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Selective decay in a helicity-injected spheromak

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Abstract. The non-linear evolution of several unstable equilibria, representative of helicity-injected spheromak configurations inside a cylindrical flux conserver, is studied by means of three dimensional resistive MHD simulations. These equilibria are force-free ($\nabla \times \mathbf{B} = \lambda(\psi)\mathbf{B}$) but do not correspond to minimum energy states, having linear $\lambda(\psi)$ profiles with negative slope. Several aspects of this process are studied (magnetic energy relaxation, selective helicity decay, relaxed profiles) for different initial λ slopes. The stability threshold predicted by linear theory is recovered. The results show that complete plasma relaxation leading to a uniform λ , is achieved only if the initial profile is hollow enough. The evolution for cases just above the stability threshold is more gentle and does not end in a Taylor state. The final state in these cases has a linear $\lambda(\psi)$ profile, as the initial condition, but with a smaller slope.

1. Introduction

A spheromak is a toroidal confinement configuration having a simply connected bounding surface inside which the magnetic field is produced almost entirely by currents flowing in the plasma. A distinctive feature of spheromaks is that the toroidal magnetic field is nonzero on internal flux surfaces but vanishes at the bounding surface, which implies that no external coils link the plasma [1, 2]. This represents an important advantage from an engineering and economical point of view, but introduces some difficulties related to the sustainment of the configuration against resistive dissipation. Most of the steady-state driven spheromaks have currents flowing through plasma-facing electrodes which are orthogonal to the magnetic axis. In order to drive toroidal current along the magnetic axis, the system relies on magnetic relaxation [3] which necessarily involves non-axisymmetric (three-dimensional) MHD mode activity. According to the theory of magnetic relaxation [3], the configuration should relax to a state of minimum magnetic energy subject to the conservation of magnetic helicity. While this theory qualitatively predicts the observed configurations [4, 5, 6] it does not provide any explanation on how the toroidal current is driven and on the details of the dynamics of the process. Furthermore, in practice the observed configurations are not fully relaxed, but are maintained by the external energy source in a steady state whose energy is larger than the minimum [4].

The physical mechanism underlying Taylor's hypothesis is called "selective decay" [7, 8]. It is an MHD turbulence situation in which, under certain conditions, a system with initial magnetic helicity present evolves with a rapid decay of total energy relative to the magnetic helicity.

In this work, the non-linear evolution of several unstable equilibria, representative of a helicity-injected spheromak inside a cylindrical flux conserver, is studied by means of three dimensional resistive MHD simulations. Through relaxation the system evolves from a state representative of a driven configuration, like the one shown in Fig. 1 (a), to a relaxed (or partially

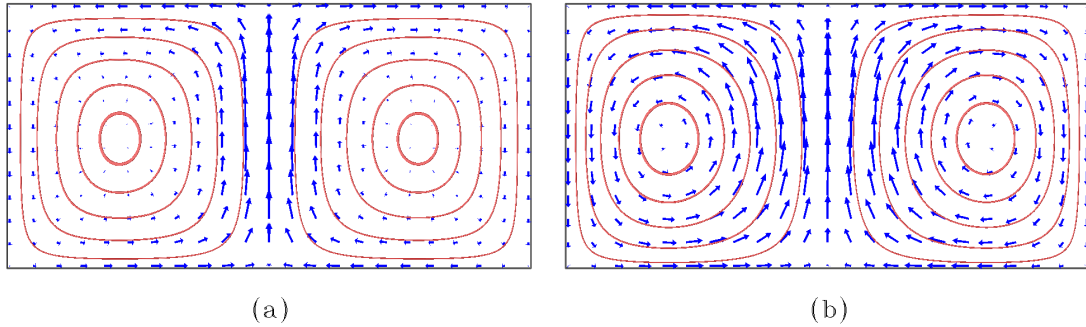


Figure 1. Magnetic field vectors and poloidal flux contours for a driven configuration with a linear $\lambda(\psi)$ profile (a) and for a relaxed configuration with uniform λ (b).

relaxed) state, more similar to the one shown in Fig. 1 (b), via a selective decay process. It has to be emphasized that no helicity is injected to the system during these simulations. We study the evolution of an isolated configuration from an initial condition that mimics the real spheromak plasma when helicity is being injected to it (sustainment phase).

