

10. DIAGNOSTICS

The diagnostics of PROTO-SPHERA will include the essential diagnostics used in small tokamak experiments. In the equatorial plane of the machine will be present a **CO₂ interferometer** able to follow the electron density n_e during the ultra-fast breakdown (time-scale $\sim 1\mu s$) of the central screw pinch and during the fast formation (time-scale ~ 1 ms) of the spherical torus. Also in the equatorial plane a multi-point Thomson **scattering** system will measure the electron temperature T_e . **Spectroscopic and visible light measurements** will look at the screw pinch, at the spherical torus and at the X-point region from the equatorial ports and at the screw pinch along the symmetry axis of the machine. **Soft-X ray tomographic arrays** should monitor the MHD activity connected with the helicity injection. **Thermography** will measure the temperature of the cathode, of the anode and of the protection plates.

At present only the **magnetic measurements** have been detailed, as an obvious question comes to mind: which information can be derived from the magnetic measurements, in absence of magnetic probes in the plasma-filled spherical torus hole of PROTO-SPHERA? The answer is that, in presence of a non-magnetic measurement (spectroscopic or interferometric) of the radius of the interface between the spherical torus and the screw pinch on the equatorial plane, a reasonable magnetic reconstruction of the PROTO-SPHERA plasma is feasible.

10.1. Magnetic Reconstruction

In a flux-core-spheromak configuration like PROTO-SPHERA, the magnetic probes cannot be present in the hole of the ST, so loop voltages and poloidal pick-up coils must be located only around the plasma sphere (see Fig. 10.1). With this geometry of the magnetic sensors it has to be clarified whether a magnetic reconstruction is still possible: in particular will the toroidal plasma current I_p be measurable, in absence of a Rogowsky coil around the ST cross-section?

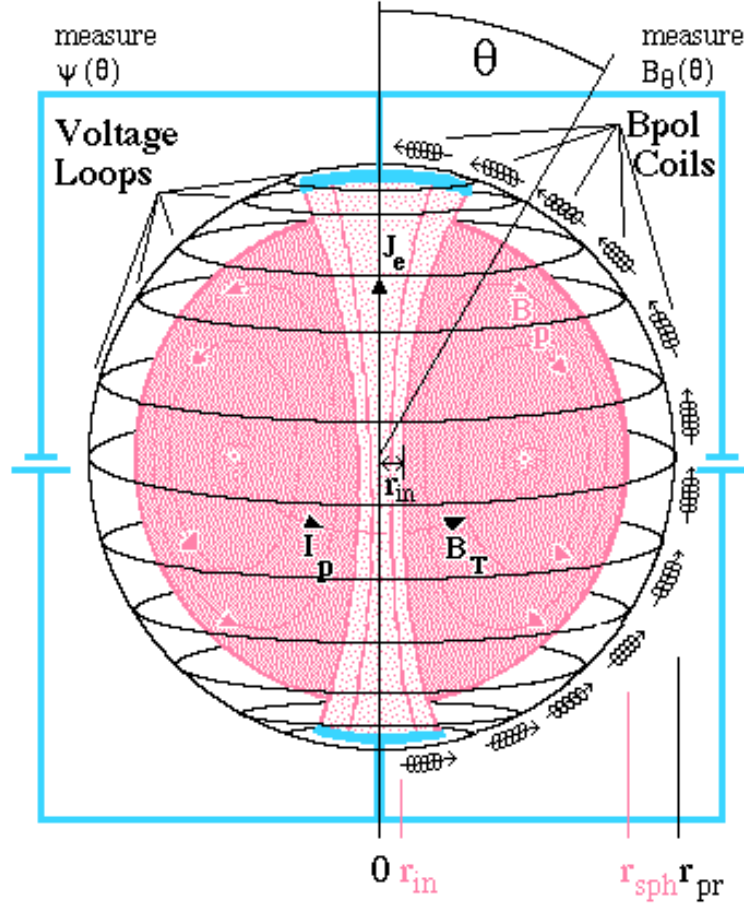


Fig. 10.1. Schematic of the magnetic sensors for a flux-core spheromak configuration.

