

Nonlinear benchmark for TAEs: HMGC results

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HMGC

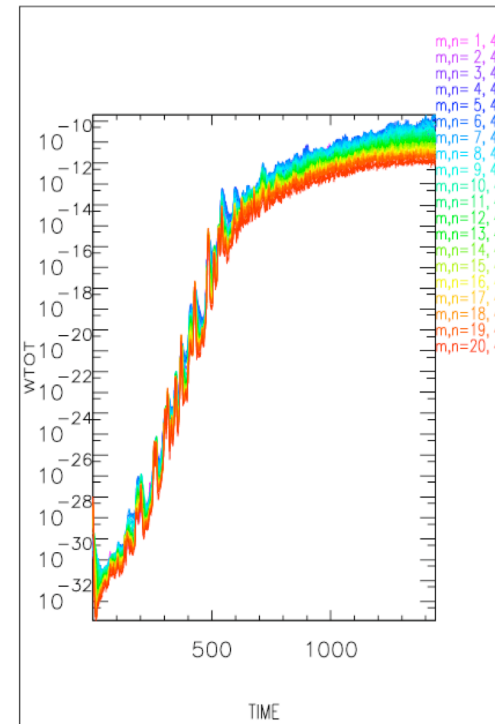
- Hybrid MHD-Gyrokinetic Code (HMGC):
 - thermal plasma described by $O(\epsilon^3)$ reduced-MHD equations
 - energetic particles (EP) described by nonlinear guiding-center Vlasov equation (Particle-in-cell technique)
 - self-consistent simulations (particles treated non-perturbatively: they contribute to MHD eqs. through pressure)
 - mode-mode coupling neglected in single- n simulations (but particle nonlinearities fully retained)

Nonlinear benchmark case

- $R_0=2.53\text{m}$, $a=0.88\text{m}$, $B_0=5.1\text{T}$
- $\beta(r) = 0.01 (1-\psi^{0.8})^{2.7}$
- $q(0)=1.6$, $q(1)=5$, $q'(0)=0$, $q'(1)=9.8$
- $q(r)=q(0)+\psi [q(1)-q(0)+(q'(1)-q(1)+q(0))(1-\psi_s)(\psi-1)/(\psi-\psi_s)]$
- $\psi_s=[q'(1)-q(1)+q(0)] / [q'(0)+q'(1)-2(q(1)-q(0))]$
- $n_D = 3.85 \cdot 10^{13} (1.-0.75 \psi^{0.74})^{1.13} \text{ cm}^{-3}$
- $n_e = 4. \cdot 10^{13} (1.-0.75 \psi^{0.74})^{1.13} \text{ cm}^{-3}$
- $Z_{\text{eff}} = 2.44$
- $T_i = 11.26 (1.-0.5 \psi)^{5.5} \text{ keV}$
- $T_e = 6. (1.-0.5 \psi)^{5.5} \text{ keV}$
- $E_0=3.52 \text{ MeV}$, $v_\alpha = 1.3 \cdot 10^9 \text{ cm/sec}$
- $v_A = 1.27 \cdot 10^9 \text{ cm/sec}$
- $m_\alpha = 4 m_p$
- $z_\alpha = 2 z_p$
- $\beta_\alpha = 0.00083 \exp[-(\psi^{1/2}/0.42)^{2.55}]$
- Slowing down distribution function for EP

