Theoretical Aspects of Collective Mode Excitations by Energetic Ions in Tokamaks

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Abstract

A general review of our present understanding of MeV ion dynamics in magnetically confined plasmas and their connection with collective mode excitations is presented, with particular emphasis on aspects that involve open questions. Both linear and non-linear theory issues are addressed, and the possible overlap with more general non-linear dynamics problems is discussed.

1. Introduction

The existence of collective modes in the gaps of the shear Alfvén continuous spectrum is widely recognized to be a concern for the good confinement of MeV energetic ions in toroidal plasmas. Among these modes, particular attention has been devoted to the so called Toroidal Alfvén Eigenmodes [1] (TAE) and their Kinetic counterpart [2] (KTAE), with frequencies in the toroidal gap of the Alfvén continuum, as well as to Kinetic Ballooning Modes [3] (KBM) and Beta induced Alfvén Eigenmodes [4] (BAE), associated with the ion diamagnetic and/or finite-β (compression) gap. In a tokamak plasma of major radius R_0 and minor radius a, the most unstable among the above mentioned modes are characterized by high toroidal mode numbers n such that $a/\rho_{LE} \gtrsim n \gtrsim a^2/R_0\rho_{LE} \gg 1$, with ρ_{LE} the energetic particle Larmor radius.

In addition to Alfvén gap modes, which are normal modes of the thermal plasma, also *strongly driven oscillations* may be excited in the presence of a sufficiently intense energetic particle free energy source. These oscillations, called Energetic Particle Modes [5] (EPM), are excited via wave-particle resonant interactions at the characteristic frequencies of the energetic ions [5], ω_{tE}, ω_{BE} and/or $\overline{\omega}_{dE}$, *i.e.*, respectively the transit frequency for circulating particles and the bounce and toroidal precession frequencies for trapped ions. As in the case of TAE, KTAE, KBM and BAE, most unstable Energetic Particle Modes are generally expected to be characterized by large toroidal mode numbers. However, low-n EPMs can also be unstable, as in the case of *fishbones*, which can be viewed [5] by all means as $n = 1$ Energetic Particle Modes.

In the presence of weak free energy sources, EPMs are not excited. In fact, their frequency is typically inside the shear Alfvén continuous spectrum and as a result, they are more strongly damped than usual Alfvén gap modes. Above a critical threshold, however, the energetic ion drive ($\alpha_E \equiv -R_0 q^2 \beta_E'$, with q the safety factor, $\beta_E = 8\pi P_E/B^2$ and "prime" denoting radial derivation) is sufficiently strong to overcome all background dampings (essentially *continuum damping* [5, 6]) and the mode is strongly unstable. Usually, above the critical EPM excitation threshold, the fast ion energy density is comparable with (higher than) that of the thermal plasma and compression effects (finite-β_E) are strong enough to force a frequency shift of the usual Alfvén gap modes out of the frequency gap itself, *i.e.*, a strong stabilization of those via radiation (continuum) damping [6, 7]. A sharp transition in the plasma stability at the critical EPM excitation threshold has been indeed observed by *nonperturbative* gyrokinetic codes in terms of changes in normalized growth rate (ω_i/ω_A, with $\omega_A = v_A/qR_0$ and v_A the Alfvén speed), real frequency (ω_r/ω_A) and parallel wave vector ($k_\| qR_0$) both as α_E [8, 9, 10] and $\alpha = -R_0 q^2 \beta'$ [10, 11] are varied. These strong dependencies of both EPM frequency and growth rate on thermal plasma as well as energetic particle pressure profiles are in good agreement with the experimental observations of Beta induced Alfvén Eigenmodes (BAE) [4] and also suggest another explanation [10] for the existence of *frequency chirping* modes observed in most large tokamaks.

The present work further explores theoretical aspects of EPM excitations by spatially localized particle sources, possibly associated with frequency chirping, which can *radially trap* the EPM in the region where the free energy source is strongest. Results of a *nonperturbative* 3D Hybrid MHD Gyrokinetic code [8] are also presented to emphasize that nonlinear behaviors of EPMs are different from those of TAE and KTAE.

2. EPM localization by strong energetic particle gradients

In the present paper, we assume a low-β ($\beta = 8\pi P/B^2$) toroidal equilibrium with typically $R_0/a \gg 1$ and use (r, φ, ϑ) toroidal coordinates. For simplicity, we also take shifted circular magnetic flux surfaces. In this case, we can describe electromagnetic wave dynamics in terms of two scalar fields: the scalar potential $\delta\phi$ and the parallel vector potential $\delta A_\|$ fluctuations.

In this Section, we analyze the problem of EPM excitation by precessional resonance in the limit $L_{pE} \ll \Delta_s \equiv 1/nq'$ and for $s^2 \approx \alpha \approx \theta_b \alpha_E \approx (r/R_0)$. Here, L_{pE} is the typical scale length of the energetic particle pressure profile, $s = rq'/q$ is the magnetic shear, θ_b is the poloidal bounce angle of a magnetically trapped fast ion, and Δ_s is the typical distance between adjacent rational surfaces for the toroidal mode number n. The $L_{pE} \ll \Delta_s$ assumption [12] is what really makes the present approach different from the previous ones. Even if this limit may seem one of purely academic interest, it is likely that it may have practical implications for actual experimental senarii characterized by low magnetic shear in a relatively extended region in the plasma interior, e.g., as those of JET optimized shear discharges [13, 14]. For $L_{pE} \ll \Delta_s$, the mode structure and stability properties are *entirely determined* by the *energetic particle source*. In fact, energetic particles provide an *effective potential well* that radially (globally) traps the EPM. This problem is analyzed in detail in Ref. [12] and, thus, we will recall here only the main results obtained therein.

