Physics of Burning Plasmas in Toroidal Magnetic Field Devices

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Nonlinear Dynamics of Burning Plasmas I

A burning plasma is a complex self-organized system where among the crucial processes to understand there are (turbulent) transport and fast ion/fusion product induced collective effects.
Nonlinear Dynamics of Burning Plasmas II

- Reactor relevant conditions require fast ion (MeV energies) and charged fusion products good confinement:
  - Identification of **burning plasma stability boundaries** with respect to energetic ion collective mode excitations and their **nonlinear dynamic behaviors** above the stability thresholds
  - Obvious impact on the **operation-space boundaries**, since collective losses may lead to significant wall loading and damaging of plasma facing materials in addition to degrading fusion performance

- Mutual interactions between collective modes and energetic ion dynamics with drift wave turbulence and turbulent transport should not deteriorate the thermonuclear efficiency:
  - MeV ion energy tails introduce a **dominant electron heating** and different weighting of the electron driven micro-turbulence w.r.t. present experiments
  - They also generate **long time-scale nonlinear behaviors** typical of self-organized complex systems
The roles of simulation and theory

- These phenomena can be analyzed, at least in part, in present day experiments and provide nice examples of mutual positive feedbacks between theory, simulation and experiment.

- In a burning plasma, however, unique features not reproducible in existing experiments are:
  
  - energetic ion power density profiles and characteristic wavelengths of the collective modes
  - local power balance dominated by electron heating (fast ions) and self-organization of radial profiles of the relevant quantities: consequence on turbulence spectra and turbulent transport

- Crucial roles of predictive capabilities based on numerical simulations as well as of fundamental theories for developing simplified yet relevant models, needed for insights into the basic processes

- Importance of using existing and future experimental evidences for modeling verification and validation
Outline

- Collective behaviors and fast ion transport: (see also B.N. Breizman Topical, Wed. 9/26)
  - The shear Alfvén fluctuation spectrum: Alfvén Eigenmodes and resonant modes
  - Fast ion transport: diffusion and avalanches

- Turbulence and Turbulent transport: (see also J.W. Connor Tutorial, Tue. 9/25)
  - Thermal ion transport: zonal flows/GAM and turbulence spreading
  - Angular momentum transport: spontaneous rotation (not this talk)
  - Thermal electron transport: dominant electron heating (not this talk)

- Mutual interactions between collective modes and energetic ion dynamics with drift wave turbulence and turbulent transport

- Examples of broader applications of fundamental physics in fusion science
Collective behaviors and fast ion transport
The role of shear Alfvén waves

- Collective behaviors due to energetic ions in burning plasmas: shear Alfvén (SA) waves play a crucial role:
  - Resonant wave particle interaction of ≈ MeV ions with SA inst. due to \( v_E \approx v_A \) (\( k_\parallel v_A \approx \omega_E \))
  - Group velocity is along B-field lines (\( \omega = k_\parallel v_A \)): particles stay in resonance

- Toroidal geometry plays a crucial role: SA waves propagate along B as in a 1D lattice and sample periodic potential structures with influence on SA spectrum and linear as well as non-linear dispersion

- Focus on non-linear dynamics and fast ion transport: conclusions largely apply to MHD modes
Shear Alfvén spectrum: continuum with gaps

- Frequency gaps are due to lattice symmetry breaking
- Linear theory reasonably well understood: few technical aspects need to be refined for more realistic comparisons with EXP
- Unified description: discrete gap modes vs. resonant (driven) continuum modes.
- Alfvén Eigenmodes (AE): weakly damped gap modes excited by fast ions; fixed frequency
- Energetic Particle Modes (EPM): fast ions drive overcomes continuum damping; resonant particle characteristic frequency
- Nonlinear dynamics and fast ion transport: reflect different nature of AE and EPM

Fast ion transports in burning plasmas

Phase space structures: fast ion resonant interactions with AE


- Transient losses $\approx \delta B_r / B$: resonant drift motion across the orbit-loss boundaries in phase space
- Diffusive losses $\approx (\delta B_r / B)^2$ above a stochastic threshold, due to stochastic diffusion in phase space across orbit-loss boundaries.