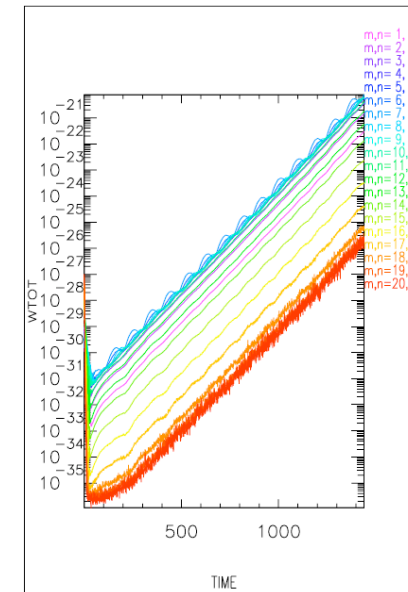
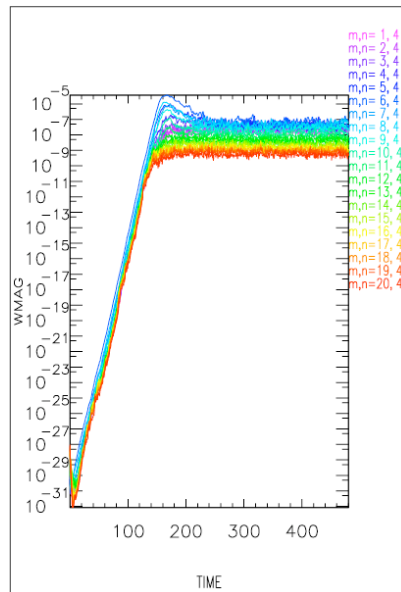
HMGC simulations

- Note: EP distribution function is not a function of unperturbed motion invariants (ψ is not a constant of motion)
- Simulations do not describe the evolution around a true equilibrium
- The lowest order distribution function evolves with time scales shorter than that needed for the formation of the radial structure of the mode
- The growth observed is due to the succession of transient wave packet growths
- No mode growth rate can be defined for low drive cases



HMGC simulations

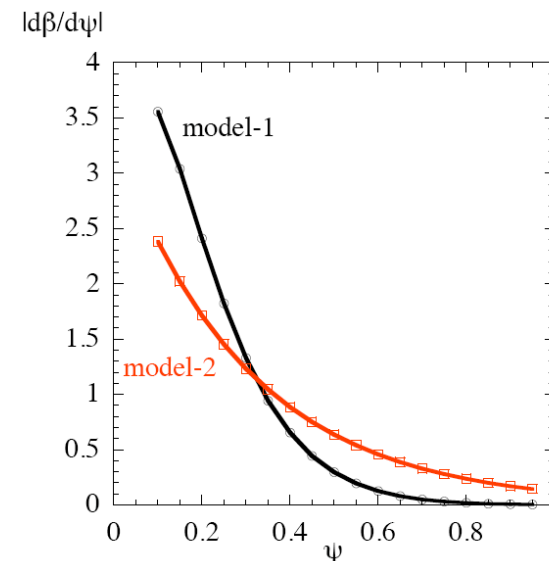
- An exponential growth can be observed only for
 - ✓ Large drive (the relevant time scale is not that of the radial structure formation, but that of the single-packet growth, shorter than the unperturbed-motion evolution of the distribution function)
 - ✓ Small orbit widths (low ρ_α/a): ψ is an almost conserved quantity in the unperturbed motion



Modified model: equilibrium distribution function

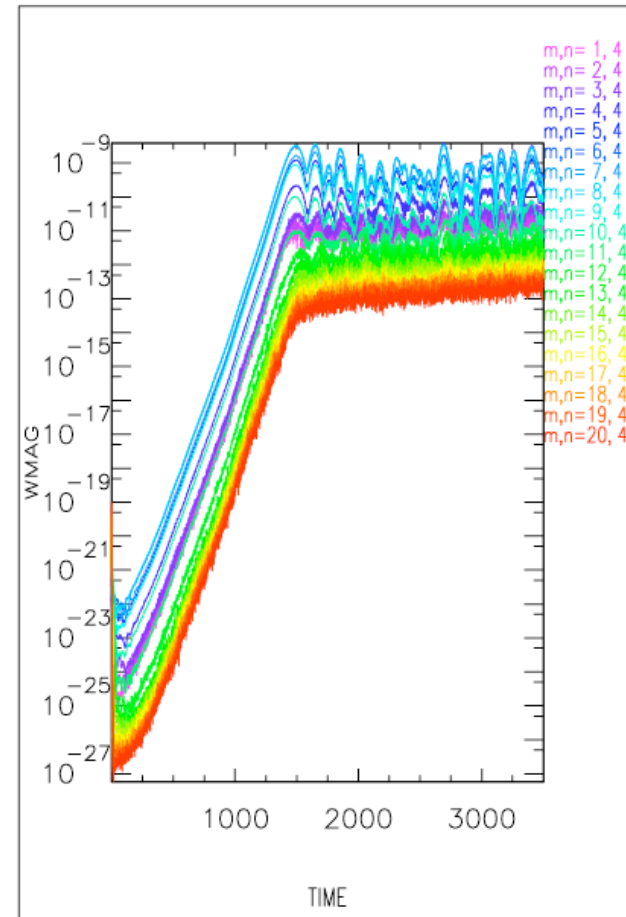
- $n = n_0 \exp(-k\psi) \Rightarrow n_0 \exp(kP_\varphi)$
with $P_\varphi = -\psi + (mc/q) R v_\varphi$
 ψ dependence in E_{crit} neglected
- Similar models adopted by Fu, Lin, etc.
- Note: arbitrary choice; the linear dependence on P_φ allows use to factorize ψ and velocity dependences
- In general, no unique way to infer the equilibrium distribution function in the phase space from its velocity-space momenta