Assuming $\omega \ll \omega_A$ (for $\omega \approx \omega_A/2$ see [12]) and highly localized Ion Cyclotron Resonant Heating around $r = r_0$, which locally creates an energetic tail of trapped ions, the vorticity equation becomes [12]:

$$(\mathbf{e}_\theta - \mathbf{e_r}\xi) \cdot \left[\Omega^2 - (nq-m)^2\right] (\mathbf{e}_\theta - \mathbf{e_r}\xi)\, \delta\psi_m + \left(s^2 D_I + \hat{\alpha}_E\right) \delta\psi_m = 0 \ , \qquad (1)$$

where $\mathbf{b} \cdot \nabla\delta\psi = (-1/c)\partial_t \delta A_\|$, $\Omega \equiv \omega/\omega_A$, $\xi \equiv \mathrm{i}(r/m)\partial_r$, $\mathbf{k}_\perp/k_\theta = -(\mathbf{e}_\theta - \mathbf{e_r}\xi)$, D_I is the Mercier index [15] and $\hat{\alpha}_E \simeq E_0(1 - \rho^2/\Delta^2)$, with $\rho = r - r_0$. For simplicity, we also take a single pitch angle analytic distribution function for the description of the energetic ion tail [16] and assume the simple limit where the fast particle temperature is a constant and the particle localization is only due to density variations. In this particular case, we obtain [12]

$$E_0 \simeq \left(\frac{\theta_b \alpha_{E0}}{2\pi}\right)\left(-\frac{4}{3}\frac{\Omega}{\bar{\Omega}_{dE}}\right)\int_0^\infty \frac{x^{3/4}}{\Gamma(3/4)} J_0^2(\sqrt{2x}k_\theta \rho_{LE}) \frac{\exp(-x)}{x - \Omega/\bar{\Omega}_{dE}} dx \ , \qquad (2)$$

where α_{E0} is the bounce averaged value of α_E at r_0 and $\bar{\Omega}_{dE} = \bar{\omega}_{dE}/\omega_A$. Under these approximations, Eq. (1) reduces to the form of a Weber's equation by assuming that $\varepsilon_A \equiv \Omega^2 - (nq-m)^2$ is nonvanishing at r_0. Then, recalling that we are presently treating a case with $L_{pE} \sim \Delta \ll \Delta_s$, this implies that *continuum damping* is *exponentially small* for such a mode, which is trapped in a narrow radial region around r_0, and is described by the following dispersion relation [12]:

$$\left(1 + \frac{E_0 + s^2 D_I}{\varepsilon_{A0}}\right) = -\frac{r_0}{m\Delta}(1 + 2\ell) \quad . \tag{3}$$

Here, ℓ is the radial mode number. Furthermore, the typical radial width, w, of the eigenfunctions is $w^2 \simeq 2(r_0/m)\Delta$. Incidentally, we note that Eq. (3) indicates that EPM excitations by localized particle sources can be easier for low magnetic shear since, as stated above, continuum damping is exponentially small.

3. Hybrid MHD Gyrokinetic simulations of EPM

Results of a *nonperturbative* 3D Hybrid MHD Gyrokinetic code [8] are presented in this Section in order to emphasize that nonlinear behaviors of EPMs are different from those of TAE and KTAE. In particular, we confirm previous findings that strong radial redistributions in the energetic particle source take place when the EPM excitation threshold is exceeded, yielding potentially large particle losses and, eventually, mode saturation [9]. Such a threshold may occur at experimentally accessible values of β_E, *e.g.*, as low as $\beta_{E0}^{th} = 0.75\%$ (on axis value) for $n = 8$ EPM excitation by Maxwellian energetic ions with $\rho_{LE}/a = 0.01$ and a pressure profile, $\beta_E = \beta_{E0}\exp(-r^2/L_{pE}^2)$, with $L_{pE}/R_0 \simeq 0.075$ and $a/R_0 = 0.1$ [17] (cf. Fig. 1). With the same parameters and profiles, Fig. 2 shows new simulation results which illustrate the nonlinear dynamic evolution of an $n = 8$ EPM. After the first phase in which the eigenmode structure forms (up to $t = 36R_0/v_A$, Fig. 2A), it appears clearly - from the modifications in the fast ion line density - that strong particle redistributions take place from $t = 36R_0/v_A$ (Fig. 2A) up to $t = 72R_0/v_A$ (Fig. 2B), which are consistent with the *mode-particle pumping* model [18] (*particle radial convection*). However, while it was shown that these nonlinear dynamics dominate particle losses and mode saturation al low-n above the EPM excitation threshold [8, 9, 17], Fig. 2 indicates a new dynamical process that becomes important for nonlinear EPM evolution already at moderate n. In Fig. 2B, evident radial fragmentation of the EPM coherent eddies ($k_\theta = k_\parallel = 0$, $k_r \neq 0$) is present and it is visible both in the contour-plots and in the radial variation of the various poloidal harmonics in which the eigenmode is decomposed. This fragmentation, meanwhile, is associated with a diffusive transport of fast ions, as it may be inferred from modifications in the fast particle density profile up to $t = 144R_0/v_A$ (Fig. 2C). The diffusive nature of both particle and

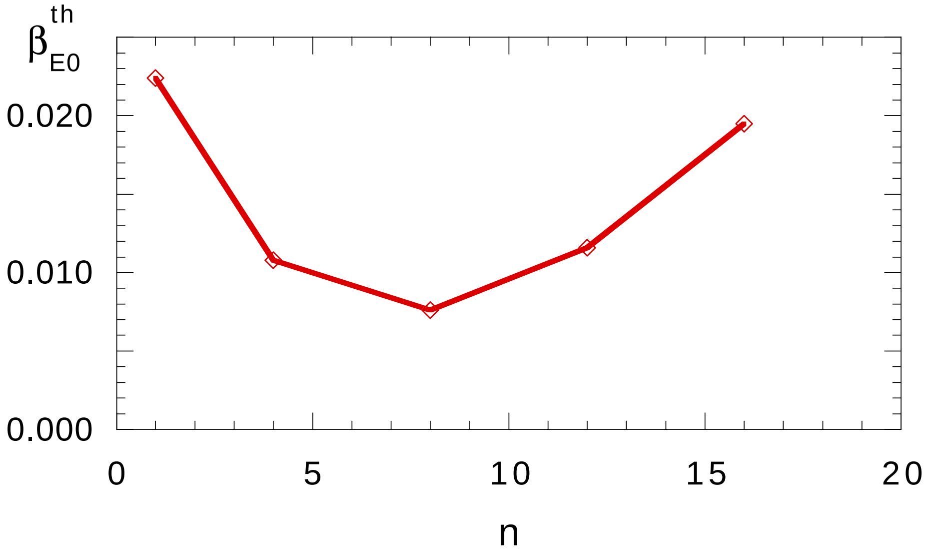

Figure 1: On axis value of the critical β_E, $\beta_{E0}^{\rm th}$, for EPM excitation as n changes. Values as low as $\beta_{E0}^{\rm th} = 0.75\%$ are obtained for $n = 8$ EPM excitation with $\rho_{LE}/a = 0.01$ and a pressure profile, $\beta_E = \beta_{E0}\exp(-r^2/L_{pE}^2)$, with $L_{pE}/R_0 \simeq 0.075$ and $a/R_0 = 0.1$ [17].

energy fluxes associated with fast ions is confirmed by analytical studies, discussed later in Section 5, which yield explicit expressions for fast ion transports [see Eq. (17)].

