Nonlinear wave-particle interactions in rising tone chorus generation

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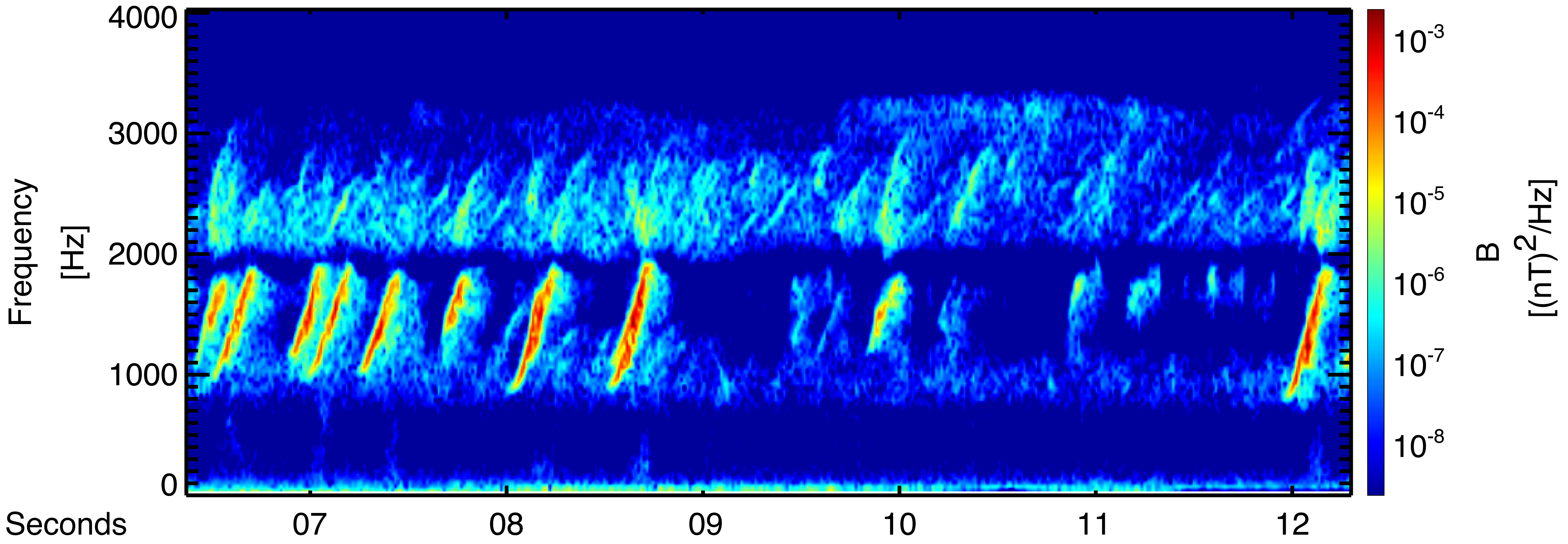
A theoretical framework for chorus excitation
(2017 Fall Meeting, AGU, New Orleans)



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Observation (Van Allen probes) of Whistler waves excited by anisotropic supra-thermal electrons.
GRL 44, GL075824 (2017).

Previous theories rely on the following equations for chorus excitation.

$$\left(\frac{\partial}{\partial t} + v_g \frac{\partial}{\partial z}\right)\delta B = -\frac{\mu_0 v_g}{2} j_E$$

Wave amplitude variation by j_E.

$$c^2 k^2 - \omega^2 - \frac{\omega \omega_{pe}^2}{\Omega_e - \omega} = \mu_0 c^2 k j_B / \delta B$$

Wave frequency variation by j_B.

Omura et al., JGR, 2009

Current densities j_E and j_B are calculated using the phase space density, *f*.

There is no equation for evolution of *f*, which is needed to study the generation of chorus.

The now famous frequency chirping rate equation was obtained assuming a form of *f*.

Assume a function form of *f*
with a phase space hole for simplicity,

$$f(v_{\parallel}, v_{\perp}, \zeta) = g(v_{\parallel}, \zeta)\delta(v_{\perp} - v_0)$$

Calculate j_E using *f*, and maximization of j_E gives,

$$\frac{\partial \omega}{\partial t} = 0.4\left(1 - \frac{v_r}{v_g}\right)^{-2} \omega_{tr}^2, \text{ with } \omega_{tr}^2 \equiv \frac{e}{m}\frac{k^2 v_{\perp}}{\omega}|\delta E|$$

