Alfvén Instabilities Effects on Energetic Particles

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Abstract. In the present work, a review of shear Alfvén instabilities effects on energetic particles is presented and important results of both theory and experiments are emphasized. Open problems and possible topics for future discussions are also indicated, such as those related to the observation of "frequency chirping modes" associated with fast minority ion tails produced during ICRF heating on TFTR [1], which may be interpreted as Energetic Particle Modes (EPM) [2].

I INTRODUCTION

The possibility of reaching the ignition condition in a thermonuclear (magnetically confined) plasma, crucially depends on the possibility that charged fusion products remain well confined in the plasma core until they are completely thermalized by *slowing down* on electrons.

Since the fusion alphas ($D + T \to n + \alpha$) are born at 3.52 MeV mainly in the plasma center, the corresponding pressure profile is peaked. As a consequence, the energetic-particle pressure gradient is a free energy source that can destabilize waves which resonantly interact with the periodic motion of the energetic particles [3–5]. As typically in the case of pressure-gradient driven modes, the instability growth rates increase with the energetic-particle diamagnetic drift frequency which is proportional to the toroidal mode number n. On the other hand, characteristic frequencies of the energetic-particle motion (*e.g.*, transit and bounce) are estimated to be in the MHz range; similar to that of shear Alfvén (s.A.) waves. This observations thus suggest that high-n ($n \gg 1$) s.A. waves are the prime candidate for the instabilities.

Shear Alfvén waves in a laboratory plasma are, however, difficult to excite, since energy is needed to bend the magnetic field lines. Moreover, in a sheared magnetic field, the s.A. waves are characterized by a continuous spectrum [6]. Thus, these waves are highly localized around the surface where $\omega = k_\parallel v_A$ (ω is the mode frequency, $k_\parallel$ is the wave vector parallel to the magnetic field and $v_A = B/\sqrt{4\pi\varrho}$ the Alfvén speed, ϱ being the plasma mass density), and strongly stabilized because of phase mixing [6].

This situation, strictly valid in a slab, is qualitatively modified in toroidal confinement devices due to the poloidal symmetry breaking associated with toroidal magnetic field inhomogeneities over a magnetic surface. The resultant couplings between neighboring poloidal harmonics produces not only frequency gaps [7] in the continuous s.A. spectrum, but also discrete Alfvén eigenmodes. These discrete modes, known as Toroidal Alfvén Eigenmodes [8,9] (TAE's), are localized in the forbidden frequency window (*gap*) of the s.A. continuum. As a consequence, TAE's are undamped, to the lowest order, due to their negligible coupling to the continuum.

That TAE's are marginally stable naturally suggests that energetic particles can resonantly destabilize these modes. Furthermore, these resonant energetic particles could also be effectively scattered by the resultant Alfvénic fluctuations. Indeed, it has been shown that even low-amplitude TAE's, with $\delta B/B \approx 5 \times 10^{-4}$, can cause severe fusion alpha particles losses [10].

The study of the linear stability of TAE's is, therefore, an important issue for tokamak fusion research, and has attracted increasing theoretical as well as experimental interest [11–22].

The linear TAE drive, due to the resonant interaction of the mode with the periodic transit of the passing energetic particles has been extensively studied [23–26]. The weakening effect on the linear drive because of resonance detuning due to finite particle drift orbits has also been considered [27–29]. Furthermore, it has been pointed out [27,29,30] that both passing and trapped (between magnetic mirror points) particles play important roles in determining the linear drive.

A number of damping mechanisms have been suggested by various authors to balance the energetic-particle linear drive and, hence, to determine the marginal stability threshold for TAE's. Electron Landau damping [23–26] is typically negligible, while ion Landau damping [26] and trapped electron collisional damping [31,32] could be important depending on the plasma parameters [29]. Also non-ideal effects of electron inertia and finite ion Larmor radius may significantly enhance the damping rate [27,33–38] and even yield a new kinetic branch of the TAE modes, the Kinetic Toroidal Alfvén Eigenmodes [33] (KTAE's). Another effective damping mechanism, due to the coupling of the TAE mode to the continuous s.A. spectrum, was suggested first in ref. [23] and studied in detail in refs. [39–42] for the high-n case, and in ref. [43] for low-n. In this case, the damping is a consequence of the toroidal mode coupling, which renders the TAE global radial mode width much broader than the typical radial extent of a single poloidal harmonic [40].

