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Theory and simulation of discrete kinetic beta induced Alfvén eigenmode in tokamak plasmas

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Abstract

It is shown, both analytically and by numerical simulations, that, in the presence of thermal ion kinetic effects, the beta induced Alfvén eigenmode (BAE)–shear Alfvén wave continuous spectrum can be discretized into radially trapped eigenstates known as kinetic BAE (KBAE). While thermal ion compressibility gives rise to finite BAE accumulation point frequency, the discretization occurs via the finite Larmor radius and finite orbit width effects. Simulations and analytical theories agree both qualitatively and quantitatively. Simulations also demonstrate that KBAE can be readily excited by the finite radial gradients of energetic particles.

(Some figures in this article are in colour only in the electronic version)

1. Introduction

Early and recent experimental observations of the beta Alfvén eigenmode (BAE) have attracted significant attention on the investigations of the low-frequency Alfvénic fluctuations in tokamaks [1–4]. These electromagnetic modes may destroy a particle’s constants of motion, and, thus, lead to radial redistribution of energetic particles (EPs) produced either as fusion products and/or by auxiliary heating systems, such as ion cyclotron resonance heating (ICRH) and neutral beam injection (NBI). At the time of the first observation of BAE [1], the existence of a plasma compressibility induced gap in the shear Alfvén continuous spectrum was demonstrated analytically and numerically using magnetohydrodynamic (MHD) codes [5]. While whether a fully kinetic treatment of thermal ions is really necessary for a proper description of the low-frequency Alfvénic fluctuation spectrum, i.e. whether MHD/fluid theories are sufficient, is still currently being debated [6], there is an increasing consensus that key physics ingredients do indeed require a kinetic approach [7–14]. In this work,
by including the thermal ion compressibility effect, our numerical simulations do show the existence of a finite-frequency BAE accumulation point in the shear Alfvén wave (SAW) continuum. No discrete eigenmode, however, is observed within the frequency gap when the tokamak plasma is ideal MHD stable. These simulation results are consistent with theoretical predictions based on the generalized fishbone-like dispersion relation (GFLLDR) [7]. On the other hand, when the effects due to finite ion drift orbit widths (FOW) and/or finite Larmor radius (FLR) are included, our simulations clearly demonstrate that the BAE–SAW continuum becomes discretized, yielding a series of discrete kinetic eigenmodes with a small frequency separation [9, 11]. This point clearly illustrates that kinetic physics such as the FLR and FOW effects do play important roles in low-frequency Alfvénic fluctuations.

Kinetic theory has been applied extensively to discuss both wave–particle resonance as well as FLR/FOW effects, on modes such as Alfvén ion temperature gradient driven mode (AITG) [10] and kinetic geodesic acoustic mode (KGAM) [15, 16]. As to BAE, which can be considered as the long wavelength counterpart of AITG [10], it has, however, been applied to a somewhat lesser extent. In this work, we demonstrate the existence of KBAE based on the theoretical framework presented in [9] and investigate its properties. For this purpose, we have extended the hybrid MHD-gyrokinetic code HMGc [17] in order to be able to properly treat thermal ion kinetic compression effects [18, 19]. The new eXtended HMGc code (XHMGc) [18, 19], thus, has the capability to investigate the KBAE dynamics, both as an initial value problem by prescribing the initial perturbations, as well as a driven resonant-cavity problem via ‘internal antenna’ [20] excitations. Numerical simulation results from XHMGc are shown to illustrate KBAE peculiar features with and without an EP drive.

The paper is organized as follows. Section 2 presents the eigenmode equations and the linear dispersion relation of KBAE. In section 3, the initial value and ‘antenna’ excitation simulation results are presented. In section 4, simulation results of modes driven by purely circulating slowing-down EPs are given. Finally, section 5 contains a brief summary and discussions.