Spheromak relaxation has been studied using numerical simulations in the past. Katayama and Katsurai [9] simulated a decaying spheromak in a cylindrical flux conserver for several q -profiles and observed the growth of an $n = 1$ mode and poloidal flux generation for the hollow λ profile situation (see section 2). Their study was restricted to unstable modes with relatively high growth rates (larger than $0.01/\tau_A$, where τ_A is the Alfvén time) due to computer limitations, and all the final states obtained in their simulations were quite near the Taylor state. The decay of an unstable spheromak equilibrium in a “bowtie” flux conserver was simulated by Izzo and Jarboe [10]. Using the same initial condition, they varied the Lundquist number over one order of magnitude and identified the minimum value needed for poloidal flux generation. Formation and sustainment in both a cylindrical “flux core” configuration and a coaxial gun were studied by Sovinec *et al* [11].

In this paper, we study the magnetic relaxation event for different values of α , the slope of the linear $\lambda(\psi)$ profile imposed at the beginning of the simulation (see Sec. 2). This slope has a threshold value above which the configuration becomes unstable to kink modes. For unstable cases well above the stability threshold, we observe, as expected, a rapid evolution of the system towards a Taylor state. However, for cases just above the stability threshold the evolution is more gentle and does not end in a uniform λ profile (Taylor state) but in a state having a linear $\lambda(\psi)$ profile with a lower slope, below the stability threshold.

In Section 2, we describe the initial conditions of the simulations. The physical model and the details of the numerical scheme employed are presented in Sec. 3. The simulation results are discussed in Sec. 4 and, finally, the conclusion are summarized in Sec. 5.

2. Initial condition

The initial condition is an axisymmetric force-free ($\mathbf{J} \times \mathbf{B} = 0$), pure magnetic ($\mathbf{u} = 0$) state. When the magnetic field is axisymmetric, the poloidal flux ψ acts as a stream function for the poloidal magnetic field and the force-free condition may be expressed in terms of the Grad-Shafranov equation for ψ ,

$$\Delta^* \psi + \lambda(\psi) \int_0^\psi \lambda(\hat{\psi}) \partial \hat{\psi} = 0, \quad (1)$$

where $\Delta^* = \frac{\partial^2}{\partial r^2} - \frac{1}{r} \frac{\partial}{\partial r} + \frac{\partial^2}{\partial z^2}$, and $\lambda = \mu_0 \mathbf{J} \cdot \mathbf{B} / B^2$. The special case $\lambda(\psi) = \lambda_0$ (constant λ -profile) corresponds to the Taylor state which has the minimum magnetic energy for a given helicity. Experimental observations showed that driven spheromaks are well described by solutions of eq. (1) using the linear relationship,

$$\lambda(\psi) = \bar{\lambda}[1 + \alpha(2\tilde{\psi} - 1)], \quad (2)$$

where $\bar{\lambda}$ is the average of λ , $\tilde{\psi} = \psi/\psi_{max}$ is the normalized poloidal flux, the constant α controls the slope of the λ -profile and its sign determines whether the current profile is hollow (negative α) or peaked (positive α). Hollow current profiles are representative of the sustainment phase of a helicity-injected spheromak having a $\lambda(\psi)$ profile peaked towards the outer flux surfaces. The linear ideal MHD stability analysis shows that this type of configurations becomes unstable to an $n = 1$ mode (kink) when α is below -0.5 [4, 12].

