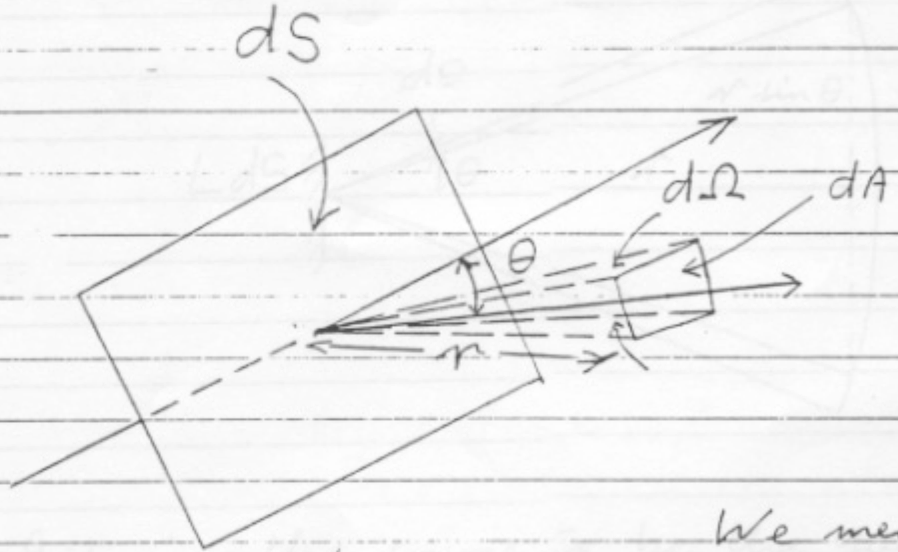
Methodology of Calibration Using an Irradiance Standard and Lambert Diffuser

The diffuser is equivalent to an extended source. We firstly consider a number of photometric concepts and units.



We measure the radiant power $d^2\phi$ (in W) emitted by a small plane source of area dS into a small solid angle $d\Omega$. The angle $d\Omega$ is given by

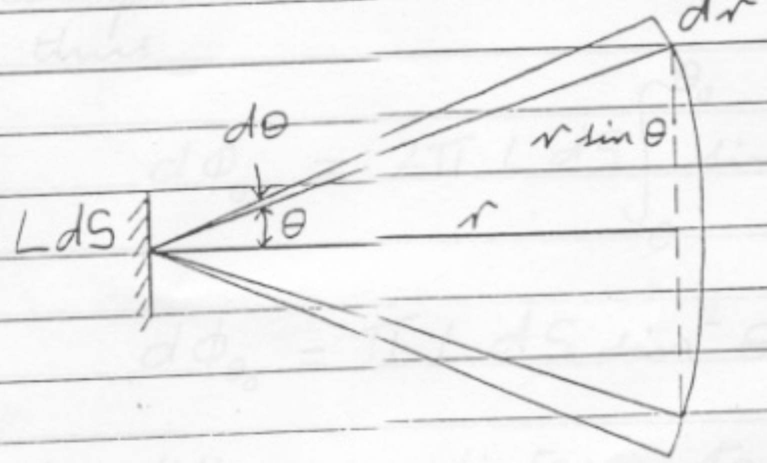
$$d\Omega = dA / r^2.$$

The radiance L (in $W\,m^{-2}\,sr^{-1}$) of the source is then defined by

$$d^2\phi = L\,dS \cos\theta\, d\Omega \quad \ldots \quad (1).$$

If the radiance of the source is independent of the angle of observation, it appears equally bright at all angles. Such a source is known as a Lambert source; the diffuser made of Russian Opal Glass is one example

We next calculate the total power radiated by a small Lambert source, with the aid of the following construction:-



Assuming the source to be the surface of an opaque body, we calculate the power radiated into a hemisphere. Firstly, the power radiated into a thin conical shell is derived. The solid angle corresponding to that thin shell is found by constructing a hemisphere of radius r. The shell intersects the sphere and defines an annulus on the sphere's surface. The width of the annulus is $dr = r\,d\theta$ and its radius is $r \sin\theta$. Thus, the area of the annulus is

$$dA = 2\pi r \sin\theta \, r \, d\theta .$$

The solid angle subtended by the annulus is $d\Omega = dA/r^2$,

$$d\Omega = 2\pi \sin\theta \, d\theta \quad \ldots \quad (2).$$

Combining equations 2 and 1, the power radiated into the solid angle $d\Omega$ is