- k adjusted to yield approximately the same β_E' as the former model at the expected TAE radius: $k=3.3$



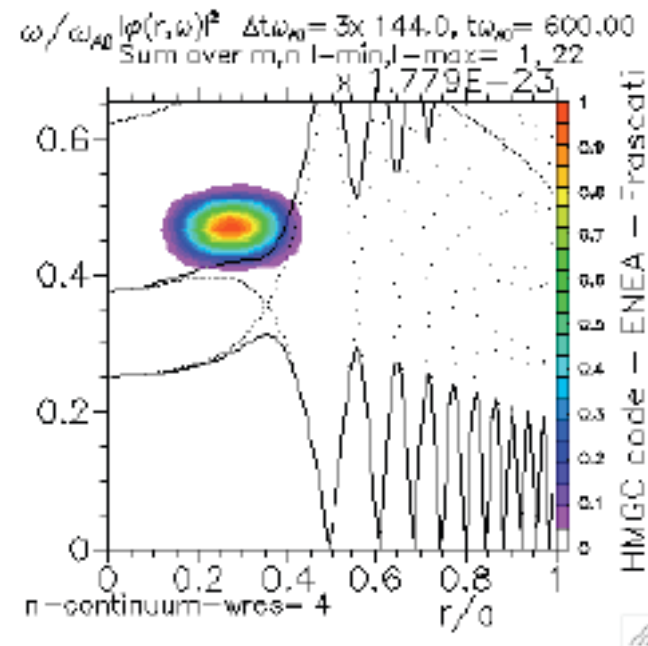
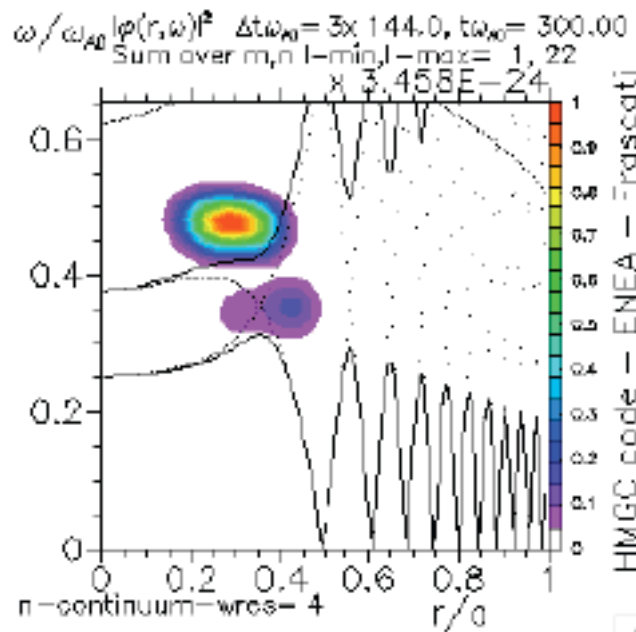
Modified model: equilibrium distribution function