In addition to *ideal* (TAE) and *kinetic* (KTAE) discrete *plasma eigenmodes* of the s.A. spectrum, tokamak plasma may be characterized by *forced oscillations* in the presence of a sufficiently strong energetic particle free energy source. These forced oscillations, called Energetic Particle Modes [2] (EPM's), are excited via resonant interactions with fast ions [2,44] and, thus, have the typical frequencies of particle motions, *i.e.*, ω_{tE} (the "transit" frequency of fast ions around the toroidal plasma column), ω_{bE} (the "bounce" frequency associated with their periodic motion between two magnetic mirror points for "trapped" energetic particles), and/or ω_{dE} (the precession rate in the toroidal direction of the trapped particle orbits). Since these forced oscillations have nothing to do with the presence of a frequency gap in the s.A. continuous spectrum, it is readily demonstrated that the threshold in the energetic particle free energy in order to excite EPM's is associated with the necessity to balance (at least) the *continuum damping* due to coupling to the Alfvén continuum [2]. The EPM frequency is usually located in the s.A. continuum [2], it is typically smaller than that of of TAE and KTAE [27,45] and it is essentially determined by the typical frequencies of particle motions. In this sense, EPM's *continuously cover* the s.A. continuum and can be looked at as a *bridge* between Alfvén gap modes and Kinetic Ballooning Modes (KBM) [46,47] in the ion diamagnetic frequency range. Similarly, EPM's can be viewed as the high-n equivalent [2] of the *fishbone* oscillations [48–50].

Considering that small fluctuation levels of s.A. oscillations in a tokamak plasma can cause severe fusion alpha particles losses [10], it is not only important to analyze the global stability properties of those waves, but also to understand their non-linear dynamics and the fundamental physical processes which, eventually, yield to saturation of these modes. Essentially, it is possible to classify the saturation mechanisms of Alfvén modes in tokamaks in two major categories: those related to energetic particle non-linearities, *i.e.*, to the non-linear modifications of particle motions; and those associated with mode-mode couplings. The former of these processes are the most studied in the literature, since the first work on non-linear dynamics of s.A. waves in tokamaks [51], and are expected to be the most important for weakly unstable modes. However, it has been recently pointed out that also the latter processes may be important in determining non-linear mode saturation, depending on the considered plasma equilibrium (see, *e.g.*, ref. [52]). In any event, it is difficult to identify, in general, a *single* specific physical process – falling in one of the two categories mentioned above – that may be thought to be the most relevant in determining the non-linear evolution of s.A. waves in tokamaks and their effect on the energetic particle population. In fact, there may exist other non-linear dynamics, even stronger than the effects discussed above, which may determine the saturated level of these instabilities. This is, *e.g.*, the typical case of EPM's, as it may have been expected from the very definition of these modes as *forced oscillations.*

II EXCITATION OF ALFVÉN GAP MODES

As mentioned in the Introduction, the fact that Alfvén gap modes are weakly damped makes it possible to drive them unstable via resonant interactions with the characteristic motions of energetic particles [27,23–26,28–30]. Incidentally, it is also worth pointing out that, similarly to what happens for the formation of the toroidicity induced frequency gap [7] in the shear Aflvén continuum at $\omega \simeq v_A/(2qR_0)$ (q being the safety factor and R_0 the plasma major radius), other gaps in the continuous spectrum are also formed at $\omega \simeq \ell v_A/(2qR_0)$, for $\ell = 2, 3, \ldots$, due to either non-circularity of magnetic flux surfaces ($\ell = 2, 3, \ldots$) [53], to anisotropic trapped energetic ion population ($\ell = 2, 3, \ldots$) [54] or to finite-β (mainly $\ell = 2$) [55,56], where β is the ratio of kinetic to magnetic field pressure. Discrete plasma eigenmodes, similar to TAE and KTAE, are associated with these *higher order* gaps. These modes have received less attention than TAE and KTAE essentially because, being characterized by higher frequencies with respect to the modes associated with the toroidal gap ($\ell = 1$), they need higher energy particles to be resonantly excited. Furthermore, their dynamic properties are not expected to be qualitatively different from those of TAE/KTAE. For this reasons, these *higher order* gap modes will not be further mentioned in this paper.