The magnetic signals have been calculated as an output of the free boundary predictive equilibrium code (see Sect. 4.5). The reconstruction algorithm is based upon the expansion of the flux function ψ in spherical coordinates (r, ϑ, ϕ) :

$$\psi = \sum_{n=1}^{N_{\max}} \left[M_n^i(r) r^{-n} + M_n^e(r) r^{n+1} \right] \sin \vartheta P_n^1(\cos \vartheta) \quad ;$$

where $M_n^i(r)$ and $M_n^e(r)$ are the internal and external spherical multipolar moments. The magnetic signals (typically the flux function and the B_{pol} measurements) are best-fitted through an iterative equilibrium solution, by using a functional parameterization of the sources of the Grad-Shafranov equation: the plasma pressure $p(\psi)$ and the diamagnetic current $I_{\text{dia}}(\psi)$.

The same coefficients are used both for the ST as well as for the Pinch:

$$\begin{aligned}
p(\psi) &= A_1 \left(\frac{\psi}{\psi_x} \right)^{\alpha_1} && \text{Inside the ST,} \\
p(\psi) &= A_1 && \text{Inside the force-free pinch,} \\
I_{\text{dia}}^2(\psi) &= \sum_{i=1}^{N_F} B_i \left(\frac{\psi}{\psi_x} \right)^{\beta_i} && \text{Inside the ST,} \\
I_{\text{dia}}^2(\psi) &= \sum_{i=1}^{N_F} B_i \left(\frac{\psi}{\psi_x} \right)^2 && \text{Inside the force-free pinch,} \\
\text{with the obvious constraint } \sum_{i=1}^{N_F} B_i &= I_e^2.
\end{aligned}$$

Moreover it has to be remarked that:

- The best-fit of the data can determine the multipolar moments up to $N_{\text{max}}=7$ [89].
- The number of $p(\psi)$ and $I_{\text{dia}}^2(\psi)$ functional parameters must be kept as low as possible in order to avoid numerical instabilities during the iterations ($N_F=3$ has been chosen).
- As the plasma pressure $p(\psi)$ can be measured on an experiment, the exponent of the pressure has been fixed to $\alpha_1=1.1$, just the same value used in the predictive equilibrium code. On the other hand the squared diamagnetic current $I_{\text{dia}}^2(\psi)$ cannot be directly measured, so the arbitrary choice $\beta_1=0.5$, $\beta_2=1.0$ and $\beta_3=1.5$ has been made.

The most accurate magnetic reconstruction is obtained by putting the magnetic sensors on a constant $r=r_{\text{pr}}$ surface. With this choice it is easy to separate the external and internal current density contributions to the flux function expansion: the values of $M_n^i(r_{\text{pr}})$ and $M_n^e(r_{\text{pr}})$ at $r=r_{\text{pr}}$ are computed from the best-fit of the magnetic measurements once for all, before starting the iterative solution. It comes out that the magnetic probes around the spherical plasma are not sufficient for obtaining an equilibrium reconstruction, so an additional constraint is needed [89]. In particular the addition of the radius of the ST-SP interface r_{in} on the equatorial plane is required. This datum should be derived from non-magnetic measurements (e.g. spectroscopy or interferometry).

Therefore there are 4 unknowns $[A_1, B_1, B_2, B_3]$ and 6 data: $[\sum_{i=1}^{N_F} B_i = I_e^2, r_{\text{in}}, M_1^i(r_{\text{pr}}), M_3^i(r_{\text{pr}}), M_5^i(r_{\text{pr}}), M_7^i(r_{\text{pr}})]$. The Grad-Shafranov equation is iteratively solved by adopting the following scheme. A tentative Multipolar Moments $\tilde{M}_n^{\{i\}}(r)$ expansion is calculated from the tentative $\tilde{\psi}, \tilde{A}_1, \tilde{B}_1, \tilde{B}_2, \tilde{B}_3$; then a linear overdetermined system (6 equations, 4 unknowns) finds the correcting factors $[\tilde{A}_1', \tilde{B}_1', \tilde{B}_2', \tilde{B}_3']$ by matching: $\tilde{M}_n^i(r_{\text{pr}}) = M_n^i(r_{\text{pr}})$ for $n=1,3,5,7$; $\psi(r_{\text{in}}, \pi) = \psi_x$ and $\sum_{i=1}^{N_F} B_i = I_e^2$ (exactly). At this point a new set of $\tilde{\psi}, [\tilde{A}_1, \tilde{B}_1, \tilde{B}_2, \tilde{B}_3]$ is calculated and the process is iterated up to convergence.

