [keV]	$\langle n_e \rangle 10^{20}$ [m ⁻³]	B_e^m [T]	$\square$	P_{charge} [MW]	$\langle j \rangle_{ST}, \langle j \rangle_P$ [MA/m ²]
SMALL SIZE									
D-T	1.09	2	17.5, 2.7	12, 2.6	4.6	2.4	0.99	117	4.2, 6.5
D- ³ He	1.09	4	47.5, 7.2	76, 14.6	4.0	6.5	0.61	149	11.4, 17.6
MEDIUM SIZE									
D-T	1.54	1	35, 5.3	11, 2.6	4.4	3.4	0.43	261	4.2, 6.5
D- ³ He	1.54	2	95, 14.4	70, 14.6	3.9	9.2	0.27	336	11.4, 17.6
LARGE SIZE									
D-T	2.18	0.5	70, 10.6	10, 2.7	4.0	4.8	0.17	505	4.2, 6.5
D- ³ He	2.18	1	190, 29	64, 15.1	3.6	13	0.11	671	11.4, 17.6

Table 12.9. Comparison between D-T and D-³He burners of three sizes.

The surface averaged value $\langle \square \rangle = \square_0 \langle \vec{j} \cdot \vec{B} / B^2 \rangle$ will decrease from the edge of the plasma to the axis of the main spherical torus (see Fig. 12.3): if the spheromak discharge can be sustained by driving current on its closed flux surfaces, magnetic helicity, flowing down the $\langle \square \rangle$ gradient, will be injected into the main spherical torus, through magnetic reconnections at the X-points. At present a method for injecting currents into the surrounding spheromak plasma in order to start-up and to sustain these burners has still to be developed. Having in mind the high $\square$ possibility of the CKF configurations, one can imagine that plasma motions, i.e. radial electric field, could create the magnetic field. In this case the surrounding spheromak discharge could be sustained by inducing axisymmetric torque on it, with opposite directions in the surrounding plasma on the inboard of the Spherical Torus and on the outboard of it. In the case of a reactor the radial electric field could even be the natural result of losses of charged fusion products.

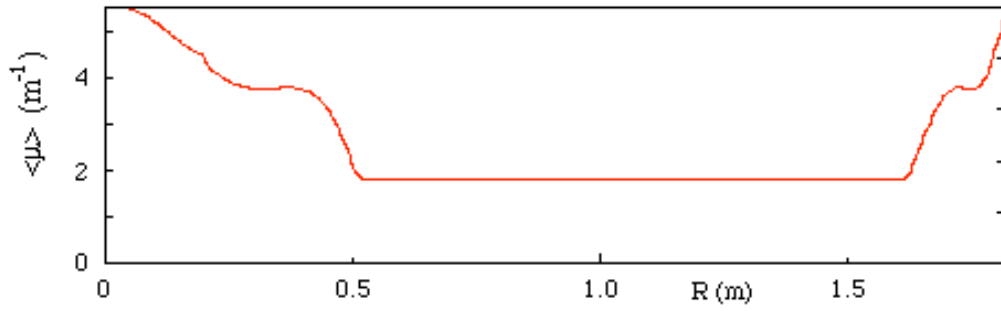


Fig 12.3. Profile of $\langle \mu \rangle$ on the equator for $\square L=1$ standard size SPHERA burner.

The plasma motion will drive the magnetic field of the surrounding discharge until magnetic reconnections will occur at the X-points of the configuration, injecting magnetic helicity into the main Spherical Torus. The two secondary tori, present on top and bottom of the configuration, will be produced by the same magnetic reconnections that sustain the main Spherical Torus.