In section 4 we present results from the non-linear evolution of solutions to eq. (1), using the linear (in ψ) λ -profile (2) and the values given in table 1 for α and $\bar{\lambda}$. The domain is a cylindrical flux conserver ($\psi|_{\partial\Omega} = 0$, which implies $(\mathbf{B} \cdot \mathbf{n})|_{\partial\Omega} = 0$) with $h/a = 1$ (where, h is the cylinder height, and a its radius). To this pure magnetic initial condition we add a small perturbation to the axial component of the velocity (u_z), having toroidal wavenumber $n = 1$ and an energy $\sim 5 \times 10^{-4} W_0$, where W_0 is the initial magnetic energy.

Table 1. α and $\bar{\lambda}$ values for the simulations presented in Sec. 4.

α	$\bar{\lambda}$
0.0	4.955
-0.5	5.320
-0.6	5.510
-0.7	5.785
-0.8	6.230

3. Model description and computational approach

Simulations in this study are performed with the VAC (Versatile Advection Code) [13, 14], in which several numerical methods for solving compressible hydrodynamics and magnetohydrodynamics (MHD) equations are implemented. In particular, this code can be used to solve the isothermal resistive MHD model, which is the one used in this work. Defining a length scale a (the cylinder radius), a density scale ρ_0 and a magnetic field scale B_0 , we can write the equations in dimensionless units,

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{u}) = 0 \quad (3)$$

$$\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} + \left(\frac{c_s^2}{c_A^2} \right) \frac{1}{\rho} \nabla \rho = \frac{1}{\rho} (\nabla \times \mathbf{B}) \times \mathbf{B} + \left(\frac{\nu}{ac_A} \right) \nabla \cdot \hat{\Pi} \quad (4)$$

$$\frac{\partial \mathbf{B}}{\partial t} + \nabla \times (-\mathbf{u} \times \mathbf{B}) = - \left(\frac{\eta}{\mu_0 ac_A} \right) \nabla \times (\nabla \times \mathbf{B}), \quad (5)$$

where $c_A = B_0 / \sqrt{\mu_0 \rho_0}$ is the Alfvén speed, $\hat{\Pi} = (\nabla \mathbf{u} + \nabla \mathbf{u}^T) - \frac{2}{3} (\nabla \cdot \mathbf{u}) \mathbf{I}$, and the pressure was eliminated using $p = c_s^2 \rho$, where c_s is the sound speed. The three dimensionless parameters

appearing in eq. (3)-(5) can be identified in the following way. The parameter $\beta = (c_s/c_A)^2 = \mu_0 p_0/B_0^2$, in the left hand side of eq.(4), quantifies the relative importance of hydrodynamics forces (coming from pressure gradients) and magnetic forces (Lorenz's force). Spheromaks are low β plasmas and thus to first order approximation, forces due to hydrodynamic pressure gradients may be ignored. For this reason, we do not advance the continuity equation, eq. (3), and we set a small value for c_s , compared to c_A . Since the initial condition has uniform density (and consequently pressure), the pressure forces vanish in our simulations. This simplification is usually referred to as the zero- β (or zero-pressure) approximation and is widely used when modelling low- β plasmas [10, 11]. The second dimensionless parameter is the dimensionless kinematic viscosity $\tilde{\nu} = \nu/(ac_A)$, the last one is the dimensionless resistivity, $\tilde{\eta} = \eta/(\mu_0 ac_A)$. The parameter $\tilde{\eta}$ may also be interpreted as the ratio of two time scales: $\tau_A = a/c_A$, the Alfvén time (representative of MHD propagation) and $\tau_r = \mu_0 a^2/\eta$, the resistive time. The inverse of this quantity is the Lundquist number $S = \tau_r/\tau_A$. In this work we use $\beta = 0$, $P_m = \tilde{\nu}/\tilde{\eta} = 1$ (magnetic Prandtl), and $\tilde{\eta} = 0.5 \times 10^{-4}$.