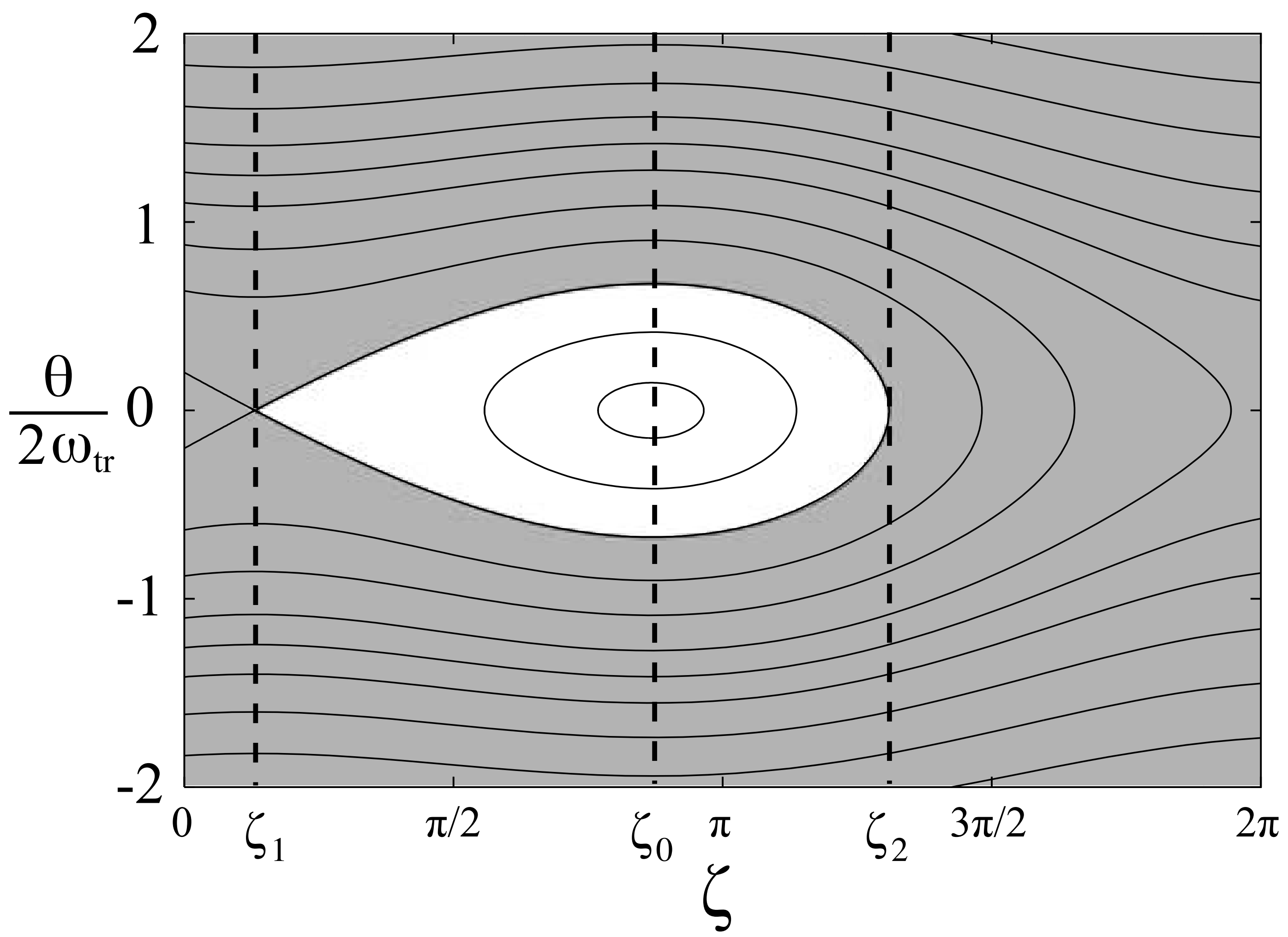


Figure 1. Trajectories of resonant electrons in the $(\theta - \zeta)$ phase space for the inhomogeneity ratio $S = -0.4$. The phase angle ζ_0 is the center of trapping motion, while ζ_1 and ζ_2 indicate the boundaries of the trapping region at $\theta = 0$.

Omura et al, JGR, 2008

A very successful relation. But we still need the evolution equation of *f* to make PREDICTIONS.