Lichtenberg & Lieberman 1983, Sp.-Ver. NY
Simulation results: strongly unstable 1D system

Creation of phase space structures changes the distribution function thereby permitting otherwise disallowed modes to grow (R.G.L. Vann, et al. 2005 Intl. Sherwood Conf.)
Fast ion transports in burning plasmas


- **Strong energetic particle redistributions** are predicted to **occur above the EPM excitation threshold** in 3D Hybrid MHD-Gyrokinetic simulations: S. Briguglio, F. Zonca and G. Vlad, Phys. Plasmas 5, 1321, (1998).
Avalanches and NL EPM dynamics

Zonca et al. IAEA, (2002)

Importance of toroidal geometry on wave-packet propagation and shape
Propagation of the unstable front

Gradient steepening and relaxation: spreading ... similar to turbulence
Fast ion transports in burning plasmas


- **Nonlinear Dynamics of Burning Plasmas**: energetic ion transport in burning plasmas has two components:
  
  - slow diffusive processes due to weakly unstable AEs and a residual component possibly due to plasma turbulence (Vlad et al. PPCF 47 1015 (2005); Estrada-Mila et al., submitted to Nucl. Fusion 2006).
  
  - rapid transport processes with ballistic nature due to coherent nonlinear interactions with EPM and/or low-frequency long-wavelength MHD: fast ion avalanches & experimental observation of Abrupt Large amplitude Events (ALE) on JT60-U (K. Shinohara et al. PPCF 46, S31 (2004))
Abrupt Large amplitude Events (ALE) in JT60-U


Courtesy of M. Ishikawa, K. Shinohara and JT60-U
Fast ion transport: 3D simulation and experiment

- Abrupt Large amplitude Events (ALE) in JT60-U:
  - $n = 1$ mode, $\beta_{H0} = 8\pi P_{H0}/B^2 \approx 3\%$;
  - linear growth rate $\gamma \approx 0.106\tau_{A0}^{-1}$;
  - half width of the pulse $\Delta t_{ALE} \approx 64.5\mu s$;
  - experimental range $\Delta t_{ALE} \approx 50 \div 200\mu s$;
  - energetic particle profiles compare well before and after ALE burst

Fast ion transport: some broader applications

Convective amplification of the EPM wave-packet and ballistic particle transport in Avalanche process is described by the complex Ginzburg-Landau equation (GLE)

\[ \partial^2_{\xi} A_n = i \frac{\gamma_L}{D} \left( \frac{\Delta \gamma_L}{\gamma_L} + \frac{L_{NL}^2}{\gamma_L} \partial^2_{\xi} (\gamma_L |A_n|^2) \right) A_n + \frac{\Delta \omega}{D} A_n \]

\[ \xi = r - v_{gr} t \]

\[ \gamma_L \propto \alpha_H = -R_0 q^2 (d \beta_H / dr) \]

For Gaussian source function \( \alpha_H = \alpha_{H0} \exp[-(\xi - \xi_0)^2] \), the GLE reduces to its canonical form; for generalized stretched Gaussian distribution, i.e., \( \alpha_H = \alpha_{H0} \exp[-|\xi - \xi_0|^\mu] \) (\( 1 < \mu < 2 \)), the GLE is rewritten in terms of fractional derivative operators:

\[ \nabla^2 A = q^2 A - p^2 A \nabla^{2-\mu}|A|^2 \]

Fractional derivative Riesz Operator

\[ \nabla^{2-\mu}|A|^2 = \frac{1}{\Gamma(1-\mu)} \nabla \int_{-\infty}^{x} \frac{|A|^2(x_0)}{(x - x_0)^{2-\mu}} dx_0 \]

The fractional derivative GLE incorporates the key features of non-Gaussianity and long-range dependence in thresholded nonlinear dynamical systems.
Turbulence and Turbulent transport
Thermal ion transport
Thermal ion transport: role of $E \times B$ flows

- Better understood, compared to other transport channels
- **Ion Temperature Gradient** (ITG) Turbulence: Best Candidate for $\chi_i \gg \chi_{i,NEO}$
  - Internal Transport Barrier (ITB) when (roughly) $\omega_{E \times B} \gtrsim \gamma_L$
- With recent advances in **gyrokinetic codes**, simulation results begin to converge for simple cases, not only in numbers, but also in **underlying physics**.