4. Nonlinear Equations for EPM dynamics

The *radial fragmentation* of EPM coherent eddies has a clear analogy and possible overlaps with more general nonlinear dynamics problems, and specifically with the spontaneous excitation of *zonal flows* by drift-Alfvén turbulence [19]. Within this framework, we have recently demonstrated that EPM may yield to *spontaneous excitation of zonal flows* [20] since they are modulationally unstable above a given amplitude threshold of the coherent eddies which they form above their excitation threshold. The EPM nonlinear (NL) evolution is dominated by fast ion nonlinearities (which enter in the ballooning interchange term in the vorticity equation [19]) for $(\alpha_E/\beta_i)(R_0/r)^{1/2}(T_i/T_E)\epsilon_0^{-1} \gg 1$, with $\epsilon_0 \equiv 2(r/R_0 + \Delta')$, which is typical for unstable EPMs. Meanwhile, fast ion nonlinearities play a role via NL modifications of their nonadiabatic response, $\delta\overline{H}_k \sim (\delta\phi - v_\parallel \delta A_\parallel/c)_{k'}\delta\overline{H}_{\rm z}$, where k, k' subscripts refer to the high frequency EPMs and sidebands, generated via NL interaction with the low frequency zonal field (subscript ${\rm z}$). Thus, the NL fast ion response is formally equivalent to a *quasi-linear diffusion*, consistently with numerical simulations.

To see this in detail, we strictly follow Refs. [19, 20, 21] and, for both scalar potential $\delta\phi$ and parallel vector potential $\delta A_\parallel$ fluctuations, we describe the NL dynamic evolution in terms of a four-mode coupling scheme, *i.e.*, each electromagnetic fluctuation is taken to be coherent and composed of a single $n \neq 0$ mode $(\delta\phi_{EPM}, \delta A_{\parallel EPM})$ and a zonal perturbation $(\delta\phi_{\rm z}, \delta A_{\parallel {\rm z}})$; *e.g.*, for scalar potential fluctuations we take

$$
\begin{aligned}
\delta\phi_{EPM} &= \delta\phi_0 + \delta\phi_+ + \delta\phi_- \\
\delta\phi_0 &= {\rm e}^{{\rm i}\int n\theta_k dq + {\rm i}n\varphi} \sum_m {\rm e}^{-im\vartheta} \phi_0(nq-m) + c.c. \quad , \\
\delta\phi_\pm &= \begin{pmatrix} {\rm e}^{{\rm i}\int n\theta_k dq} \\ {\rm e}^{-{\rm i}\int n\theta_k^* dq} \end{pmatrix} {\rm e}^{\pm {\rm i}n\varphi + {\rm i}\int k_{\rm z} dr} \sum_m {\rm e}^{\mp im\vartheta} \phi_\pm(nq-m) + c.c. \quad , \\
\delta\phi_{\rm z} &= {\rm e}^{{\rm i}\int k_{\rm z} dr} \phi_z + c.c. \quad , \qquad (4)
\end{aligned}
$$

where an analogue decomposition is assumed for fluctuating parallel vector potentials. Here, θ_k is the eikonal describing the radial structure of the wave radial envelope. Thus, Eq. (4) suggests that zonal fields may be actually considered as radial modulations of the wave envelope, while the $(\pm)$ modes are simply upper and lower sidebands due to zonal field modulations of the mode [21]. Furthermore, we have adopted the convention that, in the expressions involving the $(\pm)$ sidebands, the first row in a two component array will refer to the $(+)$ while the second row will refer to the $(-)$ sideband. The same notation will be used throughout.

Introducing the notation $\mathbf{b} \cdot \nabla \delta\psi \equiv -(1/c)\partial_t \delta A_\parallel$, the $n \neq 0$ wave NL equations are the quasineutrality condition [19]

$$
\frac{ne^2}{T_i}\left(1 + \frac{T_i}{T_e}\right)\delta\phi_k = \left\langle eJ_0(\gamma_i)\overline{\delta H}_i \right\rangle_k - \left\langle e\overline{\delta H}_e \right\rangle_k \quad , \qquad (5)
$$

and the vorticity equation [19]

$$
\begin{aligned}
& B\partial_\ell\left(k_\perp^2 \frac{\partial_\ell \delta\psi_k}{B}\right) + \frac{\omega^2}{v_A^2}\frac{k_\perp^2}{b_i}\left[\left(1 - \frac{\omega_{*ni}}{\omega}\right)(1 - \Gamma_0(b_i)) - \frac{\omega_{*Ti}}{\omega} b_i(\Gamma_0(b_i) - \Gamma_1(b_i))\right]\delta\phi_k \\
&= \frac{4\pi}{c^2}\sum_s \langle e\omega\omega_d J_0 \overline{\delta H}\rangle_k + \frac{\mathbf{b}\cdot(\mathbf{k}_\perp'' \times \mathbf{k}_\perp')}{cB}\partial_t\left(\delta A_{\parallel,k'} \nabla_\perp^2 \delta A_{\parallel k''}\right)_k \\
&\quad + \frac{4\pi}{c^2}\partial_t\langle e\frac{c}{B}\mathbf{b}\cdot(\mathbf{k}_\perp'' \times \mathbf{k}_\perp')(J_0(\gamma)J_0(\gamma') - J_0(\gamma''))\delta L_{k'}\overline{\delta H}_{ik''}\rangle_k \quad . \qquad (6)
\end{aligned}
$$