2. Linear dispersion relation of KBAE

With the inclusion of small but finite FLR and FOW effects, the generalized form of vorticity equation in the SAW inertial layer can be written as (see equation (7) in [9]),

\[ \partial_t^2 \delta \Psi + \Lambda^2 \delta \Phi = \left[ \frac{3}{4} \left( 1 - \frac{\omega_B}{\omega} + \frac{\omega_A \omega_T}{\omega} \right) b_i + q^2 \frac{\omega_B}{\omega} \delta \Psi \right] \frac{\omega_B^2}{\omega_A} \delta \Phi = 0, \quad (1) \]

where the functions \( \Lambda^2 \) and \( S(\omega) \) are defined in equation (3) and equation (B28) of [9]. The relation between \( \delta \Psi \) and \( \delta \Phi \), meanwhile, is given via the quasi-neutrality condition (see equation (8) in [9])

\[ \left( 1 + \frac{\omega_B}{\omega} \right) (\delta \Phi - \delta \Psi) + \Lambda^2 \frac{\omega_B^2}{\omega_A} b_i \delta \Phi = 0, \quad (2) \]

where the well-known ballooning mode representation has been used [21]. Furthermore, as described in [9], \( |\eta| \gg 1 \) corresponds to the SAW inertial layer, \( \delta \Phi \equiv (k L / k \eta) \omega \delta \phi \), \( \delta \Psi \equiv (k L / k \eta) \delta \psi \), \( \delta \phi \) is the perturbed scalar potential, \( \delta \psi \) is the induced potential related to the perturbed parallel vector potential \( \delta A_L \) by \( \delta A_L \equiv -i(c / \omega) \vec{b} \cdot \nabla \delta \psi \), \( \omega_A \equiv v_A / q R_0 \) is the Alfvén frequency where \( v_A = B / \sqrt{4 \pi n e m_i} \), \( \omega_B = \omega_m + \omega_A T \), \( \omega_m = (T_e / e B)(k \times \vec{b}) \cdot (\nabla n_i) / n_i \), \( \omega_A T \equiv (T_e / e B)(k \times \vec{b}) \cdot (\nabla T_e) / T_e \), \( \omega_B = v_B / q R_0 = \sqrt{2 T_e / m_i} q R_0 \), \( \tau = T_e / T_i \) and \( b_i = k_L^* / \rho_L^* \). In this work, we take the \( \omega_B / \omega \rightarrow 0 \) limit. Thus, for \( \omega \gg \omega_B \), we note that \( \Lambda^2 = (q R_0 k_i)^2 = (\omega^2 - \omega_{BAE}^2) / \omega_A^2 \) gives the local dispersion
relation [8], where $\omega_{BAE} = q\omega_i(7/4 + \tau)^{1/2}$ is defined as the BAE frequency. Combining (1) and (2) one obtains

\[
\left(1 + \left(1 - \frac{\omega_{BAE}^2}{\omega_i^2}\right)\tau b_i \right)\partial_{\theta}^2 \Phi + \frac{\omega^2}{\omega_i^2} \left(1 - \frac{3}{4} b_i \right) \delta \Phi - \frac{\omega_{BAE}^2}{\omega_i^2} \left(1 + q^2 \frac{\omega_i^2}{\omega_{BAE}^2} S(\omega) b_i \right) \delta \Phi = 0.
\]

(3)

Here, the first term on the lhs contains the $\tau b_i$ term due to the usual electron pressure corrections to the ideal Ohm’s law which is negligible near the BAE accumulation point, as discussed in [9, 10]. The second term includes the thermal ion FLR correction to the charge density due to the polarization current, while the third term contains the thermal ion FLR/FOW corrections to the charge density due to the perturbed diamagnetic current. Note that it is this latter term that accounts for peculiar features associated with geodesic curvature in toroidal geometry [9, 10]. The FLR/FOW correction is important when one considers mode conversion due to radial singular structures associated with resonant excitation of the SAW continuous spectrum. Equation (3) reduces to the well-known result for kinetic Alfvén waves (KAW) [22, 23] by taking the limit $\omega_{BAE}/\omega \to 0$. We can cast (3) into the following standard form:

\[
\partial_{\theta}^2 \Phi + \Lambda^2 \delta \Phi - \theta^2 Q^2(\omega) \delta \Phi = 0,
\]