- The effective up-shift of onset condition for large ion heat flux is caused by **Zonal Flows**

from Cyclone project
Zonal Flows are common in plasmas

Zonal Flows on Jupiter

Paradigm Change
P.H. Diamond, et al. 2005
PPCF 47, R35

ZFs peculiarities
- No direct radial transport
- No linear instability
- Turbulence driven
Zonal Flows regulate turbulence: effect on transport


- Transport is a local process which is predicted to scale as $\propto I$: confirmed by numerical simulations (Z. Lin, et al. 1999, PRL 83, 3645; ... 2004, PoP 11, 1099)


- Any size scaling of turbulent transport can be reduced to dependence of $I$ on $L$. 
Turbulence spreading: size scaling of transport

- Turbulence spreading: local level of turbulence fluctuation may not be due to local free energy source
- Turbulent transport depends on local turbulence intensity but it is intrinsically non-local: inadequacy of the mixing length paradigm $\chi \approx \gamma_L / k_\perp^2$
- Impact on machine operations and burning plasma performance

Z. Lin, et al. 04 *PRL* 88, 195004

- Turbulence spreading is mediated by nonlinear interactions:
  - Zonal Flows: importance of toroidal geometry (L. Chen, *et al.* 04, *PRL* 92, 075004; *PoP*’04; *PoP*’05)
- Importance of toroidal geometry and global equilibria (R.E. Waltz and J. Candy 04, *PoP* 12, 072303)
Turbulence spreading: some broader applications

- Turbulence spreading can be described by turbulence intensity evolution equations in the form (T.S. Hahm, P.H. Diamond, Z. Lin, et al. 04, PPCF 46, A323)

\[
\frac{\partial}{\partial t} I = \Gamma I + \frac{\partial}{\partial r} \left( D \frac{\partial}{\partial r} I \right) \quad \text{where} \quad \begin{cases} 
\Gamma = \gamma(r) - \alpha I^\beta \\
D = D_0 I^\beta 
\end{cases} \quad \text{with} \quad \begin{cases} 
\beta = 1 \text{ (weak turb.)} \\
\beta = 1/2 \text{ (strong turb.)}
\end{cases}
\]

- This equation can be derived from various types of closure theories (O.D. Gürcan, et al. 05, PoP 12, 032303): the form is that of a generalized Fisher equation.

- Turbulence spreading is ballistic for both weak and strong turbulence regimes: turbulence front speed scales like \((\Gamma D)^{1/2}\) (M. Yagi, et al. 06, PPCF 48, A409)

- Spreading is mediated by avalanches via gradient steepening and relaxation (L Villard, et al. 04, NF 42, 172; PPCF 46, B51; M. Yagi, et al. 06, PPCF 48, A409): qualitatively similar to fast ion avalanches.
High frequency $\mathbf{E} \times \mathbf{B}$ flows: Geodesic Acoustic Modes

- Toroidal geometry affects toroidally and poloidally symmetric $\mathbf{E} \times \mathbf{B}$ flows via geodesic curvature (finite compressibility): Geodesic Acoustic Mode (GAM) excitation at finite frequency (N. Winsor, et al. 1968, *PF* 11, 2448): $\omega \simeq (7/4 + T_e/T_i)^{1/2}(2T_i/m_i)^{1/2}/R$

- GAM are excited by drift wave (DW) turbulence and compete with Zonal Flows. However, they much weaker de-correlation effect on DW turbulence, due to finite frequency (T.S. Hahm, et al. 1999, *PoP* 6, 922)

- In low $q$ region, stationary Zonal Flows persist (higher GAM damping). Transport is consequently lower for lower $q$ value from gyro-fluid simulations (N. Miyato and Y. Kishimoto 04, *PoP* 11, 5557)

- Role of geodesics curvature in inhibiting turbulence suppression by ZFs has been pointed out for edge turbulence (B. Scott 03, *PLA* 320, 53)

- Linear dependence of average $\chi_i$ on inverse plasma current is observed in global GK PIC simulations (as in EXP!)

\[ \chi_i/\chi_{gB} = f(I) \]

\[ \chi_i^{\text{ml}}/\chi_{gB} = f(I) \]

\[ I_{N0}/I_N \]

P. Angelino, et al. 06, *PPCF* 48, 557
GAM continuous spectrum

In realistic plasmas: $T_e(r), T_i(r), q(r)$

- $\Rightarrow \omega^2_{GAM} \simeq 2T_i(r)/(m_iR_0^2) \left(7/4 + T_e(r)/T_i(r)\right) = \omega^2_{GAM}(r)$

