Study of Magnetic Helicity and Relaxation Phenomena in Laboratory Plasmas

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With recent growing interest in magnetic helicity in the solar physics community, the present paper reviews experimental results on evolution of magnetic helicity during the formation and relaxation of laboratory plasmas. Measurements made in spheromaks, reversed field pinch (RFP) and merging plasma experiments are reviewed. In laboratory plasmas boundaries are generally well defined thus it is straightforward to define a gauge invariant relative helicity. It was found by J.B. Taylor that in the formation process laboratory plasmas seek the minimum energy state, while keeping the relative magnetic helicity roughly constant. This state can be described as a force free state with an additional constraint $\mu = \mathbf{j}\cdot\mathbf{B}/B^2$ = constant throughout the plasma interior. Also in the sustained state of laboratory plasmas, it has been verified that plasmas repeatedly relax to the Taylor minimum energy states while conserving the relative magnetic helicity. It is seen that the confinement of these pinch plasmas is often significantly affected by the relaxation processes.

1. INTRODUCTION

Magnetic field generated by current in a plasma can effectivelyconfine high pressure plasmas by generating a stable toroidal pinch configuration. But this configuration is often susceptible to a relaxation phenomenon which deteriorates its plasma confinement feature. In order to quantitatively describe the characteristics of current carrying laboratory plasmas, magnetic helicity has been defined and studied as an important physics parameter [*Taylor* 1986]. Recently there has been a growing interest in magnetic helicity — a global quantity to express a "knottedness" of twisted magnetic field lines---in the solar physics community[*Rust*, 1994]. A primary objective of the present paper is to review experimental studies of evolution of magnetic helicity during the formation and relaxation processes of laboratory plasmas.

A toroidal pinch is one of the simplest systems for confining plasma by magnetic field and most utilized for confining hot plasmas for fusion research. In these systems a toroidal current is induced to heat and compress the plasma through the well known pinch effects. Tokamaks, reversed field (toroidal) pinch (RFP), and spheromak configurations belong to this category. The magnetic field produced by internal current provides an inward pinch force by generating a force balance with outward plasma pressure. Fig.1 presents schematics for these three configurations. While all these configuration generate self-pinching poloidal fields, toroidal fields are supplied differently; tokamak's toroidal field is very strong and is primarily created by external coils, while the toroidal field of RFP is created by combined effect of internal current and small external field and is much weaker than that of a tokamak. The spheromak does not have external toroidal field and its internal toroidal field is

Magnetic Helicity in Space and Laboratory Plasmas
Geophysical Monograph 111

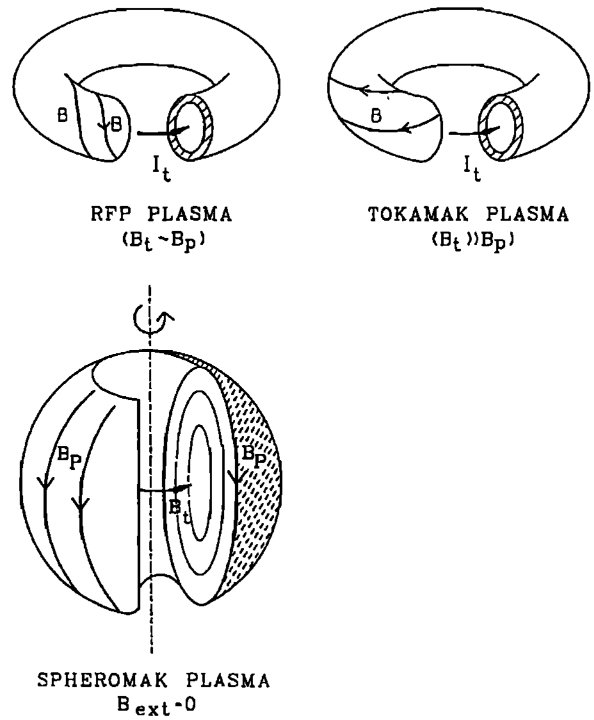

FIg.1. Schematic comparison of toroidal pinch configuration.

generated by the internal current. There is a remarkable feature common to all of these toroidal pinch discharges. It has been found that, after an initial highly turbulent state, the plasma settles into a more quiescent stable state in which the fluctuations are reduced.

To quantitatively describe the stable configuration of the twisted field lines in toroidal pinch plasmas, J.B. Taylor[1974] formulated a concept that during the relaxation of pinch plasma configuration the plasma seeks "a minimum energy state" while conserving magnetic helicity K. Here K is defined to monitor the linkage of magnetic field lines:

$$K = \int \mathbf{A} \cdot \mathbf{B} \, d^3 x,$$

where **A** is the vector potential for the magnetic field vector **B** and the integration is carried out inside the volume where the normal component of magnetic field is zero at the boundary ($\mathbf{B} \cdot \mathbf{n} = 0$). In laboratory experiments, MHD plasmas are created with variety of well defined boundary conditions which can be controlled externally.

During the formation of RFP's and spheromaks it was found that relaxation to the Taylor (minimum-energy, force-free)state was an important mechanism. In these pinch configurations, spatial profiles of $\mu = \mathbf{j} \cdot \mathbf{B}/B^2$ have been measured from internal probe data to verify the force-free minimum energy Taylor state which is characterized by μ = constant. Magnetic fluctuations with specific toroidal and poloidal mode numbers have been identified and found to be responsible for attaining the Taylor state. It has been observed that they often cause a conversion of poloidal to toroidal flux, or toroidal to poloidal flux, to occur.

In high temperature toroidal pinch discharges such as tokamaks, RFP's and spheromaks, "sawtooth"-type relaxation events were often observed and their primary mechanism was made clear by measuring their magnetic field and spatial electron temperature profiles[*Kadomtsev*, 1975]. If the plasma confinement is good at the plasma center, a deviation of the plasma configuration from the Taylor state is seen in most tokamak discharges. The good central confinement and large resistive loss at the plasma edge is considered to be responsible for this highly peaked current profile [$\mu \neq$ constant] , a deviation from Taylor state. This configuration can be usually maintained in stable manner in tokamak discharges.

When the magnetic configuration departs from the Taylor state in a RFP or a spheromak, it easily becomes unstable to low n (n = toroidal mode number) ideal MHD modes. Through these instabilities, magnetic reconnection takes place (relaxation) and a Taylor state is recovered [μ = constant]. It is found that the confinement time of these plasmas is significantly affected by this relaxation phenomenon.