10.2. Magnetic Sensors

The magnetic sensors of PROTO-SPHERA (Fig. 10.2) will be subdivided in two groups:

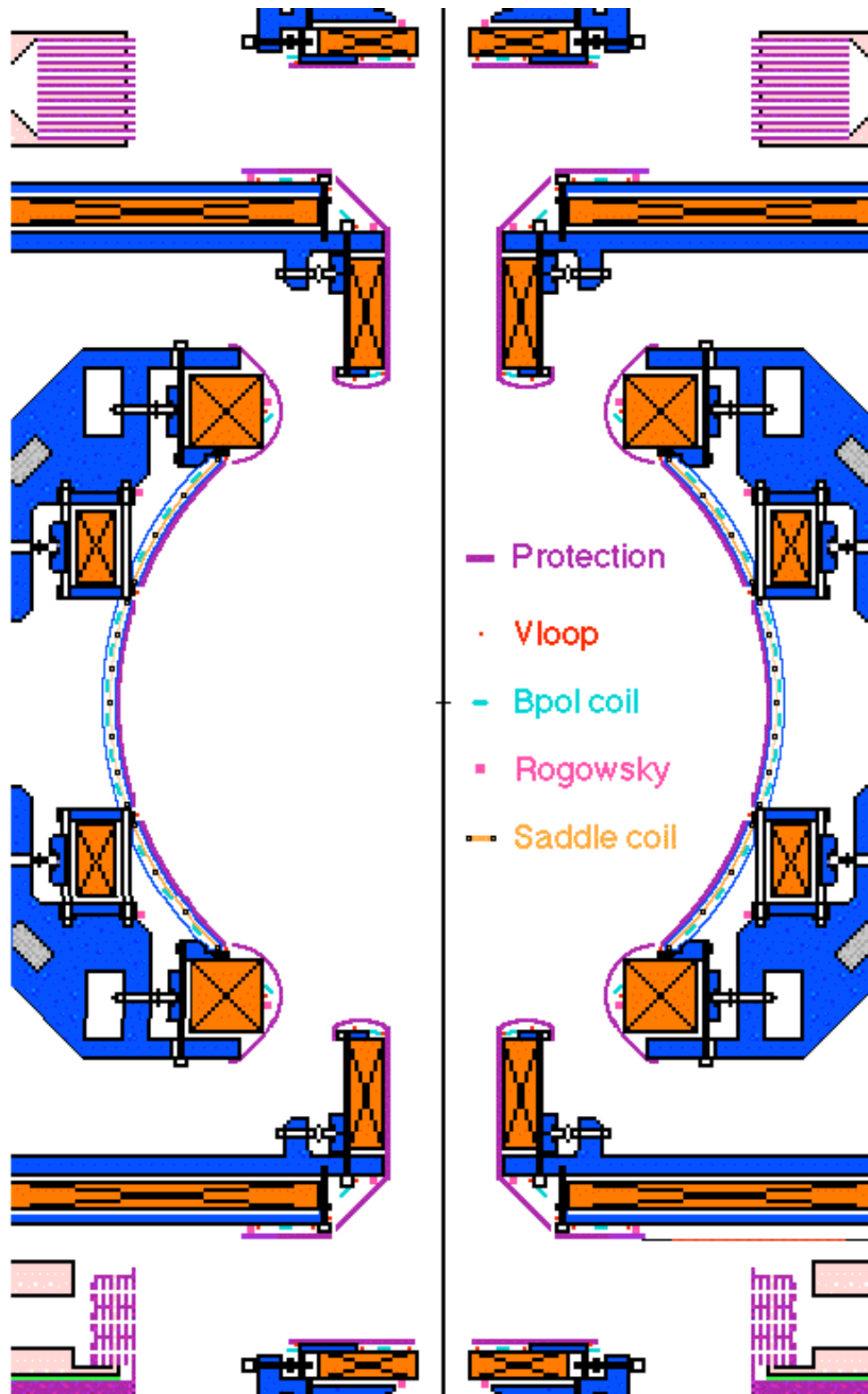


Fig. 10.2. Magnetic probes for PROTO-SPHERA.

- i) Sensors lying on a sphere ($r_{pr}=42$ cm):
 - 10 V-loops
 - 12 Saddle coils (4 inserts)
 - 16 Pick-up coils (4 inserts).
- ii) Sensors for the upper/lower pinch reconstruction:
 - 14 V-loops
 - 8 Pick-up coils (4 inserts)
 - 10 Rogowsky coils for the pinch current I_e .

