Combined action of phase-mixing and Landau damping causing strong decay of geodesic acoustic modes
Combined action of phase-mixing and Landau damping causing strong decay of geodesic acoustic modes

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Abstract – We report evidence of a new mechanism able to damp very efficiently geodesic acoustic mode (GAM) in the presence of a nonuniform temperature profile in a toroidally confined plasma. This represents a particular case of a general mechanism that we have found and that can be observed whenever the phase-mixing acts in the presence of a damping effect that depends on the wave number $k_r$. Here, in particular, the combined effect of the Landau and continuum damping is found to quickly redistribute the GAM energy in phase-space, due to the synergy of the finite orbit width of the passing ions and the cascade in wave number given by the phase-mixing. This damping mechanism is investigated analytically and numerically by means of global gyrokinetic simulations. When realistic parameter values of plasmas at the edge of a tokamak are used, damping rates up to 2 orders of magnitude higher than the Landau damping alone are obtained. We find in particular that, for temperature and density profiles characteristic of the high confinement mode, the so-called H-mode, the GAM decay time becomes comparable to or lower than the nonlinear drive time, consistently with experimental observations (Conway G. D. et al., Phys. Rev. Lett., 106 (2011) 065001).

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A crucial issue for the development of magnetic fusion is the control of transport and consequently of the turbulence suppression. The turbulence is able to self-organize in convective structures named zonal flows [1–6] characterized by a poloidal velocity flow that acts to shear and distort convective and turbulent cells leading to reduce the transport. For this reason, zonal flows are considered as one of the main saturation mechanisms for micro-instabilities such as the toroidal ion temperature gradient (ITG) instability. Due to the presence of geodesic curvature in various toroidal devices, the zonal flows contain particular oscillations named geodesic acoustic modes (GAMs) [7]. The importance of GAM oscillations resides in the different shearing efficiency that the zonal flows have in relation to their oscillatory behavior. In fact, while stationary zonal flows suppress turbulence efficiently, oscillations make the action of the zonal flow less effective [8,9]. Moreover, GAMs can also drive energy from zonal flows.

This transfer pathway goes from the electric field of the zonal flow to the pressure perturbation linked to the turbulence [10]. This effect reduces the amplitude of the zonal flow and increases the level of the turbulence fluctuations. Therefore, the understanding of GAM behavior and of the eventual mechanisms of GAM suppression can have a strong impact on the control of the turbulence. The oscillatory behavior of zonal flows depends on the value of the safety factor $q$. In tokamak plasmas, the GAMs are subject to Landau damping by the passing ions. The damping is strong for low values of the safety factor and becomes weaker for large values of the safety factor. For this reason, it is expected that GAMs are prevalent for typical values of parameters at the edge of tokamaks, while low-frequency zonal flows become dominant in the plasma core. Moreover, due to the dependence of the local GAM frequency on the plasma parameters such as temperature, a continuum spectrum of GAM exists in tokamak plasmas, where the temperature varies across the magnetic flux surfaces. As a consequence, a GAM is affected

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by phase-mixing [11,12], similarly to shear Alfvén waves (SAW), whose continuum spectrum depends mainly on the safety factor rather than on the plasma temperature [13]. The effect of the phase-mixing is to modify the radial structure of the perturbation, with a cascade from low to high radial wave numbers, with a rate in time that is proportional to the local radial derivative of the continuum frequency [13]. For SAW, phase-mixing plays a relevant role only well above the beta-induced Alfvén eigenmode (BAE), where the Landau damping is less efficient. On the contrary, for GAM, phase-mixing and Landau damping can act at the same time, with a combined action resulting in a strong damping mechanism. In fact, the Landau damping of GAM also strongly increases with the radial wave number, due to the finite orbit width of the passing ions. Consequently the efficiency of the Landau damping can be greatly amplified by phase-mixing.

This represents a particular case of a general mechanism, that we have found, and that can be observed whenever the phase-mixing acts in the presence of a damping effect that depends on the wave number \( k \). The general mechanism, which we call phase-mixing damping adjustment (PDA), can play a very important role not only in the plasma fusion domain, but also in other physical contexts, such as in the open problem of the heating of the solar corona. In this context, the wave energy can be efficiently absorbed in the presence of a continuum spectrum [14] in conjunction with other mechanisms such as reconnection driven by the generation of the magnetic field [15].