(4)

where

\[
Q^2(\omega) = \frac{s^2 \kappa^2 \rho_i \rho_i}{\omega_i^2 \omega_A^2} \left[\frac{3}{4} + q^2 \frac{\omega_i^2}{\omega} S(\omega) + \frac{\tau (\Lambda \omega_A/\omega)^4}{1 + \tau \omega_{BAE}/\omega} \right].
\]

(5)

In this work, we further consider the limit $\tau = 0$, which allows us to assume the ideal MHD Ohm’s law [18, 19], as readily seen from (2). From the expression of $S(\omega)$ given in [15], one readily obtains

\[
\frac{3}{4} + q^2 \frac{\omega_i^2}{\omega} S(\omega) \simeq \frac{3}{4} + \frac{13 \omega_{BAE}^2}{7 \omega^2} + \frac{747 \alpha_{BAE}^2}{98 \omega^4} + i\pi^{1/2} q^4 e^{-\omega_2^2/4\kappa^2} (\omega_5^5/256\alpha_5^5 + \alpha^3/32\alpha_5^3).
\]

(6)

Note that (6) represents the approximation of $(3/4 + q^2 \omega_i^2/\omega S(\omega))$ under the condition $(7/4 + \tau)q^2 \gg 1$. In this case Re$[3/4 + q^2 \omega_i^2/\omega S(\omega)] > 0$ is always satisfied. Meanwhile, as discussed in [15], the sign of Re$[3/4 + q^2 \omega_i^2/\omega S(\omega)]$ can change at low frequencies with possible interesting implications on the mode dynamics.

Equation (4) can be solved locally for the discretized KBAE spectrum near the BAE accumulation point as illustrated in figure 1. The existence of radially localized discrete modes can be understood as follows: the KAW is trapped within the potential well formed...
on the ‘high frequency side’ of the local SAW continuous spectrum [22, 23] and modified by thermal ion compressibility effects near the BAE accumulation point. Meanwhile, on the ‘low frequency side’ [22, 23], KAW is evanescent. Under this condition, analogous to that discussed by Rosenbluth and Rutherford for KAW in [24] and Mett and Mahajan for kinetic toroidal Alfvén Eigenmode (KTAE) in [25], radially bound states (discrete modes) can exist, whose energy levels correspond to those of the ‘harmonic oscillator’ described by (4), i.e.

\[ \Lambda^2 = (2\ell + 1) Q \]  

with \( \ell = 0, 1, 2, \ldots \) being the ‘radial quantum number’ [9, 10]. Note that (7) describes the energy levels of well-localized radially bound states. The finite coupling to the (radial) external region and non-uniform plasma response via longer wavelength feature of the global mode structures is described by [9–11]

\[ -2 Q^{1/2} \frac{\Gamma(3/4 - \Lambda^2/4Q)}{\Gamma(1/4 - \Lambda^2/4Q)} = \delta W_f + \delta W_k. \]  

(8)

Here, \( \delta W_f \) and \( \delta W_k \) are, respectively, the perturbed potential energies of the background MHD fluid and EPs in the ideal region. Equation (8) reduces to the GFLDR [3, 7, 10]

\[ i\Lambda = \delta W_f + \delta W_k \]  