- $\omega_{GAM}$ varies radially

- $\omega^2_{GAM}(r)$ forms a continuous spectrum

\[ \partial_r \delta J_r(r, t) = 0 \Rightarrow \text{BAE-GAM degeneracy} \]

\[ \frac{\partial}{\partial r} \left\{ N_0(r) \left[ \omega^2 - \frac{2(\gamma_iT_i + T_e)}{m_iR_0^2} (r) \right] \delta E_r \right\} = 0 \]

$\Rightarrow$ Singular solution at $\omega^2 = \omega^2_{GAM}(r)$

$\Rightarrow$ Generally $\partial_r \left( N_0(r)\Lambda^2(\omega)\delta E_r \right) = 0$ [Zonca&Chen PPCF 1996]

$\Rightarrow$ Similar to Alfvén resonance [Chen&Hasegawa POF 1974]
Kinetic GAM

- $\delta E_r$ singular at $r_0$ where $\omega^2 = \omega^{2\text{GAM}}$
  - $|k_r| \to \infty$ finite ion Larmor radius effects!
  - Linear mode conversion to Kinetic GAM (KGAM) $\Rightarrow$ propagating radially outward
  - Similar to, e.g., Kinetic Alfvén Wave (KAW) [Hasegawa & Chen POF 1976]

Dispersion relation of KGAM

- Assuming $1 \gg k_r^2 \rho_i^2 \gg 1/q^2$ and including higher order $k_r^2 \rho_i^2$ corrections in GAM
  - $\Rightarrow \delta f_i$ expansion up to order $O[(\omega_d/\omega)^4]$ terms
  - $\omega^2 = \omega_{GAM}^2(r) + Cb_i$
    - $C > 0$, $b_i = k_r^2 \rho_i^2$
\[ C > 0, \text{ complicated expression, lengthy: can be obtained from } [\text{Zonca, Chen, Santoro, Dong PPCF 1998}] \text{ as a limiting case, using the degeneracy of BAE and GAM spectra} [\text{Zonca}&\text{Chen 2006}] \text{ in the long wavelength limit} \]

\[ \Rightarrow b_i > 0 \text{ when } \omega^2 > \omega_{GAM}^2 \text{: propagation} \]

\[ \Rightarrow b_i < 0 \text{ when } \omega^2 < \omega_{GAM}^2 \text{: cut-off} \]

- Radial wave equation and mode conversion of GAM

- In nonuniform plasma \( k_r = -i \partial / \partial r \)

\[ \Rightarrow \text{Radial wave equation} \]

\[ \frac{\partial}{\partial r} \left\{ N_0(r) \left[ \rho_i^2(r) C(r) \frac{\partial^2}{\partial r^2} + \omega^2 - \omega_{GAM}^2(r) \right] \delta E_r \right\} = 0 \]

\[ \Rightarrow \text{Same as that for mode conversion of shear Alfvén wave} [\text{Hasegawa}&\text{Chen POF 1976}] \]

Evidence of outward propagating GAM in JFT-2M [Ido etal. PPCF 2006]
Nonlinear excitations of Kinetic GAM

- Coherent 3-wave interactions [Chen, Lin, White POP 2000]
  - Linear parametric instability: Pump DW $\oplus$ KGAM $\Leftrightarrow$ Lower sideband

- Pump DW (ITG) $\Rightarrow$ $\delta \Phi_0 : (\omega_0, k_0)$

- Zonal Mode (KGAM) $\Rightarrow$ $\delta \Phi_\zeta : (\omega_\zeta, k_\zeta)$ and Pump DW modulation

\[
\begin{align*}
\delta \Phi_0 &= \left[ A_0 e^{-in_0 \zeta} \sum_m e^{im\theta - i\omega_0 t} \Phi_0(n_0q - m) + c.c. \right] \\
\delta \Phi_\zeta &= \left[ A_\zeta e^{ik_\zeta r - i\omega_\zeta t} + c.c. \right]
\end{align*}
\]
Nonlinear excitation favors short (zonal) radial wavelengths ⇒ KGAM is excited  
(L. Chen et al. Sherwood and US-TTF 2007; submitted to PRL)