More specifically, in this letter, the combined effect of phase-mixing and Landau damping is proposed as a novel mechanism, which can play a crucial role in the dynamics and in the decay of GAM at the tokamak edge, where the temperature gradients are very large. Depending on the confinement regime, low confinement (L-mode), improved (low) confinement (I-mode) and high confinement (H-mode), different parameter values are achieved in the experiments, in particular changing the relative strength of temperature and density gradients at the edge of the plasma. The sequence of events in the transition phases between these regimes, particularly regarding the turbulence suppression, the development of the edge temperature and density gradients and the increase of the shear flow, is not completely established yet. GAMs are considered to be potential key players in the dynamics of the transition to I- and to H-modes [16], and their absence can enhance the effects of the shear flow on the turbulence [9]. GAMs are reported to be regularly observed in L-mode, and are also observed in I-mode, whereas they are not observed in H-mode. It will be shown that the proposed mechanism of GAM decay is consistent with the observed existence or nonexistence of GAMs in the different confinement regimes [17].

In this work we have used the numerical gyrokinetic particle-in-cell code ORB5 [18] which now includes all extensions made in the NEMORB project [19]. The ORB5 code uses a Lagrangian formulation based on the gyrokinetic Vlasov-Maxwell equations [20,21]. The code solves the full-f gyrokinetic Vlasov equation for ions. Drift-kinetic equations are solved for electrons. Energy and momentum conservations can be proved via gyrokinetic field theory [10]. To obtain the potential \( \phi \), the Vlasov equation must be coupled to the quasi-neutrality condition. With this model, in the first part of the work we investigate the Landau damping rate \( \gamma \) of GAMs as a function of several parameters, such as the safety factor \( q \) and the radial wave number \( k_r \) associated to these structures. To this purpose, we initialize a sinusoidal potential perturbation \( \phi = \sin(ak_r r) \) independent of poloidal and toroidal angles along the minor radius \( a \) with \( 0 \leq r \leq 1 \). The perturbation evolves in a linear electrostatic collisionless simulation with adiabatic electrons. The considered range is \( 0.027 \leq k_r \rho_s \leq 0.27 \). The ion Larmor radius is defined as \( \rho_i = \sqrt{2} \frac{T_i^{1/2}/T_{i,0}}{eB_0} \) with \( T_i = \sqrt{eB_0/m_i} \) and with \( T_{i,0}/T_{e,0} \) ion/electron temperature calculated in the middle of the radial domain. We choose \( m_i \) equal to hydrogen mass. The magnetic field \( B_0 \) is calculated at the axial radial position. We choose a tokamak with an inverse aspect ratio \( \epsilon = a/R_0 = 0.1 \). Hereafter, we consider circular magnetic flux surfaces. In this limit, flux surface coordinate \( r = \sqrt{\psi/\psi_{edge}} \) is a good approximation of the radial coordinate. We assume a flat safety factor profile. Temperature and density profiles are also considered flat and the diameter \( L_r = 2/\rho_s = 320 \), with \( \rho_s = \rho_s/a \). In this way we can compare the results of the ORB5 simulations with the analytical prediction of the theory of Sugama-Watanabe in which finite orbit width (FOW) effects are included to the first order [22,23] (for higher order see [24]). The reference simulation has a spatial grid of \( (r \times \theta \times \phi) = (256 \times 64 \times 4) \) and a time step of \( 100 \Omega_i^{-1} \), with \( \Omega_i \) being the ion cyclotron frequency. Hereafter, all time quantities are normalized with \( \Omega_i^{-1} \). All the simulations have been performed with \( 10^9 \) deuterium markers. The results are summarized in fig. 1 that shows the damping rate \( \gamma \) as a function of \( k_r \rho_s \) for several values of the safety factor \( q \). We find that for \( q = 1 \), all \( k_r \) are damped at the same rate. We emphasize that for large value of \( q \) the damping rate strongly increases with \( k_r \rho_s \). We recall that large \( q \) values are found at the edge of tokamaks, where GAMs can play an important role in the dynamics of the turbulence transport. As a general remark, we find a good agreement between theory and simulations.