Introducing a gaussian error on the magnetic measurements and a fixed error on the r_{in} position determination has checked the resilience of the reconstruction. The poloidal flux and the magnetic field due to the poloidal field coils has been subtracted from the measured signals, in order to avoid the use of very high order spherical harmonics.

10.3. Results of the Magnetic Reconstruction

The results of the reconstruction of $(I_p, \beta_p, I_{Pinch}^\phi)$ and (q_{95}, q_0) are shown in Tab. 10.1 and Tab. 10.2.

Time-slice	I_p [kA]	β_p	I_{Pinch}^ϕ [kA]	q_{95}	q_0
T0 ($I_e = 8.25$ kA)	0.0	---	2.87	---	---
T3 ($I_e = 60$ kA)	30.0	1.15	169	3.39	1.18
T4 ($I_e = 60$ kA)	60.0	0.50	225	2.87	1.08
T5 ($I_e = 60$ kA)	120.0	0.30	283	2.68	0.97
TF ($I_e = 60$ kA)	240.0	0.15	382	2.83	1.03

Table 10.1. Values of I_p , β_p , I_{Pinch}^ϕ , q_{95} and q_0 , from the predictive equilibrium calculation.

Time-slice	I_p [kA]	β_p	I_{Pinch}^ϕ [kA]	q_{95}	q_0
T0 ($I_e = 8.25$ kA)	0.0 ---	--- ---	2.97 $\pm 5\%$	--- ---	--- ---
T3 ($I_e = 60$ kA)	29.9 $\pm 8\%$	0.0/2.05 >100%	165 $\pm 1\%$	3.00 $\pm 17\%$	1.30 $\pm 29\%$
T4 ($I_e = 60$ kA)	58.0 $\pm 4.9\%$	0.54 $\pm 47\%$	202 $\pm 1.7\%$	2.71 $\pm 7.5\%$	0.86 $\pm 3.5\%$
T5 ($I_e = 60$ kA)	116.5 $\pm 1.3\%$	0.30 $\pm 66\%$	260 $\pm 4.8\%$	2.56 $\pm 17\%$	0.79 $\pm 11\%$
TF ($I_e = 60$ kA)	249.0 $\pm 3.6\%$	0.19 $\pm 53\%$	361 $\pm 8.5\%$	2.79 $\pm 8.3\%$	0.83 $\pm 12\%$

Table 10.2. Values of I_p , β_p , I_{Pinch}^ϕ , q_{95} and q_0 from the equilibrium reconstruction.

An error of $\pm 1\%$ has been assumed with the exception of the time-slice T3, in which the error has been increased up to $\pm 2\%$; an indetermination of $r_{in} = \pm 2.5$ mm has been used with the exception of T3, in which it has been increased up to $\pm 5/-10$ mm. The results are that:

- The ST toroidal current I_p can be measured with an error of about $\pm 3.6\%$ to $\pm 8\%$.
- The toroidal component of the screw pinch current I_{Pinch}^ϕ can be measured with an error of about $\pm 1\%$ to $\pm 8.5\%$.
- An accurate β_p estimate is impossible with magnetic measurements alone (the error is always greater than $\pm 47\%$).

The results about the estimate of the toroidal current density j_ϕ profile are shown in Fig. 10.3 and the results about the estimate of the safety factor q profile are shown in Fig. 10.4.

The strong toroidicity effects allow for a good reconstruction of the q profile, but the reconstruction of the j_ϕ profile is much less accurate.

Figures 10.5 and 10.6 show the quality of the plasma boundary reconstruction at the beginning of the spherical torus formation and at the flat-top. There is some inaccuracy in the reconstruction of the shape of the flat-top plasma (which has a very compressed pinch), probably due to the need of very high order spherical harmonics.

In order to have a better magnetic reconstruction of the plasma disks near the electrodes it would be probably necessary to switch to cylindrical coordinates.

