Topic: A1

3D Numerical Simulations of the Dynamics of a Flux Core Spheromak

P. L. García Martínez, R. Farengo
Centro Atómico Bariloche e Instituto Balseiro, 8400 Bariloche, RN, Argentina
E-mail:pablogm@cab.cnea.gov.ar

The formation and sustainment of a spheromak by d. c. helicity injection has been demonstrated in several devices [1-3]. In all these experiments a central core of magnetic flux that links the electrodes is needed during the formation and sustainment phases to inject magnetic helicity. The configurations produced are therefore flux core spheromaks (FCS).

An experiment has been proposed to employ basically the same technique (i. e. d. c. helicity injection) for the formation and sustainment of a very low aspect ratio tokamak like configuration [4]. The device will combine a sophisticated electrode design with a set of poloidal field coils for radial equilibrium (vertical field) and plasma shaping. If successful, it would result in the formation of a spherical tokamak (ST) [5] type configuration with two important advantages over more conventional formation/sustainmente methods. The first one is that the ST could be formed and sustained by helicity injection and the second one is that the central post would be replaced by a Z-pinch. Formation/sustainment and the central post are probably the main problems for the development of attractive fusion reactors based on the ST concept and the study of alternatives to solve these problems is clearly of interest.

Since the formation and sustainment of the desired configuration relies on plasma relaxation studying the dynamics of relaxation and the final state towards which the plasma evolves is of extreme importance. Relaxation and poloidal flux amplification have been associated with the presence of an n=1 kink instability in the central column that flattens the initially hollow current profile. Most spheromak experiments work at small elongation (κ~1) to avoid the dangerous tilt instability. However, STs are naturally elongated ($\kappa \geq 2$) and this results in relatively high values of the edge safety factor (q_{edge}) in spite of the relatively small values of the toroidal field, which are needed to obtain a high β. The effect of elongation, and the excitation of both kink and tilt modes, on relaxation and flux amplification has not been studied in the past. The effect of the external, open, flux on the evolution of the tilt mode has not been studied either.

The PROTO-SPHERA (PS) [4] has large annular electrodes and poloidal field coils that provide radial equilibrium and control of the plasma shape. An energy principle calculation that includes the correct shape and the contribution of the vacuum magnetic energy indicates that the confuguration should be stable against the tilt mode at the proposed elongation (κ=2.3). However, the dynamics of the relaxation has not been studied. Here we study the relaxation of a flux core spheromak inside a cylindrical vessel using 3D numerical simulations. The geometry considered is a simplified version of that proposed for the PS and differs from the one used in most spheromak experiments (plasma gun).

The force-free FCS equilibria that we employ as initial condition for the simulations are characterized by their λ profile (λ=**J.B**/B^2) and elongation and by the amount of open flux through the electrodes (ψ_e). We vary these quantities and determine the evolution of the magnetic energy content of different modes, the poloidal flux, the magnetic helicity and the λ profile. Our results show that the slope of the λ profile determines the rate and extent of the relaxation induced by the kink mode and that, for the boundary condition employed, the addition of open flux changes the dynamics of the tilt mode, which disappears after some time, at least for elongations up to 1.9.

Initial condition

The configuration considered, shown in Fig. 1, is a simplified version of the one proposed for the PS experiment. It consists of a cylindrical flux conserver and two electrodes. The top and bottom of the flux conserver are rings of inner radius b and outer radius a and the side is a cylindrical shell of radius a and height h. The electrodes are circular, with radius b, and the entire configuration is axisymmetric ($\partial/\partial\theta$=0). Finally, we assume that a set of external coils, not shown in the figure, produce an axial magnetic field ($B_{ext}\ \hat{e}_z$) on both electrodes that remains fixed.

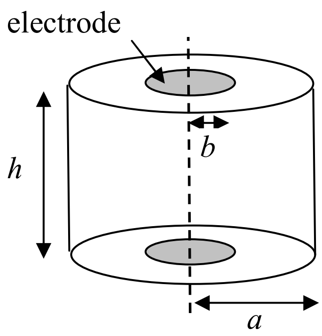

Fig. 1: Geometry considered

.