In Eqs. (5) and (6), the subscript k stands for 0 or $(\pm)$ depending on whether the EPM wave or its sidebands are considered, simple angular brackets $\langle \ldots \rangle$ denote velocity space integration, $\sum_s$ stands for summation on particle species s (e, i and E for, respectively, electrons, thermal ions and energetic particles), $\gamma_s \equiv k_\perp v_\perp/\omega_{c,s}$, $\omega_{c,s}$ is the cyclotron frequency, J_0 is the Bessel function of zero order, $\partial_\ell \equiv \mathbf{b}\cdot\nabla$, $b_i = k_\perp^2\rho_{Li}^2$, $\rho_{Ls} = (T_s/m_s)^{1/2}/\omega_{c,s}$ is the Larmor radius for the s-species, $\Gamma_{0,1}(b_i) \equiv I_{0,1}(b_i)\exp(-b_i)$, ω_{*ni} and ω_{*Ti} are the thermal ion diamagnetic frequencies associated with - respectively - density and temperature gradients, ω_d is the magnetic drift frequency, $\mathbf{k} = \mathbf{k}' + \mathbf{k}''$, $\delta L_k \equiv \delta\phi_k - (v_\parallel/c)\delta A_{\parallel k}$ and the fluctuating particle distribution functions have been decomposed in adiabatic and nonadiabatic responses as [22]

$$\delta F = \frac{e}{m}\delta\phi\frac{\partial}{\partial v^2/2}F_0 + \sum_{\mathbf{k}_\perp}\exp\left(-\mathrm{i}\mathbf{k}_\perp\cdot\mathbf{v}\times\mathbf{b}/\omega_c\right)\overline{\delta H}_k \quad . \tag{7}$$

The nonadiabatic response of the particle distribution function, $\overline{\delta H}_k$, is obtained from the NL gyrokinetic equation [22]:

$$\begin{aligned}
&\Big(\partial_t + v_\parallel\partial_\ell + \mathrm{i}\omega_d\Big)_k\overline{\delta H}_k = \mathrm{i}\frac{e}{m}QF_0J_0(\gamma)\delta L_k - \frac{c}{B}\mathbf{b}\cdot\left(\mathbf{k}''_\perp\times\mathbf{k}'_\perp\right)J_0(\gamma')\delta L_{k'}\overline{\delta H}_{k''} \quad ,\\
&QF_0 = \omega_k\frac{\partial F_0}{\partial v^2/2} + \mathbf{k}\cdot\frac{\hat{\mathbf{b}}\times\nabla}{\omega_c}F_0 \quad .
\end{aligned} \tag{8}$$

The relevant NL equations for zonal fields [19], meanwhile, can be derived from the quasineutrality condition and parallel Ampère's law. Assuming $k_\perp^2\rho_{Li}^2 \ll 1$, the NL coupling coefficients are formally of the Hasegawa-Mima type and the quasineutrality condition reads [19]:

$$\begin{aligned}
\partial_t\chi_{iz}\delta\phi_{\mathrm{z}} \;=\; &\frac{c}{B}k_\vartheta k_{\mathrm{z}}k_{\mathrm{z}}^2\rho_{Li}^2\left[\left(\alpha_0 - \left|\frac{k_\parallel v_A}{\omega_0}\right|^2\right)\langle\langle|\Psi_0|^2\rangle\rangle + 2\alpha_0\mathbb{R}\mathrm{e}\langle\langle(\Phi_0-\Psi_0)^*\Psi_0\rangle\rangle\right.\\
&\left.+\alpha_0\langle\langle|\Phi_0-\Psi_0|^2\rangle\rangle\right]\left(A_0^*A_+ - A_0A_-\right) \quad .
\end{aligned} \tag{9}$$

Here, ω_0 is the lowest order (real) EPM frequency and we have introduced the notations $\chi_{iz} \simeq 1.6q^2\epsilon^{-1/2}k_{\mathrm{z}}^2\rho_{Li}^2$ [23], $\epsilon \equiv r/R_0$, $\alpha_0 \equiv 1 + \delta P_{\perp\mathrm{i}0}/(ne\delta\phi_0)$, and Φ_0, Ψ_0 indicate the symmetric Fourier Transforms into ballooning space of ϕ_0, ψ_0 in Eq. (4). Furthermore, for simplicity, we have omitted collisional damping of $\delta\phi_{\mathrm{z}}$ [24], $\langle\langle\ldots\rangle\rangle$ stands for integration over ballooning space, $|k_\parallel|^2\langle\langle|\Psi_0|^2\rangle\rangle \equiv \langle\langle|\partial_\theta\Psi_0/qR_0|^2\rangle\rangle$, θ is the "angle-like" coordinate in ballooning space, and A_0 and $A_\pm$ indicate the amplitude of radial envelopes of the EPM and sidebands at the current radial position. Similarly, from the parallel Ampère's law one can show that, typically [19],

$$\frac{\omega_0}{k_\parallel c}\delta A_{\parallel\mathrm{z}} \approx \frac{\omega_0^2}{k_\parallel^2 v_A^2}k_{\mathrm{z}}^2\delta_e^2\delta\phi_{\mathrm{z}} \ll \delta\phi_{\mathrm{z}} \quad ,$$

which makes it possible to neglect the effect of $\delta A_{\|\mathrm{z}}$, due to the strong shielding of parallel electron current on the electron collisionless skin depth $\delta_e = c/\omega_{pe}$, since zonal fields enter in Eqs. (5) and (6) in the typical form $\delta\phi_{\mathrm{z}} - (\omega_0/k_\| c)\delta A_{\|\mathrm{z}}$.