Conservation of magnetic helicity has been verified during relaxation in many laboratory pinch plasmas in which the Taylor minimum energy states are identified after the relaxation events. Recently merging spheromaks have been utilized to investigate magnetic reconnection of magnetized toroidal plasmas or helical flux ropes [*Yamada et al.*,1990, *Ono et al.,* 1993]. Experiments with fully 3 dimensional magnetic reconnection are now possible in the MRX device [*Yamada et.al.*,1997] where evolution of magnetic helicity can be studied.

In this paper experimental findings on the relaxation processes of current carrying plasmas are reviewed to elucidate the fundamental physics mechanisms. The evolution of magnetic helicity of these pinch plasmas is studied. A special focus is put on spheromak characteristics since RFP properties will be described in another paper of this monograph [*Prager*]. In the end of this paper, a possible application of the laboratory results to the observed phenomena in the solar atmosphere is presented.

2. MAGNETIC HELICITY AND TAYLOR'S MINIMUM ENERGY STATE

In order to quantitatively describe the characteristics of current carrying laboratory plasmas, magnetic helicity has been defined and studied as an important physics parameter. Magnetic helicity represents a net twist, linkage or the "knottedness" of magnetic field, and defined by

$$K \equiv \int \mathbf{A} \cdot \mathbf{B} \, dV \qquad (1)$$

in a closed boundary. **B** is the magnetic field and **A** is its vector potential. If we express K by magnetic flux;

$$K = \int \mathbf{A} \cdot d\mathbf{l} \, Bda = \int \psi \, d\Phi \qquad (2)$$

because magnetic flux can be expressed $Bda = d\Phi$, $\int \mathbf{A} \cdot d\mathbf{l} = \Psi$ (where the integrals extend over a suitable boundary), we can write the magnetic helicity as the product of linked fluxes aas shown in Fig.2, where "da" represents differential cross section of the torus. For two simply connected loops of flux, ψ_0 and Φ_0, the entire volume integration simply leads to [*Berger and Field*, 1984]

$$K = 2 \, \psi_0 \Phi_0, \qquad (2a)$$

For laboratory plasmas, we often define the magnetic helicity after subtracting the helicity associated with vacuum magnetic fields thus making it gauge-invariant definition.

The gauge-invariant definition of the magnetic helicity in toroidal geometry is expressed by

$$K \equiv \int \mathbf{A} \cdot \mathbf{B} \, dV - \Phi_t(a)\psi_p(a) = \int [\mathbf{A} - \mathbf{A}_t(a)] \cdot \mathbf{B} \, dV \qquad (3)$$

where $\Phi_t(a)$ is the total toroidal flux, $\psi_p(a)$ is the external poloidal flux threading the central hole of the torus and $A_t(a) = \psi_p(a)/(2\pi R)$. Evolution of magnetic helicity becomes an issue whenever a plasma changes its topology thru magnetic reconnection or relaxation processes.

Based on his careful examination of RFP formation processes, J. B. Taylor proposed a conjecture that K should be conserved during plasma relaxation as the plasma rapidly approaches its minimum-energy state. The essential point of his hypothesis is that magnetic energy dissipates much faster than helicity during plasma relaxation inside of a boundary with $\delta\mathbf{B} \cdot \mathbf{n} = 0$.

With this assumption, Taylor theory predicts the minimum-energy state decribed by

$$\nabla \times \mathbf{B} = \mu \mathbf{B} \qquad (4)$$

$$\text{with } \mu = \text{constant.} \qquad (4a)$$

The quantity μ (the Lagrange multiplier in the calculation) is a constant eigenvalue with units of $(\text{length})^{-1}$. While μ is defined as a measure of the ratio of current density to magnetic field, $\mathbf{j} \cdot \mathbf{B}/B^2$ in the interior of the plasma, it is also equal to the ratio of total magnetic energy to helicity, 2W/K. This finding has been very effective in describing many pinch plasma states in which internal current plays a major role in confining plasma.

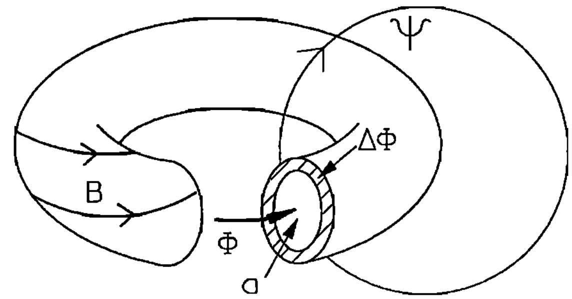

Fig. 2. Geometry of Magnetic helicity for toroidal plasma

3. FORMATION AND RELAXATION OF REVERSED FIELD PINCH

3.1 Formation of RFP Plasmas

The RFP is an axisymmetric toroidal pinch in which plasma is confined by a poloidal magnetic field Bp created by toroidal plasma current, and toroidal field B_t created by a poloidal plasma current and external coil currents. By carefully examining RFP discharges, Taylor postulated that the RFP configuration originates from a process of plasma relaxation in which plasma settles into a state of minimum energy with a constraint of constant helicity. If this state is described by simply by $\nabla \times \mathbf{B} = \mu\mathbf{B}$ with constant μ through out the large aspect ratio toroidal vacuum vessel , it would naturally lead to the well known Bessel function solution :

$$B_t(r) = B_{to} J_0(\mu r), \qquad (5a)$$

$$B_p(r) = B_{to} J_1(\mu r) \qquad (5b)$$

where B_{to} denotes toroidal field at the minor axis. This formulation is valid as long as the large aspect ratio toroidal plasma can be approximated by a cylinder of radius a. The relaxed force free states are independent of initial state and described by the dimensionless parameters, F and θ; F [$= B_t(a)/\langle B_t \rangle$] is the ratio of toroidal field at the wall to the average toroidal field and $F<0$ implies a field reversal at the edge. θ [$= B_p(a)/\langle B_t \rangle$] is called a pinch parameter defined as the ratio of poloidal field at the wall to the average toroidal field. In practice this force free state does not exactly describe RFP donfigurations since the assumption of μ= constant holds only in the inner part of the plasma and it goest down to zero at the edge. But the field profiles described by Eq.4(a,b) agree remarkably well with those with quiescent plasma obtained in RFP devices as shown in Fig. 3a. Fig. 3b shows points on the F-θ curve for several experiments together with a theoretical curve. In general, when the toroidal field reverses at the edge, $F < 0$, a more stable

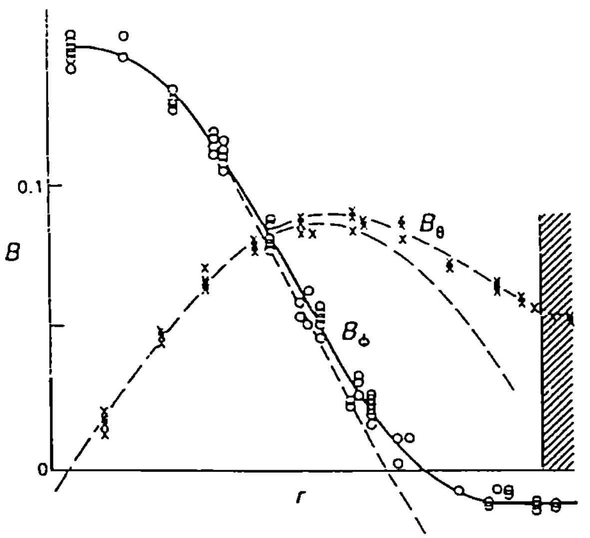

Fig. 3a. Experimental and theoretical magnetic profiles of RFP [*Bodin*, 1984].

plasma is obtained thus the configuration is maintained as "reversed field pinch" (RFP).

