Nonlinear Saturation of Toroidal Alfvén Eigenmodes

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The saturation of toroidal Alfvén eigenmodes (TAE) due to the effect of magnetohydrodynamic (MHD) nonlinearities is analyzed. The saturation level scales as $(a/R_0)^{5/2}$ (the ratio of the minor to the major radius of the torus), and for typical values of the inverse aspect ratio $\epsilon = a/R_0$, it is comparable with that predicted when the saturation mechanism is due to a nonlinear reduction in the energetic particle drive. The MHD saturation mechanism is thus expected to be potentially relevant to the nonlinear TAE dynamics.

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Toroidal Alfvén eigenmodes (TAE) have recently attracted significant attention [1,2], since they may be destabilized by the resonant interaction with energetic plasma ions with velocities of the order of the Alfvén speed $v_A = B/\sqrt{\varrho}$ ($B \equiv$ magnetic field strength, $\varrho \equiv$ plasma mass density) [1] and large energetic particle losses can be produced consequently.

A possible nonlinear saturation mechanism for TAE has been proposed by Berk and Breizman [2], who analyzed the effects of alpha particles trapped in a finite amplitude TAE wave. In this Letter we describe a different picture of the nonlinear TAE dynamics, where magnetohydrodynamic (MHD) nonlinearities are important, rather than those of the energetic particles. Our goal is to derive an equation for the time evolution of the TAE wave amplitude, assuming that the energetic particles dynamics is linear. In particular, we will show that, above a critical threshold for the mode amplitude, nonlinear saturation occurs due to the excitation of a perturbation to the nonlinear mode structure, which dissipates energy on very short scales, determined, e.g., by finite plasma resistivity. The obtained saturation levels are generally comparable with those expected as a consequence of energetic particles nonlinearities [2].

We consider a pressureless axisymmetric toroidal equilibrium with shifted circular magnetic surfaces and a cylindrical coordinate system (R,ϕ,Z) is used. For a large aspect ratio torus ($R_0/a \equiv 1/\epsilon \gg 1$, R_0 and a being major and minor radii), the magnetic field can be written as $\mathbf{B} = R_0 \nabla\Psi \times \nabla\phi + R_0 B_0 \nabla\phi + O(\epsilon^2) B_\phi$, and the component of the fluid velocity perpendicular to $\nabla\phi$ as $\boldsymbol{v}_\perp = (R^2/R_0)\nabla U \times \nabla\phi + O(\epsilon^3) v_A$, where B_0 is the vacuum magnetic field at $R = R_0$. The "magnetic flux" $\Psi = \Psi_{\rm eq} + \psi$ is given by an equilibrium term $\Psi_{\rm eq}$ and a fluctuation ψ, while U is entirely due to fluctuations, since no equilibrium flows are considered here. The fields ψ and U [3] are obtained from the parallel Ohm's law and the vorticity equation

$$\frac{\partial\psi}{\partial t} + \boldsymbol{v}_\perp \cdot \nabla_\perp \psi = \frac{R^2}{R_0^2} \mathbf{B}_{\rm eq} \cdot \nabla U + \eta \Delta^* \psi + O(\epsilon^4) v_A B_\phi , \tag{1}$$

$$\varrho_0 \left(\frac{\partial}{\partial t} + \boldsymbol{v}_\perp \cdot \nabla_\perp - \frac{2}{R_0} \frac{\partial U}{\partial Z} \right) \nabla_\perp^2 U = \mathbf{B} \cdot \nabla \Delta^* \Psi + O(\epsilon^4) \varrho \frac{v_A^2}{a^2} . \tag{2}$$

In Eq. (1), η is the plasma resistivity and $\Delta^* \equiv R\partial/\partial R(R^{-1}\partial/\partial R) + \partial^2/\partial Z^2$ is the Grad-Shafranov operator, while, in Eq. (2), $\varrho_0 = \varrho R^2/R_0^2$ is assumed constant. A curvilinear coordinate system (r,θ,ϕ) may be used, where r is a radial-like flux function, θ a poloidal anglelike coordinate, and ϕ the toroidal angle, chosen such that the inverse Jacobian $(\nabla r \times \nabla\theta) \cdot \nabla\phi = R_0/rR^2$ and the safety factor $q = q(r)$. Each fluctuation ψ, U is written in the form $F(\mathbf{x},t) = \sum_{m,n} e^{i(n\phi - m\theta)} F_{m,n}(r,t)$, with $F_{-m,-n}(r,t) = F^*_{m,n}(r,t)$ and $m(n)$ being the poloidal (toroidal) mode number.