- Exponential growth phase observed, even for low drive



Modified model: linear phase

- Both TAE and KTAE (RPSAE kinetic counterpart) observed at low drive: $(n_\alpha/n_i)_0=0.014$
- KTAE localizes in such a way to minimize continuum damping (that is, where the radial derivative of the continuum frequency is minimum)
- $(\gamma/\omega)_{\text{KTAE}} \sim 0.01$
- $\gamma_{\text{TAE}} < \gamma_{\text{KTAE}}$ (TAE observable only in the first stage of the linear growth phase)



Modified model: linear phase

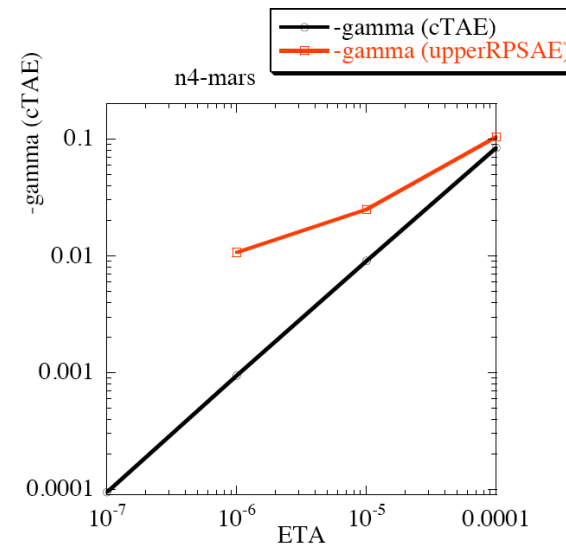
- Why does the KTAE dominate?
 - ✓ HMGC is a resistive code
 - ✓ Simulations performed at artificially large resistivity ($S=10^4$) in order to use reasonably large Δt and Δr
 - ✓ In this regime, in the MHD limit (no EP), comparable (significant) damping rates: $\gamma_d^{\text{TAE}} \approx \gamma_d^{\text{RPSAE}}$

- ✓ Decreasing η , TAE damping would decrease faster than RPSAE one, yielding

$$\gamma_d^{\text{TAE}} < \gamma_d^{\text{RPSAE}}$$

and, possibly

$$\gamma_{\text{TAE}} > \gamma_{\text{KTAE}}$$



MARS

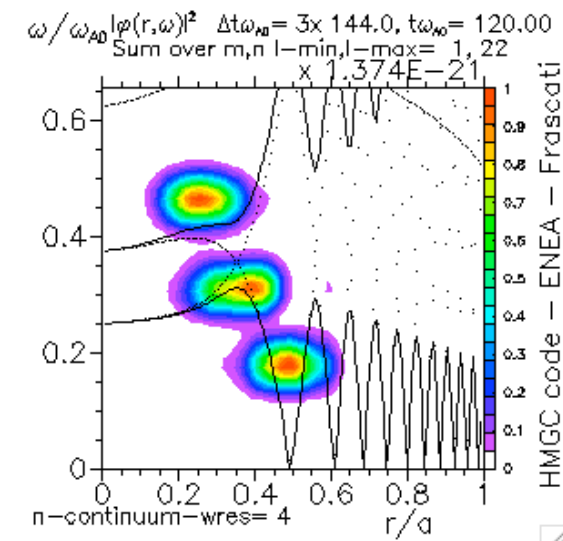
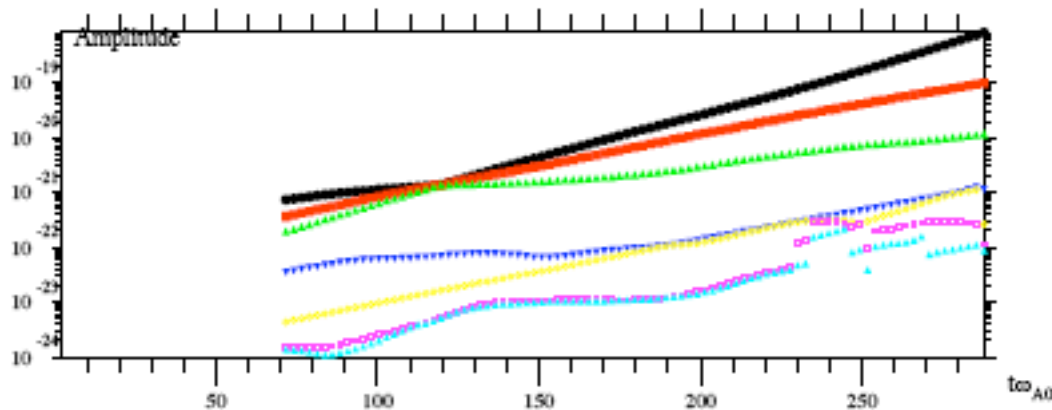
- Note however that KTAE cannot be analyzed, by a spectral code, as the perturbative modification of a continuum oscillation, whose time behavior is $1/t \exp(-i\omega t)$