A tendency of RFP plasmas to relax into the Taylor state is more clearly demonstrated by the time evolution of F-q curves taken from discharges in the HBTX [*Bodin & Newton*, 1980] and ZT-40 devices [*DiMarco* et al., 1983]. During fast pinch discharges in HBTX, the curve was temporarily forced away from the Taylor state but it quickly fell back into the relaxed state. When the discharge current rises slower, the curve always stays close to the Taylor state during the formation stage.

3.2. Sawtooth Relaxation in RFP plasmas

Once an RFP plasma configuration is established, the stable discharge can be maintained as long as the plasma current is sustained by inductive drive. Magnetic helicity is supplied by poloidal flux injection into the plasma. During the discharge the poloidal flux often becomes too excessive to maintain the Taylor state and a relaxation occurs as a sawtooth oscillation. If one draws the F-q diagram for the whole sawtooth process, the F-q curve gradually deviates from the Taylor state then quickly comes back to the relaxed state. During this relaxation process a conversion of flux from poloidal to toroidal flux has been seen many RFP discharges. It has been seen that the total helicity, K (= 2 $\Psi\Phi$) is kept roughly constant.

The magnetic helicity and energy evolution during the sawtooth cycle in RFP plasma was studied [*Ji* et al. '95] in the Madison Symmetric Torus (MST) RFP. The MST is a large RFP device (major radius = 1.50m, minor radius = 0.52m) with plasma current Ip up to 700kA. The plasma is surrounded by a 5cm-thick aluminum shell with one toroidal and one poloidal gap. The shell also acts as the vacuum vessel and a single-turn toroidal field coil.

Sawtooth oscillations consist of a fast crash phase and a slow recovery phase. The plasma rapidly relaxes towards its minimum energy state during the crash within a short time. This was also illustrated by drawing a F-θ curve. With lack of measurement of the exact magnetic field profiles, the μ profiles (and hence energy and helicity) were assumed from other measured quantities by employing equilibrium models of $\mu=\mu_0[1-(r/a)^{\alpha}]$. With this assumption the changes of K and W during a sawtooth crash was carefully monitored. Fig. 4 presents inventory of magnetic helicity, magnetic energy, poloidal flux and toroidal flux during a sawtooth oscillation. It was found that during the relaxation event the magnetic helicity decreases by 1.3-5.1 %, while the magnetic energy decreases by 4.0-10.5 %. Hence, the helicity conservation conjecture is modestly well-satisfied in that the helicity decay is less than the energy decay by a factor of2-3.

This result that the decay ratio of energy to helicity is a factor of2-3 instead of orders of magnitude, indicates that helicity conservation is only a rough approximation. The helicity change is larger than the simple MHD prediction. Determination of detailed mechanisms for possible anomalous helicity dissipation during relaxation awaits further investigations

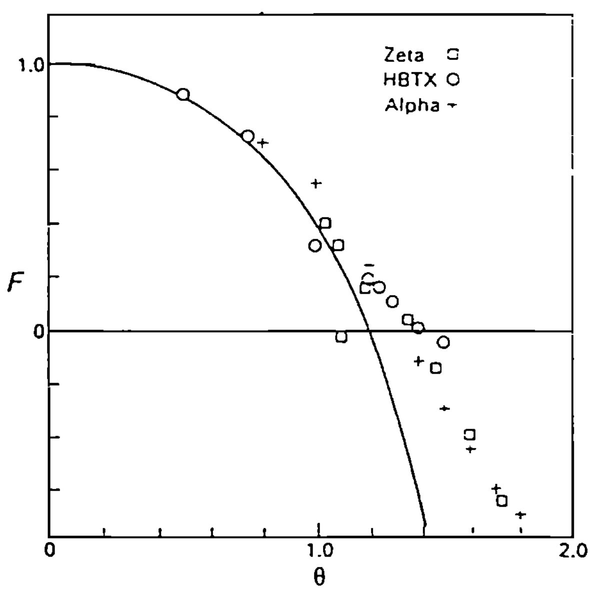

Fig. 3b. F-θ diagram. Data from HBTX, Alpha, ZETA[*Bodin and Newton,* 1984]

4. FORMATION AND RELAXATION OF SPHEROMAK PLASMAS

4.1 Spheromak Configuration

The spheromak confines a compact toroidal-shaped plasma in a spherical boundary. Poloidal fields are produced by plasma currents and external coils. Toroidal fields are produced entirely by plasma currents so that no coils link the plasma. The spheromak has many features in common with RFP including magnetic shear stabilization dynamics and confinement. The most notable difference between the RFP and the spheromak is that the spheromak does not have a reversal of the toroidal field at the outer edge of the plasma (B_t =0).

Spheromak is one type of toroidal pinch plasma which can be characterizedby a Taylor minimum energy state. This configuration is generally enclosed by a spherical boundary or a conducting shell and the plasma is confined by the magnetic field produced by its own internal current. The magnetic field profile in this configuration can be straightfowardly derived from Eq. (3) with a spherical (can be oblate or prolate) boundary condition of $\mathbf{B}\cdot\mathbf{n}$ =0 .