For the sake of simplicity, we consider a single frequency gap, due to the toroidal coupling of the (1,1) and (2,1) Fourier harmonics. From the linear theory [4,5], it is well known that a frequency gap is opened at the interaction of the (1,1) and (2,1) shear Alfvén cylindrical continua [where $q_0 = q(r_0) = 3/2$ and the frequency is $\omega_0 = v_A/2q_0R_0 = \omega_A/2$] because of the $O(\epsilon)$ toroidal coupling. The two harmonics are characterized by a typical scale length a, outside the gap, and ϵa in the gap region, because ϵa is the radial width of the gap region itself. Since the gradients of the eigenfunctions are larger in the gap region, nonlinearities will be important mostly there. Thus, the nonlinear interaction of the (1,1) and (2,1) harmonics will locally force (1,0) and (3,2) nonlinear beat components. [Other nonlinear beat components, i.e., (0,0), (2,2), and (4,2), are not present [6], since Alfvén waves are exact solutions of the cylindrical ideal MHD equations, the self-coupling nonlinearity being zero.] This four-modes coupling scheme determines the evolution of the system in our model. Two time scales exist: The faster corresponds to the Alfvén scale, while the longer is determined, in linear theory, by the $O(\epsilon)$ toroidal coupling. Therefore, the time derivative can be written as $\partial_t = \partial_{t0} + (\epsilon_0\omega_0/2)\partial_T$, with $\partial_{t0} = O(\omega_0)$, $\epsilon_0 = 2\Delta' + r_0/R_0$, Δ' being the derivative of the Shafranov shift, and $T \equiv \epsilon_0\omega_0 t/2$ being a dimensionless time variable.

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The Fourier components $F_{m,n}(r,t)$ of the fluctuations can then be expressed by means of asymptotic series $F^{(0)}_{m,n}(r,t_0,T) + \epsilon_0 F^{(1)}_{m,n}(r,t_0,T) + \cdots$.

A typical estimate for the mode amplitude is obtained from Eqs. (1) and (2) assuming that toroidicity and nonlinearities compete at the same level. This yields $U_{1,1} \approx U_{2,1} \approx \epsilon^{5/2} a v_A$ and $U_{1,0} \approx U_{3,2} \approx v_A a \epsilon^3$. Using the above derived ordering, it is possible to determine an explicit expression of the (1,0) and (3,2) components in terms of the (1,1) and (2,1) harmonics, by direct integration of Eqs. (1) and (2) [7]. The (1,0) and (3,2) components turn out to have a time dependence of the form $U_{1,0} = \hat{U}_{1,0}(r,T)$ and $U_{3,2} = e^{-2i\omega_0 t_0}\hat{U}_{3,2}(r,T)$, with the "hat" denoting variables which change only on the longer time scale.

At the lowest order in our asymptotic expansion, the dynamics of the (1,1) and (2,1) components on the faster time scale is described by the linear cylindrical equations, yielding $U^{(0)}_{m,1} = e^{-i\omega_0 t_0}\hat{U}^{(0)}_{m,1}(r,T)$ and $\hat{\psi}^{(0)}_{m,1} = -(q - m)B_0/(qR_0\omega_0)\hat{U}^{(0)}_{m,1}$ $(m = 1,2)$. The nonlinear equations valid in the gap region [7], which determine the time evolution of $\hat{U}^{(0)}_{1,1}$ and $\hat{U}^{(0)}_{2,1}$ on the longer scale, are derived from the vorticity equation and Ohm's law at the first order in the asymptotic expansion $[O(\epsilon)]$, by requiring the first order $\hat{U}^{(1)}_{1,1}$ and $\hat{U}^{(1)}_{2,1}$ components to be free of secular terms, yielding

$$\begin{aligned}(i\partial_T - i\Gamma_\alpha - 4i\nu\partial_x^2 - 2sx)\partial_x u + \partial_x v \\ - C_u(T) - 4\partial_x u \partial_x^2 |v|^2 = 0, \\ (i\partial_T - i\Gamma_\alpha - 4i\nu\partial_x^2 + 4sx)\partial_x v + \partial_x u \\ - C_v(T) - 4\partial_x v \partial_x^2 |u|^2 = 0.\end{aligned} \tag{3}$$

