Relativistic Dynamics of Stars around the Galactic Centre

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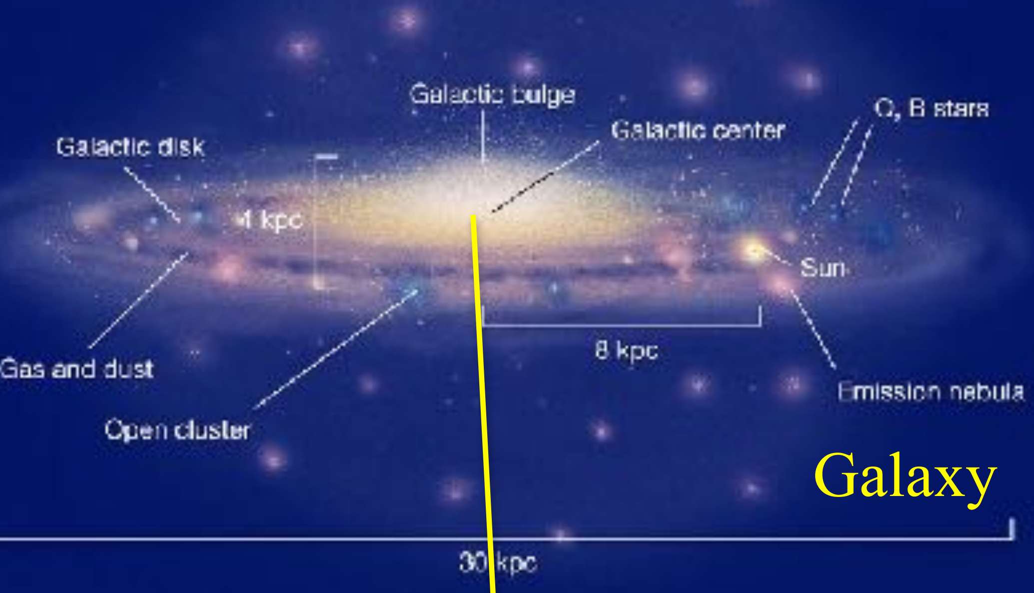
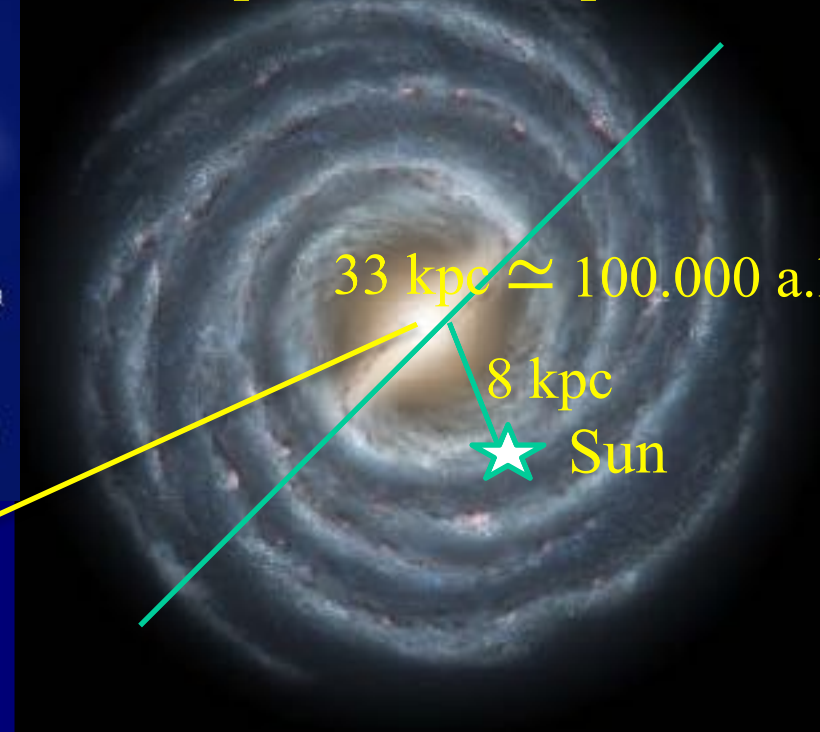
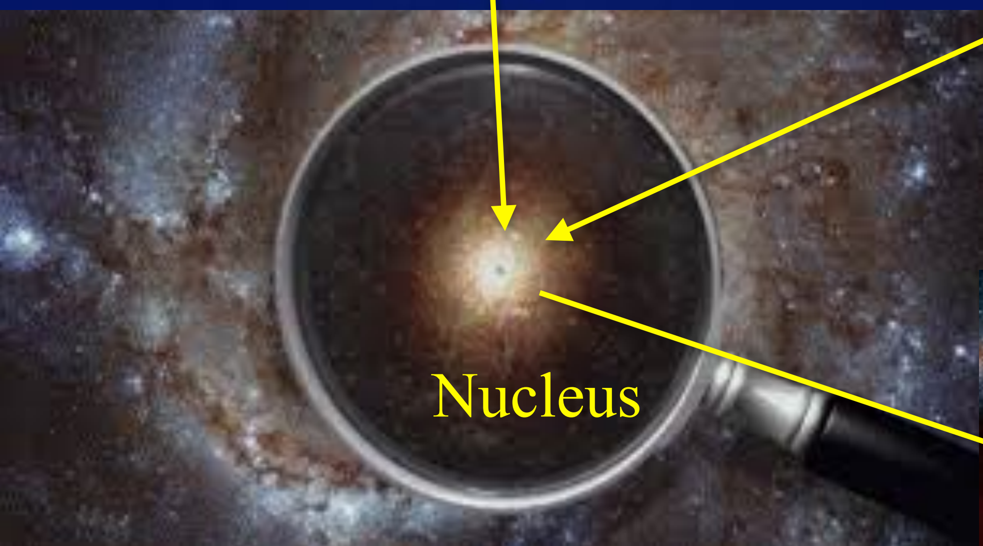
March 17 2022

