Reanalysis of the O'Neil approach of the beam-plasma interaction

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Outline

- Motivation
- Brief review of the beam plasma instability and the theory of O’Neil et al.
- An alternative model: fluid mechanics approach
- Results and comparison
- Outlook

Motivation

- Originally motivated by experimental data interpretation - e.g. see Phys. Fluids (1971), 14, 2780.
- More recently proposed as a reduced model for the coupling between energetic particles and Alfven waves near marginal stability in fusion plasmas - e.g. see Chen, Zonca: Rev. Mod. Phys. **88**, 015008 and citations therein.
- The system is related to modern plasma acceleration techniques

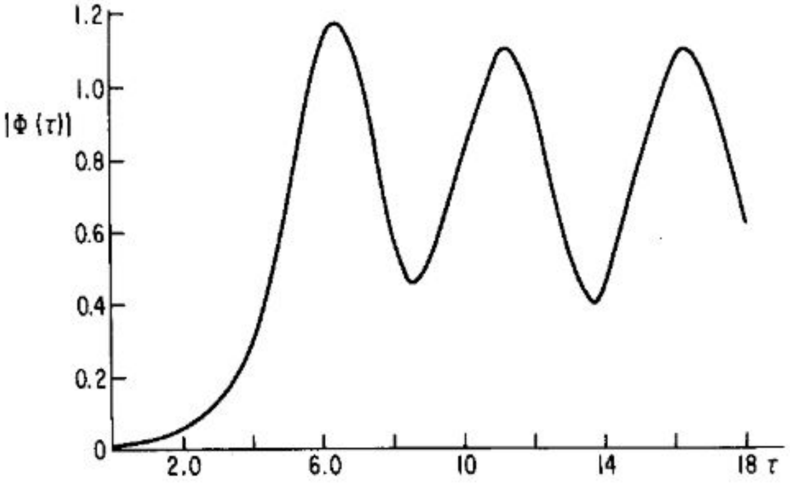
The main aim of this work is to deduce a computable expression of the plasma perturbation induced by the interaction

Beam plasma instability 101

Plasma supports a wide spectrum of electromagnetic waves. When injected with a fast, tenuous beam, particles with velocity close to the phase velocity of the oscillations will exchange energy with the field.

Firstly the field ramps up, until nonlinear saturation occurs and it starts oscillating.

The beam particles are trapped in the potential and rotate in phase space.



[Phys. Fluids (1971), 14, 1207]

The theory of O'Neil et al.

Essential literature:

- O'Neil, T.; Malmberg, J. Phys. Fluids (1968), 11, 1754–1760.
- Drummond & al. Phys. Fluids (1970), 13, 2422–2425
- O'Neil, T.; Winfrey, J.; Malmberg, J. Phys. Fluids (1971), 14, 1204–1212

Essential features:

- For a tenuous enough beam, the interaction is proven to happen with a single mode for the whole saturation process (so-called Single Wave Model)
- Extension to many modes is straightforward

The theory of O’Neil et al. - 2

Their numerical model is derived by considering the electric potential Fourier amplitudes. The cold plasma dielectric function is expanded about ω_p and the beam is discretized in N charge sheets, for which they write equations of motion.

In the end a differential equation for each component $\phi_k(t)$ is found.

After a proper definition of adimensional variables, the resulting set of equations is:

$$\boxed{\begin{aligned} u_i' &= \sum_j i l_j \phi_j e^{i l_j \bar{x}_i} + \text{c.c.}, \\ \phi_j' &= -i\phi_j + \frac{i\eta}{2 l_j^2 N} \sum_i e^{-i l_j \bar{x}_i} \end{aligned}}$$

Fluid mechanics approach

Alternative tractation employing two charged fluids. After linearization, the plasma background can be integrated out of the equations:

$$\begin{cases} \partial_t n_1 + n_0 \partial_x v_1 = 0, \\ \partial_t v_1 = -k_{\mathrm{B}} T_{\mathrm{e}} \partial_x n_1 / m n_0 + e \partial_x \varphi / m, \\ -\partial_x^2 \varphi = -4\pi e (n_1 + n_{\mathrm{B}}). \end{cases} \longrightarrow \left(\partial_t^2 - \cancel{v_{\mathrm{Te}}^2 \partial_x^2} + \omega_{\mathrm{p}}^2 \right) n_1 = -\omega_{\mathrm{p}}^2 n_{\mathrm{B}}.$$

cold plasma approx.

$$\boxed{n_1(x,t) = A(x) e^{i\omega_{\mathrm{p}} t} + \text{c.c.} - \omega_{\mathrm{p}} \int_0^t \sin\left[\omega_{\mathrm{p}} \left(t - t'\right)\right] n_B(x, t') \, \mathrm{d}t' \, .}$$

Fluid mechanics approach - 2

First notable result: the backreaction can be easily computed from

$$\partial_t n_1 + n_0 \partial_x v_1 = 0 \;\rightarrow\; v_1(x,t) = -\frac{1}{n_0}\partial_t \int n_1(x,t)\,\mathrm{d}x$$