Here, $\nu = \eta/(\omega_0 r_0^2 \epsilon_0^3)$, $s = r_0 q_0'/q_0$ is the magnetic shear, and $x \equiv 2(r - r_0)/(r_0\epsilon_0)$. A term $-i\Gamma_\alpha$ has been also inserted *ad hoc* to model a linear drive, $\gamma_\alpha = \epsilon_0\omega_0\Gamma_\alpha/2$, associated with the energetic particles. Furthermore, $u(x,T)$ and $v(x,T)$ are the Fourier components of the vorticity normalized to $U_0^{-1} \equiv (2/\epsilon_0)^{3/2} \times (1/\omega_0 r_0^2)$; $u(x,T) \equiv \hat{U}^{(0)}_{1,1}/U_0$, $v(x,T) \equiv \hat{U}^{(0)}_{2,1}/U_0$, and the quantities $C_u(T) \equiv 2q_0^2\hat{C}_{1,1}(r_0,T)/U_0$ and $C_v(T) \equiv 2q_0^2\hat{C}_{2,1}(r_0,T)/U_0$ are obtained from the dissipationless linear cylindrical equations outside the gap region [5]:

$$\begin{aligned}\partial_r[\hat{U}_{m,n}(r,T)/r] = \hat{C}_{m,n}(r,T)/[r^2 D_{m,n}(r)], \\ \partial_r[r\hat{C}_{m,n}(r,T)] = (m^2 - 1)D_{m,n}(r)\hat{U}_{m,n}(r,T),\end{aligned} \tag{4}$$

with $D_{m,n}(r) = 1/9 - [n - m/q(r)]^2$. Equations (4) need to be integrated with regularity conditions at $r = 0$ and boundary conditions at $r = a$ (e.g., that of a rigid conducting wall). Since $D_{m,n}(r_0) = 0$, $\hat{U}_{m,n}$ is discontinuous at $r = r_0$ (while $\hat{C}_{m,n}$ is continuous), with the jump in $\hat{U}_{m,n}$ determined by the matching with the solution in the gap region.

From the second of Eqs. (4), it follows that $C_u = 0$. Meanwhile, $\hat{U}_{1,1} = U_0 A(T)(r/a)$ for $r < r_0$ and $\hat{U}_{1,1} = 0$ for $r > r_0$, with $A(T)$ given by the jump condition $A(T)(r_0/a) = -\int_{-\infty}^{+\infty} dx\, \partial_x u$. As to the (2,1) component, the jump condition yields [5]

$$\begin{aligned}[\hat{U}_{2,1}(r_{0+}) - \hat{U}_{2,1}(r_{0-})]/U_0 = -KC_v = \int_{-\infty}^{+\infty} dx\, \partial_x v \\ \equiv \delta v(C_v),\end{aligned} \tag{5}$$

with K being a constant which depends only on the q profile. Such a condition determines the time behavior of C_v, and, hence, it is a nonlinear equation for the evolution of the TAE wave amplitude. Equations (3), together with Eq. (5), define, a well posed problem for studying the nonlinear dynamics of TAE models. More general nonlinear equations, valid for arbitrary mode numbers, will be given elsewhere [7].