5. Modulational Instability of EPM

From the structure of Eqs. (5) to (9), it is readily recognized that, as stated above, energetic particles contribute to the EPM NL dynamics only via the ballooning interchange term in the vorticity equation, since they *carry pressure but not inertia*. In fact, energetic particles do not appear explicitly in neither Eq. (5) nor (9). This fact allows us to easily evaluate the conditions under which fast ions dominate the EPM NL evolution. In fact, recall that the typical EPM eigenfunction has a mode structure [1, 6, 7]

$$\Psi_0 = E_0(\theta)\exp\left(\mathrm{i}/2 + \mathrm{i}\Lambda_0\right)\theta + F_0(\theta)\exp\left(-\mathrm{i}/2 + \mathrm{i}\Lambda_0\right)\theta \tag{10}$$

in ballooning space, where E_0, F_0 are the amplitudes of the $k_\| qR_0 = \pm 1/2$ Alfvén waves that form the standing EPM and $\Lambda_0^2 = (\Omega_0^2 - 1/4)^2 - \epsilon_0^2\Omega_0^4$ is the (square of) continuum damping intensity [6], with $\Omega_0 = \omega_0/\omega_A$. Meanwhile, we also have $|\Phi_0 - \Psi_0| \approx k_\perp^2\rho_{Li}^2|\Psi_0|$ [19]. Then, since $\beta_i \ll 1$ and $\omega_0 \approx \omega_A/2 \gg \omega_{*pi}$, it is readily demonstrated that $\alpha_0 \simeq 1$ in Eq. (9) and

$$\partial_t\chi_{i\mathrm{z}}\delta\phi_{\mathrm{z}} = -\frac{c}{B}k_\vartheta k_{\mathrm{z}} k_{\mathrm{z}}^2\rho_{Li}^2\left(W_-\frac{\langle\langle|E_0|^2\rangle\rangle}{\Omega_0^2} + W_+\frac{\langle\langle|F_0|^2\rangle\rangle}{\Omega_0^2}\right)\left(A_0^* A_+ - A_0 A_-\right) \quad , \tag{11}$$

where $W_\pm = |\Omega_0^2 - 1/4| \pm |\Lambda_0|$ in the small magnetic shear limit, whereas, in general $W_\pm = O(\epsilon_0)$ and they need to be determined numerically [6].

Equation (11) allows us to estimate the magnitude of $\delta\phi_{\mathrm{z}}$ and, in turn, to simplify the direct solution of Eq. (8) for the *zonal nonadiabatic response* $\overline{\delta H}_{\mathrm{z}}$ of the various species. At this level, the first crucial difference between fast ions and thermal particles emerges: for the former ones, $\overline{\delta H}_{zE}$ is dominated by the *formally nonlinear* terms in Eq. (8) [20]; while the response of the latter are essentially due to the *formally linear* term $\propto \delta\phi_{\mathrm{z}}$ [19]. The fundamental reasons for these different dynamic responses are the finite orbit width of fast ions (compared to the inverse perpendicular wavelength) and the intrinsic resonant character of their interaction with the EPMs. Using the ordering of $\delta\phi_{\mathrm{z}}$, as it results from Eq. (11), it is also readily demonstrated that the NL $n \neq 0$ nonadiabatic fast ion response is dominated by the $\overline{\delta H}_{kE} \sim (\delta\phi - v_\|\delta A_\|/c)_{k'}\overline{\delta H}_{zE}$ nonlinearity, which is an order $q^2(R_0/r)^{1/2}\epsilon_0^{-1}$ larger than that $\overline{\delta H}_{kE} \sim \delta\phi_z \overline{\delta H}_{k'E}$. Thus, as stated above, the NL fast ion response is formally equivalent to a *quasi-linear diffusion*, consistently with numerical simulations. Using these results and those of Ref. [19], which allow us to estimate thermal particles nonlinearities in the *optimal ordering* $qR_0\omega/(T_i/m_i)^{1/2} \approx k_\perp \rho_{Li}$, a lengthy but straightforward direct comparison of NL terms in Eq. (6) yields the already anticipated result that fast ions dominate the NL EPM dynamics for $(\alpha_E/\beta_i)(R_0/r)^{1/2}(T_i/T_E)\epsilon_0^{-1} \gg 1$, which is typical for unstable EPMs.

Considering a NL regime dominated by fast particle dynamics - as we will do in the following - has two main advantages: the first and obvious one is to analyze a *relatively simple* physical model; the second advantage is to explore a regime in which the zonal field $\delta\phi_z$ never enters directly in the NL EPM equations. $\delta\phi_z$ becomes a "passive scalar", entirely determined by thermal plasma nonlinearities via Eq. (11) once the NL EPM field evolution is consistently determined by fast particles. This physical picture, provides the theoretical framework which justifies using a Hybrid MHD-Gyrokinetic model for the consistent simulation of NL EPM dynamics [8, 9, 17]. The zonal field $\delta\phi_z$ can be evaluated by direct numerical solution of Eq. (11) as a *post-processor* of the Hybrid MHD-Gyrokinetic code.

Detailed analytic solutions of Eqs. (6) to (8) may be obtained in the limit $k_\theta \rho_{LE} \approx \epsilon_0^{1/2} \approx k_z \rho_{LE}$, formally corresponding to the most unstable linear mode, for which the energetic particle drive in the MHD ideal region is dominated by *geodesic curvature* [5, 25]. Following Ref. [25], we transform Eq. (8) to the drift-center reference frame, according to the invertible transform

$$
\begin{aligned}
\overline{\delta H}_k &= \left(\sum_\ell \mathrm{i}^\ell J_\ell(\lambda) \mathrm{e}^{-\mathrm{i}\ell\theta} \right)_k \overline{\delta H}_{dk} \ , \\
\overline{\delta H}_{dk} &= \sum_\ell (-\mathrm{i})^\ell J_\ell(\lambda_k) \mathrm{e}^{\mathrm{i}\ell\theta} \overline{\delta H}_{dk\ell} \ , \\
\overline{\delta H}_{dk\ell} &= \overline{\delta H}^{(+)}_{dk\ell} \mathrm{e}^{\mathrm{i}\theta/2} + \overline{\delta H}^{(-)}_{dk\ell} \mathrm{e}^{-\mathrm{i}\theta/2} \ , \qquad (12)
\end{aligned}
$$

where [25] $\lambda = (k_\perp/k_\theta)(\Omega_d/\omega_t)$ accounts for finite drift orbit effects, $\omega_t = v_\parallel/(qR_0)$, $\Omega_d = k_\theta(v_\parallel^2 + v_\perp^2/2)/(R_0\omega_c)$ and a decomposition similar to that for the eigenmode structure, Eq. (10), has been assumed for $\overline{\delta H}_{dk\ell}$.