The initial condition for our simulations is a force-free FCS equilibrium obtained by solving the equation:

$$\nabla \times \mathbf{B} = \lambda(\psi)\mathbf{B}\,, \tag{1}$$

with $\lambda(\psi) = \bar{\lambda}\left[1+\alpha(2\psi-1)\right]$, where ψ is the poloidal flux, $\bar{\lambda}$ is the average value of λ and α is the slope, whose value determines if the current profile is hollow (negative α) or peaked (positive α). When α=0, λ is uniform (Taylor state).

MHD code and boundary conditions

The equilibrium indicated above is used as initial condition for the Versatile Advection Code (VAC) [6] which solves the resistive, isothermal, 3D MHD equations using a shock-capturing scheme based on a Roe-type Riemann solver and a Woodward limiter. The divergence-free condition ($\nabla\cdot$**B**=0) is maintained using the projection method. To simplify the physics we do not advance the density. This corresponds to a constant pressure computation, usually referred to as the zero-β (or zero-pressure) approximation, widely used when modelling low-β plasmas.

All length are normalized with the plasma radius and the time with the Alfven time, defined as $\tau_A=a/v_A$. The resistive time is $\tau_r=\mu_0/(\eta\lambda^2)$ and the Lundquist number $S=\tau_r / \tau_A$ is approximatelly 400. A uniform cartesian grid is used, with $N_x\times N_y\times N_z=100\times100\times75$. The cylindrical flux conserver is constructed using appropriate values at ghost cells, i.e. knowing the solution inside the flux conserver ($r<a$) we set the values of external grid points ($r>a$) in such a way that the boundary conditions are satisfied at $r=a$. The perfectly conducting wall conditions employed are: $\mathbf{B}\cdot\hat{n}=0$ and $\mathbf{J}\times\hat{n}=0$. At the electrodes ($r<b$, $z=0$ and $r<b$, $z=h$) the normal component of the magnetic field is kept constant. Since we force the tangential current to be zero at the wall, $\lambda(a)=0$ and even the simulations initiated with $\alpha=0$ (uniform λ) develop small gradients close to $r=0$ and $r=a$. The Fourier analysis in the toroidal direction is performed after the equations have been evolved in the 3D cartesian grid.

The code was tested by recovering existing results. For example, initializing the simulations with regular spheromaks (uniform λ profile, no open flux through electrodes) of different elongations the $n=1$ tilt mode appeared when κ was increased from 1.6 to 1.7, in excellent with previous results [7]. This is shown in Fig. 2, where we ploted the ratio of the energy in the $n=1$ mode to the total energy. It can be seen that for $\kappa=1.6$ ($a=1$) the energy in the $n=1$ mode remains negligible at all times. When $\kappa=1.7$ a slow growth of $W_{n=1}$ is observed and when $\kappa=1.8$, $W_{n=1}$ grows rapidly until all the energy is on this mode.

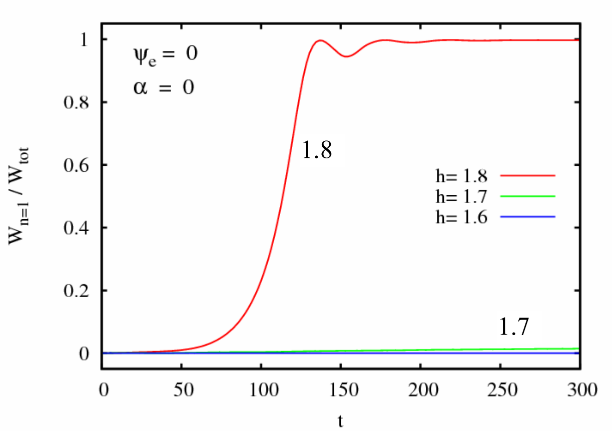

Fig. 2. Tilt mode in a regular spheromak

Results

No open flux, low elongation

In this case the only relevant parameter is the slope of the λ profile (α). The initial condition is a regular spheromak inside a perfect flux conserver ($\mathbf{B}\cdot\hat{n}=0$ everywhere). The relaxation of the non uniform λ profile is produced by an $n=1$ kink mode. Fig. 3 shows the evolution of the ratio $W_{n=1}/W_{n=0}$ for different values of α. It is clear that the amount of