In the linear limit, Eq. (3) has solutions of the form $C_v(T) = \overline{C}_v e^{-i\lambda T + \Gamma_\alpha T}$, with $\overline{C}_v$ being independent of T. In this case, $\delta v(C_v)$ is linearly proportional to C_v and Eq. (5) determines the eigenvalue λ. Following the analysis of Ref. [8], it can be shown that $\mathrm{Im}(\lambda)$ is related to the effect of dissipation, yielding $\mathrm{Im}(\lambda) \approx \nu s^2$ in the case $\nu s^2 \ll 1$. It is worthwhile recalling that $\mathrm{Im}(\lambda)$ can be significantly larger than the estimate $\mathrm{Im}(\lambda) \approx \nu s^2$ if the mode frequency is sufficiently close to the shear Alfvén continuum and/or finite ion Larmor radius and electron inertia effects are included [9]. Although the inclusion of these effects in our treatment is possible, we will nevertheless neglect them for simplicity, since the physical picture of mode saturation remains unaltered. Therefore, if $\Gamma_\alpha \gg \nu s^2$, the mode is linearly unstable.

Also at saturation, i.e., when the mode amplitude does not grow in time, solutions of the form $C_v(T) = \overline{C}_v e^{-i\lambda T + \Gamma_\alpha T}$ satisfy Eq. (3) with the nonlinear terms included, provided the condition $\mathrm{Im}(\lambda) = -\Gamma_\alpha$ is satisfied. Taking, for the sake of simplicity, the small magnetic shear limit, $s \ll 1$, the nonlinear system, Eqs. (3), yields two coupled nonlinear ordinary differential equations for the functions $f \equiv e^{i\lambda T - \Gamma_\alpha T}\partial_x u$ and $g \equiv e^{i\lambda T - \Gamma_\alpha T}\partial_x v$:

$$\begin{aligned}(\lambda - 4i\nu s^2\partial_z^2 - 2z)f + g - 8|g|^2 f = 0, \\ (\lambda - 4i\nu s^2\partial_z^2 + 4z)g + f - 8|f|^2 g = \overline{C}_v,\end{aligned} \tag{6}$$

with $z \equiv sx$. In terms of g, Eq. (5) together with the saturation condition $\mathrm{Im}(\lambda) = -\Gamma_\alpha$ yield

$$\begin{aligned}\int_{-\infty}^{+\infty} g_R\, dz = -sK\overline{C}_v, \\ \Gamma_\alpha = \frac{4\nu s^2 \int_{-\infty}^{+\infty}(|\partial_z f|^2 + |\partial_z g|^2)\, dz}{\int_{-\infty}^{+\infty}(|f|^2 + |g|^2)\, dz},\end{aligned} \tag{7}$$

with $g_R \equiv \mathrm{Re}(g)$ and $g_I \equiv \mathrm{Im}(g)$. (The second of Eqs. (7) is derived from Eqs. (6), constructing a quadratic form in which the quartic nonlinearities are eliminated by exact subtraction.) Equations (7) determine the two unknowns of the problem, namely, $\mathrm{Re}(\lambda)$ and $\overline{C}_v$ [7,10] at saturation.

Assume that the mode is linearly unstable ($\Gamma_\alpha \gg \nu s^2$). From the second of Eqs. (7), it is clear that the saturation condition can be satisfied only if very small scales are excited. In order to understand how such an excitation can occur, it is convenient to neglect the effect of resistive dissipation in Eqs. (6) first, yielding an algebraic system in the unknowns f and g:

$$(\lambda - 2z)f + g - 8|g|^2 f = 0, \quad (\lambda + 4z)g + f - 8|f|^2 g = \overline{C}_v. \tag{8}$$

Among the various roots of the above system, the physical solution is that which behaves as $g \approx \overline{C}_v/4z$ and $f \approx \overline{C}_v/8z^2$ for $|z| \to \infty$, since, far from the gap region, the mode dynamics is linear. Small scales can be excited only if f and g tend to become singular. From Eqs. (8), it is clear that this can happen only if the Jacobian $J(z, \overline{C}_v) \equiv [\lambda - 2z - 8g^2(z)][\lambda + 4z - 8f^2(z)] - [1 - 16g(z)f(z)]^2$ tends to vanish. It is interesting to note that the condition $J = 0$ defines, in the linear limit, the shear Alfvén continuum and that $J < 0$ for a linear TAE mode.