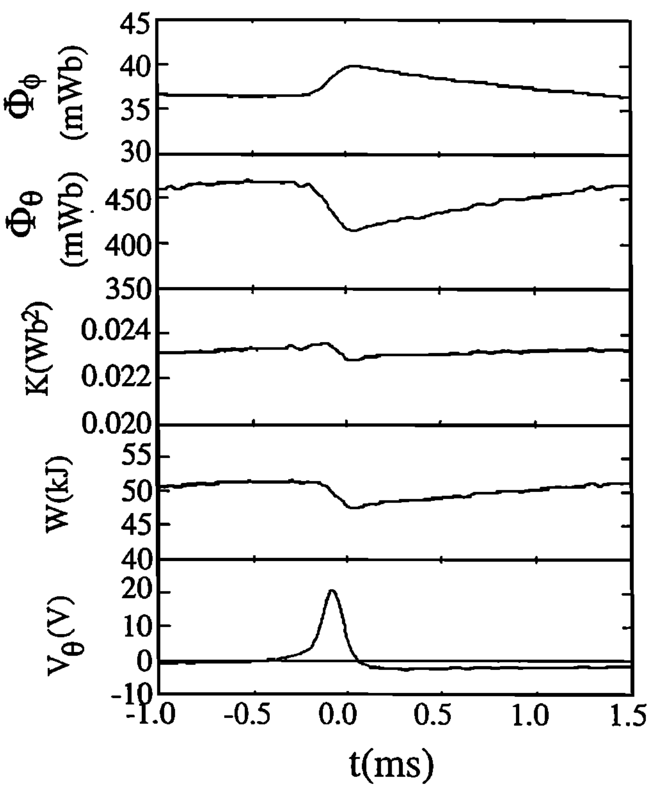

Fig. 4. Time evolution of toroidal and poloidal fluxes, the magnetic helicity K, magnetic energy W, one turn poloidal voltage V_θ during the sawtooth crash in RFP[*Ji etal,* 1995].

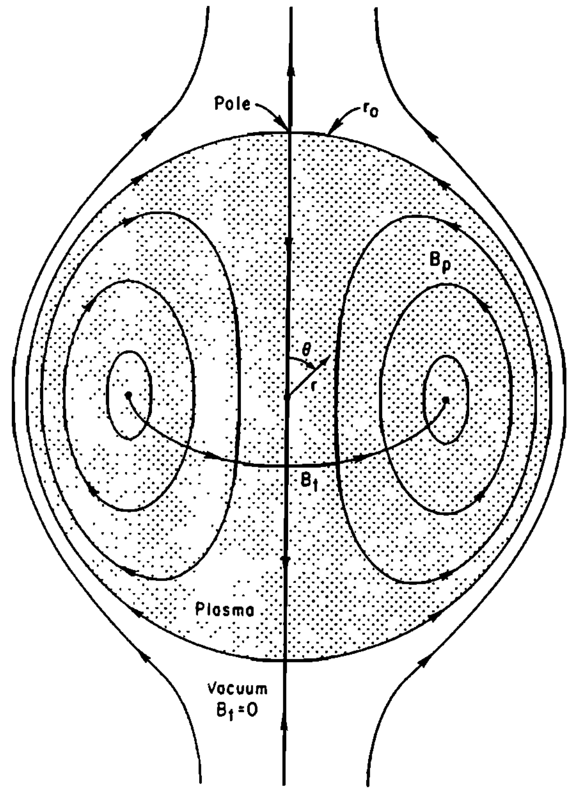

FIG. 5. Classic spheromak configuration.

The magnetic configuration of a spheromak can easily be derived by using a scaler function χ which satisfies Eq.(4),

$$\nabla^2\chi + \mu^2\chi = 0 \qquad (6)$$

Where ∇^2 is Laplacian operator. In a spherical container of radius a, the lowest eigenvalue is given by μa= 4.49, and the corresponding eigenfunction is obtained in spherical coordinates [ρ,θ,ϕ];

$$B_r = 2B_o\,[j_1(\mu\rho)/\mu\rho]\cos\theta, \qquad (6a)$$

$$B_\phi = B_o\,[j_1(\mu\rho)/\mu\rho]\sin\theta, \qquad (6b)$$

$$B_\theta = -B_o\, d/d\rho[j_1(\mu\rho)/\mu\rho]\sin\theta. \qquad (6c)$$

Where $j_1(x)$ is a spherical Bessel function defined by $j_1(x) = J_{3/2}(x) / x^{1/2}$ [*Rosenbluth and Bussac,*1979]. The flux contours of this configuration are shown in Fig. 5. The magnetic field has nested toroidal surfaces just like a toroidal pinch such as a tokamak or RFP, but the boundary is spherical instead of toroidal. The essential distinguishing feature of a spheromak is that the boundary is singly connected and there is no central conductor coils to create an external toroidal field. Therefore the toroidal

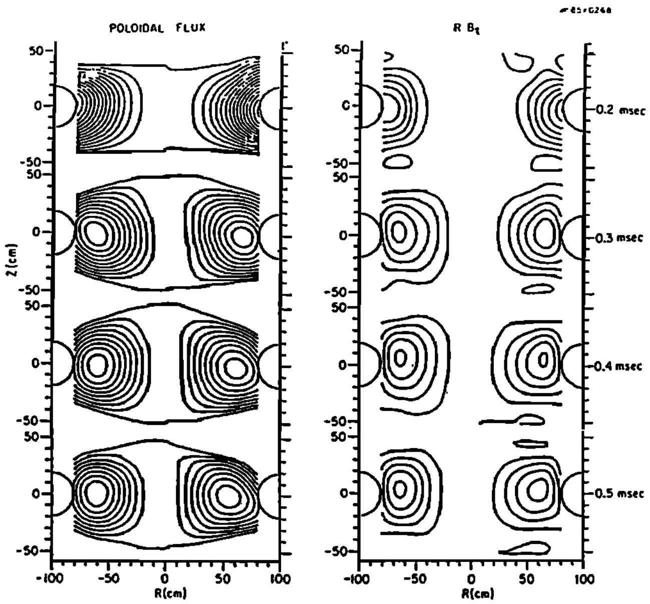

Fig. 6. Time evolution of poloidal flux and poloidal current contour plots taken 0.2, 0.3, 0.4, and 0.5 msec after the start of the S-1 discharge[*Yamada et al.,* 1985]. An excellent match of the poloidal flux ψ with the poloidal current RB_t implies the Taylor state in spheromak equilibrium. The separatrix is of the spheromak is shown by the outmost ψ contours.

field is zero everywhere on the boundary wall. It should be noted that the boundary does not have to be exactly spherical but can be oblate or prolate and even the cylindrical boundary can confine a spheromak as long as the boundary is singly connected. In an extreme case the force free minimum energy state in a doubly connected toroidal boundary, often used in describing a solar flare, can be regarded as one type of sphermak.

4-2. Spheromak Formation Process; Governed by the Taylor Principle

There are many ways of forming spheromak plasmas; plasma gun formation [*Jarboe* et al. 1980], a mixture of linear and azimuthal discharges in the theta-pinch system [*Goldenbaum* et al., 1980], and inductive formation utilizing a flux core [*Yamada* et al., 1981, 1985]. In all formation schemes, both the poloidal and toroidal currents are induced either by electrodes or electromagnetic induction. Although each formation scheme is not elaborately described in this paper, there is a remarkable and important common feature in these formation schemes. After an initial highly turbulent state, the plasma settles into a more quiescent stable state in which the fluctuations are reduced. The forming plasma approaches the minimum-energy state theorized by Taylor and a spheromak configuration is created. The final plasma state is insensitive to initial conditions or details of the formation process so long as the geometry is not changed. Sometimes, an additional external field is supplied to support the final plasma equilibrium.