Different ways of calculating *f* lead to different numerical factors in the chirping rate - amplitude relation.

$$\frac{\partial \omega}{\partial t} = 0.4 \left(1 - \frac{v_r}{v_g}\right)^{-2} \omega_{tr}^2$$

Omura et al, JGR, 2008, analytical

$$\frac{\partial \omega}{\partial t} = \frac{1}{2} \left(1 - \frac{v_r}{v_g}\right)^{-2} \omega_{tr}^2$$

Vomvoridis et al., JGR, 1982, test particle + analytical

$$\frac{\partial \omega}{\partial t} \approx \frac{\omega}{\Omega - \omega} \left(1 - \frac{v_r}{v_g}\right)^{-2} \omega_{tr}^2$$

this work, preliminary

The purpose of this work is to propose a self-consistent model for chorus excitation.

- **Assume the spectrum is nearly monochromatic with (k, ω_k).**
- **Instability causes particle redistribution in phase space.**
  - » **Growth rate spectrum changes.**
  - » **Particle distribution function needs to be computed self-consistently.**
- **A familiar model is the quasilinear theory; applicable to $\tau_{ac} \ll \tau_{NL}$.**
- **For chorus, we will need the equation of f applicable to the more general regime: $\tau_{ac} \sim \tau_{NL}$.**

A symbolic way to derive the equation of *f:*

It's convenient to write the equilibrium distribution function as

$$f_0 = f_0(\mathcal{E}, \mu) \text{ with } \mathcal{E} \equiv v^2/2, \text{ and } \mu = v_\perp^2/2B$$

Introduce the phase angle Θ between $v_\perp$ and the wave magnetic field δB:

$$\dot{\Theta} = \Omega + kv_{||} - \omega_k, \text{ where } \Omega = \dot{\alpha} \text{ and } \alpha \equiv \text{gyrophase}$$

Assume the field of the *kth* mode: $\delta E = \delta E_k(t) e^{\mathrm{i}(kz - \omega t)}$ **; therefore,**

$$\frac{\partial}{\partial t}\delta E = e^{\mathrm{i}(kz-\omega t)}\left(-\mathrm{i}\omega + \frac{\partial}{\partial t}\right)\delta E_k(t)$$

A symbolic way to derive the equation of *f:*

Considering the time variation of *$f_0(t)$* and the wave field, the Vlasov equation for the perturbed *f* is

$$\left(\mathrm{i}\dot{\Theta} - \partial_t\right)\delta f_k = \frac{e}{m} v_\perp e^{\mathrm{i}\alpha} \delta E_k \left[\frac{\partial}{\partial \mathcal{E}} + \left(1 - \frac{k v_\parallel}{\omega_k}\right)\frac{1}{B}\frac{\partial}{\partial \mu}\right] f_0(t)$$

Symbolically, the perturbed distribution function can be written as:

$$\delta f_k = \left(\mathrm{i}\dot{\Theta} - \partial_t\right)^{-1} \frac{e}{m} v_\perp e^{\mathrm{i}\alpha} \delta E_k \left[\frac{\partial}{\partial \mathcal{E}} + \left(1 - \frac{k v_\parallel}{\omega_k}\right)\frac{1}{B}\frac{\partial}{\partial \mu}\right] f_0(t)$$

A symbolic way to derive the equation of *f:*

Assuming slow time variation of the wave field compared with 1/ω => simplified but intuitive picture

$$\delta f_k = \frac{e}{m} v_\perp e^{\mathrm{i}\alpha} \delta E_k \left(\mathrm{i}\dot{\Theta} \cancel{- \partial_t}\right)^{-1} \left[\frac{\partial}{\partial \mathcal{E}} + \left(1 - \frac{k v_\parallel}{\omega_k}\right) \frac{1}{B} \frac{\partial}{\partial \mu}\right] f_0(t)$$