The resonant excitation of TAE modes, and of their kinetic counterpart, the KTAE's, depends not only on the energetic particle free energy source, but also on a number of the thermal (core) plasma equilibrium properties, *e.g.*, the (current) q profile, the pressure (β) profile, the shape of magnetic flux surfaces, *etc.*. As an example, it may be worth recalling that, for sufficiently high pressure gradients, the TAE frequency may be shifted downward and eventually out of the toroidal frequency gap, causing enhanced mode damping [24,41,57]. The critical pressure gradient for this effect to occur is close to that corresponding to the ideal MHD ballooning mode marginal stability.

Another typical example of how gap mode properties depend on plasma equilibrium is given by the sensitivity of the mode structure to changes in the q profile. Typically, for high toroidal mode numbers n, many poloidal modes are coupled together to form a *global* TAE mode, which extends across many *local* frequency gaps in the s.A. continuum at the radial positions with $q = (2m + 1)/2n$, due to the coupling of m and $m + 1$ poloidal mode numbers [39]. However, when the magnetic shear, $s = rq'/q$, becomes comparable with the *local* inverse aspect ratio, $\epsilon = r/R_0$, the situation drastically changes [58]. More specifically, when $s \sim \alpha_{core} \sim \epsilon$, where $\alpha_{core} = -R_0 q^2 \beta'$, *i.e.*, typically close to the magnetic axis, the TAE mode structure becomes so radially localized that it is essentially given by the coupling of two poloidal modes, m and $m \pm 1$, about the radial position where $q = (2m \pm 1)/2n$ [58,59]. In fact, many Low shear TAE modes (LSTAE) [59], can be shown to exist locally within the toroidal frequency gap and to be possibly driven unstable by the fusion alpha particles. These LSTAE merge into the usual TAE/KTAE modes for increasing magnetic shear.

At present, a number of numerical global stability codes [60–70] are available for the numerical computation of the Alfvén gap mode linear spectrum in general plasma equilibria and moderate ($n \lesssim 15 - 20$) toroidal mode numbers. The upper boundary on the toroidal mode numbers that can be studied is essentially set by numerical resolution (computing power) and available memory limitation, although this limit can be actually higher in specific cases such as for LSTAE (only two poloidal harmonics determine the radial mode structure) or for codes employing approximated gyrofluid descriptions of the energetic ions [66].

Numerical computations have been frequently a very helpful tool to suggest the optimal experimental conditions to observe resonantly excited Alfvén gap modes [14,15,22,71,72] or to successfully (in most cases) predict experimental observations [69,70]. The Alfvén gap mode stability has also been extensively studied for the reference plasma equilibria of the International Thermonuclear Experimental Reactor (ITER) [73]. In the case of very flat q and pressure profiles (PRETOR 1), Alfvén gap modes are expected to be stable up to $n \simeq 50$ [74]. However, for more peaked and possibly more realistic pressure profiles, TAE modes in the range $n \sim 10$ and core localized LSTAE ($n = 15 - 30$) may be unstable, depending on the specific density and temperature profiles [74]. In fact, the most relevant damping mechanisms for ITER are ion Landau damping (which increases for increasing ion pressure [1]) and trapped electron collisional (radiative) damping, which depend on both density and temperature. In general, this dependence of linear mode stability on fine details of plasma equilibrium profiles is an evidence of the fact that Alfvén gap modes tend to be only weakly stable/unstable. As a result, ITER operations at sufficiently low temperature and high density could be envisaged, which are free of Alfvén gap mode instabilities [74].