Modified model: linear phase

- Increasing drive, $(n_\alpha/n_i)_0=0.025$:
 - ✓ KTAE and TAE can still be observed
 - ✓ EPM appears

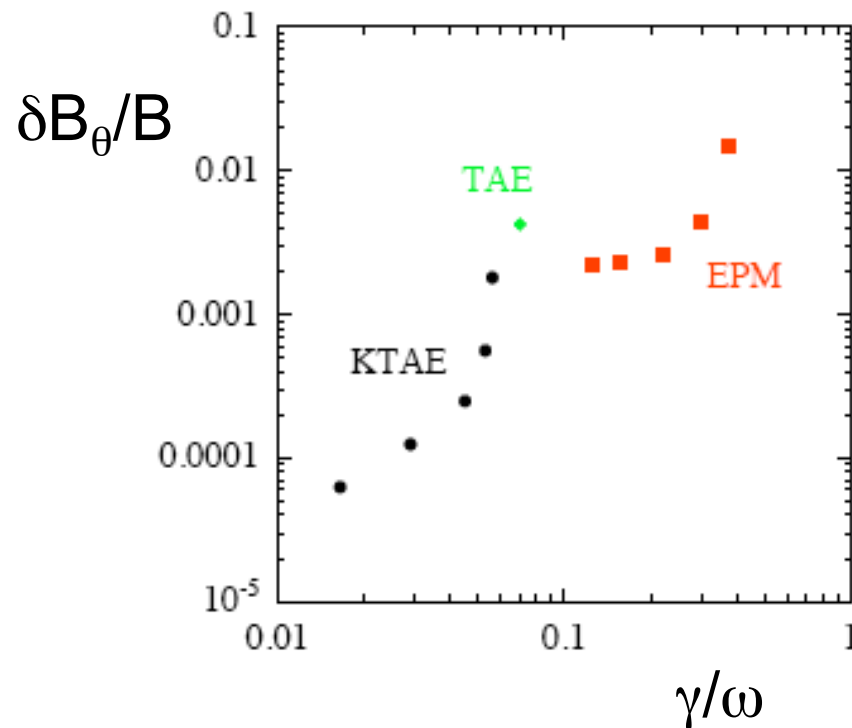
$$\begin{aligned}\gamma_{\text{TAE}} &\approx 0.006 \tau_A^{-1} \\ \gamma_{\text{KTAE}} &\approx 0.019 \tau_A^{-1} \\ \gamma_{\text{EPM}} &\approx 0.013 \tau_A^{-1}\end{aligned}$$

$$\begin{aligned}(\gamma/\omega)_{\text{TAE}} &\approx 0.02 \\ (\gamma/\omega)_{\text{KTAE}} &\approx 0.045 \\ (\gamma/\omega)_{\text{EPM}} &\approx 0.075\end{aligned}$$