Now, we discuss linear simulations considering a GAM decay that evolves in an equilibrium obtained from realistic values of parameters. To this purpose, we have selected a group of parameter values measured at the edge of ASDEX Upgrade for the shot number 20787 [25]. The considered values are summarized in table 1. Starting from these values we obtain the following quantities: an ion cyclotron frequency \( \Omega_i = 1.9 \cdot 10^8 \text{ rad/s} \), an ion thermal velocity \( v_{th,i} = \sqrt{2k_B T_i/m_i} = 1.8 \cdot 10^5 \text{ m/s} \) and a Larmor radius \( \rho_i = 9.7 \cdot 10^{-4} \text{ m} \). On the basis of these quantities we perform simulations with \( L_r = 1441 \) and
Table 1: Values of parameters for the ASDEX Upgrade experiment referred to as 20787 [25].

<table>
<thead>
<tr>
<th>$T_e$ (eV)</th>
<th>$T_i$ (eV)</th>
<th>$B_0$ (T)</th>
<th>$ak_T$</th>
<th>$a$ (m)</th>
<th>$R_0$ (m)</th>
<th>$\lambda_{GAM}$ (m)</th>
<th>$q$</th>
<th>$n$ (part/m$^3$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>170</td>
<td>170</td>
<td>2.</td>
<td>10</td>
<td>0.5</td>
<td>1.65</td>
<td>$\approx 5 \cdot 10^{-2}$</td>
<td>3.5</td>
<td>$10^{19}$</td>
</tr>
</tbody>
</table>

we assume $\tau_e = T_e/T_i = 1$ and $\epsilon = 0.3$. In order to investigate the effects of the temperature gradient we assume a flat safety factor profile $q = 3.5$ and a flat density profile. We initialize a potential perturbation with a radial gaussian profile whose width is $k_r \rho_i \approx 0.12$. Simulations have a spatial grid of $(r, \theta, \phi) = 1024 \times 64 \times 4$ and a time step of $50\Omega_i^{-1}$. The realistic conditions at the tokamak edge are reproduced in the center region of the radial domain in the simulations. By means of a hyperbolic tangent we consider two profiles of temperature with the following gradients $ak_T = a \nabla T/T = 0$ (with $T$ electron/ion temperature) and $ak_T = 10$ in the points of maximum steepness $\nabla T$. For both simulations, GAMs oscillations are observed and an overview of the results is given in fig. 2 in which we show the global electric field in the plane $(r, t)$. We observe two peaks of the electric field associated to localized potential perturbation and we note that the oscillations for $ak_T = 10$ are damped faster in time than the oscillations corresponding to $ak_T = 0$. In order to understand the GAM behavior in the presence of a nonuniform temperature profile we recall that similarly to shear Alfvén waves, GAM also constitutes a continuum spectrum described by $\omega_G(r) = \alpha + \beta r$ with $\beta = (\partial \omega_G/\partial r) = (\partial \omega_G/\partial T)(\partial T/\partial r) = 0.5 \omega_G/T(\nabla T) = 0.5 \omega_G k_T$ calculated in the middle of the radial domain in the point of maximum steepness $\nabla T$. The spatial Fourier transform $\delta E_r$ of the perturbed electric field is given by $\delta E_r = 2A_1 e^{-i\omega t} \lim_{r \to \infty} \sin[(\beta t + k_r) r]/[(\beta t + k_r)^2]$. This shows that with increasing time, energy is increasingly shifted towards high $k_r$ numbers.

We emphasize that every time that the phase-mixing acts in the presence of a damping mechanism that depends on the wave number $k_r$, the dynamics can be strongly influenced. In this letter we focus in particular on the combined effects of the phase-mixing and the Landau damping. We recall that the Landau damping is very efficient for high $k_r$. Consequently, we expect that the amplitude of these modes, excited by the continuum damping in Fourier space, decreases fast. This behavior is observed in the simulations. In fact, concerning the previous simulations, in fig. 3 we can appreciate for the case $ak_T = 10$ the generation of thin structures of $E_r$ along the radial direction at $t = 50000$ (consistently with refs. [11,12]). Moreover, the amplitude of these electric-field structures is smaller than that of the case $ak_T = 0$.