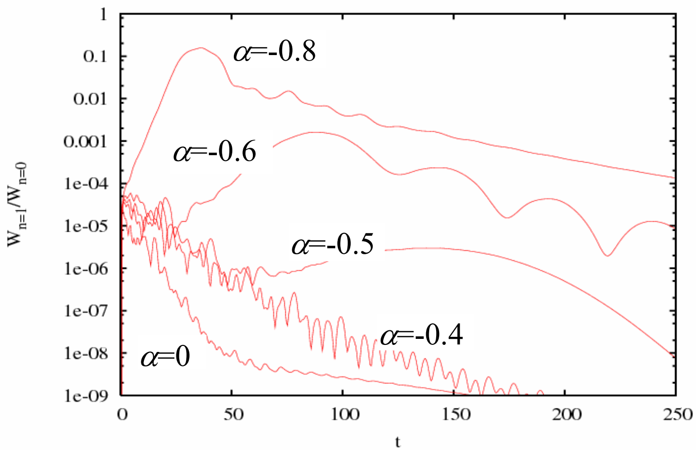

Fig. 3. Evolution of $W_{n=1}/W_{n=0}$ for different values of α.

energy in the n=1 mode increases with α. Fig. 4. shows the evolution of the ratio $W_{n=0}/K_{n=0}$ for different values of α for the same conditions as in Fig. 3. It can be seen that the relaxation rate increases with α and that for α=-0.6 and α=-0.7 the final state has a higher value of W/K than the one corresponding to α=0 (fully relaxed state) while the case with α=-0.8 achieves the same value of W/K as the case with α=0. This is further confirmed by the final λ profiles (not shown), which become more uniform as α increases. These results seem to indicate that complete relaxation, as indicated by a uniform λ in the final state, is only achieved if α is high enough.

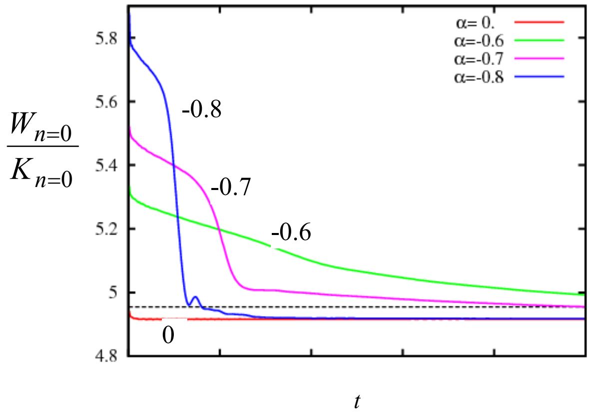

Fig. 4.Evolution of $W_{n=1}/K_{n=0}$ for different values of α

No open flux, higher elongation

When the initial λ profile is not uniform ($\alpha\neq 0$) and the elongation is increased above κ=1.66 a kink mode first appear, followed later by a tilt mode. This is shown in Fig. 5, which presents a plot of the ratio $W_{n=1}/W_{tot}$ as a function of time for different elongations and α=-0.8. The first peak corresponds to the kink and coincides for the three cases with elongation greater than 1, occuring somewhat later for the case with κ=1. The second peak (tilt) appears at approximately the same time for the three cases with κ>1 but the subsequent evolution is very different, depending on the value of κ. When κ=1.6 the energy in the n=1 mode begins to decrease until it becomes negligible. When κ=1.7 the energy in the n=1 mode increases very slowly, indicating a situation of almost marginal stability with respect to the tilt mode. Finally, when κ=1.8 the energy in the n=1 mode has a small oscillation and then increases until all the magnetic energy is stored in this mode. When κ=1 the energy in the n=1 mode becomes negligible after the initial kink.

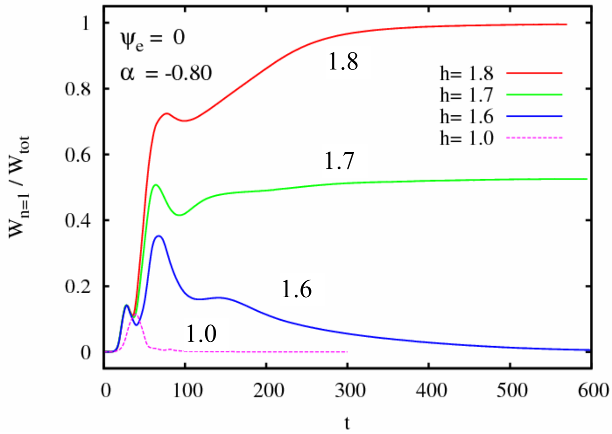

Fig. 5. Evolution of $W_{n=1}/W_{n=0}$ for different elongations

The behavior described in the preceding paragraph can be clearly observed in the sequence of vector plots shown in Figs. 6, where the vectors indicate the poloidal magnetic field.