The condition $J = 0$ is, therefore, the necessary condition for nonlinear excitation of small scales and thus for nonlinear mode saturation. To see how this results in a threshold condition on the mode amplitude, consider that $J = 0$ will occur first at the local maximum ($z = z_0$) of J, since for $|z| \to \infty$ the Jacobian is always negative ($J \approx -8z^2$). Furthermore, the first of Eqs. (7) determines λ as a function of $\overline{C}_v$ and sK. Thus, the maximum of the Jacobian $J_0 \equiv J(z_0, \overline{C}_v, \lambda)$ turns out to be a function of $\overline{C}_v$ and sK, with $J_0 < 0$ for $\overline{C}_v \to 0$. Conversely, for a given sK, specifying J_0 is equivalent to give $\overline{C}_v$, λ, and the solutions f and g. We anticipate at this point what we will be able to show later, namely, that, for any finite value of K, a finite value of $\overline{C}_v$, $\overline{C}_{v0}$, exists at which $J_0 = 0$. Thus, for any value of K, the functions f and g will tend to develop a singular behavior as a critical mode amplitude is approached. Above this critical mode amplitude $\overline{C}_{v0}$ the damping rate is expected to rapidly increase above the linear estimate $\mathrm{Im}(\lambda) = O(\nu s^2)$, and nonlinear mode saturation occurs.

A rigorous proof of the fact that the mode actually saturates above the critical amplitude threshold $\overline{C}_{v0}$ is tedious and requires the use of singular perturbation theory. For this reason, we prefer to give it elsewhere [7] with the required details. One result, however, is worth mentioning here: The actual mode saturation level $\overline{C}_{vs}$ is just slightly bigger than $\overline{C}_{v0}$ in the small dissipation limit. In fact, writing $\overline{C}_{vs} = \overline{C}_{v0} + \Delta C_v$, it is possible to show that [7]

$$\Delta C_v \simeq -0.87(4\nu s^2)^{4/5} \ln(4\nu s^2) \ln\left(\frac{(4\nu s^2)^{7/5}}{\Gamma_\alpha}\right),$$

under the condition $(4\nu s^2) \ll \Gamma_\alpha \ll (4\nu s^2)^{1/5}$. This result indicates that $\overline{C}_{v0}$ may be considered as representative for estimating the mode amplitude at saturation , since ΔC_v is actually small for $(4\nu s^2) \ll 1$. Thus, the saturated amplitude can be determined by the critical value $\overline{C}_{v0}$ at which the Jacobian of the algebraic system, Eqs. (8), vanishes. In Fig. 1, the two functions $\overline{C}_{v0}(\lambda)$ and $sK(\lambda)$ are shown. It is apparent that, for any value of K, mode saturation can occur due to the MHD nonlinearity. (Incidentally, we note that, for monotonically increasing q profiles, the constant K is negative.) For a model equilibrium $q = q(0) + [q(a) - q(0)](r/a)^2$, $q(0) = 1.1$ and $q(a) = 1.9$, it is possible to show that $\overline{C}_{v0} = -0.074$ and $\lambda = -0.578$ by numerically solving Eqs. (8). Unfolding the normalizations, this condition can be expressed as $|\hat{U}_{1,1}(r_0)|/av_A \simeq 1.58 \times 10^{-2}(a/R_0)^{5/2}$ at saturation or, equivalently, $|\delta B_{r1,1}(r_0)/B_0| \simeq 2.21 \times 10^{-2}(a/R_0)^{5/2}$.

This picture for the TAE saturation has been verified using a numerical code, described in Ref. [10], which solves the nonlinear reduced MHD equations (1) and (2). Some numerical results are given in Fig. 2, where the amplitudes of the relevant Fourier harmonics at saturation are shown vs ϵ. The saturated amplitude of the (1,1) component turns out to be $|\hat{U}_{1,1}(r_0)|/av_A \simeq 2.8 \times 10^{-2}(a/R_0)^{2.44}$, in fair agreement with the analytical results, considering that the low-shear condition is only marginally satisfied ($s = 0.53$ in the present case).