SAPIENZA
UNIVERSITÀ DI ROMA

From kpc *down* to mpc scales







Whole galaxy: 50 kpc
Bulge: few kpc, non spherical
Nucleus: few pc, spherical
SMBH: 1/10 AU

R. Penrose 50%
GB

A. Ghez 25%
USA

R. Genzel 25%
Germany

Nobel prizes for Physics 2020

Penrose
1965

[17] I. S. Shklovsky, Cosmic Radio Waves (Harvard University Press, Cambridge, Massachusetts, 1960), pp. 271-316.
[18] C. Hazard, M. B. Mackey, and A. J. Shimmins, Nature 197, 1037 (1963).
[19] M. Schmidt, Nature 197, 1040 (1963).
[20] A. T. Moffet, Science 146, 764 (1964).
[21] M. Ryle and A. Sandage, Astrophys. J. 139, 419 (1964).
[22] I. S. Shklovsky, reference 17, pp. 369, 372.
[23] E. Fermi, Phys. Rev. 75, 1169 (1949).
[24] E. N. Parker, Phys. Rev. 109, 1328 (1958).
[25] T. H. Stix, Plasma Physics Laboratory Report No. MATT-239, Princeton University, 1964 (unpublished).
[26] R. W. Fredricks, F. L. Scarf, and W. Bernstein, to be published.
[27] V. Sarabhai, J. Geophys. Res. 68, 1555 (1963).
[28] E. N. Parker, Astrophys. J. 133, 1014 (1961); Interplanetary Dynamical Processes (John Wiley & Sons, Inc., New York, 1963).
[29] T. Gold, Gas Dynamics of Cosmic Clouds, edited by H. C. van de Hulst and J. M. Burgers (North-Holland Publishing Company, Amsterdam, 1955).
[30] E. N. Parker, Space Sci. Rev. 1, 62 (1962).
[31] M. Neugebauer and C. W. Snyder, Science 138, 1095 (1962).
[32] A. Bonetti, H. S. Bridge, A. J. Lazarus, B. Rossi, and F. Scherb, J. Geophys. Res. 68, 4017 (1963).
[33] E. J. Smith, L. Davis, P. J. Coleman, and C. P. Sonett, Science 139, 1099 (1962).
[34] N. F. Ness, C. S. Scearce, and J. B. Seek, J. Geophys. Res. 69, 3531 (1964).
[35] P. J. Coleman, L. Davis, and C. P. Sonett, Phys. Rev. Letters 5, 43 (1960).
[36] D. A. Bryant, T. L. Cline, U. D. Desai, and F. B. McDonald, J. Geophys. Res. 67, 4983 (1962).
[37] A similar increase in the low-energy proton intensity, in coincidence with the passage of a blast wave, was observed on 12 November 1960 [J. F. Steljes, H. Carmichael, and K. G. McCracken, J. Geophys. Res. 66, 1363 (1961)]. The increases proton intensity may have been partly or wholly of interplanetary origin, but the increase was of such a form that it is not possible to rule out a solar origin.

GRAVITATIONAL COLLAPSE AND SPACE-TIME SINGULARITIES

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(Received 18 December 1964)

The discovery of the quasistellar radio sources has stimulated renewed interest in the question of gravitational collapse. It has been suggested by some authors[1] that the enormous amounts of energy that these objects apparently emit may result from the collapse of a mass of the order of $(10^6-10^8)M_\odot$ to the neighborhood of its Schwarzschild radius, accompanied by a violent release of energy, possibly in the form of gravitational radiation. The detailed mathematical discussion of such situations is difficult since the full complexity of general relativity is required. Consequently, most exact calculations concerned with the implications of gravitational collapse have employed the simplifying assumption of spherical symmetry. Unfortunately, this precludes any detailed discussion of gravitational radiation—which requires at least a quadripole structure.

The general situation with regard to a spherically symmetrical body is well known.[2] For a sufficiently great mass, there is no final equilibrium state. When sufficient thermal energy has been radiated away, the body contracts and continues to contract until a physical singularity is encountered at $r=0$. As measured by local comoving observers, the body passes within its Schwarzschild radius $r=2m$. (The densities at which this happens need not be enormously high if the total mass is large enough.) To an outside observer the contraction to $r=2m$ appears to take an infinite time. Nevertheless, the existence of a singularity presents a serious problem for any complete discussion of the physics of the interior region.

The question has been raised as to whether this singularity is, in fact, simply a property of the high symmetry assumed. The matter collapses radially inwards to the single point at the center, so that a resulting space-time catastrophe there is perhaps not surprising. Could not the presence of perturbations which destroy the spherical symmetry alter the situation drastically? The recent rotating solution of Kerr[3] also possesses a physical singularity, but since a high degree of symmetry is still present (and the solution is algebraically special), it might again be argued that this is not representative of the general situation.[1] Collapse without assumptions of symmetry[4] will be discussed here.

Collapse *without* assumption of *symmetry*

Curiosity: what did Penrose *really* think?

…It will be shown that, after a certain critical condition has been fulfilled, deviations from spherical symmetry cannot prevent space-time singularities from arising.