Due to limitations in the available computing power, all global stability codes are inefficient to compute high-n Alfvén gap mode spectra. For this reason, theoretical approaches based either on 2D-WKB [39,41] or on 2D-Fourier techniques [75,76] still provide valuable tools to investigate the resonant excitation of high-n Alfvén gap modes. When applied to ITER, these tools have confirmed that Alfvén gap modes are either marginally stable [76–78] or weakly unstable [75], depending on the considered plasma equilibrium.

Energetic particle losses up to 80% of the entire fast particle population have both been predicted theoretically and found experimentally. The particle loss mechanism is essentially of two types [10,79]: (1) transient losses, which scale linearly ($\approx \delta B_r/B$) with the mode amplitude, due to resonant drift motion across the orbit-loss boundaries in the particle phase space of energetic particles which are born near those boundaries; (2) diffusive losses above a stochastic threshold, which scale as $\approx (\delta B_r/B)^2$, due to energetic particle stochastic diffusion in phase space and eventually across the orbit-loss boundaries. It has been also pointed out that transient losses, scaling as $\approx (\delta B_r/B)^{1/2}$, may also occur due to those energetic particle whose drift orbit islands intersect the orbit-loss boundary [74]. Due to the large system size, mainly stochastic losses are expected to play a significant role in ITER. The stochastic threshold for a single mode is $(\delta B_r/B) \approx 10^{-3}$, although that may be greatly reduced ($(\delta B_r/B) \lesssim 10^{-4}$) in the multiple mode case [10,79].

For weakly unstable Alfvén gap modes, the most efficient nonlinear saturation mechanism is via phase-space nonlinearities (wave-particle trapping) [51,80]. This fact has been confirmed by many numerical simulations [65,81–85]. Depending on the "reconstruction rate" of the energetic particle distribution function, either steady state saturation [51] or nonlinear mode pulsations [80] may be obtained. Recently, it has been shown [86,87] that very near marginal stability, *i.e.*, for $0 < \gamma_L - \gamma_d \ll \gamma_L$ with γ_L the linear mode drive and γ_d the background dissipation rate, the mode may saturate at a steady-state level given by $\omega_B^2 \simeq 2\sqrt{2}(1 - \gamma_d/\gamma_L)^{1/2}\nu^2$, with $\omega_B \approx (\delta B_r/B)^{1/2}$ the bounce frequency of a particle trapped in the potential well of the wave, and $\nu \sim (\nu_p \omega^2)^{1/3}$ the effective collision frequency (ν_p is the pitch angle scattering rate). This saturation level is *stable* provided that $\nu > 4.38(\gamma_L - \gamma_d)$, otherwise the mode amplitude will exhibit an oscillating behavior, of magnitude $\omega_B \simeq \gamma_L(1 - \gamma_d/\gamma_L)^{5/4}$ for $\nu \ll (\gamma_L - \gamma_d)$ [86,87]. This theoretical analysis has revealed successful in "fitting" some experimental observations of mode saturation levels on TFTR [18]. Meanwhile, the same theoretical framework predicts, after a phase of explosive growth of the mode [86,87], the formation of hole-clump pairs in the particle phase space [88], which are a possible explanation of the experimental evidence of spectral line "pitchfork" splitting of Alfvén gap modes in JET [21]. The hole-clump structure in phase space has a typical life-time $t \sim \gamma_L^2/\nu^3$, during which the mode amplitude is essentially set by the condition $\omega_B \sim \gamma_L$ and the mode frequency can "chirp" (up and/or down) by an amount $\delta\omega \approx \gamma_L(\gamma_d/\gamma_L)^{1/2}(\gamma_L/\nu)^{3/2}$ [88]. The experimental observation of "chirping" modes (see, *e.g.*, ref. [1]) qualitatively fits in this theory [88], although it is worth noticing that large "chirping" can be explained only for $\gamma_L/\nu \gg 1$.

The nonlinear dynamics of Alfvén gap modes, discussed so far, refer to single wave-particle resonances. It is then not surprising that both steady-state scenarios and pulsations of Alfvén gap modes yield negligible energetic particle losses, unless phase-space stochasticity is reached, possibly via phase-space explosion ("domino effect" [89]). This fact has been confirmed also

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1) Note, however, that also the fusion α drive increases with thermal ion β.