The system of partial differential equations that composes the physics model is solved in a uniform cartesian grid, with $N_x \times N_y \times N_z = 100 \times 100 \times 50$. The cylindrical flux conserver is constructed using appropriate values at ghost cells, i.e. knowing the solution inside the flux conserver ($r < a$) we set the values of external grid points ($r > a$) in such a way that the boundary conditions are satisfied (at $r = a$), up to the interpolation error. The perfectly conducting wall conditions employed are $(\mathbf{B} \cdot \hat{\mathbf{n}})|_{\partial\Omega} = 0$ and $(\mathbf{J} \times \hat{\mathbf{n}})|_{\partial\Omega} = 0$, and the condition $\mathbf{u}|_{\partial\Omega} = 0$. Note that this choice of boundary conditions is not compatible with the force-free equilibria that we use as initial conditions. The reason is that this boundary conditions imposes $\lambda|_{\partial\Omega} = 0$, while the initial condition has $\lambda|_{\partial\Omega} = \lambda(\psi = 0) = \bar{\lambda}(1 - \alpha) \neq 0$, from eq. (5) and the condition $\alpha < 0$. In general, an ideal force-free equilibrium may have a non vanishing tangential current at the flux conserver boundary. Nevertheless, we still use the boundary conditions mentioned above for compatibility with previous works [9, 10, 11] and because they are familiar Dirichlet and Neumann condition for $\mathbf{B}$. Conditions on the tangential derivative of $\mathbf{B}$ at the wall are not easy to apply in our representation, but can be applied in the context of spectral methods [15]. We will see, in the following section, that this boundary condition only affects the outer flux surfaces, i.e. the flux surfaces that are close to the wall. We note that in real laboratory plasmas, the current always drops off near the wall because the temperature there is lower and the resistivity is higher.

4. Results

The evolution of the poloidal flux for four different initial conditions is shown in Fig. 2 (ψ is normalized with its initial value and t is in Alfvén time units). Poloidal flux amplification, which is intimately related with toroidal current drive via magnetic relaxation, is observed in all the unstable cases (cases having $\alpha < -0.5$). We can see, in Fig. 2, that the activity that produces the flux amplification develops at earlier times in the most unstable cases and that this amplification is faster in those cases (has a shorter rise time).

In Fig. 3 the decay of the total magnetic energy and the energy of the $n = 0$ component of the Fourier decomposition in the toroidal direction are shown for the cases $\alpha = -0.6$ (a) and $\alpha = -0.8$ (b). The energies are normalized with their initial values. The case $\alpha = -0.6$, in Fig. 3(a), which presents a small amount of flux generation (Fig. 2), has an almost axisymmetric evolution (the evolution of W_{Tot} and $W_{n=0}$ can not be distinguished). We can also see that the energy decay of the configuration due to resistivity that results after the appearance of a small amount of higher mode activity (Fig. 3(c)), at times greater than 150, is faster than the decay of a Taylor state (in the presence of resistivity, the Taylor state decays exponentially, $\sim \exp(-2\eta\lambda_0^2 t)$, in dimensionless units [1], as shown with a black line in Fig. 3(a) and (b)).

The evolution of the total and $n = 0$ magnetic energies for the case $\alpha = -0.8$ is shown in Fig.

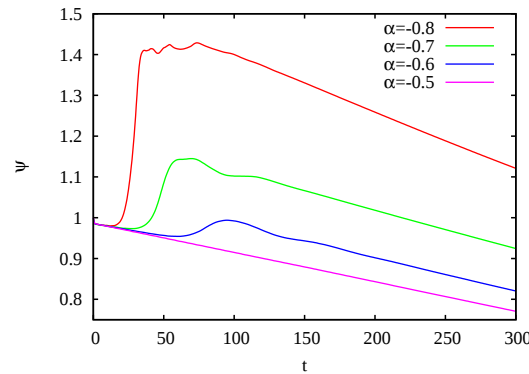


Figure 2. Poloidal flux vs time (in Alfvén time units) for four different cases. The initially unstable states (those with $\alpha < -0.5$) exhibit MHD activity resulting in poloidal flux amplification, which is directly related with toroidal current drive.