Second fact: the beam system can be closed. Let $\mathcal{M} = \partial_t + v_\mathrm{B}\partial_x$, then:

$$\left(\mathcal{M}^2 + {\omega_\mathrm{B}}^2 + v_{T_\mathrm{B}}^2\partial_x^2\right) n_\mathrm{B}(x,t) - \omega_\mathrm{p}{\omega_\mathrm{B}}^2 \int_0^t \sin\left[\omega_\mathrm{p}\left(t-t'\right)\right] n_\mathrm{B}(x,t')\,\mathrm{d}t' =$$
$$= A(x)e^{i\omega_\mathrm{p}t} + \text{c.c.}$$

Fluid mechanics approach - 3

A numerical model has been developed using the same beam discretization introduced by O'Neil. In adimensional variables the set of equations reads:

$$u_i' = \sum_j i l_j \phi_j e^{i l_j \bar{x}_i} + \text{c.c.}$$

$$G^{\pm}(0) = 0, \qquad G^{\pm\prime} = \pm i G^{\pm} + \sum_i e^{-i l_j \bar{x}_i}$$

$$\phi_j = \frac{1}{l_j^2}\left(A e^{i\tau} + \text{c.c.} + \frac{\eta}{N}\sum_i e^{-i l_j \bar{x}_i} + \frac{i\eta}{2N}(G^+ - G^-)\right)$$

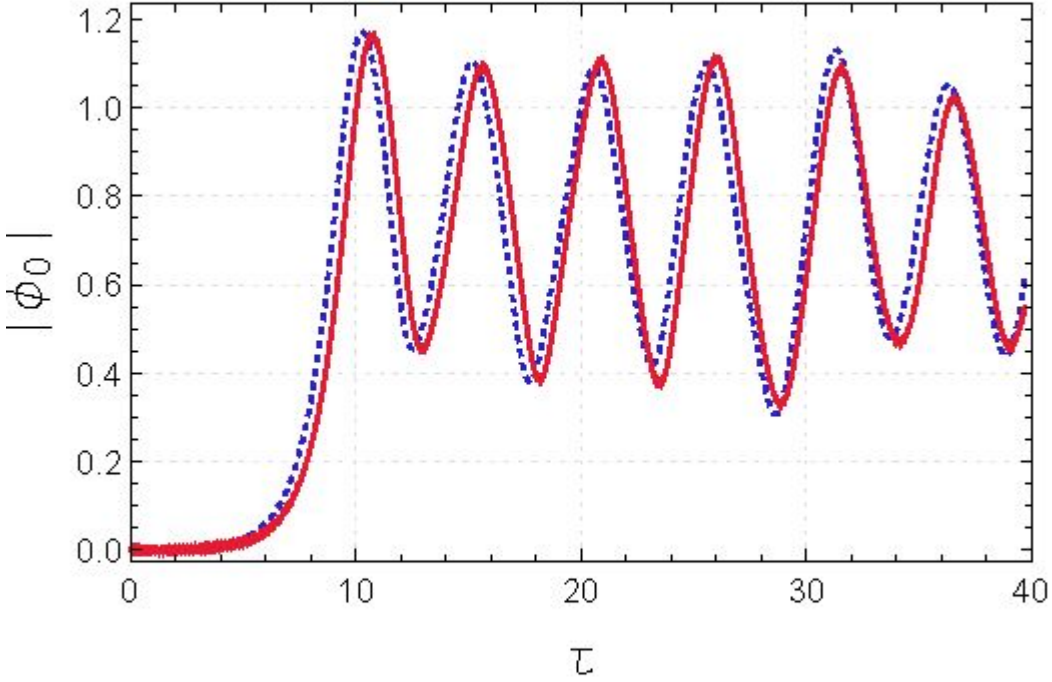
$$\bar{n}_{1j} = A e^{i\tau} + \text{c.c.} - \frac{\eta}{2iN}\left(G^+ - G^-\right)$$

$$\bar{v}_{1j} = \frac{i}{l_j}\left(i A e^{i\tau} + \text{c.c.} - \frac{\eta}{2N}(G^+ + G^-)\right)$$