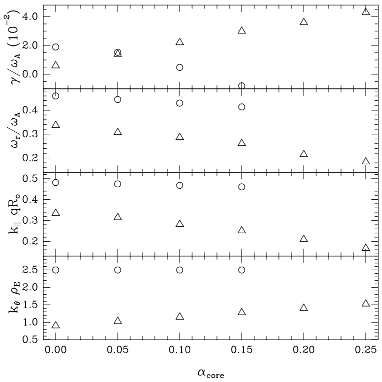

FIGURE 1. TAE (○) to EPM (△) transition for increasing α_{core} as discussed in ref. [45]. Parameters are $s = 0.32$, $\alpha_E = 0.121$, $\epsilon = 0.1$ and $v_E/v_A = 1$, and they are chosen such that, for $\alpha_{core} = -R_0 q^2 \beta' = 0$, an unstable TAE and a weakly unstable EPM are present at the same time.

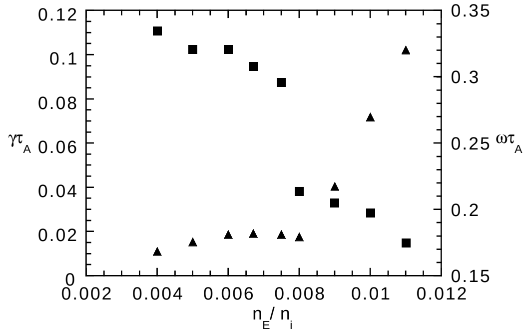

FIGURE 2. TAE to EPM transition for increasing fraction of energetic ions as discussed in ref. [98–100]. The "jump" in real frequency is what confirms the presence of two different modes, *i.e.*, the transition to an EPM dominated regime, characterized by the strong dependence of the growth rate (△) on n_E/n_i.

by recent numerical simulations of alpha particle driven Alfvén gap modes in ITER [85]. For this reason, the dominant loss mechanism below stochastic threshold is expected to be that of scattering of barely counter-passing particles into unconfined "fat" banana orbits [10,79]. Once again, this loss mechanism is expected to be unimportant in ITER, due to the small ratio of banana orbit width to system size.

Wave-particle trapping is not the only possible saturation mechanism for Alfvén gap modes. Mode-mode coupling has also been proposed by various authors [52,90–96] and it has been recently confirmed by nonlinear particle-MHD simulations [97] that, in the multi-mode case, it is difficult to rule out the possibility that MHD nonlinearities play a significant role in the nonlinear dynamics on Alfvén gap modes resonantly excited by energetic particles. The saturation levels predicted for wave-particle trapping in the single resonance case are to be compared with those expected for mode-mode couplings, *e.g.*, in the case of *ion Compton scattering*, $(\delta B_r/B) \approx \epsilon^2(\gamma_L/\omega)^{1/2}$ [92], saturation due to $\delta \mathbf{E} \times \delta \mathbf{B}^*$ [52,90], $(\delta B_r/B) \approx \epsilon^{5/2}\bar{A}/(nq)$, or saturation due to *nonlinear density cavitons* [94,96] $(\delta B_r/B) \approx \beta^{1/2}\epsilon^{3/2}\bar{A}$. Here, $\bar{A}$ is typically a small number depending on the considered equilibrium [52,96], providing a further reason, as pointed out in the Introduction, why also mode-mode couplings may be important in determining nonlinear Alfvén gap mode saturation. In general, however, the Alfvén gap mode nonlinear dynamics in the presence of stochastic diffusion and/or in the more interesting high-n multiple resonance case still remains to be investigated.

III EXCITATION OF ENERGETIC PARTICLE MODES

In addition to Alfvén gap modes, which are normal modes of the plasma, also *forced oscillations* may be excited in the presence of a sufficiently strong energetic particle free energy source. These forced oscillations, called Energetic Particle Modes [2] (EPM's), are excited via wave-particle resonant interactions at the characteristic frequencies of the energetic ions [2,44], *i.e.*, ω_{tE}, ω_{bE} and/or ω_{dE}.