Let us take an example of S-1 Sphermak discharge[*Yamada*, 1985]. In the S-1 inductive formation scheme, the formation can be considered in terms of a slow injection of magnetic helicity into the plasma from a induction coil called a flux core[*Yamada* et al. 1981] which injects both toroidal and poloidal flux into the plasma. When this toroidal shaped solenoid is energized it induces poloidal and toroidal current around the ring. If the plasma is pushed inward toward the major axis by increasing (compressing) external field, the plasma pinches off from the flux core and spheromak is formed through reconnection and relaxation into a Taylor state. Figure 6 presents a time evolution of poloidal flux and poloidal current contour plots taken 0.2, 0.3, 0.4, and 0.5 msec after the start of the S-1 discharge. An excellent match of the poloidal flux ψ with the poloidal current RB_t implies the Taylor state in spheromak equilibrium.

In S-1 the electron temperature T_e ranges from 10 to 130 eV depending on plasma current and fill pressure. The electron density n_e reaches peak values of (0.2 to 1.5) $\times\ 10^{14}\ cm^{-3}$ during the formation phase and decays thereafter with a time constant of about 1 msec. The volume-averaged beta, β, is in the range of 5-10%.

During the spheromak formation large amplitude, globally coherent modes are observed [*Janos*, 1986] in coincidence with the flux conversion phenomena, suggesting that the modes play a vital role in the relaxation process by providing a means for flux conversion. These modes are low n-number, m = 1 modes where n and m are defined by the functional dependence $\exp[i(n\phi + m\theta)]$ of the fluctuations on toroidal angle ϕ and poloidal angle θ. During formation, peak amplitudes of the low n modes relative to the unperturbed field have been observed as high as 5- 20%.

4. 3 Verification of a Taylor State in Spheromaks

All magnetic configurations in a stable equilibrium have to satisfy the static force balance of **J** x **B** = ∇p described by the well known Grad-Shafranov equation[*Wesson*, 1985]. Solving the axisymmetric Grad-Shafranov equation, it can be shown that

$$\mu = \mathbf{J}\cdot\mathbf{B}/B^2 = I'(\Psi) + p'(\Psi)\ I(\Psi) / B^2 \qquad (7)$$

where $I(\Psi) = RB_t$ is the poloidal current stream function; R is radial distance from the major axis and B_t is the local toroidal field, $p(\Psi)$ is the plasma pressure, and the prime indicates differentiation with respect to Ψ. In order for μ to be a spatial constant, the second term must vanish: [*Hart* et al., 1986]. Since its numerator is a function of Ψ only, and the B^2 in the denominator is not in general a function of Ψ only, the second term will not even be constant on a flux surface. Therefore, it cannot be constant in the whole plasma volume as required by the Taylor state. Thus the constancy of Ψ requires that p'(Ψ) = 0. The equation remaining after dropping the p' term requires that I is a linear function of Ψ in order that I'(Ψ) is constant. The slope of I vs. Ψ is equal to the eigenvalue μ. For a given plasma shape, the eigenvalue is inversely proportional to the midplane separatrix radius.

Of fundamental importance to understanding the physics of spheromak equilibrium is the concept of the Taylor minimum energy state. The best way to determine how close this relaxed plasma is to the Taylor state is to measure the μ profile, where μ is defined by $\mu = \mathbf{J}\cdot\mathbf{B}/B^2$. If μ is constant in space, then the plasma is in the Taylor state. Profiles of the μ parameter were obtained by measuring the local B inside the plasma with magnetic probes[*Hart* et al, (1986)]. In order to deduce μ, the current density must be calculated. To measure the internal magnetic structure of the plasma, a one-dimensional probe array containing coils measuring all three components of the magnetic field was scanned. This facilitates taking the curl of the fields to calculate the current density. The calculation assumes the symmetry axis of the plasma is unperturbed. The profiles of μ(R) in the midplane are shown w.r.t. major radius R for two differenttimes during the formation in Fig. 7a. Fig. 7b presents a two-dimensional contour plot of μ across the plasma cross section. At 0.2 msec, there is a deficit of current density near the symmetry axis because the driving electric fields are strongest near the plasma forming flux core. By 0.5 msec into the discharge, the current has diffused into the center to produce a flat μ profile. The variation of the t = 0.5 msec curve from the expected value is within the error bars of the measurement. Because of the limitations of the numerical differentiation, the error for μ is about 15 %. At early times, the μ profile peaks near the flux core and has a current hole near the symmetry axis. At 0.5 msec, the profile is basically flat throughout the plasma volume, with the gradients all occurring near the separatrix. This is in good agreement with the measured radius from the flux plot.

4.4 Helicity Injection and Sustainment of Spheromak Plasma by Coaxial Gun

The formation of spheromaks occurs on a wide time scale, generally longer than t_A ($t_{form} \geq 10^2\ \tau_A$), where τ_A is the Alfven transit time; therefore, it is called a slow formation. As in the case of the RFP, spheromak formation can be considered in terms of an injection of

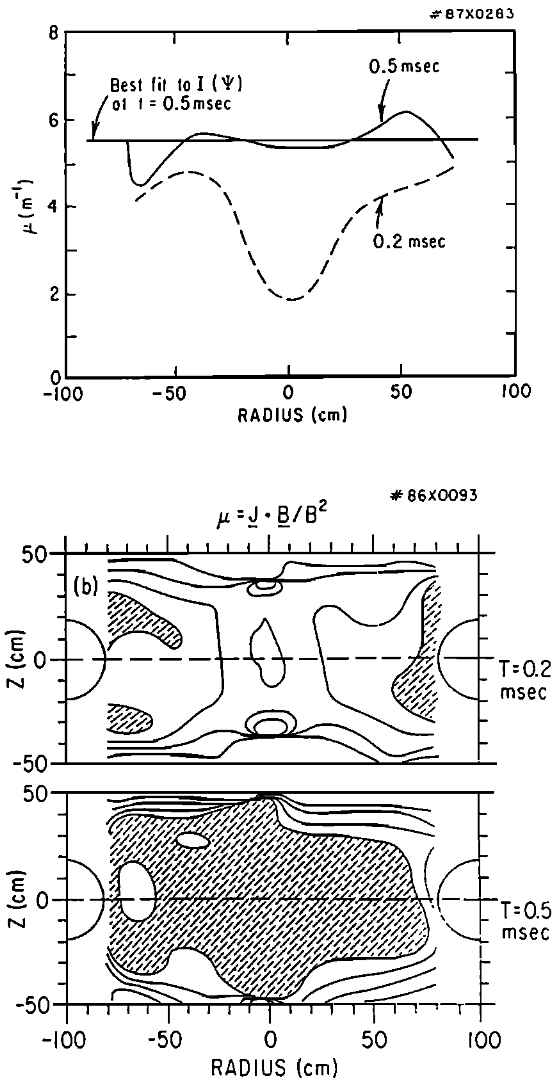

Fig. 7. 2-D contour plots, during (0.2 msec) and after (0.5 msec) formation, of μ = **j**· **B**/B2. The shaded area is the region in which μ takes the nominal value of 5.5 m -1. Fig. 7b. μ profile versus major radius in the midplane[*Hart* et al., 1986].