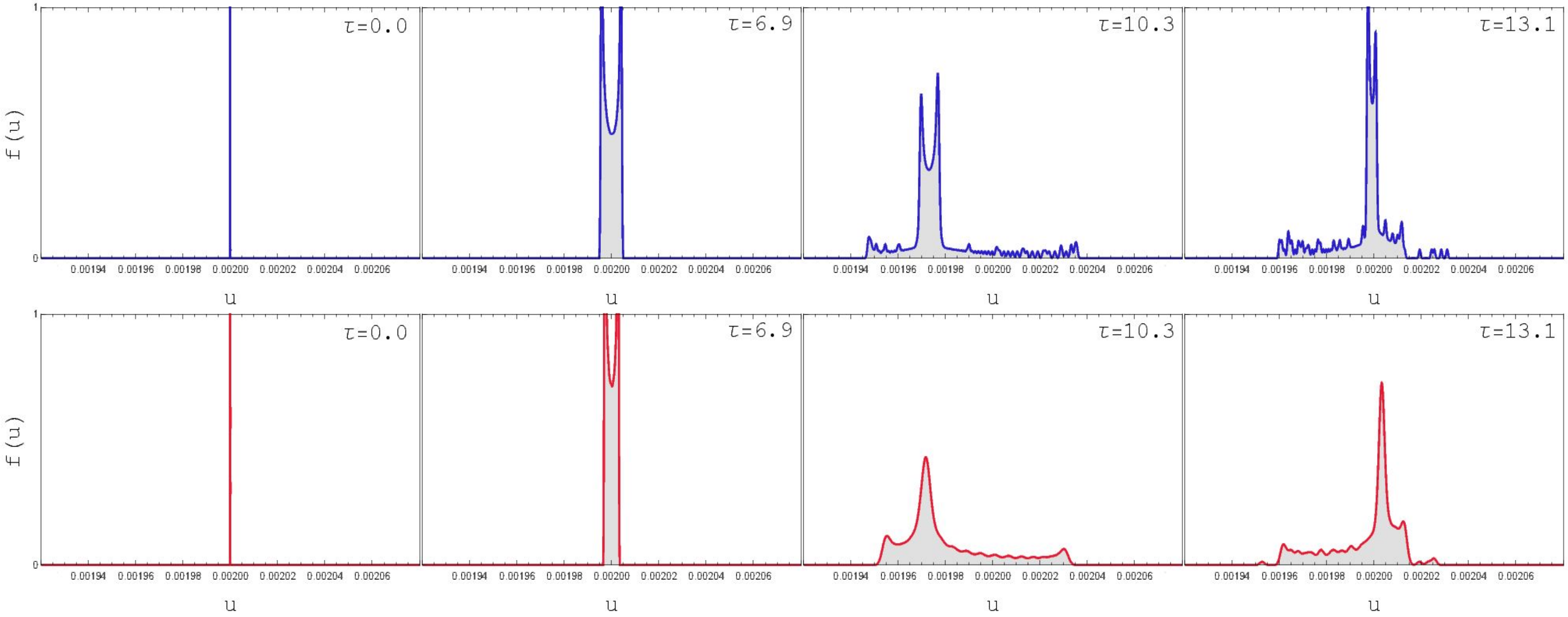
Single mode - mode comparison

Simulation parameters:

- N=10^5
- Spatially homogeneous beam
- All velocities equal to the resonant value (cold beam)



Single mode - f(u) comparison



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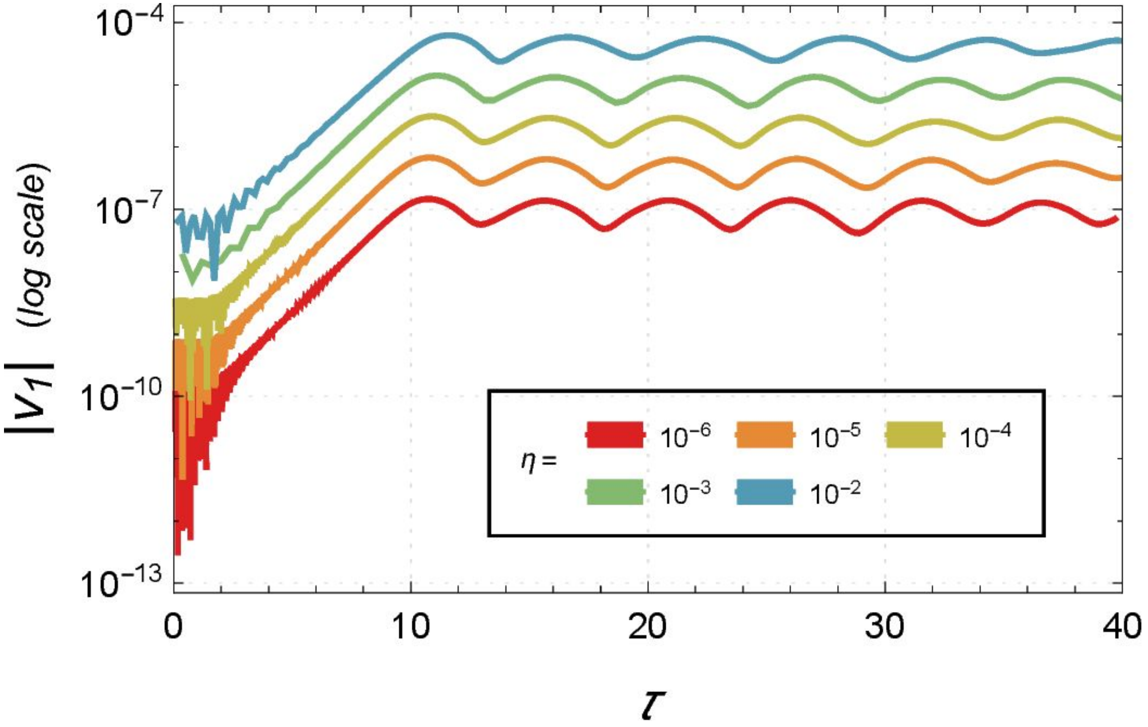
Single mode - backreaction