Nonlinear dynamics

- $\delta \Phi_-, \delta \Phi_\zeta$ growth ⇒ depletes the pump $\delta \Phi_0$

⇒ 3-wave nonlinear system with prey-predator self-regulation

\[
\begin{align*}
(d/dt - \gamma_0 n) \delta A_{0n} &= -\frac{c}{B} k_{\theta n} k_g \delta A_{-n}^* \delta A_\zeta \\
(d/dt + \gamma_{-n}) \delta A_{-n} &= \frac{c}{B} k_{\theta n} k_g \delta A_{0n}^* \delta A_\zeta \\
(d/dt + \gamma_g) \delta A_\zeta &= \frac{c}{2B} \alpha_i k_{\theta n} k_g \delta A_{0n} \delta A_{-n}
\end{align*}
\]

⇒ Driven-dissipative system: limit cycle, period doubling, route to chaos, strange attractor [Wersinger et al. PRL 1980].

Broader Implications of plasma physics for non-linear dynamics
Mutual interactions of collective modes with drift wave turbulence
Physics issues behind fluctuations non-linear interactions

- Interaction between collective modes and thermal plasma turbulence:
  - collective modes due to energetic particles
  - plasma turbulence due to thermal components

- Intrinsic separation of spatial scales (orbit size) in the free energy source: interaction occurs
  - if the time scales become comparable such as for Alfvén ITG (e.m. ITG)
  - if mediated by the 3rd entities such as zonal structures: zonal flows, fields, corrugations of radial profiles
Mutual interactions of collective modes with DW turbulence

- **E.m. plasma turbulence:** theory predicts excitation of Alfénic fluctuations in a wide range of mode numbers near the low frequency accumulation point of s.A. continuum, \( \omega \simeq \left( \frac{7}{4} + \frac{T_e}{T_i} \right)^{1/2} \left( \frac{2T_i}{m_i} \right)^{1/2} / R \) (F. Zonca, L. Chen, *et al.* 96, *PPCF* 38, 2011; ... 99, *PoP* 6, 1917):
  - by energetic ions at long wavelength: finite Beta AE (BAE)/EPM
  - by thermal ions at short wavelength: Alfvén ITG

- **Magnetic flutter:** may be relevant for electron transport (B.D. Scott 2005, *NJP* 7, 92; V. Naulin, *et al.* 2005, *PoP* 12, 052515)

- Recent observations on DIII-D confirm these predictions (R. Nazikian, *et al.* 06, *PRL* 96, 105006)
A “Sea of Core Localized Alfvén Eigenmodes” Observed in DIII-D Quiescent Double Barrier (QDB) plasmas

- Bands of modes $m=n+l$, $l=1, 2, \ldots$
- Neutral beam injection opposite to plasma current: $V_{||} \approx 0.3V_A$

R. Nazikian, et al. 06, PRL 96, 105006

Long time scale behaviors

- Depending on proximity to marginal stability, AE and EPM nonlinear evolutions can be predominantly affected by


- AITG and strongly driven MHD modes behave similarly
Zonal Flows and Zonal Structures

- Very disparate space-time scales of AE/EPM, MHD modes and plasma turbulence: complex self-organized behaviors of burning plasmas will be likely dominated by their nonlinear interplay via zonal flows and fields.


- Long time scale behaviors of zonal structures are important for the overall burning plasma performance: generators of nonlinear equilibria.

- The corresponding stability determines the dynamics underlying the dissipation of zonal structures in collision-less plasmas and the nonlinear up-shift of thresholds for turbulent transport (L. Chen, *et al.* 2006).

- Impact on burning plasma performance
Conclusions

- Burning plasmas are complex self-organized systems, whose investigation requires a conceptual step in the analysis of magnetically confined plasmas.

- Integrated numerical simulations are crucial to investigate these new physics; while fundamental theories provide the conceptual framework and the necessary insights.

- Verification against experimental observations in present day machines is a necessary step for the validation of physical models and numerical codes for reliable extrapolations to burning plasmas.

- Lack of understanding of some complex burning plasma behaviors can be likely filled in by increasingly complicated and more realistic modeling of plasma conditions as computing performances improve.

- However, some other unexplained behaviors may be just indications of fundamental conceptual problems: mutual positive feedbacks between theory, simulation and experiment will be necessary.

- Burning plasma physics is an exciting and challenging field: many examples of fundamental problems with broader applications and implications.