$$d^2\phi = L\,dS\cos\theta \, . \, 2\pi \sin\theta \, d\theta .$$

The total power radiated into a cone of half angle θ_0 is thus

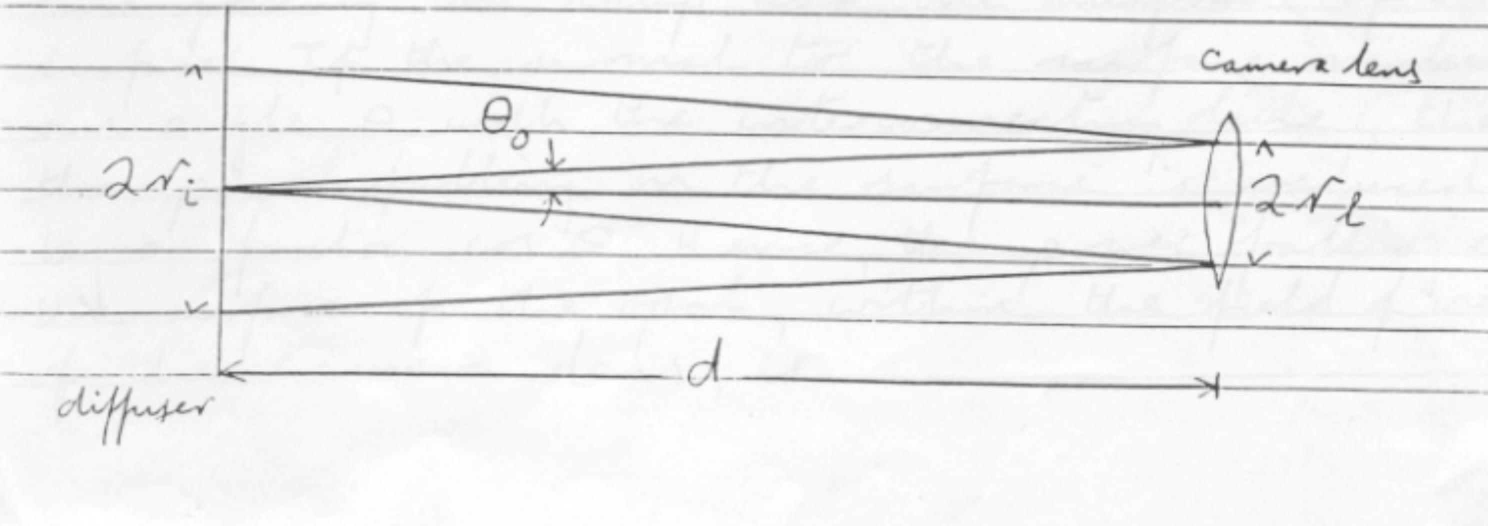
$$d\phi_{\theta_0} = 2\pi L\,dS \int_0^{\theta_0} \sin\theta \cos\theta \, d\theta ,$$

$$d\phi_{\theta_0} = \pi L\,dS \sin^2\theta_0 \quad . \quad . \quad . \quad . \quad (3).$$