(9)

in the limit \( |\Lambda^2/4Q| \gg 1 \), i.e. when the fine structures of the ‘discretized’ SAW continuous spectrum are unimportant due to the intrinsic (linear) frequency line width connected with the EP drive or time coherence of the external source [15, 26, 27]. This can be easily done by taking the large argument expansion of the Euler gamma functions. Global plasma properties, thus, affect the BAE/KBAE dispersion relations [9–11] via \( \delta W_f \) and \( \delta W_k \). It is worth noting that, for the most unstable modes that are relevant for the description of EP transport in burning plasmas [3, 7, 28], EP dynamics enter only via \( \delta W_k \) and never contribute to the inertial layer. The dispersion relation for radially localized KBAE modes, (7), is readily obtained from (8) for either \( |\delta W_f + \delta W_k| \ll |Q|^{1/2} \) (even modes, \( \ell = 0, 2, 4, \ldots \)) or \( |\delta W_f + \delta W_k| \gg |Q|^{1/2} \) (odd modes, \( \ell = 1, 3, 5, \ldots \)). Equation (8) has recently been used for analyses of BAE excited in Tore Supra discharges heated by ICRH [29, 30].

3. Simulation results of KBAE

The theoretical framework for GFLDR [7] has been adopted to extend the hybrid model used in HMGC [17]. The corresponding set of equations for studying low-\( \beta \) (\( \beta \sim O(\epsilon^\delta) < 1 \), where \( \epsilon = a/R \) and \( a(R) \) are the tokamak aspect ratio and the minor(major) radius of the plasma) plasma dynamics with the frequency ranging from kinetic ballooning mode (KBM)/BAE to toroidal Alfvén eigenmode (TAE) are given in [18, 19]. The extended hybrid MHD-gyrokinetic simulation model is derived analytically based upon this set of equations by taking into account both thermal ion kinetic compressibility and diamagnetic effects in addition to EP kinetic dynamics. Within this hybrid model, the fluid response of the thermal background plasma is described by a set of \( O(\epsilon^\delta) \)-reduced MHD equations for a low-\( \beta \) plasma, and the EP and thermal ions kinetic dynamics enter via the pressure tensors, which are computed by solving the gyrokinetic equations with the particle-in-cell technique. While the FLR effects are ignored in the current model, the FOW effects are fully taken into account for correctly describing mode dynamics associated with guiding-center magnetic curvature drift in a toroidal geometry. The extended model has been developed assuming \( \tau = 0 \) (ideal Ohm’s law) as well as ignoring the FLR effects in order to simplify the technical complications while still maintaining all essential physics ingredients. The model has been successfully implemented into the eXtended version
Figure 2. Alfvén continuum for \( n = 3 \) and \( m = 1–12 \) with the \( q \) profile specified in the text. Here, the continuum is corresponding to a uniform cold plasma equilibrium (without kinetic thermal ions).

For convenience of implementing the new numerical scheme, the total perturbed distribution \( \delta f \) has been solved instead of the non-adiabatic particle response \( \delta K \), as in the original theoretical treatments [7, 31]. Meanwhile, assuming cold electrons and ignoring thermal ion FLR, ideal MHD parallel Ohm’s law \( (E_\parallel = 0) \) can be readily recovered.