With present assumptions and within the framework of weak turbulence theory, Eqs. (8) may be solved for the linear and NL nonadiabatic fast ion responses. The actual calculations are cumbersome but straightforward and, thus, they will be omitted here for brevity. Results are most easily expressed introducing the new notation

$$\overline{\delta H}^{(\pm)}_{dk\ell} = \left(\frac{\ell \omega_t}{\omega_0} \right) J_0(\gamma_k) \left(\begin{array}{c} E_k \\ F_k \end{array} \right) \mathrm{e}^{\mathrm{i}\Lambda_k \theta} \delta \tilde{H}^{(\pm)}_{dk\ell} \quad , \tag{13}$$

where $E_k,\ F_k, \Lambda_k$ have the same meaning of $E_0,\ F_0, \Lambda_0$ in Eq. (10), but extended to the case where k stands for $(\pm)$ sidebands of the pump EPM. Dropping the subscript E for shortness, we finally obtain:

$$\begin{aligned} \delta \tilde{H}^{(\pm)}_{d+\ell} &= \frac{e}{m} \frac{\omega_+}{\mathcal{L}^{(\pm)}_{+\ell}} \left(\frac{QF_0}{\omega} \right)_+ \left\{ A_+ + \frac{k_\theta^2 c^2}{B^2} k_\mathrm{z}^2 J_0^2(\lambda_\mathrm{z}) \frac{M^{(\pm)}_\ell}{\omega_\mathrm{z}} A_0 \left[\frac{A_0^* A_+}{\mathcal{L}^{(\pm)*}_{0\ell}} \right. \right. \\ & \qquad \left. \left. - A_0 A_- \left(\frac{1}{\mathcal{L}^{(\pm)}_{0\ell}} + \frac{1}{\mathcal{L}^{(\mp)}_{-(-\ell)}} \right) \right] \right\} \quad , \\ \delta \tilde{H}^{(\mp)}_{d-(-\ell)} &= \frac{e}{m} \frac{\omega_-}{\mathcal{L}^{(\mp)}_{-(-\ell)}} \left(\frac{QF_0}{\omega} \right)_- \left\{ A_- - \frac{k_\theta^2 c^2}{B^2} k_\mathrm{z}^2 J_0^2(\lambda_\mathrm{z}) \frac{M^{(\pm)}_\ell}{\omega_\mathrm{z}} A_0^* \left[\frac{A_0 A_-}{\mathcal{L}^{(\pm)}_{0\ell}} \right. \right. \\ & \qquad \left. \left. - A_0^* A_+ \left(\frac{1}{\mathcal{L}^{(\pm)*}_{0\ell}} - \frac{1}{\mathcal{L}^{(\pm)}_{+\ell}} \right) \right] \right\} \quad . \end{aligned} \tag{14}$$

In Eq. (14), we have introduced the definition of the *nonlinear propagators* and of $M^{(\pm)}_\ell$:

$$\begin{aligned} \mathcal{L}^{(\pm)}_{+\ell} &= \omega_t \left(\ell + \Lambda_+ \pm 1/2 \right) - \omega_+ + \frac{k_\theta^2 c^2}{B^2} k_\mathrm{z}^2 J_0^2(\lambda_\mathrm{z}) \frac{M^{(\pm)}_\ell}{\omega_\mathrm{z}} |A_0|^2 \quad , \\ \mathcal{L}^{(\mp)}_{-(-\ell)} &= \omega_t \left(-\ell + \Lambda_- \mp 1/2 \right) - \omega_- - \frac{k_\theta^2 c^2}{B^2} k_\mathrm{z}^2 J_0^2(\lambda_\mathrm{z}) \frac{M^{(\pm)}_\ell}{\omega_\mathrm{z}} |A_0|^2 \quad , \\ \mathcal{L}^{(\pm)}_{0\ell} &= \omega_t \left(\ell + \Lambda_0 \pm 1/2 \right) - \omega_0 + \frac{k_\theta^2 c^2}{B^2} k_\mathrm{z}^2 J_0^2(\lambda_\mathrm{z}) \frac{M^{(\pm)}_\ell}{\omega_\mathrm{z}} \left(|A_+|^2 + |A_-|^2 \right) \quad , \\ M^{(\pm)}_\ell &= \frac{|\Omega_0^2 - 1/4| \pm |\Lambda_0|}{2} \left\langle \left\langle J_0^2(\gamma_0) J_\ell^2(\lambda_0) \frac{\ell^2 \omega_t^2/\omega_0^2}{k_\perp^2/k_\theta^2} \right\rangle \right\rangle \quad . \end{aligned} \tag{15}$$

Here, we have also assumed that time dependencies in the solutions of the NL equation may be taken as complex exponentials, which is justified assuming proximity to marginal stability for the pump EPM mode, *i.e.*, $\Gamma_\mathrm{z} \equiv \mathbb{I}\mathrm{m}\omega_\mathrm{z} \gg \mathbb{I}\mathrm{m}\omega_0$, and considering the early destabilization phase of the EPM sidebands, *i.e.*, $|A_\pm| \ll |A_0|$. Note that Eq. (15) includes both *turbulent broadening* and real *frequency shift* of wave-particle resonances. In what follows, we will derive analytic expressions in which the terms $\propto M^{(\pm)}_\ell$ in the propagators can be *consistently* neglected. However, their formal presence in the NL wave particle resonances is what guarantees regularity of these contributions; thus, we have explicitly indicated them in Eq. (15).