If, as seems justifiable, actual physica1 singularities in space-time ***are not to be permitted to occur,*** *the conclusion would appear inescapable that inside such a collapsing object at least one of the following holds:*

(a) Negative local energy occurs.
(b) Einstein's equations are violated.
(c) The spacetime manifold is incomplete.
(d) The concept of space-time loses its meaning at very high curvatures — possibly because of quantum phenomena.

In fact (a), (b), (c), and (d) are somewhat interrelated, the distinction being partly one of attitude of mind. …

The SgrA* $4x10^6 M_\odot$ Central "Massive Black Hole"

S14: 47.3 yr period,
per. dist. 90 AU ~ 1050 R_S

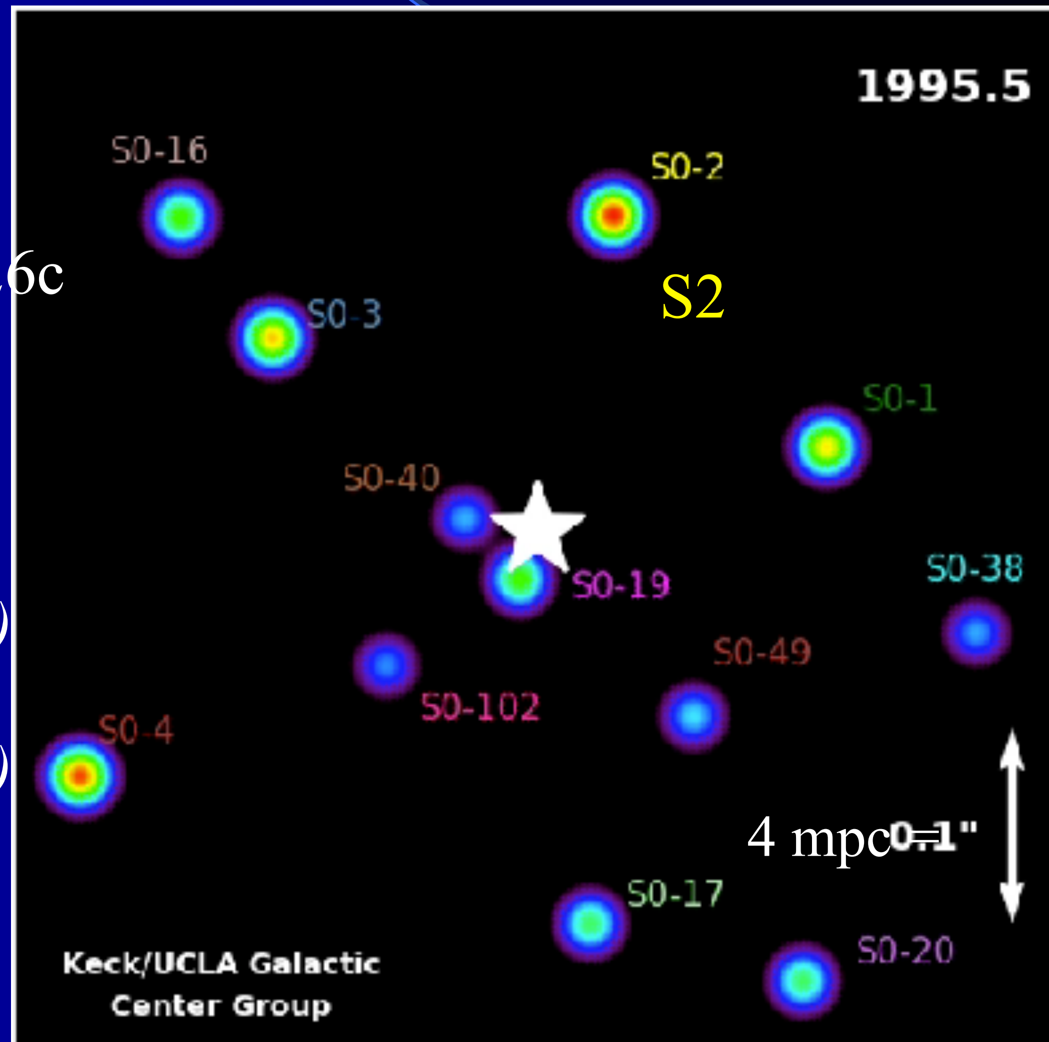
S2: 15.56 yr period;
per. dist. 120 AU ~ 1400 R_S
per. speed ~ 7650 km/s ~ 0.026c