Linear Response

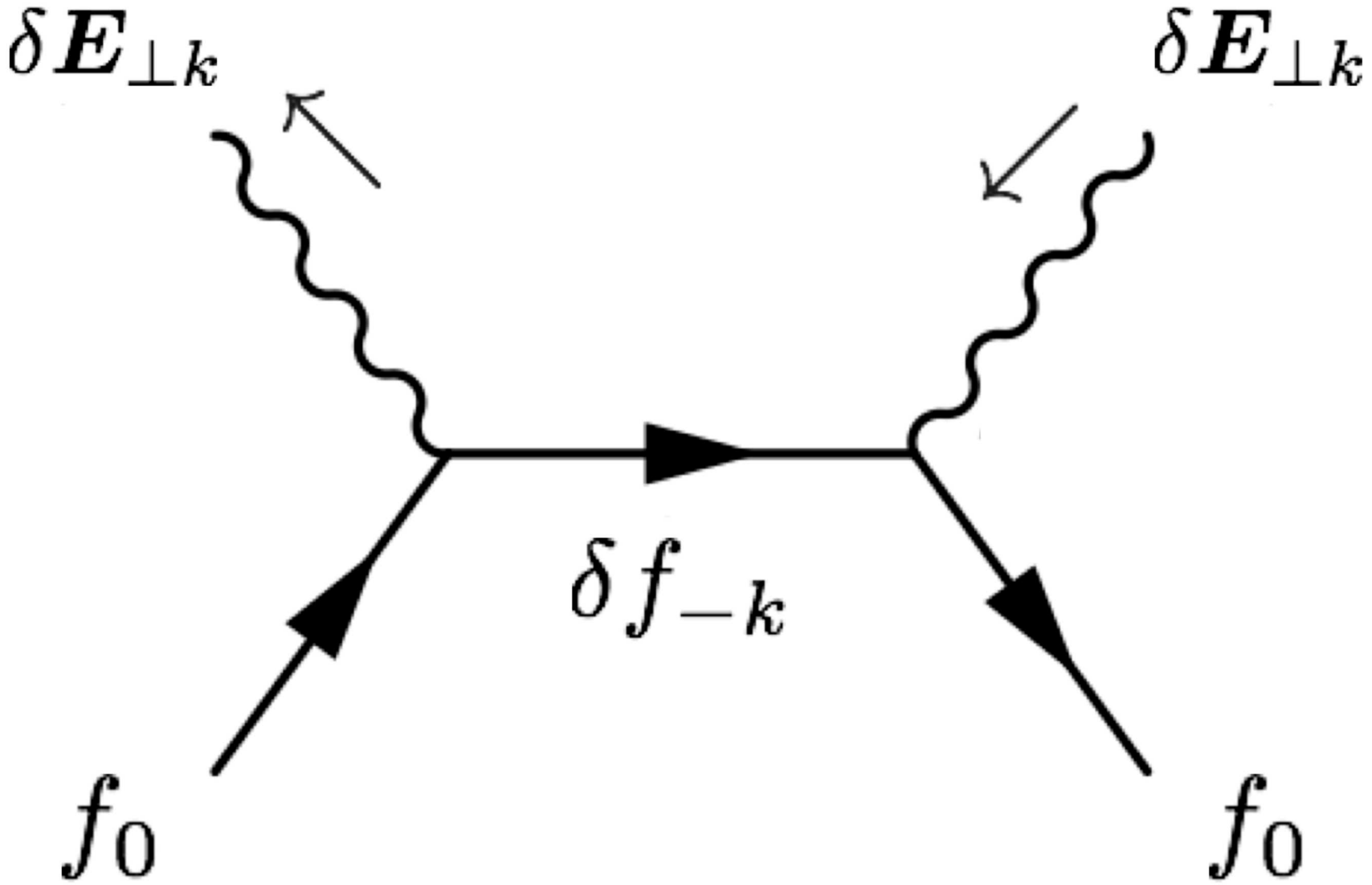
The evolution equation for $f_0(t)$ consistent with δE_k and δf_k is

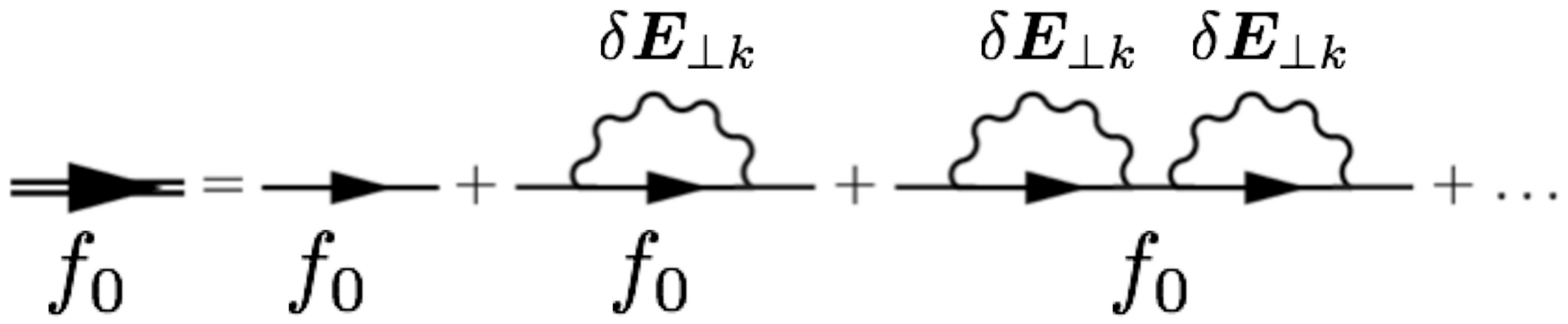
$$\frac{\partial f_0}{\partial t} = \frac{e}{m} v_\perp e^{-\mathrm{i}\alpha} \delta E_{-k} \left[\frac{\partial}{\partial \mathcal{E}} + \left(1 - \frac{k v_\parallel}{\omega_k}\right) \frac{1}{B} \frac{\partial}{\partial \mu}\right] \delta f_k$$

Plug the equation of δf_k into the equation of f_0 => the Dyson equation.