We have considered a range for $q$ and for $k_r$ for which the theory of Sugama-Watanabe is a good approximation. To this purpose, we consider the Landau damping rate...
the case ak = 0 (dash-dotted line) and for the case ak = 10 (continuous line) at t = 50000.

\[ \gamma = -f(v_{Ti}, q, \tau_i) + k_{r0}^2 g(v_{Ti}, q, \tau_i) \]

for a GAM with a fixed radial wave number \( k_{r0} \). The \( f \) and \( g \) expressions can be found in eq. (3) of ref. [23]. Considering that in the presence of a continuum spectrum the radial wave number \( k_r \) is shifted in time with \( k_r = (k_{r0} + \beta t) \), we write the damping rate due to the combined action of phase-mixing and Landau (PL) damping as follows:

\[ \gamma_{PL}(t, k_T) = -f(v_{Ti}, q, \tau_i) + (k_{r0} + \beta t)^2 g(v_{Ti}, q, \tau_i). \]  

(1)

The evolution of the electric field is described at each time by:

\[ \gamma_{PL}(t, k_T) = \frac{1}{E_r} \frac{\partial E_r}{\partial t}. \]  

(2)

Equation (2) is solved with an initial-value code in order to calculate theoretically the evolution in time of the GAM electric field. This analytical evolution is then compared with the results of simulations for several values of \( ak_T \) and \( k_r \). In fig. 4 we show the electric field in the middle of the radial domain as a function of the time \( t \) in \( \Omega_r^{-1} \) units. We observe that the envelop \( E_r(t) \) deduced by eq. (2) evolves in agreement with the electric field numerically obtained for the case \( ak_T = 10 \). In fig. 5 we show the damping rate \( \gamma_{PL} \) as a function of \( ak_T \) in the range \( 0 \leq ak_T \leq 18 \). This instantaneous damping rate \( \gamma_{PL} \) is measured at the characteristic time \( t_{PL} \) in which the electric-field amplitude of GAM is half of its initial value. For \( ak_T = 0 \) we find the value \( \gamma_{PL} \approx 6.0 \cdot 10^{-6} \) estimated by the theory [22, 23]. We observe that the damping strongly increases with the increase of the temperature gradient. For these simulations we have considered the parameters of table 1, with a monochromatic signal with \( k_r \rho_i = 0.0511 \) and \( q = 3 \). The theory well describes the strong damping with respect to the case \( ak_T = 0 \) in which only Landau damping acts. We note that for large values of \( ak_T \), the damping in the simulations is a little bit smaller than the damping expected by the theory. We recall that the temperature varies with an hyperbolic tangent function. By increasing the temperature gradient \( ak_T \), the temperature radial scale length becomes shorter and eventually comparable with the fluctuation wavelength. These aspects will be further analyzed in future work. From these results, we deduce that the PL mechanism can play an important role in the suppression of GAM oscillations in the regions characterized by a strongly nonuniform temperature profile such as in the H and I modes. It is important to note that simulations performed with a density gradient different from zero have given results very close to the simulations performed with a flat density profile. This is in agreement with eq. (1) that does not depend on the density gradient. Moreover, we note that \( \gamma_{PL} \) in eq. (1) depends also on \( q \). However, the gradient of \( q \) has a weak influence on the phase-mixing and consequently on \( \gamma_{PL} \). This aspect has been verified by numerical simulations showing that the main parameter in the PL mechanism is the temperature gradient.

In order to investigate and to quantify the importance of the PL damping mechanism on these regions, we compare it with the characteristic drive given by the nonlinear coupling with the ITG mode. To this end, we consider again a characteristic time \( t_{PL} \) in which the electric-field amplitude of GAM is half of its initial value. We consider simulations with the values of table 1. The temperature gradient has been varied in the range \( 0 \leq k_{T \rho} \leq 18 \). A monochromatic signal with \( k_r = 2\pi/\lambda_{GAM} \) and a safety factor \( q = 3 \) have been chosen. The drive is characterized by the power \( \gamma_{RD} \) defined as [11]

\[ \gamma_{RD}^2 = \frac{\tau_i (k_{r0} k_r c_i)^2}{2\omega_0 (\partial D_0/\partial \omega_0)^2} \langle |\phi_i|^2 \rangle \frac{e}{T_e} A_0^2 \]  