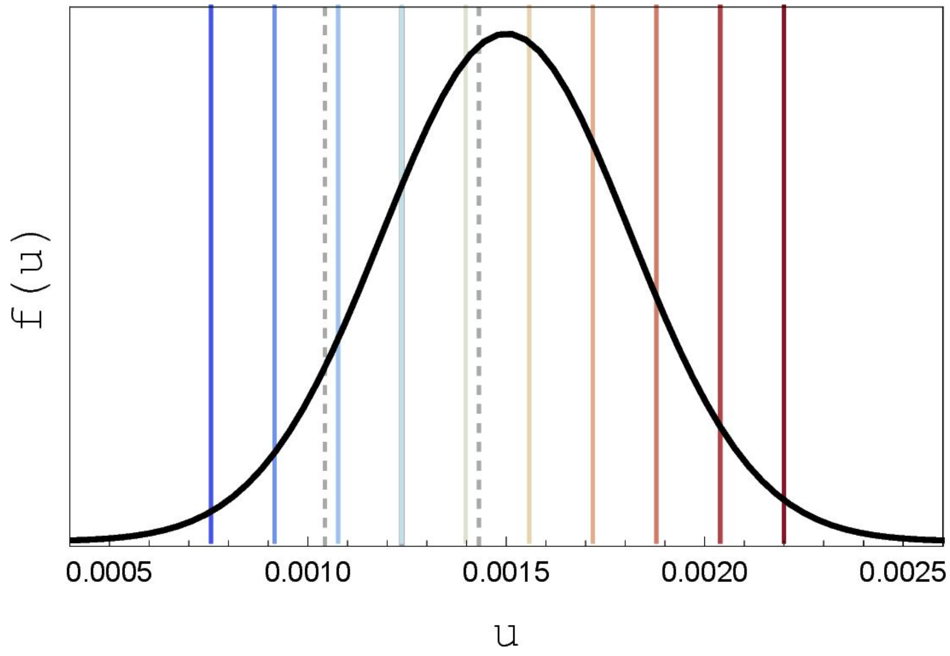
Perturbation to background plasma velocity computed at different values of the density ratio