Theoretical aspects of linear excitation of EPM's have been extensively analyzed [2,27,44,101] and the properties of these modes have been studied in a variety of model plasma equilibria [45,75–78,98–100]. Due to EPM's very nature of *forced oscillations*, most global stability codes, which are based on a perturbative approach to the energetic particle effects, are in-

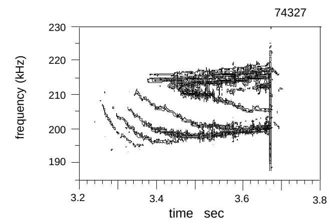

FIGURE 3. Dependence of the critical on-axis energetic article β for EPM excitation, β^{th}_{E0}, on the toroidal mode number n. [100]

FIGURE 4. Evidence of *frequency chirping* modes on TFTR. For detailed discussions see S. Bernabei, ref. [1]

adequate to study both linear and nonlinear dynamics of these modes. In fact, the linear stability of EPM's is typically studied either with one-dimensional (ballooning) gyrokinetic simulations [45] or with two-dimensional WKB-Fourier eigenvalue codes [44,75,77,78]. Meanwhile, only one global stability code [62] has dealt so far with the numerical simulation of EPM's with fully nonperturbative energetic particle dynamics [98–100], although other global stability codes could be used to study this problem (*e.g.*, [65,81]).

In the presence of weak free energy sources, EPM are not excited. In fact, their frequency is typically inside the s.A. continuous spectrum and as a result, they are more strongly damped than usual Alfvén gap modes. Above a critical threshold, however, the energetic ion drive ($\alpha_E = -R_0 q^2 \beta'_E$) is sufficiently strong to overcome all background dampings (essentially *continuum damping* [2]) and the mode is strongly unstable. Usually, above the critical EPM excitation threshold, the fast ion energy density is comparable with (higher than) that of the thermal plasma and compression effects (finite-β) are strong enough to force a frequency shift of the usual Alfvén gap modes out of the frequency gap itself, *i.e.*, a strong stabilization of those via radiation (continuum) damping [27,101]. From this discussion, a sharp transition is to be expected in the plasma stability at the critical EPM excitation threshold (for an analytic derivation see ref. [2]). This is indeed what clearly emerges from Fig. 1, where the transition from a TAE to an EPM dominated scenario is shown by the changes in normalized growth rate (γ/ω_A), real frequency (ω_r/ω_A), parallel wave vector ($k_\| q R_0$) and poloidal mode number ($k_\theta \rho_E$, with ρ_E the energetic ion Larmor radius). The "qualitatively same" transition takes place for an $n = 1$ mode in Fig. 2, where the relative density of energetic (n_E) and thermal (n_i) ions is changed at fixed $\alpha_{core} = 0$. Results are obtained with a global *nonperturbative* stability code [60–62] and show the sharp transition to an EPM dominated regime, characterized both by the strong dependence of the growth rate (full triangles) on n_E/n_i and by the "jump" in real frequency (full squares), which is located in the Alfvén continuum ($\omega_r/\omega_A < 0.5$), as in the high-n case of Fig. 1.[2] These strong dependencies of both EPM frequency and growth rate on thermal plasma as well as energetic particle pressure profiles are in good agreement with the experimental observations of Beta induced Alfvén Eigenmodes (BAE) [102] and also suggest another explanation for the existence of *frequency chirping* modes, as those of Fig. 4 [1], observed in most large tokamaks. In fact, this phenomenology can be accounted for within the theoretical framework described in refs. [2,27,47,101].

High-n WKB-Fourier stability studies of EPM's for ITER, have shown that these mode are either weakly stable [76–78] or weakly unstable [75], depending on the considered plasma equilibria (especially on pressure profile peakedness). This studies, are essentially in agreement with

2) Note that, in Fig. 2, $\omega\tau_A = \omega/(q_0\omega_A)$ with $q_0 = 3/2$ for an $m = 1, 2$ and $n = 1$ mode.