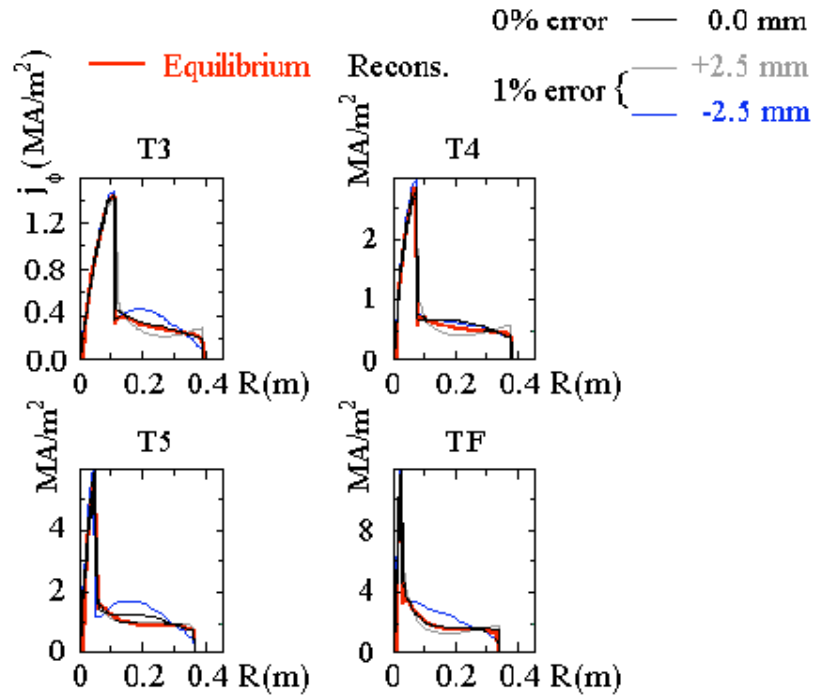


Fig. 10.3. Comparison among the j_ϕ profiles calculated by the predictive equilibrium code and the ones obtained from the magnetic reconstruction for the time-slices T3, T4, T5 and TF.

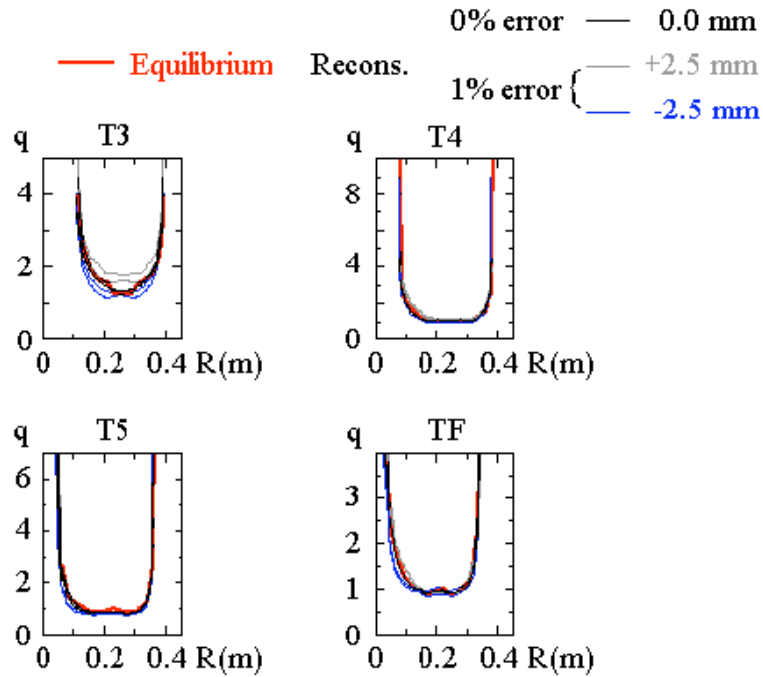


Fig. 10.4. Comparison among the q profiles calculated by the predictive equilibrium code and the ones obtained from the magnetic reconstruction for the time-slices T3, T4, T5 and TF.