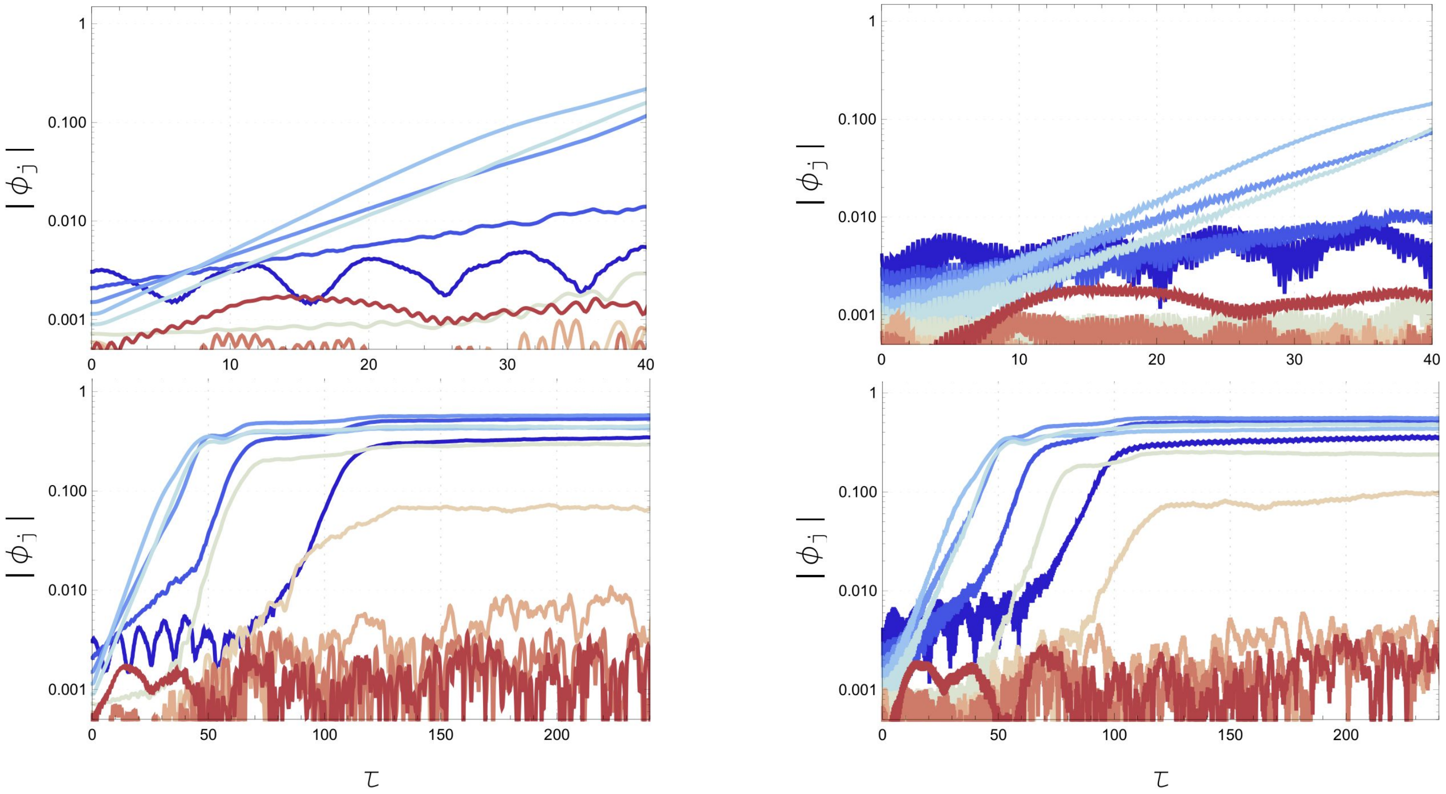
Multimode: quasi-linear regime

Velocity spread corresponding to the most resonant mode nearly overlaps with adjacent modes

→ particles diffuse in velocity space



Multimode: quasi-linear regime - modes comparison



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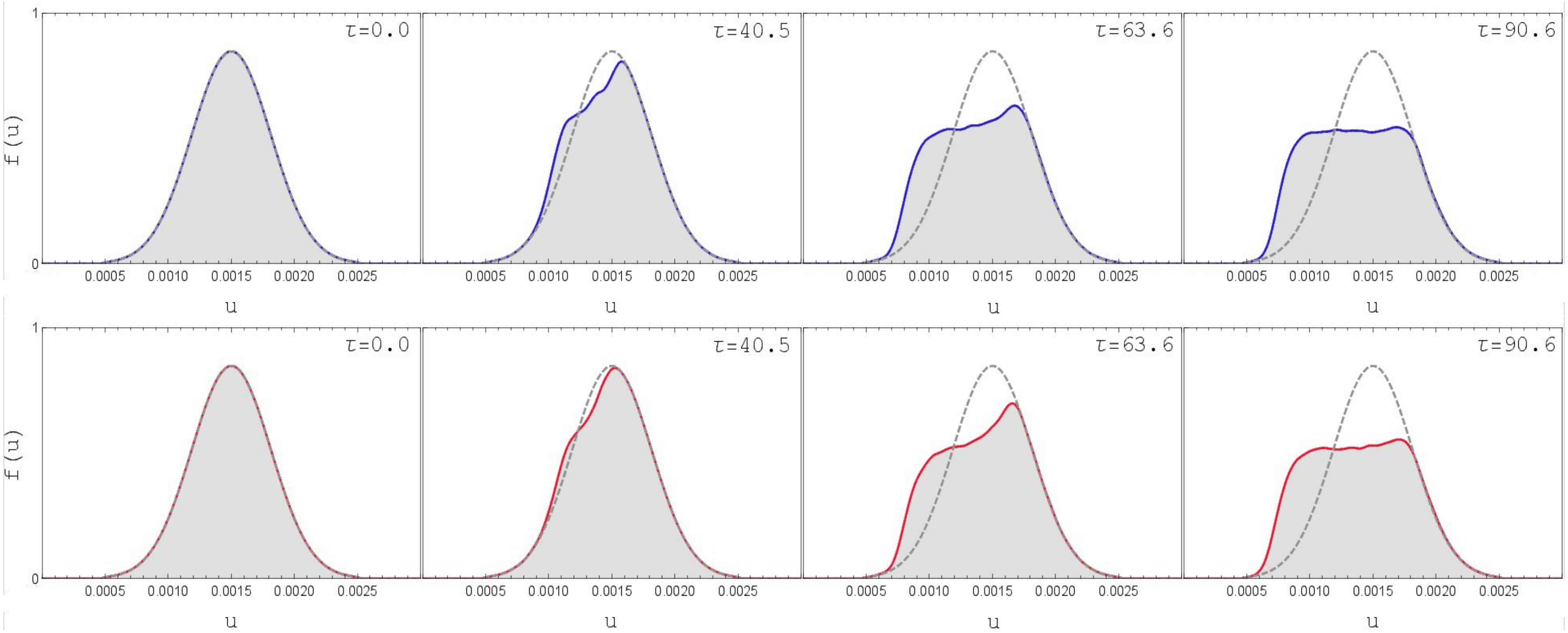
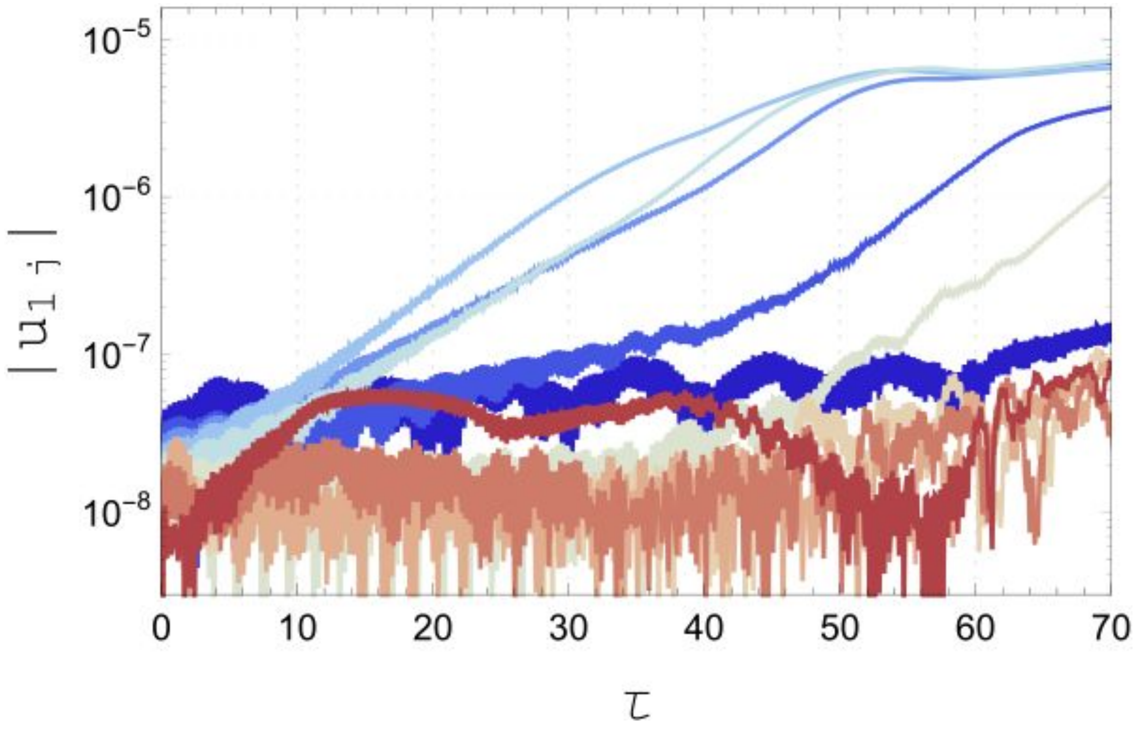
Multimode: quasi-linear regime - f(u)