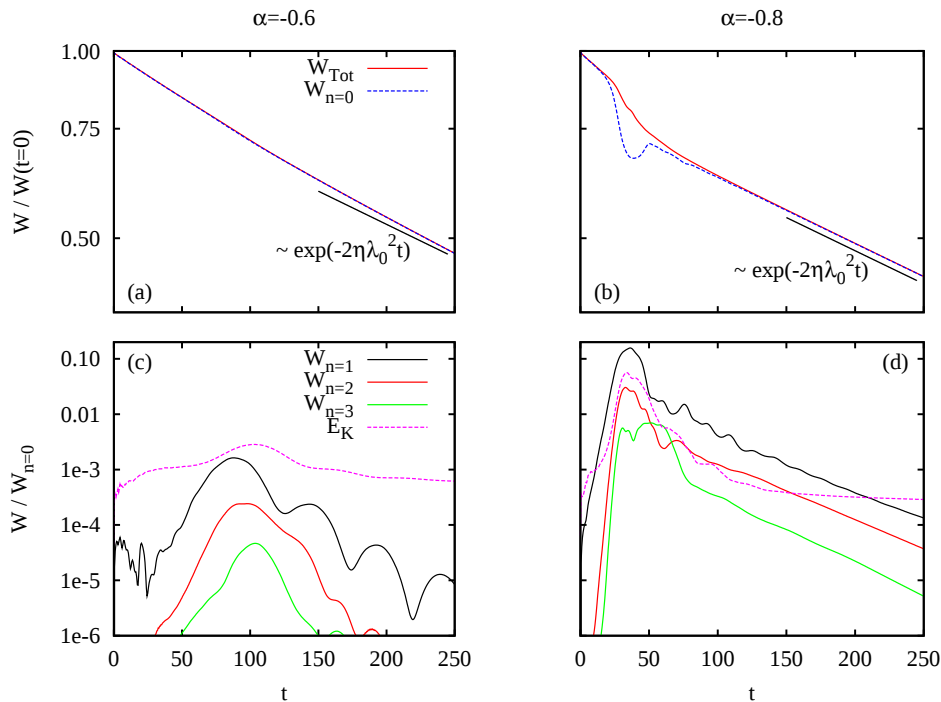


Figure 3. Evolution of the total (W) and the $n = 0$ ($W_{n=0}$) magnetic energies for the cases $\alpha = -0.6$ (a) and $\alpha = -0.8$ (b), relative to their initial value. Higher modes magnetic energies ($W_{n=1,2,3}$) and kinetic energy (E_K) relative to $W_{n=0}$ for the same cases ((c) and (d), respectively).

3(b). In this case, the system undergoes a stepwise relaxation event with a significant deviation from the axisymmetric situation. During this strong mode activity the total magnetic energy decays faster until axisymmetry is recovered and finally the magnetic energy of the system decays with the same slope than the Taylor state.

The higher mode activity can be seen in Fig. 3 (c) and (d), along with the kinetic energy, for the cases $\alpha = -0.6$ and $\alpha = -0.8$ respectively. The energies in this case are divided by

$W_{n=0}$ (at each time). In both cases, the initial perturbation in the velocity is translated almost instantaneously to $W_{n=1}$. In the $\alpha = -0.6$ case, this perturbation is not enough to trigger the instability, and the system resistively decays without activity until approximately $t = 25\tau_A$, when the instability begins to develop. This time delay is not observed in the more unstable case having $\alpha = -0.8$, where the magnetic energy of the $n = 1$ component grows exponentially from the beginning of the simulation (Fig. 3 (d)). In both cases the dominant mode is the $n = 1$ and the saturation of this mode is in temporal coincidence with the poloidal flux amplification.