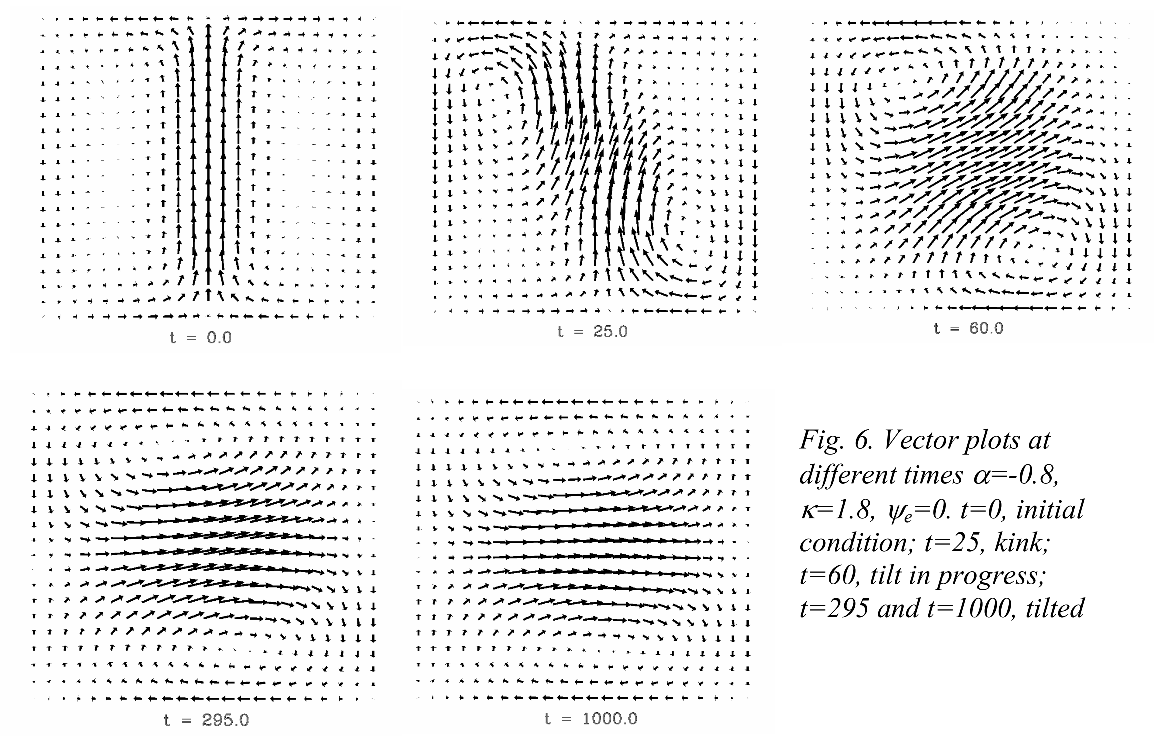

Fig. 6. Vector plots at different times α=-0.8, κ=1.8, ψ_e=0. t=0, initial condition; t=25, kink; t=60, tilt in progress; t=295 and t=1000, tilted

Open flux, high elongation

When a finite amount of open flux, quantified as a percentage of the initial poloidal flux, is added the dynamics of the tilt mode changes completely. Fig. 7 shows the evolution of $W_{n=1}/W_{n=0}$ for α=-0.8, ψ_e=10% and different elongations. The kink mode appears earlier but has basically the same behavior as in the case without flux (note that the scale in Figs. 5 and 7 is different). The initial evolution of the tilt is also similar but it saturates at a smaller amplitude. The long term evolution for κ≥1.7 is completely different. For κ.≥1.7 the energy in the n=1 mode increases, reaches a maximum, decreases abruptly, has a second (smaller) peak and finally becomes negligible. Although the qualitative behavior is the same for κ=1.7; 1.8 and 1.9, the maximum amplitude and the time elapsed before the sharp reduction increases with κ. Figure 8 shows the evolution of higher order modes for κ=1.8. It is intresting to note that during the time the energy in the n=1 mode remains almost constant (100<t<300) the energy in the other modes increases.