Multimode: quasi-linear regime - backreaction

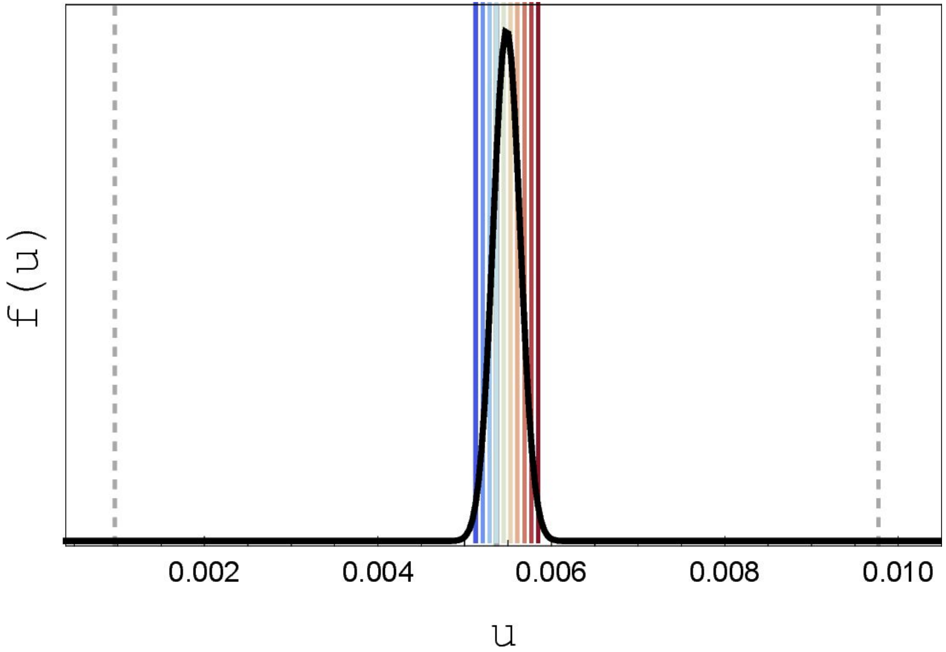
Fourier components of the background plasma excited velocity