The initial and final B_z and B_θ radial profiles at $z = h/2$ are shown in Fig. 4(a) for the $\alpha = -0.6$ case, and in 4(b) for the $\alpha = -0.8$ case. The profiles for the corresponding Taylor states are also shown. While in both cases the final state is more similar to the Taylor solution

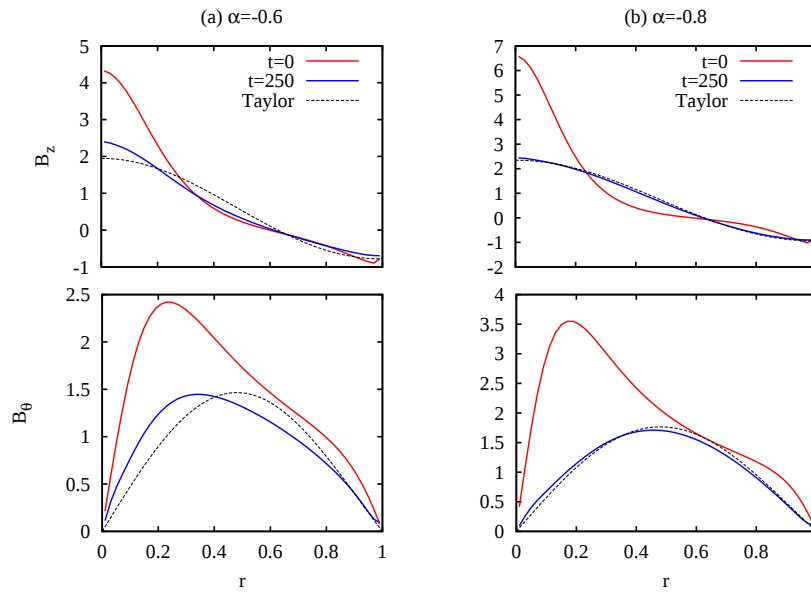


Figure 4. Initial, final and Taylor magnetic field radial profiles at $z = h/2$. Axial and toroidal components are shown for the cases $\alpha = -0.6$ and $\alpha = -0.8$. Note that the y-axis scale is different in each column.

than the initial state, the final profiles in case (b), with $\alpha = -0.8$, are much closer to the fully relaxed configuration described by the Taylor state than the final profiles in case (a).

The effect of the magnetic relaxation on the λ profiles can be observed in Fig. 5, for the case $\alpha = 0$ (a), and for the cases $\alpha = -0.6$ (b) and $\alpha = -0.8$ (c). As in the case of Fig. 4, these radial profiles correspond to $z = h/2$. The $\alpha = 0$ case shown in Fig. 5(a) does not present MHD activity because it is indeed the relaxed state (it is initially the Taylor state). However, the $\lambda(r)$ profile for this case is shown in order to see the effect of the boundary condition applied. Note that the boundary condition $(\mathbf{J} \times \mathbf{n})|_{\partial\Omega} = 0$ imposed on the flux conserver makes λ vanish at $r = 1$, so it is inconsistent with the uniform λ of the relaxed state. This perturbation introduced on the outer flux surfaces near the cylinder wall propagates, as time evolves, along these flux surfaces producing deviations from the uniform λ profile near the symmetry axis of the cylinder. Nevertheless, internal flux surfaces remain unaffected by this perturbation. A similar effect of this perturbation on the cases $\alpha = -0.6$ and $\alpha = -0.8$ can be observed in Fig. 5 (b) and (c), respectively.