Fig. 9 shows the evolution of the poloidal and toroidal fluxes for κ=1.8. The kink produces a rapid increase in the poloidal flux and a rapid decrease in the toroidal flux. During the tilt phase both fluxes decay resistively and both increase when the tilt disappears. To conclude this section, we show in Fig. 10, the same sequence of vector plots as in Fig. 6 for the case with open flux. Although at t=295 the configuration is tilted at approximately 45 degrees at t=560 the tilt has disapeared.

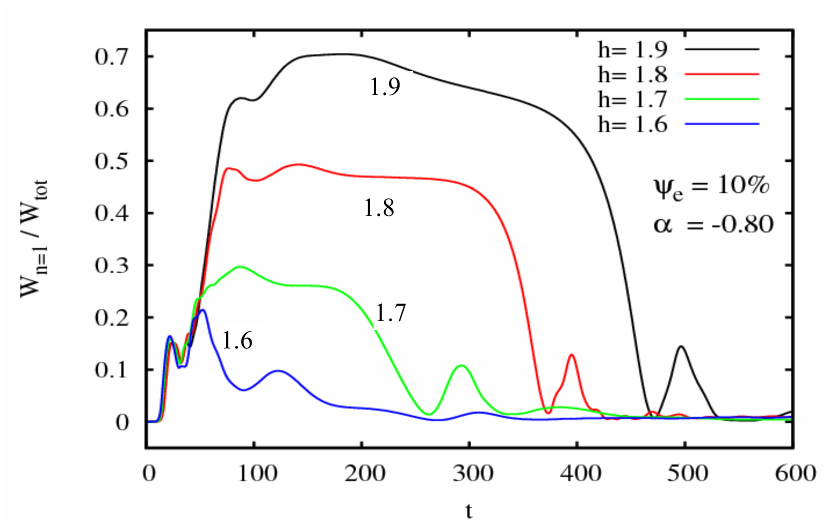

Fig. 7. Evolution of the energyin the n=1 mode for different elongations

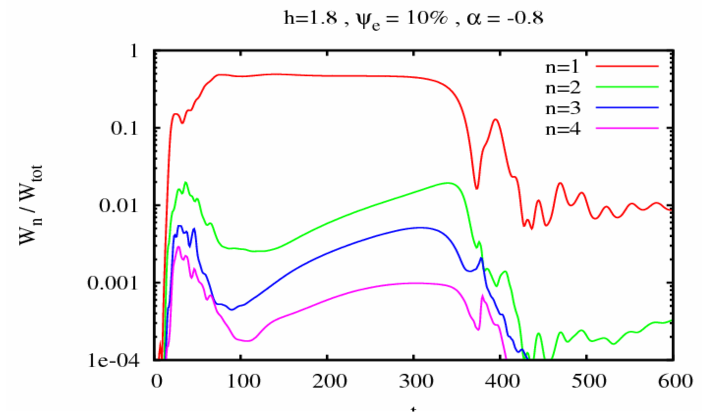

Fig. 8. Evolution of the energy in higher order modes for h=1.8.