Both initial perturbations and ‘antenna’ excitations have been used to simulate BAE or KBAE with kinetic thermal ion dynamics. Simulations refer to an equilibrium magnetic field characterized by shifted circular magnetic surfaces with inverse aspect ratio \( a/R_0 = 0.1 \) and the \( q \)-profile given, in the cylindrical approximation, by \( q(r/a) = q(0) + [q(a) - q(0)]r^2/a^2 \), with \( q(0) = 2.7 \) and \( q(a) = 3.9 \). Firstly, in the simulations of initial perturbations, modes with toroidal mode number \( n = 3 \) and poloidal mode numbers \( m = 1–12 \) are investigated. Given the \( q \) profile, the frequency gaps in the SAW continuous spectrum are properly kept within the plasma volume. One typical continuous spectrum is shown in figure 2. The accumulation point for \( n = 3, m = 9 \) is localized at \( r = 0.5a \) since \( q = 3 \) and, correspondingly, \( k_\parallel = 0 \) at that point. In the initial-value simulations, \( n = 3 \) and \( m = 9 \) perturbation in the electrostatic potential is introduced around the \( q = 3 \) location initially. In the current simulations, the equilibrium thermal ion density and temperature are kept uniform to neglect the diamagnetic drift. Figures 3(a) and (b) plot the time histories of, respectively, the electrostatic potentials without and with the thermal ion kinetic effects. Figure 3(a) shows that, in the case without the thermal ion kinetic effects, the perturbed field is purely damped due to phase mixing at the accumulation point, where the accumulation point frequency is \( \omega_{acp} = 0 \). When thermal ion kinetic effects are taken into account, figure 3(b) shows that the perturbed field oscillates with a finite frequency and damps exponentially due to the ion Landau damping, as demonstrated from the semi-log plot shown in figure 3(c). The simulation shown in figures 3(b) and (c) refers
Figure 3. Time history of the electrostatic potential (normalized) on the accumulation point: (a) without kinetic thermal ions; (b) with kinetic thermal ions for $\beta_{ic} = 0.0072$, where the black line is the real part of the electrostatic potential, the blue line is the imaginary part and the red line corresponds to an exponential function as $e^{\gamma t}$; (c) $|\phi|$ in log scale.

Figure 4. Maximum amplitude for different antenna frequencies. $\ast$ is the amplitude (a.u.); red dashed line corresponds to the fitting function.

to $\beta_{ic} = 0.0072$. Here $\beta_{ic}$ is defined as $\beta_{ic} = 8\pi n_{ic} T_{ic} / B^2$, where the subscript $ic$ denotes the core ions.

Secondly, we use ‘antenna’ excitation [32] to investigate the eigenmode frequencies, damping rate and the mode structures. Here ‘antenna’ means artificially introducing a source with a fixed amplitude and a tunable frequency. When the eigenmode is in resonance with the ‘antenna’ frequency, the response is maximized as in a resonant cavity. In the simulations, thus, we vary the ‘antenna’ frequencies and look for the frequency which gives the maximal response, and it is then the eigenmode frequency. Typically, ‘antenna’ excitation is a good tool to investigate the eigenmode frequency, damping rate and mode structures. To demonstrate more precisely, we show how to measure the eigenmode frequency and damping rate for $\beta_{ic} = 0.0072$ as an example. In figure 4, for different ‘antenna’ frequencies, the maximum amplitudes at $r = 0.5a$ of the electrostatic potential response are plotted. The damping rate is
related to the maximum wave response amplitude by \[33\]
\[
\delta \phi_{\text{max}} \propto \frac{1}{\sqrt{(\omega_0^2 - \omega_{\text{ant}}^2) + 4\gamma^2 \omega_{\text{ant}}^2}}.
\]

(10)

Here \(\delta \phi_{\text{max}}\) is the maximum amplitude, \(\omega_{\text{ant}}\) is the ‘antenna’ frequency, \(\omega_0^2 = \omega_r^2 + \gamma^2\) where \(\omega_r\) is the eigenmode real frequency given by the frequency corresponding to the maximum \(\delta \phi_{\text{max}}\) and \(\gamma\) is the damping rate. Thus, the eigenmode frequency and damping rate can be measured. Figure 5(a) shows the time evolution of the perturbed potential. It can be seen that, when ‘antenna’ frequency matches the eigenfrequency, the perturbed potential reaches a steady state due to the ‘antenna’ drive balancing the damping mechanism. Figures 5(b) and (d) show, respectively, the radial and the poloidal mode structures. Figure 5(c) shows that SAW has accumulation points at the finite BAE frequencies and the eigenmode frequency is above

Figure 5. Simulation results of ‘antenna’ excitation with \(\omega_{\text{ant}} = 0.13\). (a) Time evolution of the normalized electrostatic potential at \(r = 0.5a\); (b) radial mode structure; (c) Alfvén continua and radial frequency spectrum; (d) poloidal mode structure.
Table 1. Frequency comparison.