The cases showed in Fig. 5(b) and 5(c) begin with a hollow λ profile indicating that the current is concentrated in the outer flux surfaces. After the magnetic relaxation, the current

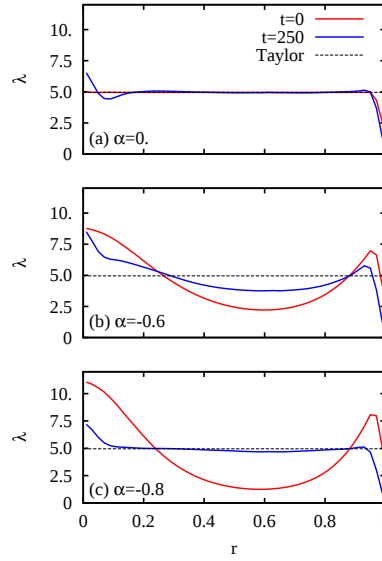


Figure 5. Radial λ profiles at $z = h/2$. The boundary conditions introduces a small perturbation that affects the outer flux surfaces in the three cases. The most unstable case $\alpha = -0.8$ (c) exhibits a complete λ uniformization.

is redistributed and the λ profile becomes more uniform. In particular, at the magnetic axis position ($r \sim 0.65$), the value of λ grows in these cases implying that some toroidal current has been driven along the magnetic axis. The uniformization of the λ profile in the case $\alpha = -0.6$ is incomplete, in contrast with the more unstable case $\alpha = -0.8$ which exhibits an almost uniform final λ profile.

In Fig. 6 we show the evolution of the ratio $2W_{n=0}/K_{n=0}$ for four cases, ranging from the initial Taylor state $\alpha = 0$, to the most unstable case $\alpha = -0.8$. The case $\alpha = 0$ evolves close to

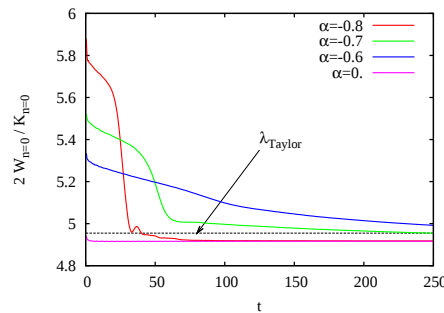


Figure 6. Selective decay of magnetic energy ($W_{n=0}$) relative to the magnetic helicity ($K_{n=0}$), in four cases. As the initial condition is more unstable, Taylor's hypothesis is better fulfilled during relaxation.

the theoretical value for the Taylor state (indicated as λ_{Taylor} in Fig. 6). The small discrepancy is due to the physical boundary condition imposed at the flux conserver walls ($(\mathbf{J} \times \mathbf{n})|_{\partial\Omega} = 0$), and possibly to numerical inaccuracies introduced by the discretization. In the cases with $\alpha < 0$ the ratio $2W_{n=0}/K_{n=0}$ decays. During the relaxation event itself, when MHD activity arises, the

decay rate grows in the three cases, indicating that the energy is dissipated faster than helicity specially at that moment. After the activity, the decay rate of the energy becomes more similar to that of the magnetic helicity, specially for the initially more unstable cases.

From the three cases presented in Fig. 6 with $\alpha < 0$, only the case $\alpha = -0.8$ reduces its magnetic energy to the minimum value allowed by the helicity present in the system, satisfying the Taylor hypothesis. In the other cases the relaxation is incomplete, in the sense that there is still some magnetic energy above the minimum compatible with the helicity present.