Meanwhile, the solution of Eq. (8) for the *zonal* nonadiabatic particle response yields:

$$\overline{\delta H}_{\rm z} \simeq \frac{k_\theta c}{B}\frac{k_{\rm z}}{\omega_{\rm z}} J_0^2(\lambda_{\rm z}) \sum_\ell \sum_{(\pm)} \quad M_\ell^{(\pm)} \quad \left[A_0^* A_+ \mathbb{I}{\rm m} \left(\frac{\delta \tilde{H}_{d0\ell}^{(\pm)*}}{A_0^*} - \frac{\delta \tilde{H}_{d+\ell}^{(\pm)}}{A_+} \right) \right.$$
$$\left. - A_0 A_- \mathbb{I}{\rm m} \left(\frac{\delta \tilde{H}_{d0\ell}^{(\pm)}}{A_0} - \frac{\delta \tilde{H}_{d-(-\ell)}^{(\mp)}}{A_-} \right) \right] \quad , \qquad (16)$$

where the explicit appearance of $\mathbb{I}{\rm m}\delta\tilde{H}_{dk\ell}$ only emphasizes the fundamental role played by *resonant particles*, formally resembling the *quasi-linear diffusion* effect on the particle equilibrium distribution function. This analogy is only formal since, here, we have a single n coherent EPM eddy that nonlinearly produces $\overline{\delta H}_{\rm z}$ via modulational instability. From Eq. (16), the low frequency particle and energy density transport equation for fast ions is readily obtained as

$$\begin{aligned}
&\frac{\partial}{\partial t}\delta n_{\rm z} = -\frac{\partial}{\partial r}\Gamma_E \quad , \\
&\frac{\partial}{\partial t}\left(\frac{\delta P_{\parallel {\rm z}}}{2} + \delta P_{\perp {\rm z}}\right) = -\frac{\partial}{\partial r} Q_E \quad , \\
&(\Gamma_E, Q_E) \simeq 2\frac{k_\theta c}{B}\frac{e}{m}\left\langle J_0^2(\lambda_{\rm z}) \left[1, m(v_\parallel^2 + v_\perp^2)/2\right] \left(-\frac{QF_0}{\omega_0}\right) \right. \\
&\qquad \left. \times \sum_\ell \sum_{(\pm)} \delta\left(\mathcal{L}_{0\ell}^{(\pm)}/\omega_0\right) M_\ell^{(\pm)} \left(A_0^* A_+ + A_0 A_-\right) \right\rangle \quad , \qquad (17)
\end{aligned}$$

where Γ_E, Q_E are, respectively, NL particle and energy fluxes of fast ions. Considering that $\omega_{*pE}/\omega_0 \approx k_\theta \rho_{LE} q R_0 / L_{pE} \gg 1$, Eq. (17) demonstrates that energetic particle transports are dominated by *diffusive processes* after radial fragmentation of the coherent EPM eddies sets in.

Equations (14) can be used to explicitly derive the NL contribution of fast ion dynamics to the vorticity equation, Eq. (6), specialized to the case of EPM sidebands. Multiplying both sides by $\Psi_\pm$ for the $(\pm)$ sidebands and integrating over the ballooning space, we obtain, after some algebra:

$$\begin{aligned}
D_+ A_0^* A_+ &= {\rm i}\frac{\gamma_M^2}{\omega_{\rm z}^2}\left(2A_0^* A_+ + A_0 A_-\right) \quad , \\
D_- A_0 A_- &= -{\rm i}\frac{\gamma_M^2}{\omega_{\rm z}^2}\left(A_0^* A_+ + 2A_0 A_-\right) \quad . \qquad (18)
\end{aligned}$$

Here, $D_\pm$ are the linear EPM sideband dispersion functions [6, 7] and

$$\gamma_M^2 \quad = \quad \pi\epsilon_0 \frac{v_E^2}{v_A^2}\frac{\beta_E q^2}{8} q^2 k_\theta^4 \rho_{LE}^4 k_{\rm z}^2 v_E^2 \left|\frac{e_E A_0}{T_E}\right|^2 \sum_\ell \sum_{(\pm)} \left\langle \frac{QF_{0E}}{\omega_0}\frac{T_E}{m_E n_{0E}} \delta\left(\frac{\mathcal{L}_{0\ell}^{(\pm)}}{\omega_0}\right)\right.$$

$$\times\left(\left|\Omega_0^2-\frac{1}{4}\right|\pm|\Lambda_0|\right)\left\langle\left\langle J_0^2(\gamma_0)\frac{\ell^2 J_\ell^2(\lambda_0)}{\lambda_0^2}\right\rangle\right\rangle^2 J_0^2(\lambda_{\rm z})\left(\frac{v_\perp^2}{2v_E^2}+\frac{v_\parallel^2}{v_E^2}\right)^4\Bigg\rangle \quad , \tag{19}$$

with $v_E^2 = T_E/m_E$ and the E subscripts explicitly indicated for clarity.

Equations (18) can be viewed as 2D partial differential equations for the slow radial and time evolution of the EPM sideband envelopes. In this respect, and since they include fast particle NL dynamics treated within the framework of weak turbulence theory, they may be considered as the generalization of the linear formalism developed in Refs. [6, 7] for a self-consistent calculation of the radial envelope of the mode, which was later applied to the case of electrostatic drift waves driven by ion temperature gradients [26]. Full exploitation of the present formalism is beyond our scope. Here, we will consider the local limit of Eqs. (18) to demonstrate that EPMs are modulationally unstable and to derive the growth rate of the low frequency zonal field perturbation, $\delta\phi_{\rm z}$, which is associated to their radial fragmentation.

Following Refs. [6, 7], we readily see that, in the local limit,

$$\begin{aligned} D_+ &\simeq \frac{\partial D_0}{\partial\omega_0}\omega_z - \frac{\partial \mathbb{R}{\rm e}D_0}{\partial\omega_0}\Delta_L \quad , \\ D_- &\simeq -\frac{\partial D_0^*}{\partial\omega_0}\omega_z - \frac{\partial \mathbb{R}{\rm e}D_0}{\partial\omega_0}\Delta_L \quad , \end{aligned} \tag{20}$$

where D_0 is the EPM linear dispersion function [6, 7] and

$$\Delta_L = (nq')^2\frac{\partial^2\mathbb{R}{\rm e}D_0/\partial k_r^2}{\partial\mathbb{R}{\rm e}D_0/\partial\omega_0}\left[1-\cos\left(\frac{k_{\rm z}}{nq'}\right)\right] \quad . \tag{21}$$