Energetic particle trapping and mode-mode coupling, as saturation mechanisms, are characterized by different dependences of the saturated amplitudes on the relevant experimental parameters. In fact, the mechanism proposed in Refs. [4,11] gives $\delta B/B \approx \epsilon(a/\rho_\alpha)(\gamma_\alpha/\omega)^2$ at saturation, where ρ_α is the energetic particles Larmor radius. The MHD saturation level, instead, scales as $\delta B/B \approx \overline{C}_{v0}\epsilon^{5/2}$. Thus, energetic particle trapping is expected to be more important for weak energetic particle drive. However, as the TAE frequency approaches the shear Alfvén continuum accumulation points for $\lambda \to \pm 2\sqrt{2/3}$ (cf. Fig. 1), the critical threshold $\overline{C}_{v0}$ gets smaller, and the present mechanism may become

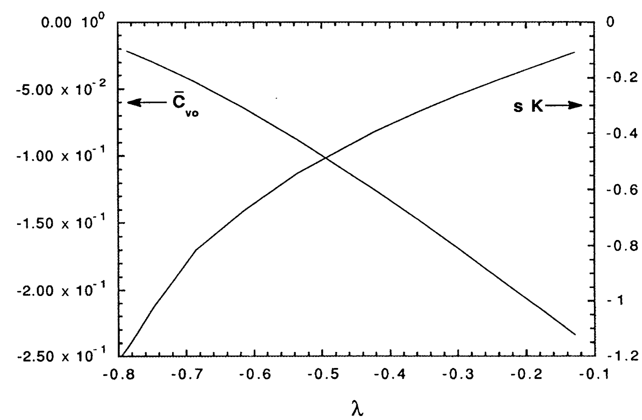

FIG. 1. Numerical results are shown for $sK = sK(\lambda)$, obtained from the first of Eqs. (7), and for $\overline{C}_{v0}(\lambda)$, obtained from the condition $J_0 = 0$. The choice of q profile determines sK, and, hence, the eigenvalue λ. The corresponding saturation amplitude $\overline{C}_{v0}$ may be determined graphically.

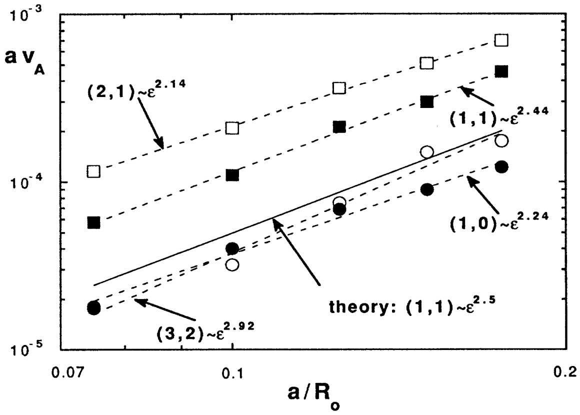

FIG. 2. Saturated amplitudes vs ϵ for the relevant TAE Fourier harmonics. Full circles refer to the (1,0) component, full squares to the (1,1) component, open squares to the (2,1) component, and open circles to the (3,2) component. Broken lines are best fits to the numerical results. Analytical results for the (1,1) component are also shown as a solid line.

relevant. Taking $\epsilon \approx 1/3$ and $(a/\rho_\alpha) \approx 30$ as typical parameters, and considering $\lambda = -0.5 \rightarrow \overline{C}_{v0} \approx -0.1$ for a TAE mode not too close to the frequency gap center, it emerges that the MHD nonlinearity becomes more important for $(\gamma_\alpha/\omega) > 2-3 \times 10^{-2}$. We may then conclude that both MHD and energetic particle nonlinearities, depending on the considered scenario, can be important for TAE modes saturation. The former tends to be more relevant for strong mode drive or for peaked mode structures (TAE's close to the continuum accumulation points), while the latter is predominant for weak mode drive, i.e., close to marginal stability. In general, however, they may be operative at the same time.

The nonlinear TAE saturation, as presented here, predicts the formation of a constant amplitude wave when the nonlinearly enhanced dissipation balances the linear mode drive. For simplicity, the latter is kept constant, but, in a realistic situation, it may vary due to energetic particles diffusion. As a consequence, a TAE wave might collapse after saturation, showing that the scenario considered here does not exclude the possibility of nonlinear TAE pulsations [11], which may be caused by energetic particle losses.

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