magnetic helicity into the plasma from electrodes or induction coils. The CTX group at Los Alamos developed a magnetized, coaxial plasma gun to form and sustain spheromaks. The characteristics of the coaxial plasma gun plasma are well documented to optimize the efficiency of this formation[*Barnes* et al., 1986, *Wysocki* et al, 1988]. The gun consists of an inner electrode which is magnetized by an external coil as shown in Fig. 8 [*Jarboe* et al., 1980]. The initial poloidal flux links the inner to the outer electrode. Gas is puffed into the annular gap and high voltage is applied between the electrodes breaking down the gas. Current flows from the inner to the outer

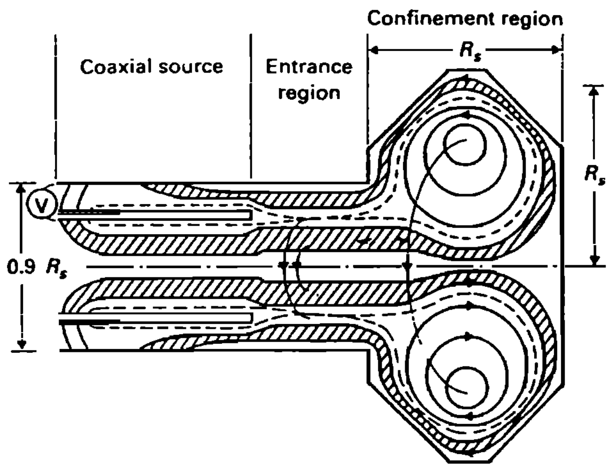

Fig. 8. Sustained configuration of CTX experiment[Jarboe et al. 1985]

electrode generating toroidal magnetic flux which encircles the inner electrode. If the toroidal field pressure force is sufficient then the plasma distends the gun flux and a spheromak is formed with the initial poloidal gun flux ψ_{gun} trapped in the plasma. During formation, helicity is injected at a rate $dK/dt = 2\ \psi_{gun}(d\Phi/dt)$. The voltage applied between the inner and outer electrode determines the rate at which toroidal flux is injected from the gun, $d\Phi/dt = V_{gun}$.

The helicity in the final spheromak has been compared to the injected helicity produced by a wide range of voltages on the poloidal- and toroidal field capacitor banks(or power supply). The global magnetic helicity in the plasma after formation was found to be proportional to the helicity injected by the gun, regardless of how that helicity was produced by the gun. Fig. 9 depicts the value of magnetic helicity measured in the plasma at 50 μsec after the initiation of the discharge versus the helcity injection from the gun based on the relationship shown above. This close relationship confirms conservation of magnetic helicity during the plasma formation.

Relaxation is also an essential requirement for this coaxial gun formation scheme. As the spheromak relaxes to the Taylor state, some of the injected toroidal flux is often converted to poloidal flux. The final plasma state is insensitive to the ratio of the poloidal to toroidal flux or to the details of the formation process so long as the geometry is not changed. Helicity balance between gun and spheromak has been verified in detail by the CTX group [*Barnes* et al. ,1986, 1990]. The efficiency of this process depends on the relative sizes of the spheromak and the gun, helicity injector. It has been shown that energy coupling efficiency approaches unity if the gun and spheromak are of comparable size.

Spheromaks can be sustained near the Taylor state by continuously injecting helicity and energy through magnetic reconnection at a rate that balances helicity and energy dissipation. Steady-state spheromak sustainment (current drive) was attained by applying a sufficient voltage across the gun terminals while maintaining the linked gun flux[*Jarboe* et al., 1983; *Browning*, et al., 1992]. During the gun-formation process of spheromak, the gun flux must detach from the gun by magnetic reconnection in order to form the spheromak. In a similar way, spheromak sustainment requires continuous reconnection processes as the poloidal flux is replenished.

4.5 Plasma Relaxation During the Decay Phase Spheromak Plasmas

When a spheromak discharge is optimized its central temperature rises due to better confinement without radiation losses. With an enhanced pressure gradient the magnetic activity during the decay phase is observed to increase and the relaxation oscillations become pronounced. High T_e operation often allowed us to obtain a better picture of sawtooth-type relaxation events [*Ono* et al., 1988].

Figure 10a shows a time evolution of T_e profile measured in the S-1 spheromak by a multipoint Thomson scattering system. After the formation of the spheromak, the T_e profile gradually changes to a peaked profile up to 360 μsec. The maximum peak temperature (T_e = 80 eV) is obtained at 360 μsec when the peaking of T_e becomes maximum; afterwards, a large drop in T_e is observed and the T_e profile changes from peaked to broad. Simultaneously the measured poloidal and toroidal magnetic field profiles gradually become more peaked as

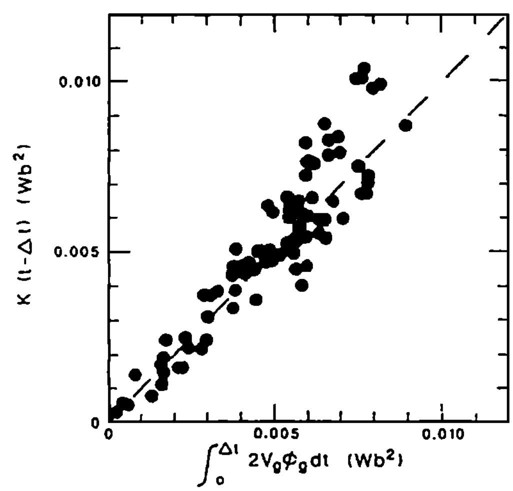

Fig. 9. Helicity conservation in CTX. Initial increase of helicity content of the plasma for δt= 50 μsec versus helicity injected from the gun[*Barnes*, et al., 1986].