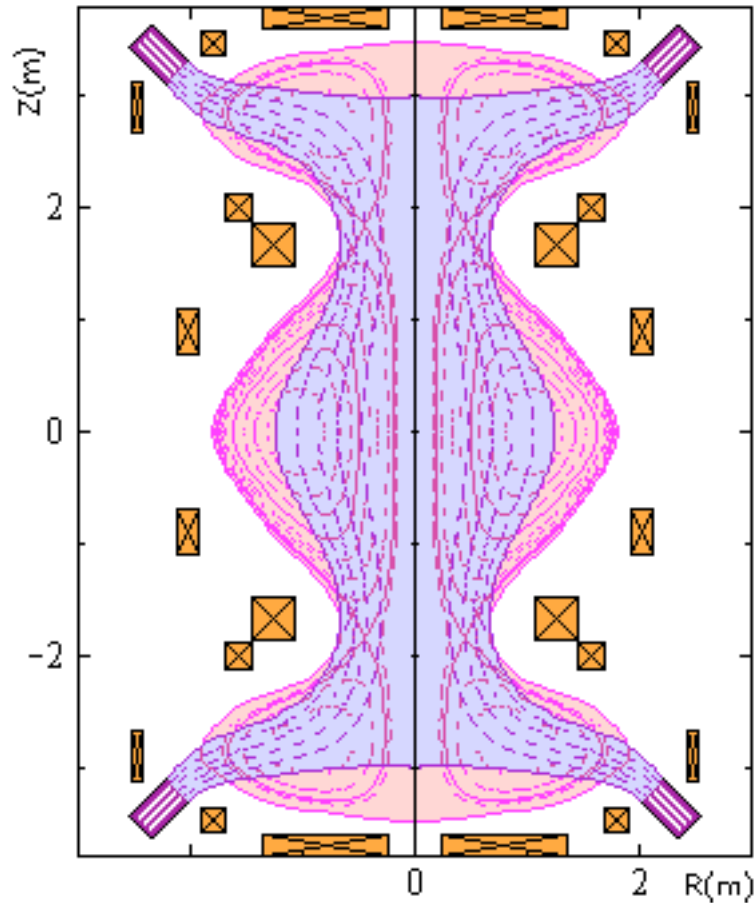


Fig. 12.4. Superposition of the initial electrode discharge (open field lines) and of the final configuration (closed field lines) for the $\square L=1$ standard size SPHERA burner. The electrodes are sketched in the corners of the figure.

However the current density that can be injected by electrodes at the beginning of the startup of the D-T burners, before the inductive formation of the Spherical Torus, is roughly adequate, even with the present electrode technology (i.e. using the same

current density obtained in front of the electrodes of PROTO-PINCH: $j_e=0.8 \text{ MA/m}^2$). Fig 12.4 shows that, for the $\square L=1$ standard size burner, the plasma facing the electrodes is a ribbon with radius $R_{el}=2.12 \text{ m}$ and width $\square Z_{el}=0.28 \text{ m}$: that means that a longitudinal pinch current up to $I_e=3 \text{ MA}$ can be driven from the electrodes at the beginning of the start-up of the configuration.

The 3 MA have to be compared with the poloidal current inside the surrounding spheromak discharge at the end of the formation for the $\square L=1$ standard size D-T burner, which varies from $I_e=4.5 \text{ MA}$ for $H_{ST}=0.5$ (Table 12.5), to $I_e=3.4 \text{ MA}$ for $H_{ST}=1$ (Table 12.6) and decreases to $I_e=2.7 \text{ MA}$ for $H_{ST}=2$ (Table 12.7).

Even for D- ^3He burners the comparison is not so unfavorable, as the poloidal current inside the surrounding spheromak discharge at the end of the formation for the $\square L=1$ standard size will vary from $I_e=12.1 \text{ MA}$ for $H_{ST}=1$, to $I_e=9.2 \text{ MA}$ for $H_{ST}=2$ and will decrease to $I_e=7.3 \text{ MA}$ for $H_{ST}=4$.

So it should not be impossible to drive the currents required for the burners on open field lines, but it is completely uncertain whether these currents could then be rerouted on closed field lines during the inductive formation of the Spherical Torus and finally maintained during the sustainment of the magnetic configuration.

13. COSTS AND TIME SCHEDULE

Vessel , including: tiles, supports, baking system and base	570,000 Euro
Design contract	77,500 Euro
Subtotal	647,500 Euro
Poloidal field coils , including: feedthroughs and cooling system	620,000 Euro
Design contract	77,500 Euro
Subtotal	697,500 Euro
Anode and Cathode , including feedthroughs and cooling system	230,000 Euro
Design contract	50,000 Euro
Subtotal	280,000 Euro
Assembly contract	90,000 Euro
Pumping, gas feeding & cntrl.sys.	180,000 Euro
<u>TOTAL</u>	<u>1,895,000 Euro</u>

Power supply:	
Pinch feeder 'P'	825,000 Euro
Cathode feeder 'K'	102,000 Euro
PF feeder 'A'	181,000 Euro
PF feeder 'B'	92,000 Euro
Subtotal	1200,000 Euro
Electrical work contract	140,000 Euro
<u>TOTAL</u>	<u>1,340,000 Euro</u>

GRAND TOTAL **3,235,000 Eur**

	TOTALS	Jun-01	Oct-01	Mar-02	May-02	Sep-02	Dec-02	Jul-03	Jan-04	Apr-04	Jul-04
Money spent	3,235,000		60,000	165,000	382,000	849,000	1017,000	379,000	181,000	131,000	71,000
LOAD ASSEMBLY	648,000	Design	Contract	Tender	Construction	Check	Assembly			Final check	Guarantee
			20,000	57,500	57,000	171,000	285,000			28,500	28,500
ASSEMBLY WORK	90,000				Tender	Orders	Work		Final check		
					10,000	60,000			20,000		
PUMP,GAS,CNTRL	180,000				Tender	Orders	Assembly		Final check		
					20,000	120,000			40,000		
PF COILS	697,000	Design	Contract	Tender	Construction		Check	Assembly		Final check	Guarantee
			20,000	57,500	62,000		372,000	124,000		31,000	31,000
ELECTRODES	280,000	Design	Contract	Tender	Construction	Check		Assembly	Check	Final check	Guarantee
			20,000	30,000	23,000	138,000			46,000	11,500	11,500
POWER SUPPLY	1,200,000	Design		Tender	Construction	Check	Check	Assembly	Final check	Guarantee	
					120,000	360,000	360,000	240,000	60,000	60,000	
ELECTRICAL WRK	140,000	Design		Tender	Work			Final check	Guarantee		
				20,000	90,000			15,000	15,000		