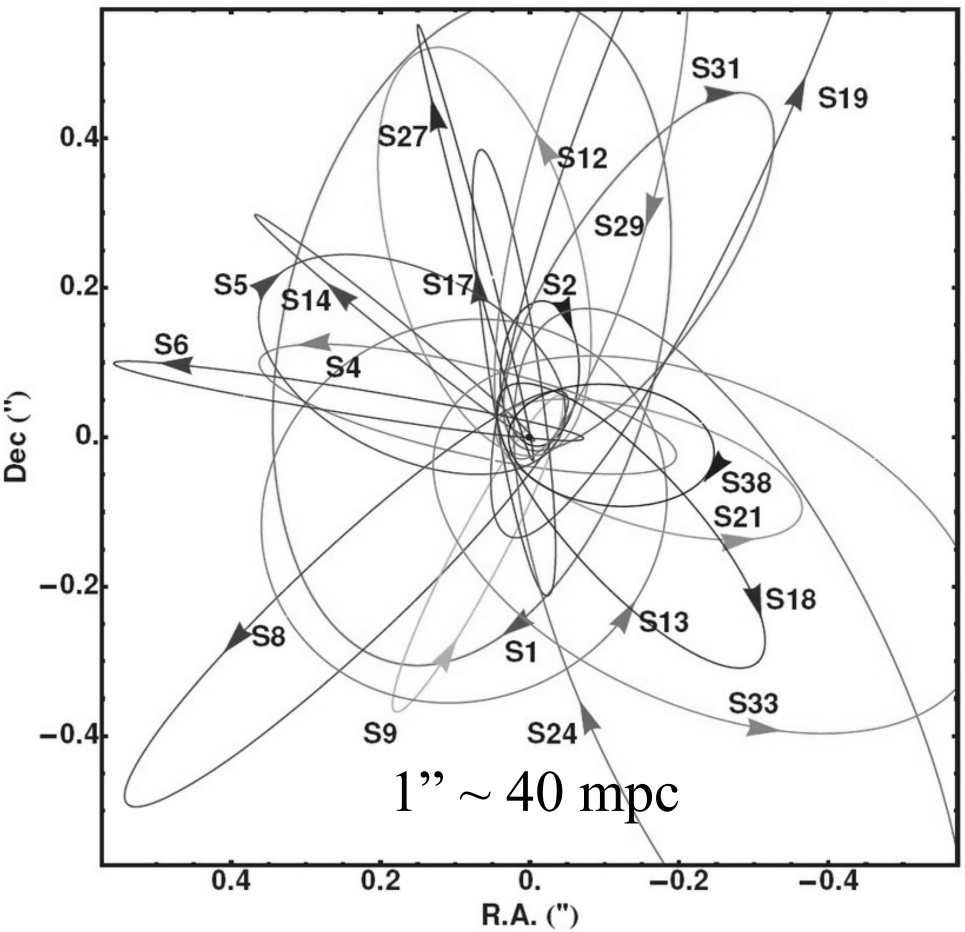
$\alpha \sim 0.0009$; $\beta \sim 0.026$

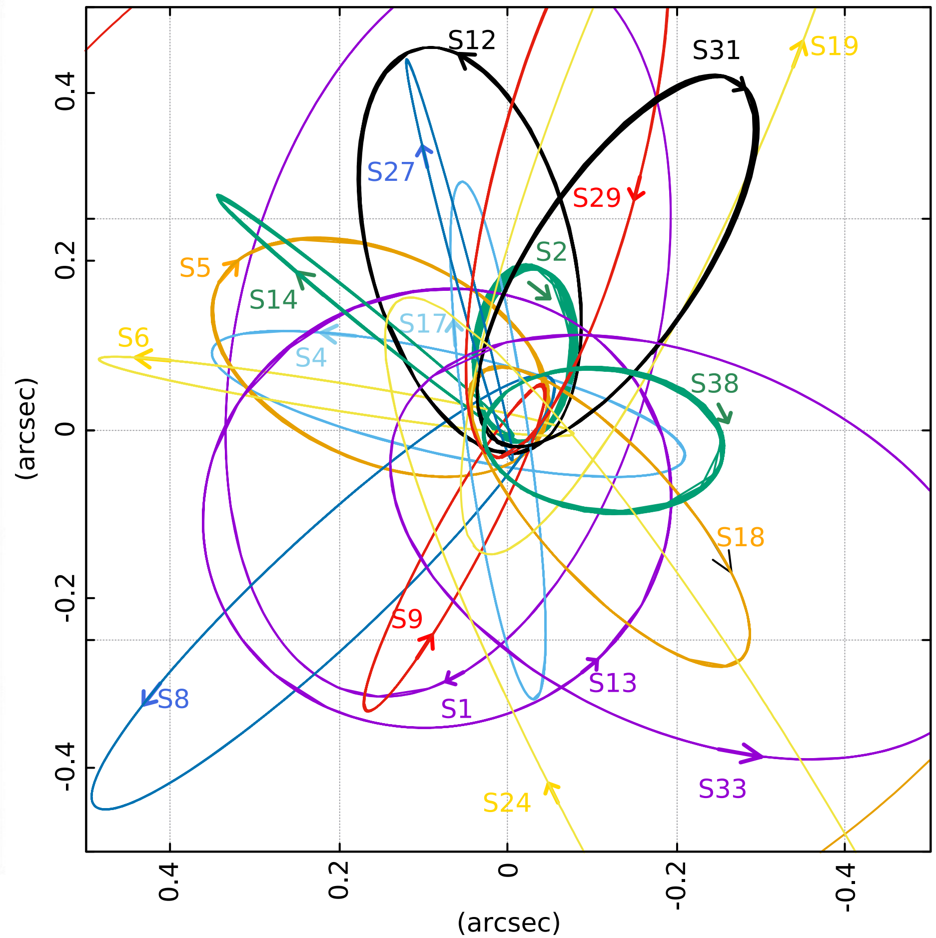
Evidence of gravitational red shift for S2
(*Gravity Collaboration, 2018*)
and of orbital precession
(*Gravity Collaboration, 2020*)

In a PN scheme $f = 0.9$
(f =0 Newtonian; f =1 GR)

About 20 yr data







real observations

(*Gravity collaboration*)

numerical integrations

Relativistic dynamics around the Galactic nucleus

from Merritt's book: Dynamics and Evolution of Galactic Nuclei

Table 4.1 Orbital parameters of the S-stars [192, 193]. The reference plane is the plane of the sky; the nodal line is the north celestial pole. A distance 8.33 kpc to Sgr A* is assumed, i.e., 1″ corresponds to 40.4 mpc (milliparsecs). The assumed mass of the SBH is $M_\bullet = 4.3 \times 10^6 M_\odot$.