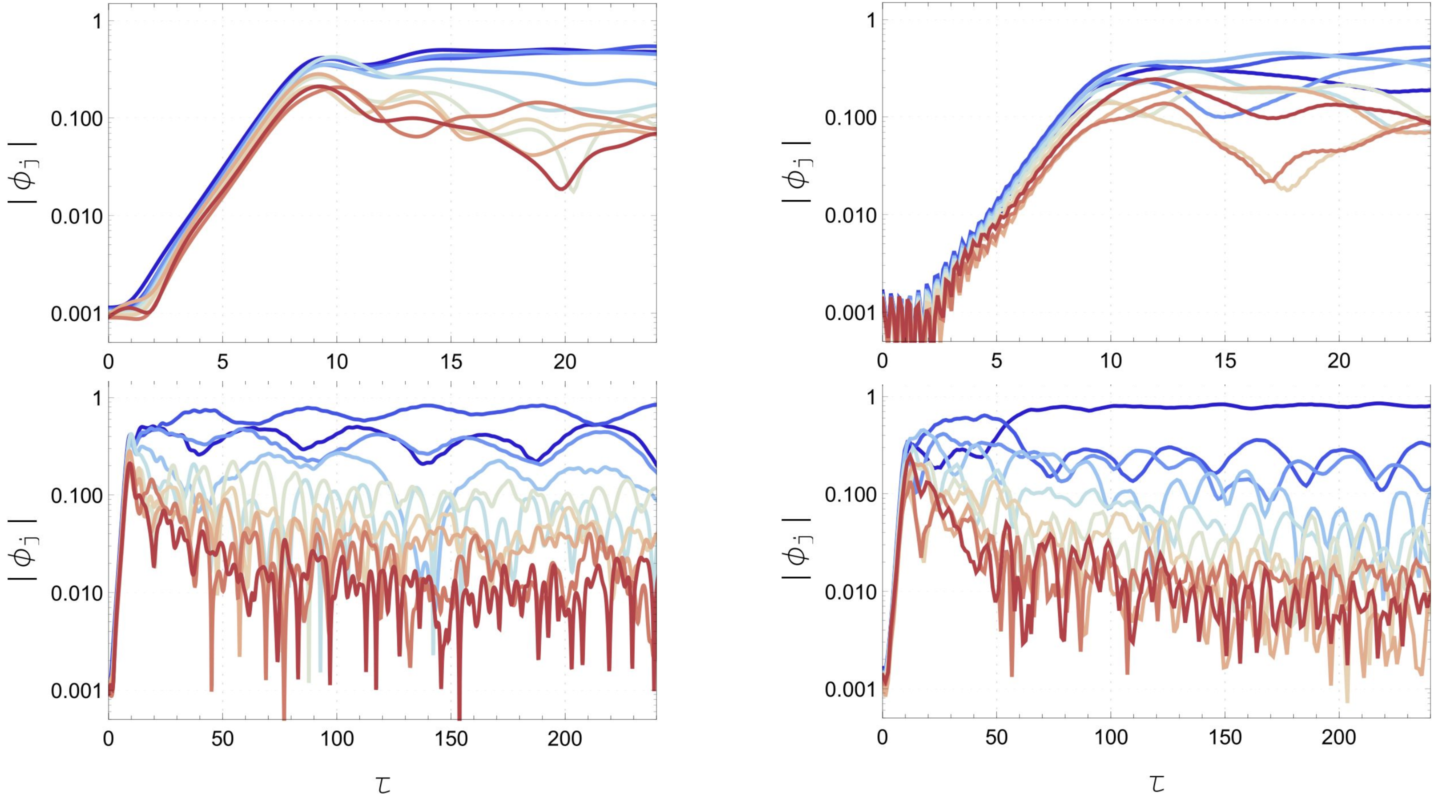
Multimode: convective regime

Velocity spread is so wide that all modes overlap.