Modified model: saturation

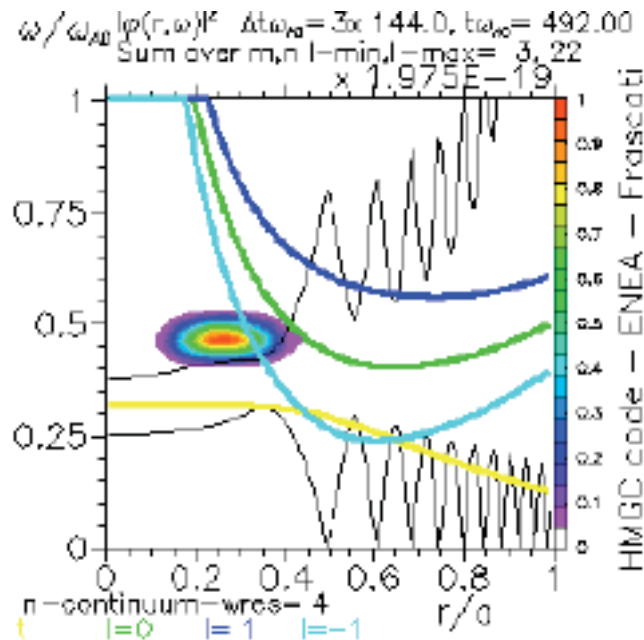
- Different saturation regimes observed for KTAE and EPM
- In the transition regime, both modes coexist with comparable relevance (and TAE as well): possible interference between saturation mechanisms
- Note that the $\delta B_\theta/B$ dependence on γ/ω is different, even for low drive, from the quadratic one usually obtained close to marginal stability