yellow = min; green = max

Star	a[mpc]	e	i [°]	Ω [°]	ω [°]	P[yr]
S1	20.5 ± 1.13	0.496 ± 0.028	120.82 ± 0.46	341.61 ± 0.51	115.3 ± 2.5	132 ± 11
S2	5.03 ± 0.04	0.883 ± 0.003	134.87 ± 0.78	226.53 ± 0.72	64.98 ± 0.81	15.8 ± 0.11
S4	12.0 ± 0.77	0.406 ± 0.022	77.83 ± 0.32	258.11 ± 0.30	316.4 ± 2.9	59.5 ± 2.6
S5	10.1 ± 1.70	0.842 ± 0.017	143.7 ± 4.7	109 ± 10	236.3 ± 8.2	45.7 ± 6.9
S6	17.6 ± 6.18	0.886 ± 0.026	86.44 ± 0.59	83.46 ± 0.69	129.5 ± 3.1	105 ± 34
S8	16.6 ± 0.16	0.824 ± 0.014	74.01 ± 0.73	315.90 ± 0.50	345.2 ± 1.1	96.1 ± 1.6
S9	11.8 ± 2.10	0.825 ± 0.020	81.00 ± 0.70	147.58 ± 0.44	225.2 ± 2.3	58 ± 9.5
S12	12.4 ± 0.32	0.900 ± 0.003	31.61 ± 0.76	240.4 ± 4.6	308.8 ± 3.8	62.5 ± 2.3
S13	12.0 ± 0.48	0.490 ± 0.023	25.5 ± 1.6	73.1 ± 4.1	248.2 ± 5.4	59.2 ± 3.8
S14	10.3 ± 0.40	0.963 ± 0.006	99.4 ± 1.0	227.74 ± 0.70	339.0 ± 1.6	47.3 ± 2.9
S17	12.6 ± 0.16	0.364 ± 0.015	96.44 ± 0.18	188.06 ± 0.32	31945 ± 3.2	63.2 ± 2.0
S18	10.7 ± 3.23	0.759 ± 0.052	116.0 ± 2.7	215.2 ± 3.6	151.7 ± 2.9	50 ± 16
S19	32.2 ± 2.59	0.844 ± 0.062	73.58 ± 0.61	342.9 ± 1.2	153.3 ± 3.0	260 ± 31
S21	8.61 ± 1.66	0.784 ± 0.028	54.8 ± 2.7	252.7 ± 4.2	182.6 ± 8.2	35.8 ± 6.9
S24	42.8 ± 7.19	0.933 ± 0.010	106.30 ± 0.93	4.2 ± 1.3	291.5 ± 1.5	398 ± 73
S27	18.3 ± 3.15	0.952 ± 0.006	92.91 ± 0.73	191.90 ± 0.92	308.2 ± 1.8	112 ± 18
S29	16.0 ± 13.53	0.916 ± 0.048	122 ± 11	157.2 ± 2.5	343.3 ± 5.7	91 ± 79
S31	12.0 ± 1.78	0.934 ± 0.007	153.8 ± 5.8	103 ± 11	314 ± 10	59.4 ± 9.2
S33	16.6 ± 3.56	0.731 ± 0.039	42.9 ± 4.5	82.9 ± 5.9	328.1 ± 4.5	96 ± 21
S38	5.62 ± 1.66	0.802 ± 0.041	166 ± 22	286 ± 68	203 ± 68	18.9 ± 5.8
S66	48.9 ± 5.09	0.178 ± 0.039	135.4 ± 2.6	96.8 ± 2.9	106 ± 6.3	486 ± 41
S67	44.2 ± 4.12	0.368 ± 0.041	139.9 ± 2.3	106.0 ± 6.1	215.2 ± 4.8	419 ± 19
S71	42.9 ± 30.9	0.844 ± 0.075	76.3 ± 3.6	34.6 ± 1.5	331.4 ± 7.1	399 ± 283
S83	113. ± 9.45	0.657 ± 0.096	123.8 ± 1.3	73.6 ± 2.1	197.2 ± 3.5	1700 ± 205
S87	50.9 ± 6.50	0.423 ± 0.036	142.7 ± 4.4	109.9 ± 2.9	41.5 ± 3.7	516 ± 44

Some reference scales

$M_{BH} \sim 4.4\text{x}10^6\, M_\odot$

Pericenters r_p: ~ from less than 1 mpc to few mpc
Schwarschild radius $r_S \sim 4.3\text{x}10^{-7}$ pc ~ 0.08 AU
BH Influence radius $r_i \sim 2.5$ pc ~ $6\text{x}10^6\, r_S$

$$r_S < r_p << r_i$$

Are we really watching a SMBH?