time increases to 360 μsec, after which some redistribution restores a broad profile. The ratio of the poloidal flux Ψ to the toroidal flux Φ, relative flux inventory Ψ'/Ψ - Φ'/Φ, and q_O, the safety factor at the magnetic axis, are shown in Fig. 10b with respect to time during the relaxation period. The safety factor q represents the winding number of the field lines on the toroidal flux surface[*Wesson*, 1985] Initially, at about 300 μsec, Ψ/Φ is about 2.7 and the q value at the magnetic axis is 0.5, which are the expected values for the Taylor state. In the peaking phase, from 300 μsec to 360 μsec, Ψ/Φ increases to 4 and q_O decreases to 0.35 which indicates that the spheromak becomes poloidal-flux rich and therefore deviates from the Taylor state. The Ψ'/Ψ - Φ'/Φ curve shows a preferential decay of Φ over Ψ until 360 μsec after which the relaxation occurs. Once the spheromak begins to relax from 360 μsec, Ψ/Φ drops to 3.0 and q_O value comes back to 0.5. The Ψ'/Ψ - Φ'/Φ curve reverses its sign during relaxation from 360 μsec to 390 μsec, which suggests flux conversion from poloidal flux to toroidal flux (ΔΨ < 0, ΔΦ > 0). The flux conversion brings the spheromak from a non-Taylor state with excessive poloidal flux to the Taylor state[*Hart et al,* 1986].

According to the result of toroidal mode analysis using external pick-up coils, the n = 2, m = 1 mode starts growing just before the onset of the relaxation and becomes dominant during the relaxation. This suggests that the n = 2, m = 1 kink mode plays an important role in the flux conversion as conceived in an MHD simulation [*Knox* et al., 1986].

5. EVOLUTION OF MAGNETIC FLUXES AND HELICITIES DURING PLASMA MERGING

Merging of two toroidal plasmas [*Katsurai and Yamada*, 1982] can provide fundamental data on the behavior of magnetic helicity during magnetic reconnection or topology changes of plasma configuration. In this set-up, the evolution of both global and local magnetic helicities can be studied to investigate helicity and dissipation during merging. Several experiments have been carried out in the past decade on this concept [*Ono* et al, 1993; *Yamada* et al., 1990, 1997; *Kornack et al,* 1998]. It is very important to know how three dimensional relaxation effects affect the helicity evolution. In some cases the plasma kinetic energy increases dramatically at the expense of magnetic energy and conversion of kinetic helicity to magnetic helicity and vice versa occur. In the TS-3 device at the University Tokyo, it was found that the helicity is conserved much better than the magnetic energy during plasma merging [*Ono* et al, 1993].

Let us consider how the magnetic fluxes and helicities evolve during plasma merging. We consider the co-

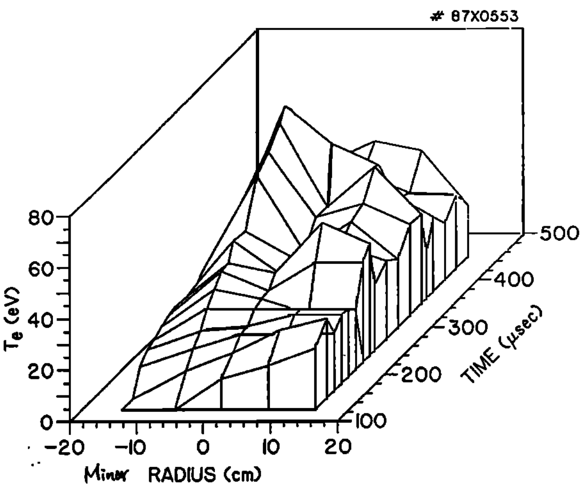

Fig. 10a. Time evolution Te (r) profile during sawtooth relaxation of S-1 Spheromak.

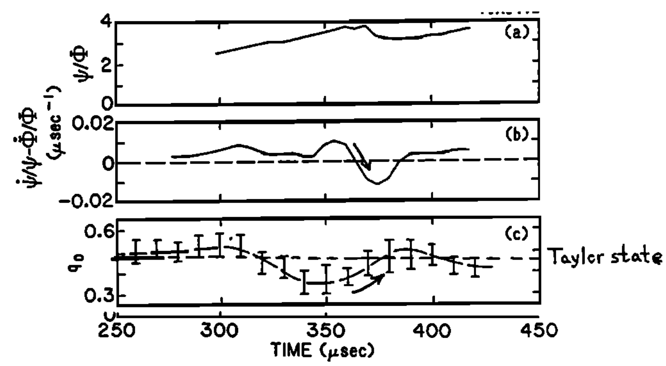

Fig. 10 b. Time evolution of Ψ/ϕ , Ψ'/Ψ - ϕ'/ϕ, and the q value* at the magnetic axis. Ψ/ϕ increases and q_O decreases to 0.35 which indicates that the spheromak becomes poloidal-flux rich and therefore deviates from the Taylor state. The curve shows a preferential decay of ϕ over Ψ until 360 μsec after which the relaxation occurs.

helicity merging case in which two toroidal plasmas with equal helicity K_0, poloidal flux ψ_0, and toroidal flux Φ_0, are merging to each other, as shown in Fig.11. Since it is expected that the toroidal fluxes and heliciites add while the poloidal fluxes do not, the magnetic fluxes after the merging(ψ_m, Φ_m) and helicity can be expressed utilizing the relatinship derived in Eq. (2a),

$$\Psi_m = \Psi_o, \quad (8a)$$

$$\Phi_m = 2\Phi_o , \quad (8b)$$

$$K_1 = 2c_o \psi_o \Phi_o , \quad (8c)$$

$$K_m = 2c_m \psi_m \Phi_m . \quad (8d)$$

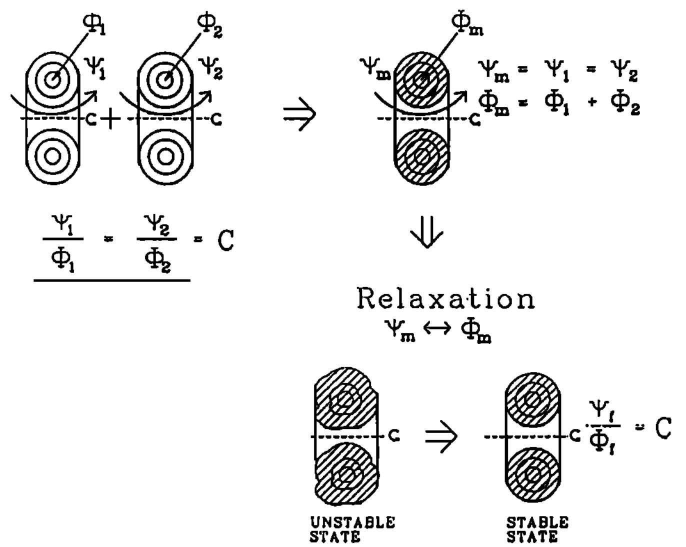

Fig. 11. Merging process of two identical spheromak plasmas

where (c_0 & c_m are the numerical coefficient (~1) based on current profiles of the plasmas before and after merging) and both plasmas are assumed in the minimum energy state(ψ_0/Φ_0= const.= α) before merging. If the same current profile is recovered after merging($c_m = c_0$), the total magnetic helicity is expected to be conserved by the magnetic reconnection process as shown by