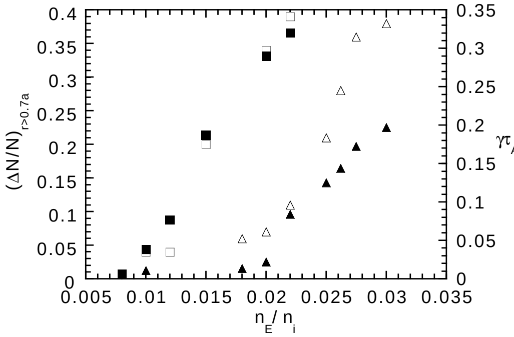

FIGURE 5. Evidence of strong spatial redistributions in the energetic particle source (△ for $n=1$ and □ for $n=4$) above the linear excitation threshold of EPM, indicated by the rapid increase in the mode linear growth rate (△ for $n=1$ and □ for $n=4$).

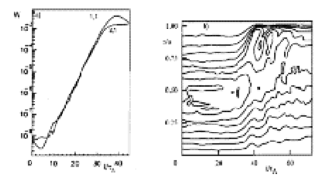

FIGURE 6. Time evolution of the equi-line-density lines (rn_E = const.) and of the total energy in the case of an unstable $n=1$ mode.

global EPM stability analyses of moderate n ($n \leq 16$) modes [100], which show that a minimum of the critical on-axis energetic article β for EPM excitation, $\beta^{th}_{E0,\min} \simeq 0.7\%$, is reached for $n=8$ and a pressure profile $\beta_E = \beta_{E0}\exp(-r^2/L^2_{pE})$, with $L_{pE}/R_0 \simeq 0.075$ [100] (see Fig. 3).

The possible practical impact of this relatively low β^{th}_{E0} w.r.t. experimentally achievable values of β_{E0}, is evident in Fig. 5. There, in fact, it is shown that strong spatial redistributions in the energetic particle source take place right above the EPM linear excitation threshold, indicated by the sharp change in the growth rate dependence on n_E/n_i. More precisely, Fig. 5 shows the fraction $\Delta N/N$ of energetic particles displaced out of a reference flux surface, chosen there at $r/a = 0.7$. As predicted in Fig. 3, the effect is more pronounced for $n=4$ than for $n=1$. Above the EPM excitation threshold, particle losses and mode saturation are consistent with the picture of *mode-particle pumping* [103] (*particle radial convection*) rather than with that of wave-particle trapping. This fact is clearly shown in Fig. 6, where the time history of the equi-line-density contours is shown along with that of the total mode energy for an $n=1$ EPM mode [99].

IV OPEN PROBLEMS AND DISCUSSIONS

The investigation of the effects of Alfvén instabilities on energetic particles is an example of successful interaction/collaboration between experiment and theory, with continuous feed-backs yielding to deeper insights and understanding of the fundamental physics process involved in this very rich research area.

Linear theory is essentially established and compares generally well with experimental observations. Nonetheless, more effort is needed for linear s.A. wave spectra computations in realistic equilibria and for high mode numbers. In fact, at present, s.A. wave spectra are computed in realistic equilibria for moderate n ($n \lesssim 15-20$) due to computation power limitations, whereas calculations of high-n spectra rely on WKB-Fourier techniques.

Nonlinear theory has also progressed and it has been successful in explaining some experimentally observed mode saturation levels and spectral line splitting. However, it is not ready yet for detailed comparisons with experiments, *e.g.*, detailed predictions of energetic particle particle losses. Experimental observations of frequency-chirping and mode bursting can provide useful insights into relevant nonlinear dynamics and a challenge to theorists. In fact, these are generally very complex phenomena, which may be partly attributed to local plasma equilibrium changes but which also clearly involve nonlinear wave and particle dynamics that are still a controversial issue.

Finally, among the most important open issues that still need to be addressed, both experimentally and theoretically, there is the description of the nonlinear s.A. wave dynamics in the multiple mode (multiple resonance) case, possibly above the stochasticity threshold, and including the possibility of strong particle redistributions, *i.e.*, particle convection.

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