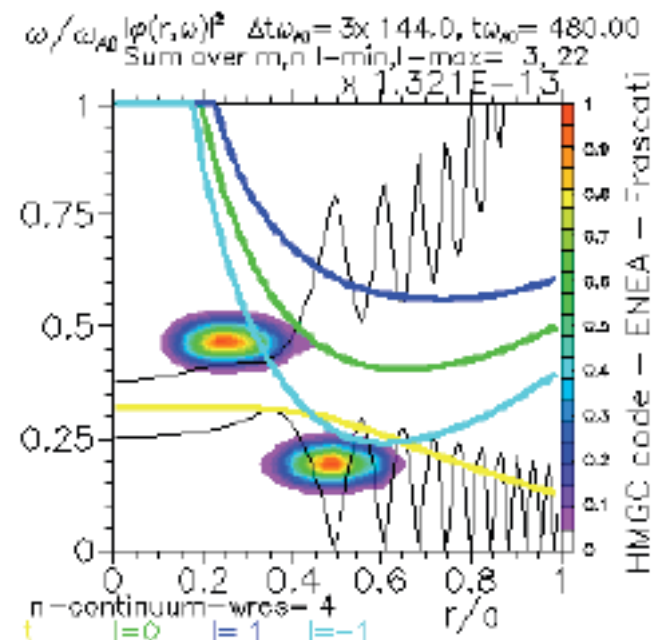
Modified model: saturation

- Saturation mechanisms: comparison between low and high drive regimes

low drive: KTAE

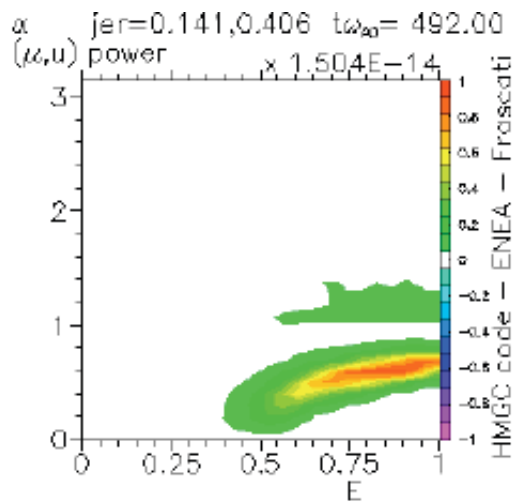


high drive: KTAE and EPM

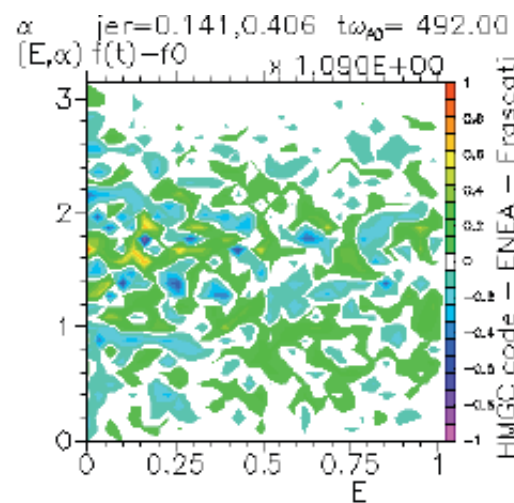


Low drive regime saturation

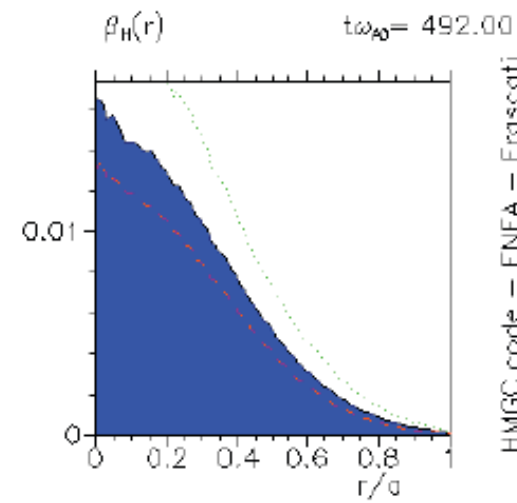
Power transfer



δF

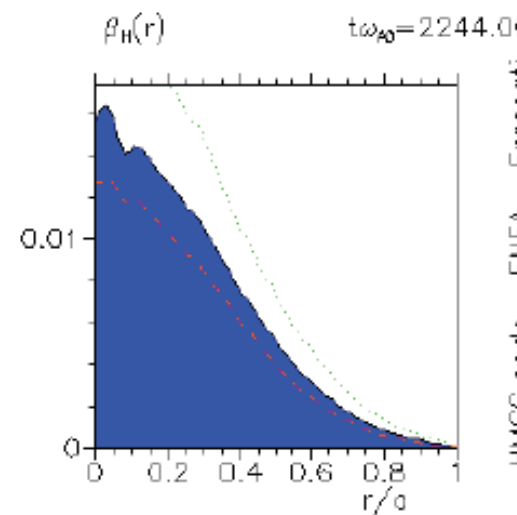
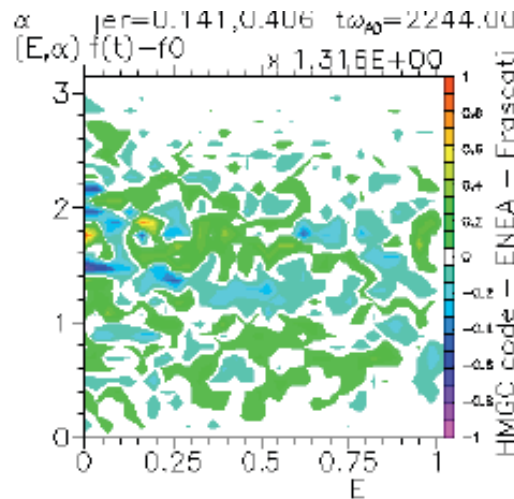


β_α



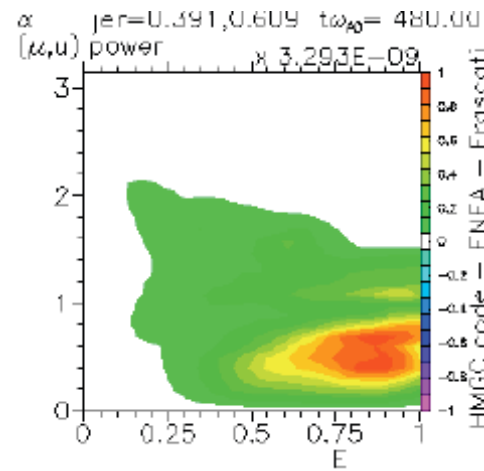
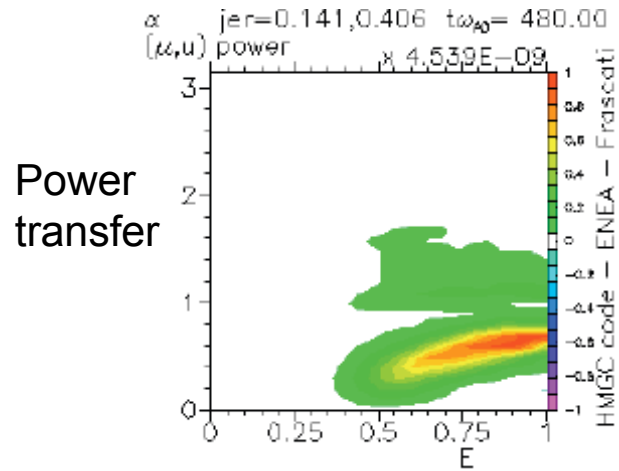
Linear phase