14. CONCLUSIONS

Chandrasekhar-Kendall-Furth (CKF) magnetic configurations are a novel approach to magnetic confinement. They contain a magnetic separatrix, which divides a main spherical torus, two secondary tori on top and bottom of the main torus and a spheromak discharge surrounding the three tori. A CKF equilibrium can be defined as a spherical torus ($q_0^{ST} \sim 1.0$, $q_{95}^{ST} \sim 2.0$) enclosed within a spheromak endowed with high elongation and therefore with high winding number ($q_0^P \sim 3$, $q_{95}^P \sim 5$). Whereas CKF force-free fields cannot sustain any pressure gradient ($\vec{\nabla}p=0$) and have a relaxation parameter $\alpha=\alpha_0 \vec{j} \cdot \vec{B}/B^2$ constant all over the plasma, unrelaxed ($\vec{\nabla} \alpha \neq 0$, $\vec{\nabla} p \neq 0$) CKF equilibria can be calculated with the boundary condition that $\alpha=\alpha_0 \vec{j} \cdot \vec{B}/B^2$ is constant only at the edge of the plasma. The surface averaged value $\langle \alpha \rangle = \alpha_0 \langle \vec{j} \cdot \vec{B}/B^2 \rangle$ will decrease from the edge of the surrounding spheromak to the axis of the main spherical torus. If the spheromak discharge can be sustained by driving current on its closed flux surfaces, magnetic helicity, flowing down the $\langle \alpha \rangle$ gradient, will be injected into the main spherical torus, through magnetic reconnections at the X-points. The gradient of the pressure profile will presumably be concentrated in the same region where the gradient of $\langle \alpha \rangle$ has the largest variation. Unrelaxed CKF equilibria, with this kind of $\langle \alpha \rangle$ and pressure profiles, can be stable to all ideal MHD perturbations with low toroidal mode number ($n=1,2,3$), up to unity beta values, $\beta=1$. This high β value opens the possibility that plasma motions, i.e. radial electric fields, can sustain the magnetic field of CKF configurations. In the case of a reactor the radial electric field could even be the natural result of losses of charged fusion products. Unrelaxed CKF fusion reactors with the right helicity injection, β limit and energy confinement, will allow for an unimpeded outflow of the high energy charged fusion products, easing direct energy conversion and the use of the burner as a space thruster.

However a method for injecting current (or torque) into a CKF configuration has not yet been developed. In order to begin an experimental study of unrelaxed CKF configurations, a preliminary experiment is being proposed. The PROTO-SPHERA experiment will study the properties of a CKF configuration where a Hydrogen force-free screw pinch, fed by electrodes, replaces in part the surrounding spheromak discharge, while poloidal field coils replace the secondary tori. PROTO-SPHERA, with a longitudinal pinch current $I_e=60$ kA, will produce a spherical torus of diameter $2 \cdot R_{sph}=70$ cm, aspect ratio $A=1.2 \div 1.3$, carrying a toroidal current $I_p=120 \div 240$ kA.

The PROTO-SPHERA project is in the framework of the research on Compact Tori (ST, spheromaks, FRC) and has the capability of exploring the connections between the three concepts. In particular its goal is to form and to sustain a flux-core-spheromak with a new technique. The magnetic configuration of the experiment has been designed aiming at a safety factor profile that is similar to the ones obtained in spherical tori with metal centerpost. The compression of the central pinch, while decreasing the total longitudinal pinch current, would lead, if successful, to the formation of an FRC. So PROTO-SPHERA could also explore a new technique for setting up an FRC.

Looking at the world program on compact tori, results from PROTO-SPHERA, if obtained as early as in 2004, should be relevant and timely for this research line.

Moreover PROTO-SPHERA contains elements of general interest in plasma physics:

- To form and sustain a magnetic confinement configuration through the non-linear saturation of an instability (self-organization).
- To investigate the coexistence between the dynamo effect (reconnections and axisymmetry breaking) and magnetic confinement.
- To simulate in laboratory plasma the solar and the protostellar flares.
- To assess the fusion relevance of simply connected magnetic confinement configurations.

PROTO-SPHERA is an experiment containing a component of scientific risk, but its success could lead to a larger size and more fusion oriented experiment.

The three major points that have to be demonstrated on PROTO-SPHERA are that the formation scheme is effective and reliable, that the combined configuration can be sustained in 'steady-state' by DC helicity injection and that the energy confinement is not worse than the one measured on spherical tori.

Later, with slight modifications in the load assembly, the PROTO-SPHERA experiment could host an unrelaxed Chandrasekhar-Kendall-Furth (CKF) configuration. Whereas the breakdown and the inductive formation of the CKF in the modified PROTO-SPHERA seem feasible, a method for sustaining it, through current or torque injection, remains to be developed.

If all the major points of PROTO-SPHERA are successfully met, and if, in the meantime, a method for injecting current (or torque) into a CKF configuration is developed, the road toward small, compact, low field and simple maintenance fusion reactor (particularly suitable to direct energy conversion and to the use as space thrusters) could be possible.

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