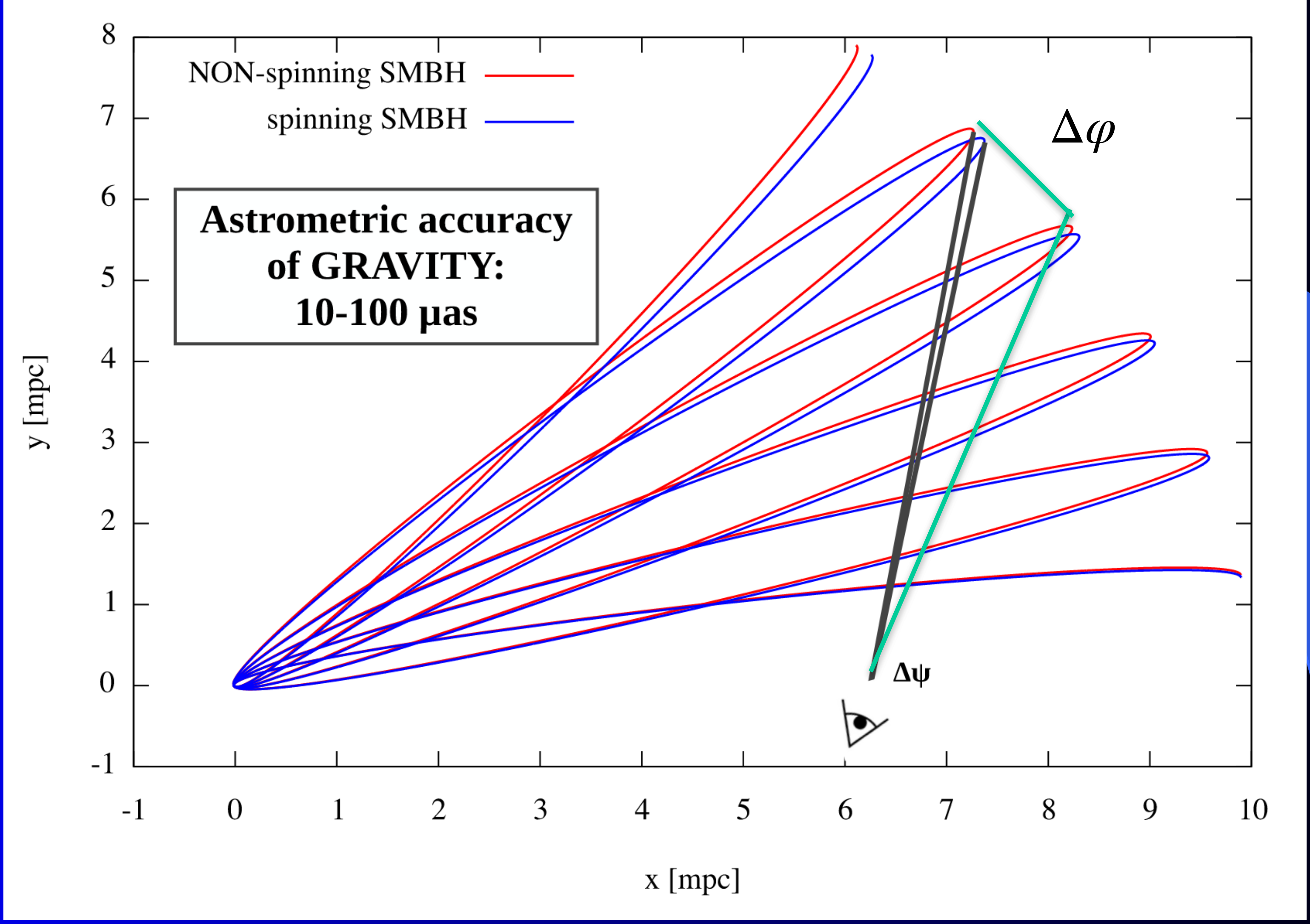
Very likely, but not certain, in principle.

Alternatives:

- an ultra dense star cluster
- a boson star or a gravastar
- A strong cusp of dark matter

In strong gravitational field both gravitational *redshift* and orbital *precession* are expected

For the S2 star both *redshift* and *precession* actually found



Non rotating (Schwarzschild) black hole

$$\delta\varphi = \frac{4}{\sqrt{r_S(u_3 - u_1)}} \int_0^{\pi/2} \frac{d\varphi}{\sqrt{1 - k^2 \sin^2 \varphi}} - 2\pi$$

$$k^2 = \frac{u_2 - u_1}{u_3 - u_1} \geq 0,$$

depends on the orbit.

At lowest order, the elliptic integral leads to the known Einstein-Schwarzschild precession per orbit