No observable modification in the velocity space distribution function and in the β_α profile

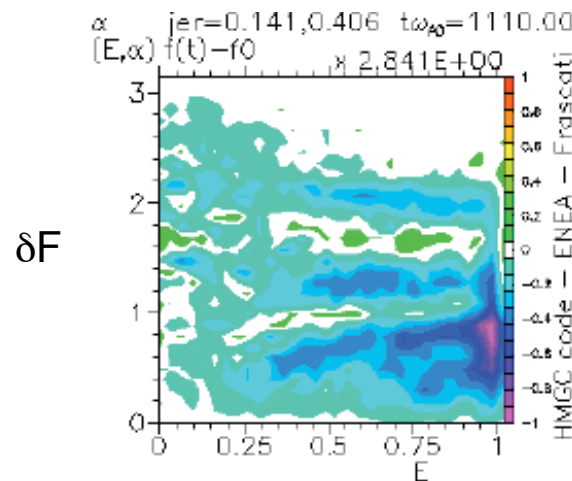


Saturated phase

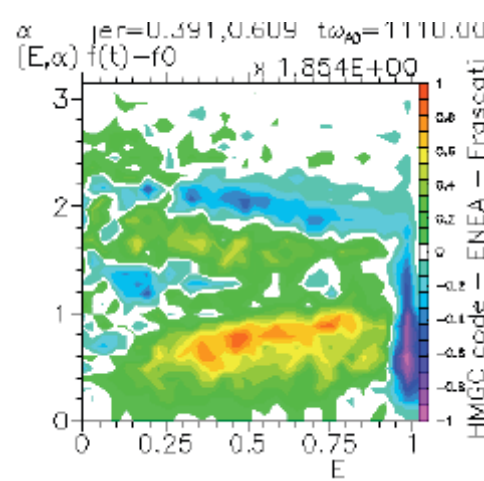
High drive regime saturation



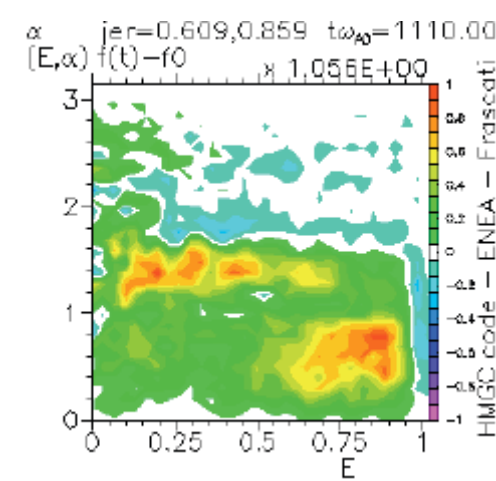
- Resonance region in velocity space is depleted
- Resonant particles are displaced outward



around KTAE



around EPM



outside EPM

Conclusions

- Equilibrium distribution function (function of unperturbed-motion invariants) needed
- Coexistence of TAE, KTAE and EPM observed even in moderate drive regimes
- Different saturation regimes observed in low and high drive limits
 - ✓ in the former, it is hard to detect numerically significant effects on the EP distribution function
 - ✓ in the latter, saturation occurs via an outward radial displacement of resonant particles
- Intrinsic limitations of HMGC: it requires relatively large η and low ε