Substituting these expressions back into Eqs. (18), we obtain the NL dispersion relation for $\omega_{\rm z}$, which is easily solved in the limit $|\omega_{\rm z}| \gg |\Delta_L|$ and yields

$$\omega_{\rm z}^3 = 2{\rm i}\gamma_M^2\frac{\partial\mathbb{R}{\rm e}D_0/\partial\omega_0}{|\partial D_0/\partial\omega_0|^2}\pm{\rm i}\frac{\gamma_M^2}{|\partial D_0/\partial\omega_0|^2}\left[(\partial\mathbb{R}{\rm e}D_0/\partial\omega_0)^2-3\left(\partial\mathbb{I}{\rm m}D_0/\partial\omega_0\right)^2\right]^{1/2} \quad . \tag{22}$$

Generally, three of the six solutions indicated by Eq. (22) are unstable. However, for the condition $|\partial\mathbb{R}{\rm e}D_0/\partial\omega_0| \ll |\partial\mathbb{I}{\rm m}D_0/\partial\omega_0|$, typically verified near the toroidal Alfvén gap [7], two of the solutions are typically most unstable

$$\omega_{\rm z} = \pm\frac{3^{1/6}{\rm e}^{\pm{\rm i}\pi/3}}{|\partial\mathbb{I}{\rm m}D_0/\partial\omega_0|^{1/3}}\gamma_M^{2/3} \quad , \tag{23}$$

which correspond to growing $\delta\phi_{\rm z}$ with positive or negative real frequency.

Solutions of the NL dispersion relation for $\omega_{\rm z}$ can be also explicitly obtained in the limit $|\omega_{\rm z}| \ll |\Delta_L|$. In this case, two growing modes are found with either positive or negative frequency:

$$\omega_{\rm z} = \pm\frac{3^{1/4}{\rm e}^{\pm{\rm i}\pi/4}}{|\Delta_L\partial\mathbb{R}{\rm e}D_0/\partial\omega_0|^{1/2}}\gamma_M \quad . \tag{24}$$

Equations (23) and (24) do not show explicitly the presence of a finite EPM threshold for the onset of the modulational instability. That would be brought back into our model provided that dissipation effects are considered on the zonal field generation [24]. Further analyses are in progress to quantitatively compare our theoretical estimates of ω_z with numerical results.

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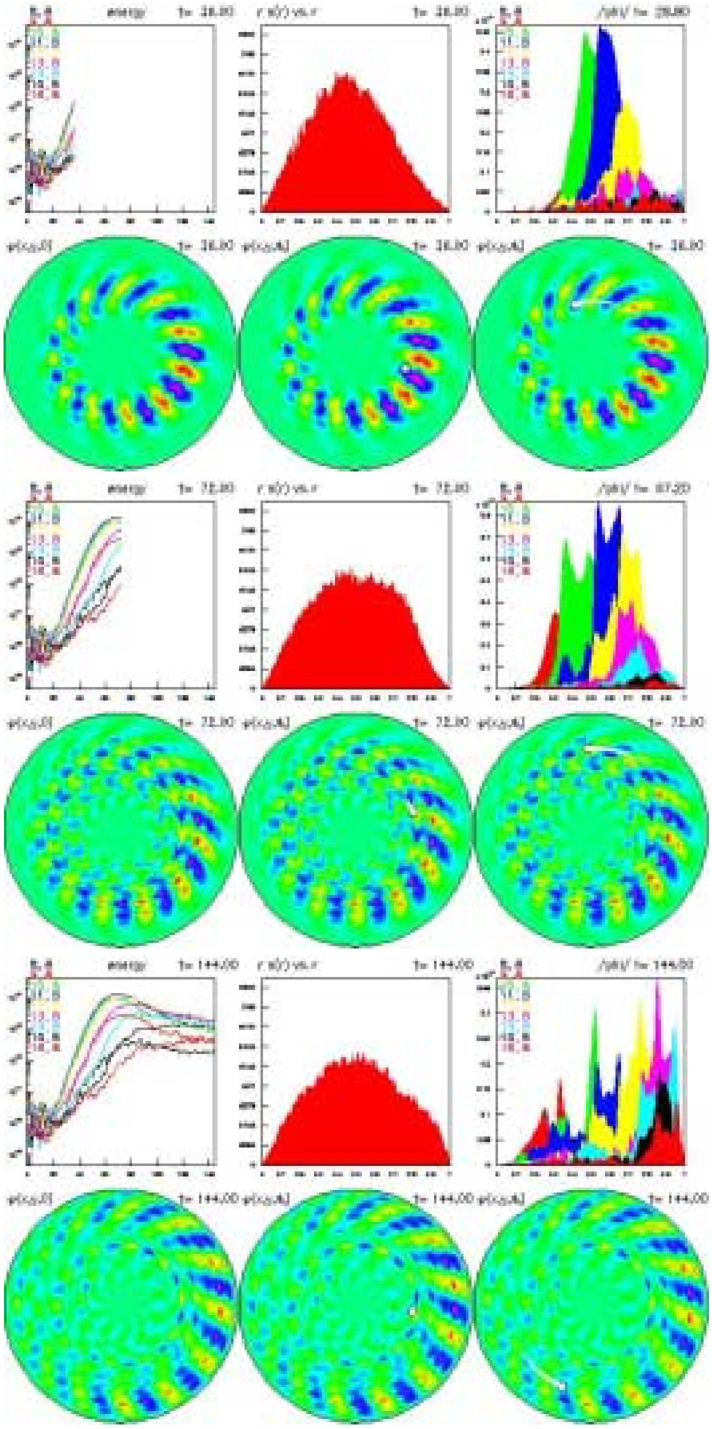

(A)

(B)

(C)

Figure 2: Nonlinear evolution of an $n = 8$ EPM at $t = 36R_0/v_A$ (A), $t = 72R_0/v_A$ (B) and $t = 144R_0/v_A$ (C). In each figure, six frames are visible. On the first row, from the left to the right: the wave energy in each poloidal component, the line density $rn_E(r)$ of energetic ions, and the radial mode structure. On the second row: the contour plot for the scalar potential fluctuation in the laboratory frame and at, respectively, the toroidal angles of a magnetically trapped and of a circulating particle (white bullet).