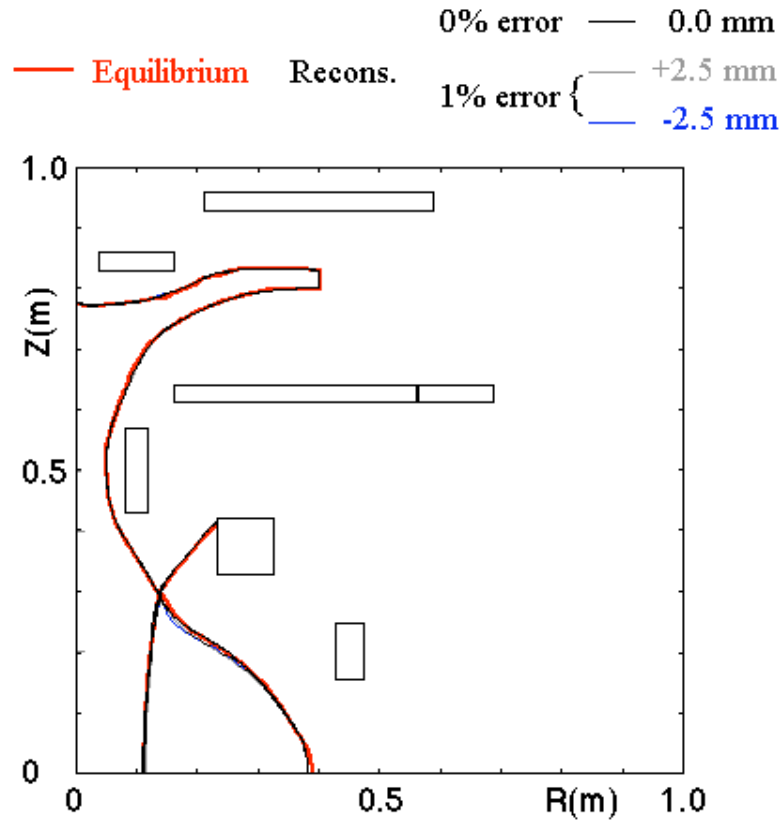


Fig. 10.5. Reconstruction of the plasma boundary at time-slice T3 ($I_p=30$ kA, $I_c=60$ kA).

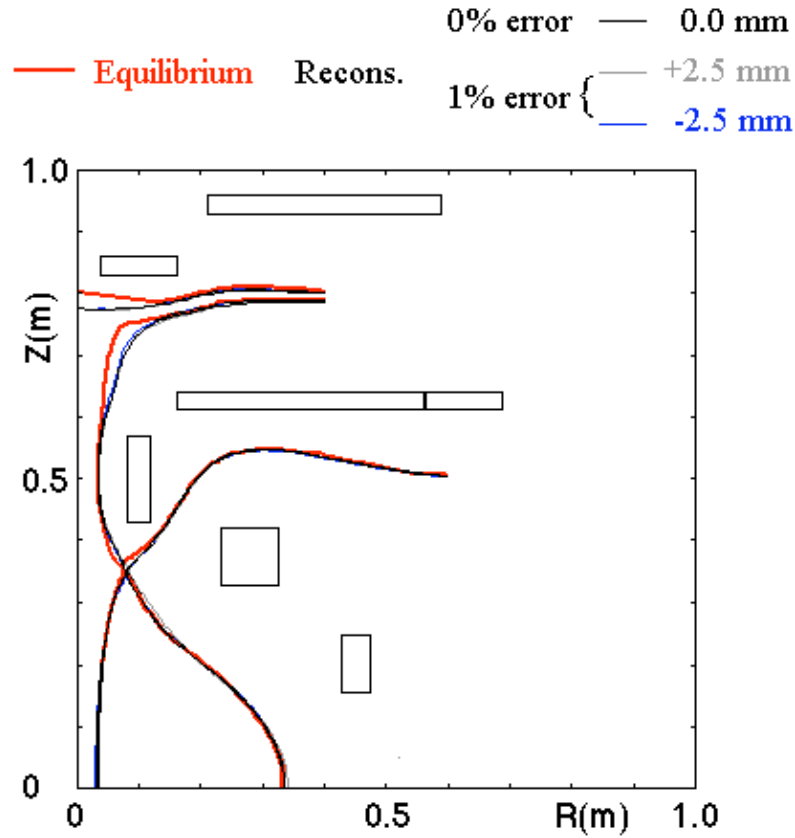


Fig. 10.6. Reconstruction of the plasma boundary at time-slice TF ($I_p=240$ kA, $I_c=60$ kA).

So, although the magnetic sensors cannot (obviously) surround the toroidal plasma (as in a standard tokamak), it is possible to reconstruct the overall configuration of PROTO-SPHERA by using standard magnetic measurement, if:

- The sensors are located on a sphere.
- The information about the inboard plasma boundary r_{in} (from non-magnetic measurements) is added.
- Care is taken to subtract the equilibrium coils contributions from the measured signals.

The ST toroidal current I_p can be detected with an error ranging from $\pm 3.6\%$ to $\pm 8\%$. The toroidal component of the screw pinch current I_{Pinch}^ϕ can be measured with an error of about $\pm 1\%$ to $\pm 8.5\%$.

An accurate β_p measurement is impossible with magnetic measurements alone (the error is always greater than $\pm 47\%$).

The strong toroidicity effects allow for a good reconstruction of the q profile, but the reconstruction of the j_ϕ profile is much less accurate.