(3)
This equation shows a drive for GAM based on a preferential three-wave interaction that emerges from the ITG turbulence dynamics. This is in agreement with recent experimental observations on H-L-2A [26]. However, because of the general statistical behavior of the turbulence dynamics, numerical studies on this subject can be addressed in the presence of a well-developed ITG turbulence. This will be object of a future work. The terms in eq. (3) are evaluated by considering $k_0\rho_i \approx k_i\rho_i \approx 0.1$, $\Omega_i = 1.9 \cdot 10^8$ rad/s. We estimate $(k_0\rho_i/k_i\rho_i)^3 = (k_0\rho_i/2^{1/2}k_i\rho_i/2^{1/2}\Omega_i)^2 \approx 9.2 \cdot 10^{11}$ (rad/s)$^2$, $\omega_i(\partial D_{\Omega_i}/\partial \omega) \approx \tau_i = T_i/T_e$, and $\langle \vert \phi_0 \vert^2 \rangle = \int_{-\infty}^{+\infty} \Phi_0(d\eta) d\eta \approx 0.3$ in which $\Phi_0(\eta) = (2\pi)^{-1/2} \int_{-\infty}^{+\infty} e^{i(nq-m)c} \Phi_0(nq - m) d(nq - m)$ with $n_0$, $m$ toroidal and poloidal numbers and the maximum of $\Phi_0 = 1$ [11, 27]. For ITG in the fluid ion local approximation, $\alpha_i \approx \tau_i(1 + \eta_i)/(3\tau_i - 1)\eta_i + R/(2L_T) + 1$ with $\eta_i = d\ln T_i/d\ln n = L_n/L_T$ [27]. We consider typical values of $\eta_i$ in the defined confinement regimes: $\eta_i \approx 2.5-8$ in the L-mode, $\eta_i = 2.5-5$ in I-mode and $\eta_i \approx 0.5-1$ in the H-mode. Finally in order to give an estimate of eq. (3), we write $\int A_0 \approx \delta n/n \approx 0.03$ in L-mode and $\delta n/n \approx 0.01$ in H-mode [28]. We note that eq. (3) depends on $k_Ta$ and $\eta_i$. We recall that $t_{PL}$ depends on $k_Ta$. The characteristic time associated to the drive reported in eq. (3) can be defined as $t_{RD} = 1/\gamma_{RD}$. When $t_{PL} < t_{RD}$, the PL damping rate exceeds the energy transfer rate from the ITG turbulence to the GAM. Both the time $t_{RD}$ and $t_{PL}$ are plotted in fig. 6 as a function of the gradient of the temperature $k_Ta$. The values $\eta_i = 1$, $\delta n/n = 0.01$ and $\eta_i = 5$, $\delta n/n = 0.03$ associated to H-mode and I-mode, respectively, have been considered for $t_{RD}$. For the first case, we observe that there is a threshold in $k_Ta$, above which $t_{RD} < t_{PL}$. Therefore, we have demonstrated that this mechanism can play a very important role in the suppression of GAM oscillations in the H-mode regime. Moreover, the competition between these two times opens new possible scenarios close the H-mode transition, such as the intermittent behavior of the GAM characteristic of the prey/predator dynamics. Instead, for the second case that reproduces the physical conditions found in the I-mode regime, we have $t_{RD} < t_{PL}$ for each value of $k_Ta$, as shown in fig. 6. This is also in agreement with experimental results that show the existence of GAM in the I-mode regime in spite of the strong temperature gradient comparable to that of the H-mode [16, 17].

It is important to note that $\eta_i = 1$ and $\eta_i = 5$ are the maximum values of the considered ranges associated to H-mode and I-mode, respectively. Moving towards lower values of $\eta_i$ in the respective ranges, the two red $t_{RD}$ curves in fig. 6 are slowly shifted towards higher time values leaving unchanged the discussed results. Considerations similar to those of the I-mode can be found for larger $\eta_i$ and larger $\delta n/n$ values representative of the L-mode regime (not shown in fig. 6), for which the characteristic times of the drive are even lower of $t_{PL}$. Therefore, the damping mechanism is less efficient and GAM suppression is not evident. These results are also in agreement with the dynamics of GAM observed at the I-H transition [16]. In the C-Mod experiments, during this transition, the temperature and the density gradient increase. In parallel to this, GAM amplitude becomes weaker, disappearing when the H-mode is reached and showing that the damping rate exceeds the transfer rate of energy on GAM. In conclusion, the results reported in this letter represent a useful piece in the complex important jigsaw puzzle of the the L-H/L-I transitions. Moreover, as we have explained, for its generality the mechanism described here can be applied also in other different physical contexts.

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