Non-perturbative approach to f_0 (single particle Green's function). Related with the Functional Renormalization Group

These equations describe the wave particle interaction (emission & absorption) process.





generally requires numerical solution

A symbolic way to derive the equation of *f:*

Assuming slow time variation of wave field compared with 1/ω => simplified but intuitive picture

$$\delta f_k = \frac{e}{m} v_\perp e^{\mathrm{i}\alpha} \delta E_k \left(\cancel{\mathrm{i}\dot{\Theta}} - \partial_t\right)^{-1} \left[\frac{\partial}{\partial \mathcal{E}} + \left(1 - \frac{k v_\parallel}{\omega_k}\right) \frac{1}{B} \frac{\partial}{\partial \mu}\right] f_0(t)$$

In limit of resonant & non-adiabatic dynamics $\Rightarrow |\dot{\Theta}| \lesssim \partial_t$

Tao et al., GRL, 2017; Tao et al., PPCF, 2017

The evolution equation for *f₀(t)* consistent with *δE$_k$* and *δf$_k$* is

$$\frac{\partial f_0}{\partial t} = \frac{e}{m} v_\perp e^{-\mathrm{i}\alpha} \delta E_{-k} \left[\frac{\partial}{\partial \mathcal{E}} + \left(1 - \frac{k v_\parallel}{\omega_k} \frac{1}{B} \frac{\partial}{\partial \mu}\right)\right] \delta f_k$$

For coherent non-adiabatic resonant w/p interaction, the equation is wave-like in phase space.

Considering resonant particles only and combining the two equations give

$$\frac{\partial^2 f_0}{\partial t^2} = \frac{e^2}{m^2} v_\perp |\delta E_k|^2 \left[\frac{\partial}{\partial \mathcal{E}} + \left(1 - \frac{k v_\parallel}{\omega_k}\right)\frac{1}{B}\frac{\partial}{\partial \mu}\right] v_\perp \left[\frac{\partial}{\partial \mathcal{E}} + \left(1 - \frac{k v_\parallel}{\omega_k}\right)\frac{1}{B}\frac{\partial}{\partial \mu}\right] f_0$$

$$= \frac{e^2}{m^2} v_\perp |\delta E_k|^2 \left.\frac{\partial}{\partial \mathcal{E}}\right|_C v_\perp \left.\frac{\partial}{\partial \mathcal{E}}\right|_C f_0, \text{ where } C \equiv \mathcal{E} - \mu B(\omega_k/\Omega)$$

Notes:

- **The equation demonstrates the ballistic propagation of the resonant structure.**
- **Maximization of the power transfer gives, with preliminary approximations,**

$$\frac{\mathrm{d}}{\mathrm{d}t} v_r \approx \frac{\omega_k}{k^2 |v_r|} \omega_{tr}^2 \Rightarrow \frac{\partial \omega_k}{\partial t} \approx \frac{\omega_k}{\Omega_0 - \omega_k}\left(1 - \frac{v_r}{v_g}\right)^{-2} \omega_{tr}^2$$

Summary

- **We derived the Dyson equation for the evolution of *f* in case of coherent wave particle interaction.**
- **Combined with wave evolution equations, this provides a self-consistent theoretical framework for chorus excitation.**
- **Preliminary results based on maximization of power transfer gives the chirping rate — amplitude relation that is qualitatively consistent with previous ones.**
- **Future work is needed to solve these equations numerically.**

For other applications of the Dyson equation, see

Chen and Zonca, Rev. Mod. Phys., 2016; Zonca et al., EPS, 2017

Discussion and Future Work

- **Work in progress with N. Carlevaro, M. Falessi and G. Montani**
- **Chorus generation analogous to the beam-plasma system**
- **Transport problem in magnetized plasmas aims at predicting f_0 evolution due to self-consistently evolving fluctuation spectrum**
- **Generalized "Dyson-like equation" (non-perturbative renormalized equation for f_0 in general geometry)**
- **Theoretical method for developing first principle based reduced transport models.**