$$K_m = K_0+K_0 = 2K_0 \quad (8e)$$

However, if Taylor's conjecture is applicable, the plasma should relax into the minimum energy state in which $\psi_m/\Phi_m = \psi_0/\Phi_0 = \alpha$. Thus, a flux conversion has to occur to create a final stable state(ψ_f,Φ_f) by reducing toroidal flux from $\Phi_m = 2\Phi_0$ to $\Phi_f = \sqrt{2}\ \Phi_0$, and increasing poloidal flux from $\psi_m = \psi_0$ to $\psi_f = \sqrt{2}\ \psi_0$. Whether this relaxation process occurs or not is an important question. Although detailed study has not been carried out yet, recent data from TS-3 and MRX suggest that this relaxation process into a new Taylor state is occurring during co-helicity merging. As in other spheromak formation schemes, this process invokes flux conversion between the toroidal and poloidal fluxes during and after the merging of the plasmas.

6. IMPACT OF EXPERIMENTAL DATA ON SOLAR FLARE RESEARCH

What impact will laboratory research have on our understanding of phenomena where magnetic helicity is crucial? Solar flares are the best-known example of rapid conversion of magnetic energy into heat [*Shibata et al.*, 1995; *Tsuneta*, 1996]. The above mentioned laboratory experiments have provided important data for systems where magnetic helicity is stored in a force-free equilibrium configuration and then via slow adjustment of an external parameter (such as in the current in external coils), brought to a situation of ideal-MHD instability which forces a topology change. However, it is not clear whether the total magnetic helicity, without a well defined (conductor shell) boundary with **B◊n** = 0, would be conserved or not during evolution of solar flares and their reconnection process.

Recent theoretical work has focused on two-dimensional models of the evolution of force-freemagnetic arcades [*Low*, 1987] whose field-line foot points are advected by flows in the solar photosphere. It gave an example of an evolution which produces a situation analogous to the antiparallel-toroidal-field double spheromak merging. Hence the merging or reconnection experiments bear directly on the physics of solar flares. One can measure how and at what rate magnetic helicity changes and magnetic energy is converted into heat.

Vector magnetogragh measurements of the solar surface show that fields in active region generally exhibit positive helicity in the south hemisphere and negative helicity in the north. D. Rust[1994] considered that helical magnetic field structures are in a force-freestate and interaction or merging of helical ropes can result in eruptive events and ejection of magnetic flux and energy. This event is analogous to the spheromak merging experiment mentioned in the previous section. It would be very important to find out how helicity evolves during the merging processes. A detailed study of evolution of the magnetic helicity and energy, of flux conversion processes, plasma flow and heating during the merging of two toroidal plasma could make an important impact on the analysis of solar flare interaction.

In general toroidal pinch plasmas can provide physics data for the role of helicity in energy conversion to directed flows, the role of turbulence in thermalizing directed-flow energies, and the role of turbulence in changing the reconnection region from the x-point-line of single two-dimensional (2-D) pictures to volume-filling processes. The spatial extent, intensity, and scale of departures from 2-D symmetry can be measured.

7. SUMMARY

It was found that relaxation to the Taylor (force-free, minimum energy) state was an important mechanism for the formation processes of RFP and spheromak plasmas [Taylor 1986]. There is a remarkable feature common to all of these toroidal pinch discharges. It was found that, after an initial highly turbulent state, the plasma settled into a more quiescent stable state in which the fluctuations were reduced. The concept of magnetic helicity has been proven to be very effectivein describing magnetic characteristics of these pasmas. Taylor's conjecture, conservation of magnetic helicity, $K \equiv \int \mathbf{A}\cdot\mathbf{B}dV$, has been verified during the relaxation of laboratory plasmas, in which the plasmas rapidly approach a minimum-energy state. There has been an evidence that the magnetic energy dissipates much faster than the helicity during plasma relaxation.

In some cases two-dimensional spatial profiles of $\mu = \mathbf{j}\cdot\mathbf{B}/B^2$, where $\nabla \times \mathbf{B} = \mu\mathbf{B}$ for a force-freestate, were deduced from internal probe data to verify the Taylor state (with μ = constant). Another important finding was the identification of flux conversion between the toroidal and poloidal fluxes of the spheromak during and after the formation of the plasma. The correlation between the MHD mode behavior and the Taylor relaxation process has been described.

In the high temperature plasmas, a "sawtooth-type" relaxation cycle was observed both in the magnetic configuration and electron temperature profile. It was found that the confinement characteristics of the spheromak were significantly affected by the relaxation phenomena.

When the magnetic configuration departs from the Taylor state in a RFP or a spheromak, it easily becomes unstable to low n (n = toroidal mode number) ideal MHD modes. Through these instabilities, magnetic reconnection takes place (relaxation) and a Taylor state is recovered. It is found that the confinement time of these plasmas is significantly affected by this relaxation phenomenon.

Conservation of magnetic helicity has been verified during relaxation in many laboratory pinch plasmas in which the Taylor minimum energy states are identified after the relaxation events. Recently merging spheromaks are utilized to investigate a magnetic reconnection of magnetized toroidal plasmas or helical flux ropes. Experiments with fully 3 dimensional magnetic reconnection are now possible in the MRX device where evolution of magnetic helicity can be studied.

In recent years, theoretical work for solar flares [*Parker*(1973), *Priest*(1984), *Low*(1992] has shown that a flux-conserving sequence of ideal-MHD equilibria must develop singular current sheets. Laboratory experiments have been investigating how theoretical singular current sheets are manifested in an often turbulent laboratory plasmas, as well as the relation between global forcing, the extent of current sheets, and the reconnection rate. To date an important finding in the current carrying laboratory plasma has been a conservation of magnetic helicity during its relaxation. A detailed experiment-theory-computation comparison will test our ability to describe nontrivial reconnection configurations with the same codes used to simulate MHD processes in magnetospheric and solar physics. Close collaborations of laboratory experiments with solar and magnetospheric research would bring a better understanding of essential mechanism of relaxation phenomena in plasmas.

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