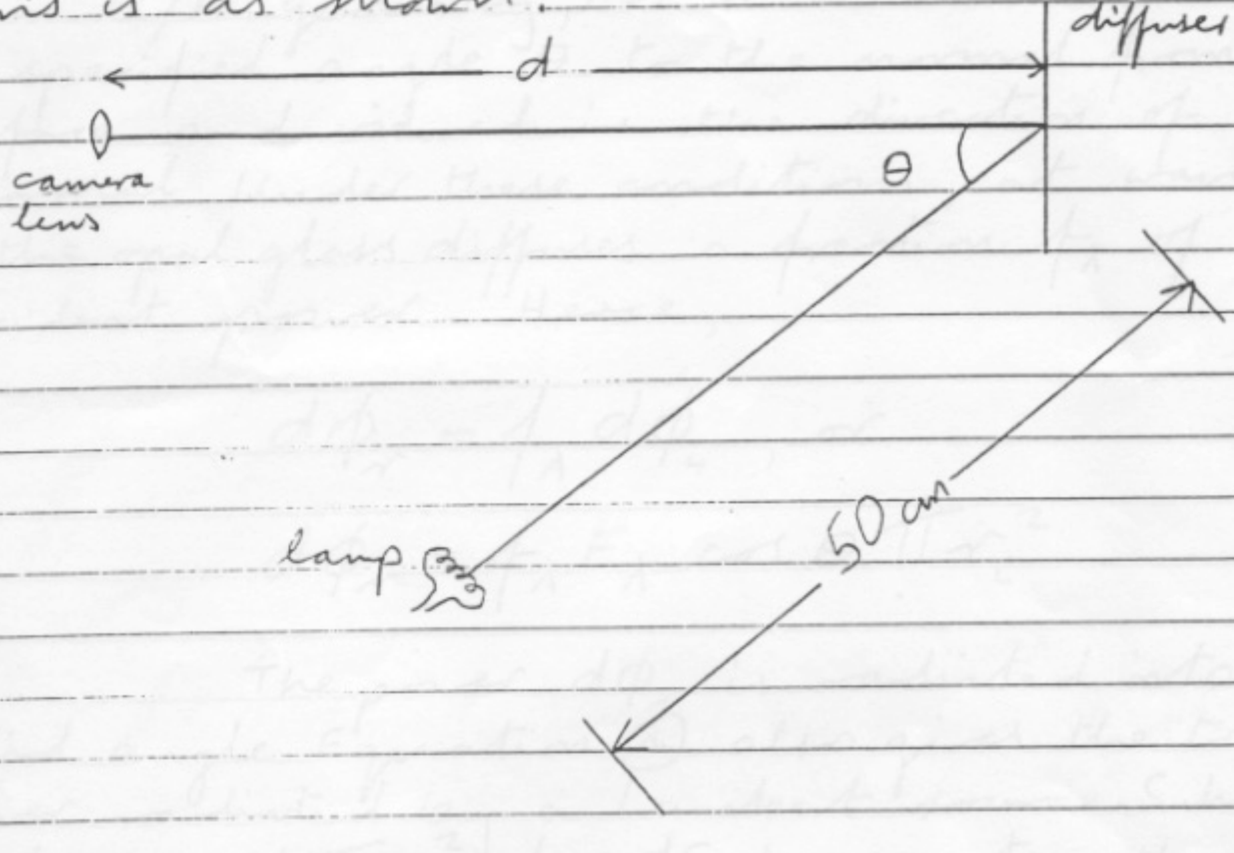
Since the diffuser radiates into a hemisphere, $\theta_0 = 90^\circ$. Thus the total power radiated by a Lambert source is

$$\boxed{d\phi_L = \pi L\,dS} \quad . \quad . \quad . \quad . \quad (4).$$

We now turn our attention to some details of the calibration system, in particular the camera lens and diffuser.



The camera lens images a portion of the diffuser of diameter $2r_i$ and collects light through an aperture of diameter $2r_L$. The layout of the lamp, opal glass and camera lens is as shown.



We can now calculate the radiance of the MS-20 opal glass. At wavelength λ, the calibrated lamp irradiates a surface 50 cm away with an irradiance E_λ ($W\,m^{-2}\,nm^{-1}$) – provided the normal to the surface is coincident with a line joining the lamp and the midpoint of the surface. If the normal to the surface makes an angle θ with the interconnecting line, then the power falling on the surface is reduced by a factor $\cos\theta$. Hence, the power falling on the surface of the opal, within the field of view of the camera lens, is

$$d\phi_i = E_\lambda \cos\theta \, \pi r_i^2 \quad \ldots \quad (5).$$

In general, the opal diffuser is calibrated – in terms of its reflectance properties – in 0°/θ° geometry, i.e. it is illuminated at a specified angle θ to the normal from its surface and viewed in the direction of the normal. Under these conditions, at wavelength λ the opal glass diffuses a fraction f_λ of the incident power. Hence,

$$d\phi_r = f_\lambda \, d\phi_i, \text{ or}$$

$$d\phi_r = f_\lambda E_\lambda \cos\theta \, \pi r_i^2 \quad \ldots \quad (6).$$

The power $d\phi_r$ is radiated into 2π solid angle. Equation (4) also gives the total power radiated by a Lambert source. Substituting L_λ for L and πr_i^2 for dS, by equating the two expressions in equations (6) and (4) we obtain the radiance of the opal, which is the desired calibration factor,

$$L_\lambda = f_\lambda E_\lambda \cos\theta / \pi \quad (W\,m^{-2}\,sr^{-1}\,nm^{-1})$$

Since, in our case, the diffuser has been calibrated (and is therefore used) at $\theta = 45°$.

$$L_\lambda = \frac{f_\lambda E_\lambda}{\sqrt{2}\,\pi} \quad (W\,m^{-2}\,sr^{-1}\,nm^{-1})$$

P.D.M.
(23/4/1990)