<table>
<thead>
<tr>
<th></th>
<th>$\beta_i = 0.0072$</th>
<th>$\beta_i = 0.0128$</th>
<th>$\beta_i = 0.02$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial</td>
<td>$\omega = 0.131$, $\gamma = 0.011$</td>
<td>$\omega = 0.164$, $\gamma = 0.013$</td>
<td>$\omega = 0.196$, $\gamma = 0.016$</td>
</tr>
<tr>
<td>Antenna</td>
<td>$\omega = 0.13$, $\gamma = 0.012$</td>
<td>$\omega = 0.166$, $\gamma = 0.013$</td>
<td>$\omega = 0.197$, $\gamma = 0.018$</td>
</tr>
<tr>
<td>$\omega_{BAE}$</td>
<td>$\omega = 0.112$</td>
<td>$\omega = 0.150$</td>
<td>$\omega = 0.187$</td>
</tr>
</tbody>
</table>

$\omega_{BAE}$ is the accumulation point frequency. For comparison, table 1 shows the results of initial perturbations, ‘antenna’ excitations and $\omega_{BAE}$ for, respectively, $\beta_i = 0.0072$, $0.0128$, $0.02$, and, correspondingly, $\rho L_i/a = 0.006, 0.008, 0.01$. Note that the frequencies are normalized to $v_A/R$.

The simulation results show that the observed frequencies are always slightly higher than the BAE accumulation point frequency. In the current case, assuming cold electrons and adopting thermal ions with uniform density and temperature, the plasma is, thus, ideal MHD stable, and there is negligible coupling to the sound wave and diamagnetic effects. According to the theoretical prediction of GFLDR [7], no eigenmode should exist in the low-frequency kinetic thermal ions induced gap when MHD is ideally stable; consistent with the simulation results.

In figure 6, we have plotted the BAE accumulation frequencies $\omega_{BAE}$ in the fluid limit, analytically predicted $\ell = 0$ KBAE eigenmode frequencies from (7), as well as eigenmode frequencies determined via ‘antenna’ excitation simulations versus $\beta_i$ for $n = 1$ and $n = 3$, respectively. Note that the analytically predicted KBAE frequencies are in good agreement with the simulation observations, and both are higher than $\omega_{BAE}$ the accumulation frequencies. In addition, in simulations, the observed oscillation frequencies are the same over the mode extent, further indicating the oscillations are eigenmodes. Note also that the $n = 3$ KBAE frequency is higher than the $n = 1$ KBAE frequency, consistent with the theoretical predictions of (7) and that the FLR/FOW kinetic effects increase with the toroidal mode number, $n$. Furthermore, there is no observation of the higher ($\ell \geq 1$) radial eigenstates of KBAE. This may be either because the potential well is not sufficiently deep to trap the higher eigenstates [10] or that,
4. Simulation results of excitations by EPs

In this section, the results of linear simulations with a fixed value of \( \beta_H = 0.009 \) (the on-axis EP pressure parameter) and toroidal number \( n = 3 \) are shown. The EP velocity–space equilibrium distribution function is taken to be a purely circulating slowing-down distribution [34] with birth energy \( E_0 = m_H v_H^2 \), where \( m_H = m_{ic} \). The \( q \) profile as well as thermal ions’ parameters are the same as those used in section 3. The on-axis parameters are \( n_{i0}/n_{ic} = 0.05 \), \( v_H/v_A = 0.3 \), \( \rho_{LH}/a = 0.03 \). The EP equilibrium density profile is taken to be \( n_H(r) = n_{i0}(1 + 2(r/a)^3 - 3(r/a)^5) \).