$$\delta\varphi = \frac{6\pi G}{c^2} \frac{m_\bullet}{a(1 - e^2)}$$

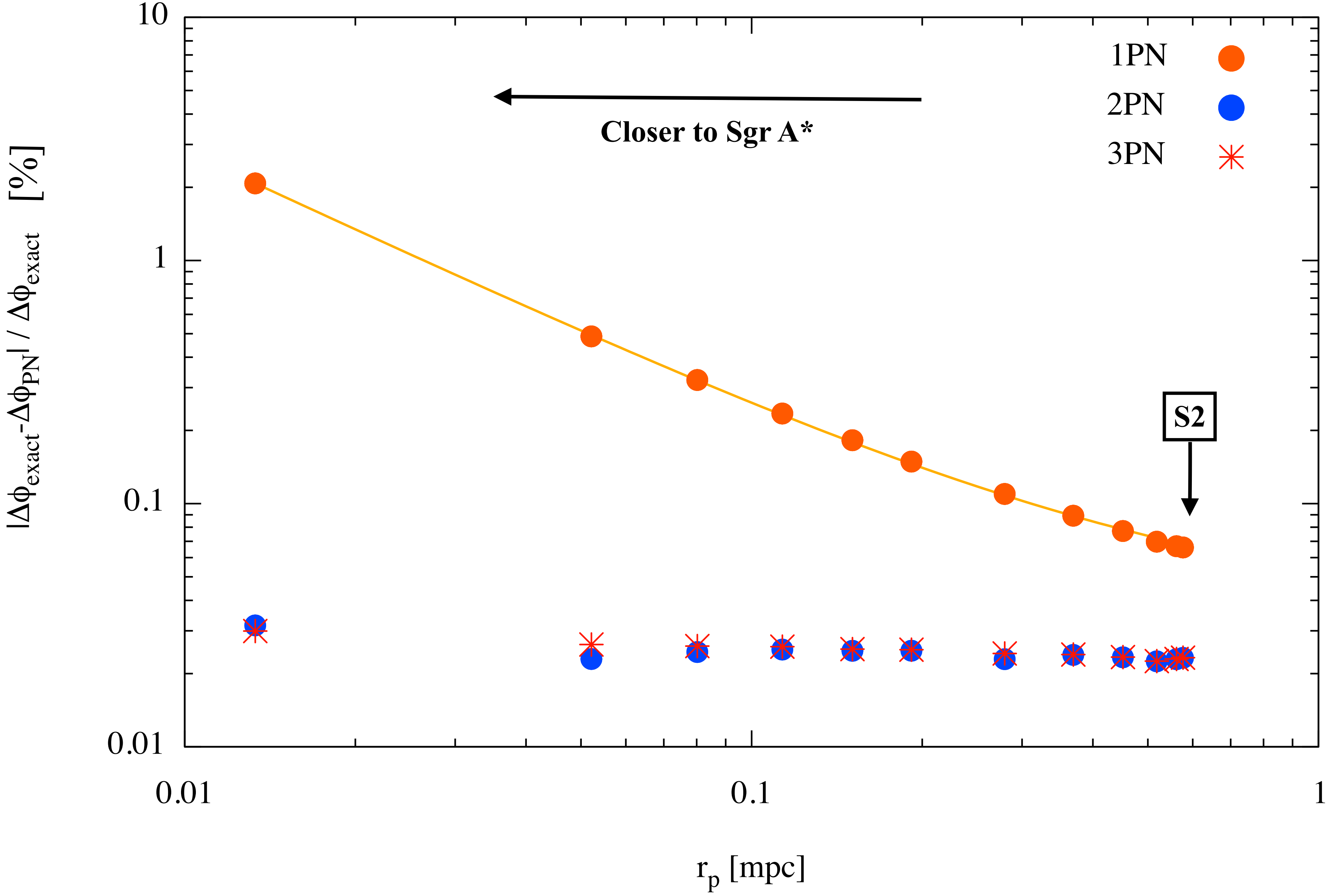
Observed value for S2 gives ~12’ compatible with 10% accuracy with the above. Can we really state about GR validity?

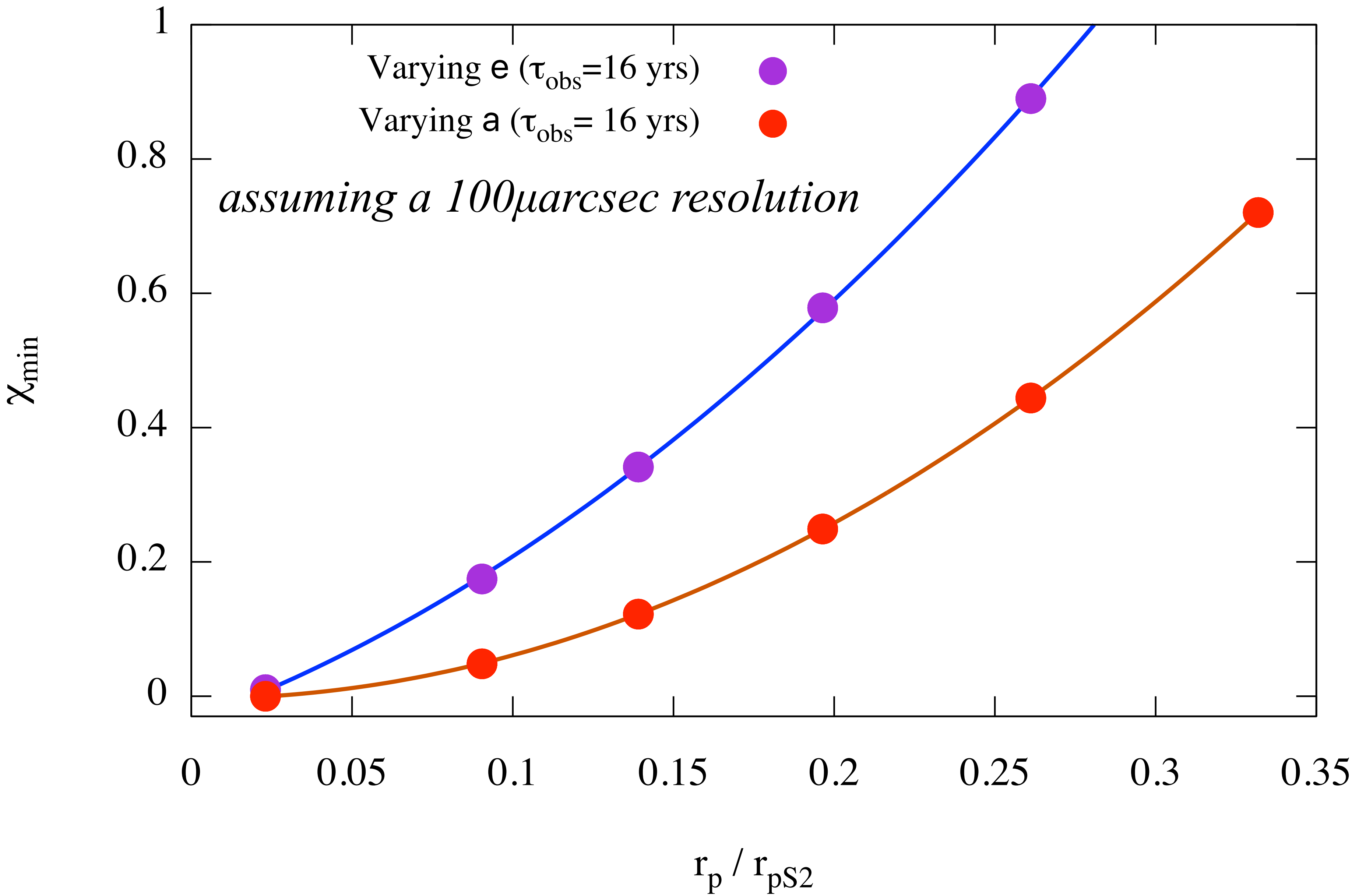
Issues:

- 1st order term is not enough
- Precession *and* red shift must be coupled
- BH may have a *spin (Kerr black hole)*

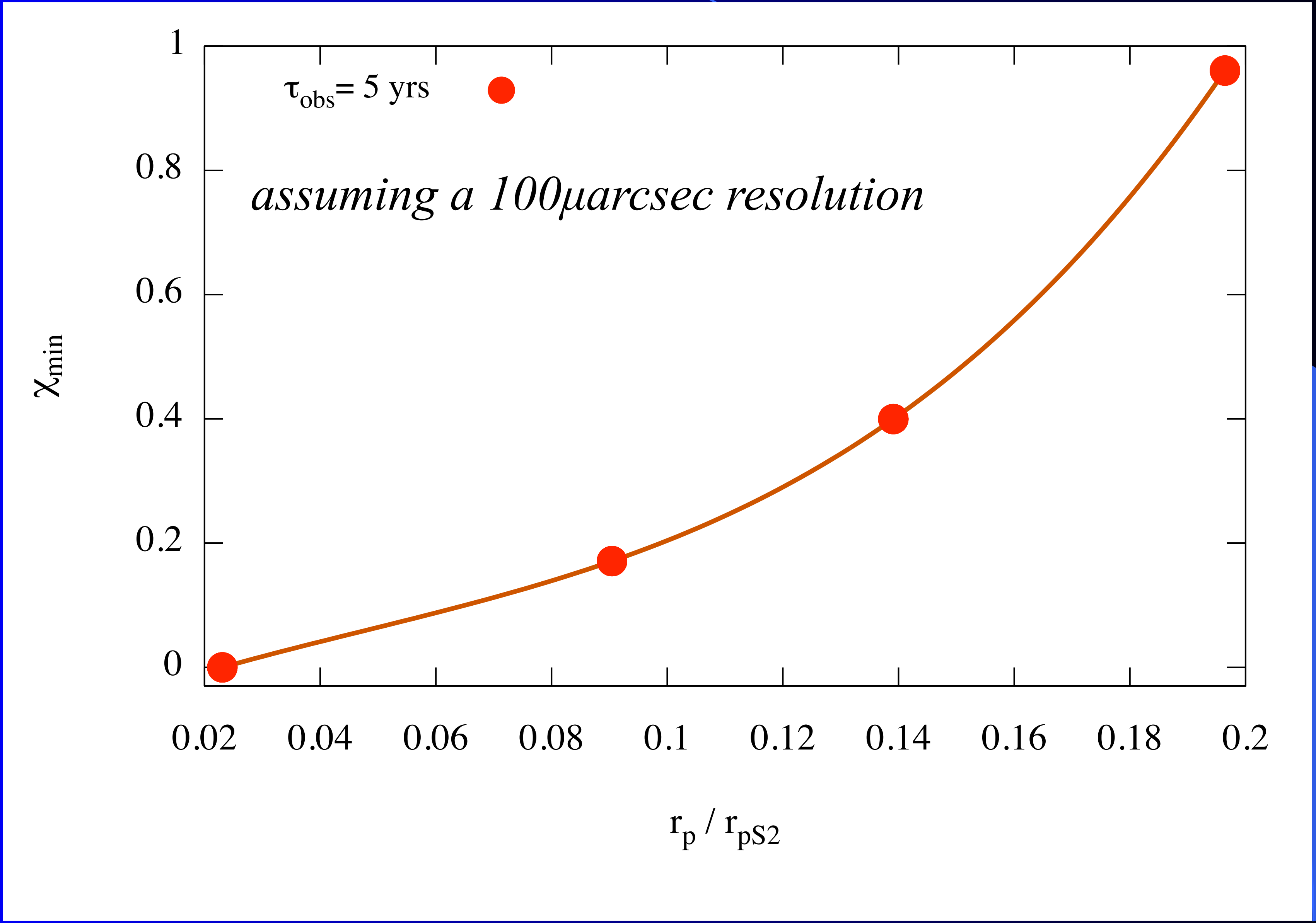
$$\mathbf{S}_\bullet = (G m_\bullet^2 / c)\chi \mathbf{s}_\bullet$$

$s_\bullet$ is the spin versor, ranging from *up* to *down*


10
1
0.1
0.01
0.01
0.1
1
$|\Delta\phi_{exact}-\Delta\phi_{PN}| / \Delta\phi_{exact}$ [%]
r_p [mpc]
1PN
2PN
3PN
Closer to Sgr A*
S2


Varying e (τ_{obs}=16 yrs)
Varying a (τ_{obs}= 16 yrs)
assuming a 100μarcsec resolution
χ_{min}
r_p / r_{pS2}
0
0.2
0.4
0.6
0.8
1
0
0.05
0.1
0.15
0.2
0.25
0.3
0.35

To measure the SgrA* on a 5 yr time basis $r_p < S\ 2/5$, $a < S\ 2/5$, $P < 2.5$ yr



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