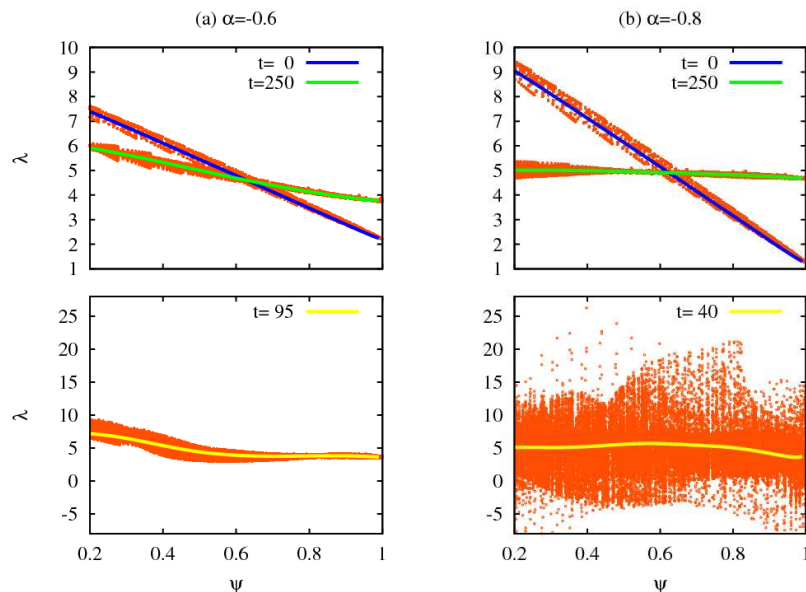


Figure 7. The values of λ and ψ are computed in each grid point and plotted in λ vs ψ space (orange points). The lines are the λ average at each ψ .

Another way to evaluate how close is a solution to the Taylor state is plotting the value of λ as a function of the poloidal flux ψ . This is done computing these two values at each grid point for the cases $\alpha = -0.6$ and $\alpha = -0.8$, for several times (Fig. 7). Each point on the graphs of Fig. 7 corresponds to one grid point and the λ average for each ψ is indicated by a line. It is observed that the net effect of the relaxation event is to reduce the slope of the $\lambda(\psi)$ profile. In the more unstable cases the MHD activity that arises produces the complete relaxation of the configuration, leading to a solution with an almost uniform λ . On the other hand, when the system is just above the stability threshold the relaxation is incomplete, and the final state after relaxation has a linear $\lambda(\psi)$ profile with a lower slope, that is below the linear stability threshold. During the relaxation event, at the time of maximum activity (when the axisymmetry is broken), the $\alpha = -0.6$ case ($t = 95$) presents a little dispersion on the λ values, concentrated at low values of ψ (outer flux surfaces). The $\alpha = -0.8$ case has a large dispersion in λ , with a flat distribution along ψ , and it presents regions with negative λ , which means that the current and the magnetic field are antiparallel. These regions may be associated with magnetic reconnection events occurring during the non-linear activity that causes the full relaxation of the system.

5. Conclusions

We have computed the non-linear evolution of several force-free equilibria representative of a helicity-injected spheromak. The cases below the linear stability threshold did not show

significant MHD activity to provide poloidal flux generation. For unstable cases poloidal flux generation was observed, as a result of the appearance of MHD activity characterized by non-axisymmetric modes. This activity is gentle in cases just above the stability threshold, leads to little flux amplification and produces an incomplete relaxation of the configuration. More unstable cases exhibit much more activity, that introduces important deviations to the axisymmetry of the configuration, produces substantial flux amplification and makes the system relax completely, as expected if the Taylor hypothesis were valid.

Measurements made in experimental spheromak equilibria indicate that during sustainment the configurations differ from the minimum-energy state and are well described by a linear $\lambda(\psi)$ profile with negative slope [4]. Although the conditions in the simulations presented here are far from the actual conditions in real experiments (because of the limited resolution, the lack of magnetic flux intercepting the boundaries, etc.), the results presented in this work suggest that the process that occurs during spheromak sustainment is an incomplete relaxation event, which leads a marginally unstable configuration to a marginally stable one. That is to say, as the power source drives poloidal current in the outer flux surfaces and the toroidal current along the magnetic axis resistively decays, α (the slope of the $\lambda(\psi)$ profile) becomes more negative, until it crosses the stability threshold. Then, some MHD activity arises producing an incomplete relaxation of the plasma, like in the case $\alpha = -0.6$ presented above. This intermittent evolution around the stability boundary has been already proposed, analyzing results from linear theory [16] and experiments [17]. Nevertheless, it has not been observed, up to our knowledge, that the non-linear evolution of a marginally unstable spheromak without sustainment ends in a marginally stable configuration, instead of fully relaxing towards a Taylor state.

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