The simulation results of Alfvénic modes excited by EPs are shown in figure 7. The top panel corresponds to excitations without the thermal ion kinetic effects. The simulation results with the kinetic thermal ion effects are presented in the lower panel. Column (a) of the top panel shows that the excited mode is localized around \( r = 0.5 \) where the EP drive is maximal and, correspondingly, the dominant poloidal mode number is \( m = nq = 9 \). Meanwhile, column (b) shows that the real frequency of the mode is around \( 0.85\omega_{LE} \), where \( \omega_{LE} = v_H/qR \) is the beam-ion transit frequency at \( q = 3 \). These simulation results are, thus, consistent with the theoretical predictions of the energetic particle mode (EPM) [35, 36]. Note that EPM exists entirely due to EPs, such that its radial localization, real frequency and linear growth rate are intrinsically determined by the EP pressure gradient drive, characteristic dynamical frequencies and competition between the EP-wave resonance drive and the BAE–SAW continuum damping.
Figure 8. The real frequency and growth rate for the $n = 3$ mode versus different thermal ion pressure parameters for $\beta_{ic} = 0.0072, 0.0128, 0.02$. ‘+’ is the mode real frequency of simulation results by EP excitations; ‘△’ is the KBAE frequencies by antenna excitations; solid line denotes the theoretical BAE accumulation point frequency; red ‘∗’ is the growth rate by EP excitation simulations.

When thermal ion kinetic effects are included, column (b) of the lower panel in figure 7 shows that the SAW has accumulation points at the finite BAE frequencies. Meanwhile, the excited modes, as shown in column (a) and (b), are localized around $r \approx 0.5a$ and $r \approx 0.7a$ corresponding, respectively, to the dominant poloidal harmonics $m = 9$ and $m = 10$. Note that, from column (b), both modes have frequencies above the BAE accumulation point, i.e. in the range of KBAE. That the $m = 9$ mode has an intensity higher than that of $m = 10$ is due to the stronger EP drive around $r = 0.5a$. We emphasize that, with the thermal ion kinetic effects, there is the frequency gap with the accumulation frequency at $\omega_{BAE}$, which either nullifies or significantly reduces the continuum damping rate. This explains why the weakly driven $m = 10$ mode is absent in the top panel without the kinetic thermal ion gap.

That the EP excited oscillations are KBAE is further demonstrated in figure 8, where the real frequencies of simulation observations by both ‘antenna’ excitations and EP excitations are plotted versus the thermal ion temperature. Figure 8 clearly shows that the frequencies scale properly with the BAE frequency. That the growth rate decreases with the thermal ion temperature can be understood as due to either the stronger damping (cf table 1) and/or the weaker EPs driven with the higher BAE frequency.

5. Conclusions and discussions

In this work, we have demonstrated, both analytically and by direct numerical simulations, that the BAE–SAW continuous spectrum can be discretized by the FLR and/or FOW kinetic effects
of the thermal ions. While the analytical theory is based on the framework of GFLDR [3, 7, 10], the simulations are carried out in XHMGC [18, 19], which has extended the capabilities of HMGC to include the thermal ion kinetic effects. Meanwhile, by adopting cold electrons and assuming spatially uniform thermal ion properties, we have neglected coupling to the acoustic branch and diamagnetic effects in order to isolate and extract the underlying physics mechanisms.

In the absence of EPs drive, our analytical results and simulations are found to be in good qualitative and quantitative agreement. Introducing the EP drive via the finite EP density gradients, numerical simulations clearly demonstrate that the Alfvénic modes can be readily excited. In the absence of thermal ion kinetic effects, the excited modes are identified as EPMs, which require a sufficiently strong drive to overcome the SAW continuous damping [37]. Including the thermal ion kinetic effects not only introduces a finite kinetic thermal ion frequency gap at the BAE accumulation frequency but also discretizes the BAE–SAW continuum. The net effects are the continuum damping is significantly reduced or nullified, and the discrete KBAEs are readily excited by the EP drive.

There are obviously many interesting issues that remain to be investigated. For example, one could introduce finite radial gradients to the thermal ions in order to examine short-wavelength instabilities such as AITG [10]. It should be interesting to explore the qualitative difference of non-linear evolution and saturation with and without thermal ion kinetic effects. These and other important issues will be systematically studied in the future.

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11
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