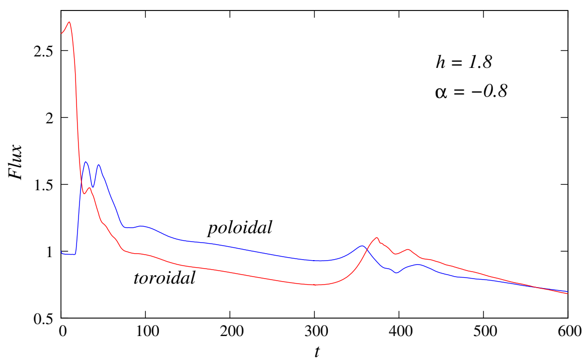

Fig. 9. Evolution of the poloidal and toroidal flux for h=1.8

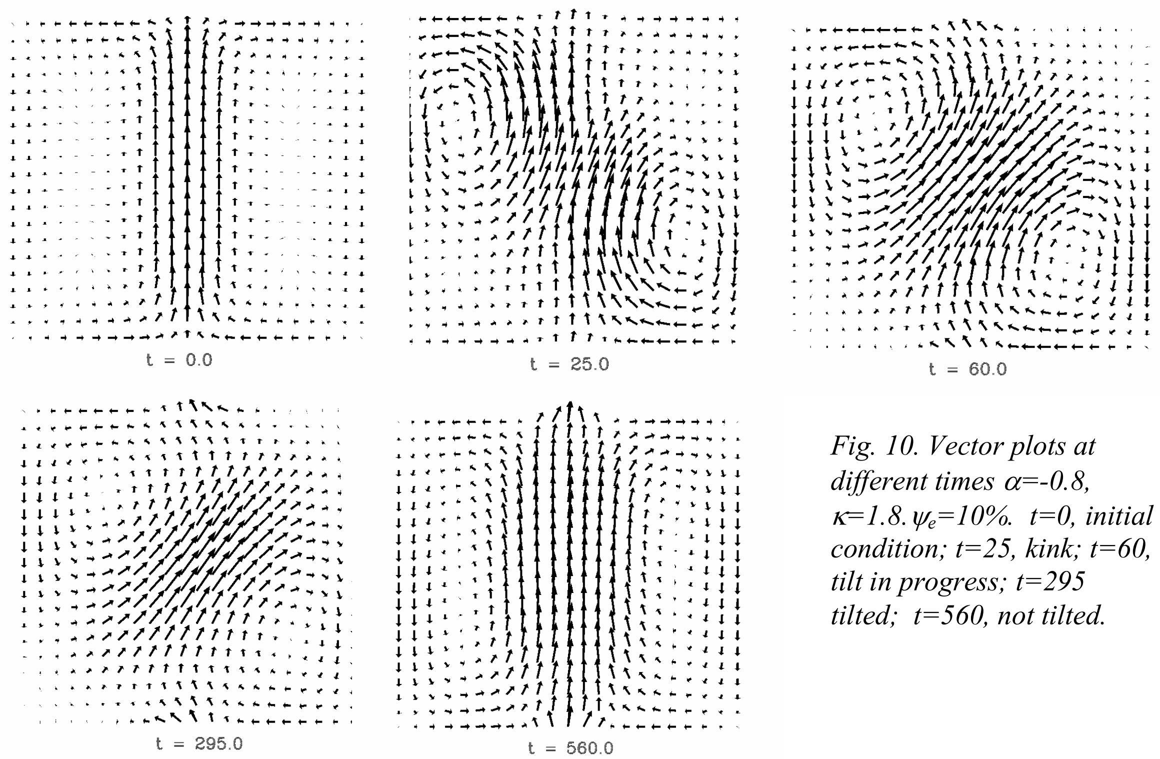

Fig. 10. Vector plots at different times α=-0.8, κ=1.8. ψ_e=10%. t=0, initial condition; t=25, kink; t=60, tilt in progress; t=295 tilted; t=560, not tilted.

Summary

We investigated the relaxation of a FCS using the VAC code, which solves the 3D, resistive MHD equations. Previous relaxation studies were limited to low elongation spheromaks, where the kink mode is responsible for the amplification of poloidal flux. We showed that the slope of the λ profile determines the rate and extent of the relaxation driven by the kink mode and that when the elongation increases beyond the critical value corresponding to the onset of the tilt mode, the initial kink is followed by a tilt and the dynamics of the relaxation changes. The addition of a small amount of open flux drastically changes the dynamics of the tilt, which eventually disappears (at least for the elongations we copnsidered).

More detailled studies are needed to further clarify the evolution of FCS with high elongation and open flux. For example, the reason for the sudden drop in the energy of the n=1 tilt mode, after a long period where it remained approximately constant, is not well understood Since the goal is steady state operation, the relaxation process must be studied in a driven situation, where helicity is injected continuously to sustain the configuration.

References

[1] T. R. Jarboe, W. C. Barnes, D. Platts and B. L. Wright. *Comment Plasma Phys. Control. Fusion* **9**, 161 (1985).
[2] P. K. Browning et al. *Phys. Rev. Lett*. **68**, 1718, 1992.
[3] E. B. Hooper, J. H. Hammer, C. W. Fernandez and F. Wysocki. *Fusion Technol*. **29**, 191 (1996).
[4] F. Alladio, P. Costa, A. Mancuso, P. Micozzi, S. Papastergiou and F. Rogier. *Nucl. Fusion* **46**, S613 (2006).
[5] Y-K. M. Peng and D. J. Strickler. *Nucl. Fusion* **26**, 1139 (1986).
[6] G. Tóth *Astrophys. Lett. Comm*. **34** 245 (1996).
[7] J. M. Finn, W. M. Manheimer and E. Ott. Phys. FLuids **24**, 1336 (1981).