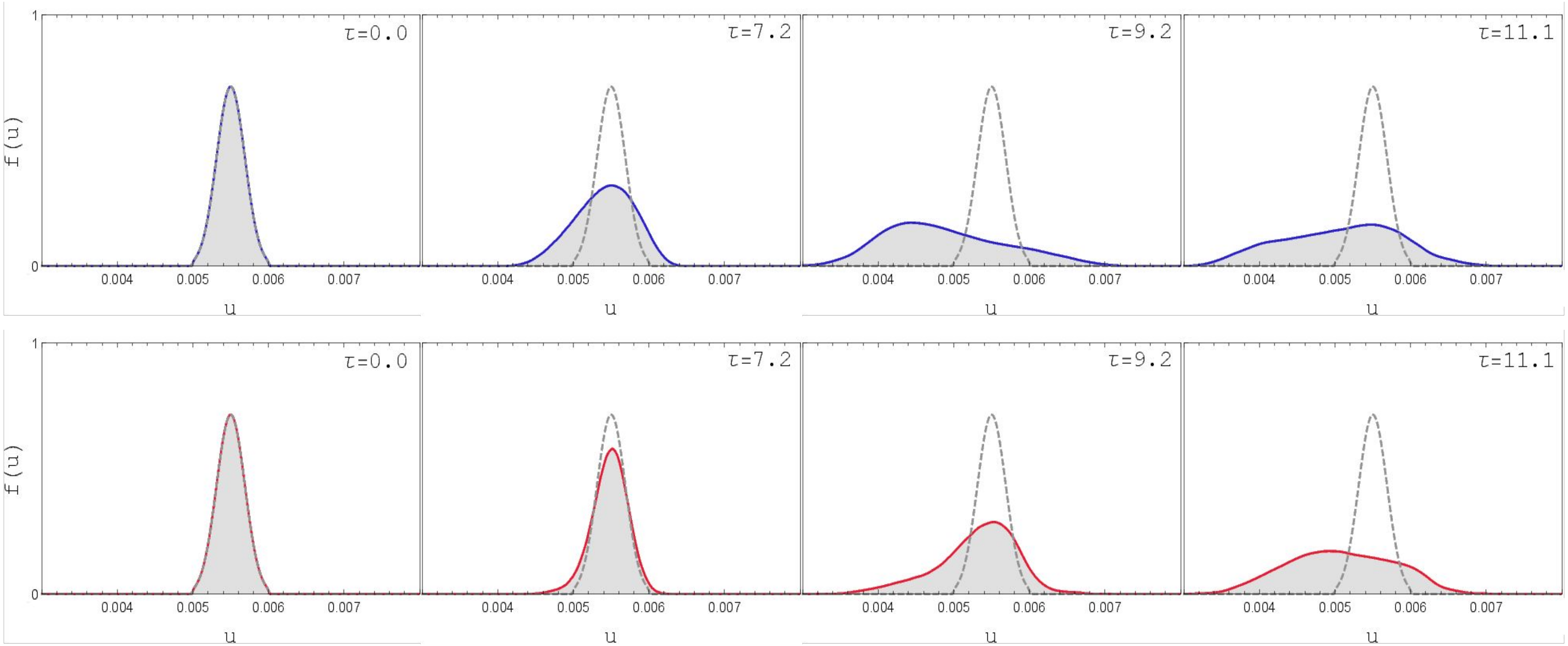
→ velocity distribution quickly relaxes



Multimode: convective regime - modes

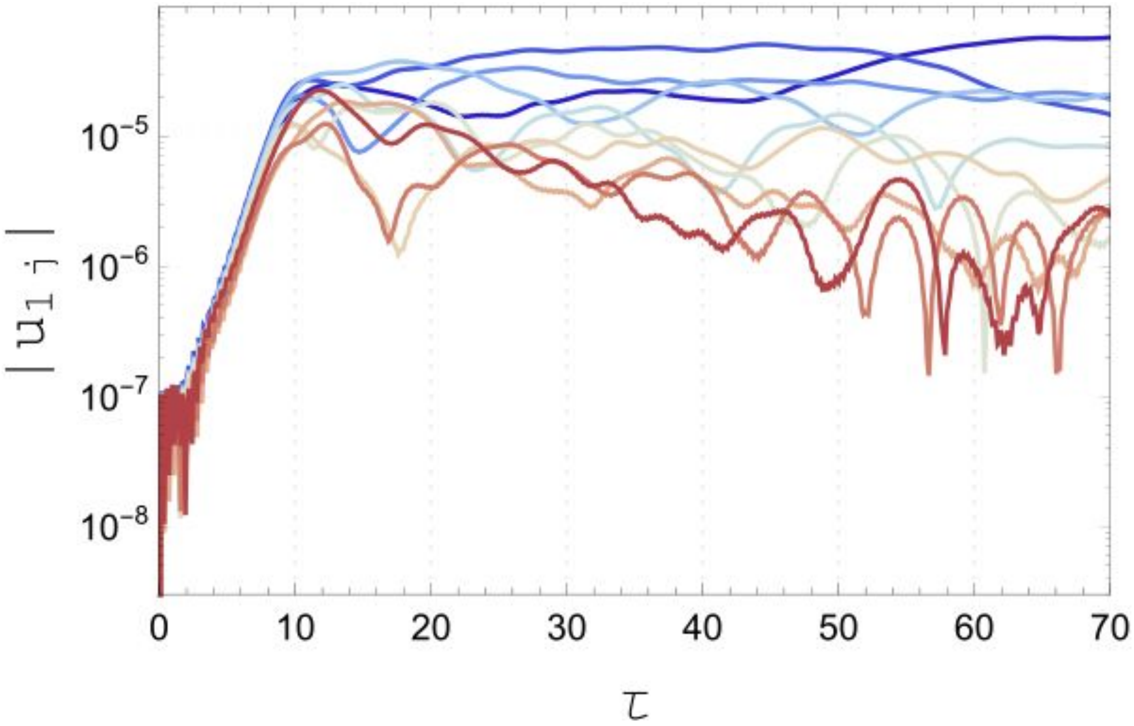


Multimode: convective regime - f(v)



Multimode: convective regime - backreaction

Fourier components of the background plasma excited velocity



Outlook

- Extend the simulations to linearly stable modes
- Different initial conditions (bunched beam?)
- Relax the cold plasma approximation
- Extension to 2D

Thank you for your attention!